A Simplified Approach for Multiple Choice Transportation Problem

Yu-Lan Wang

Abstract—**We study a multiple choice stochastic transportation problem that was proposed by Roy, Mahapatra and Biswal such that the cost coefficients of objective function have multiple choice. In this paper, we first point out that for a minimum problem, we can directly select the minimum cost among multiple choice and then we will present our simplified approach to provide the optimal solution without referring to the Lingo10 package. Our new findings will help researchers handle operation research problems in the future with respect to multiple choice objective functions.**

Index Terms—**Stochastic transportation model, Multiple choice costs, Minimum problem, Analytic method**

I. Introduction

Recently, there are several published papers to emphasize on analytical method to improve previous results. For examples, Wu et al. [1] studied the Newton method to decide the optimal replenishment policy for an economic production quantity model so that the convergence of their sequence is superior to the traditional bisection method. Wu et al. [1] also investigated the Newton method for determining the optimal replenishment policy for economic ordering quantity model with present value such that their findings are more efficient than bisection method. Hung [2] constructed inventory models with crashable lead time and present value. He derived some theoretical findings to locate optimal solutions. Hung [2] also developed continuous review inventory models with the present value of money and crashable lead time and then he obtained several lemmas and one theorem to estimate optimal solutions. Lin et al. [3] constructed inventory models from ramp type demand to a generalized setting such that the optimal solution for replenishment policy is independent of demand type. Lin et al. [3] also examined inventory models to extend them from ramp type demand to a generalized environment so that the optimal solution for their model is not related to the demand type. Lin [4] showed some improvements for the landmark paper of fuzzy sets for distributive law, convex combination and convex fuzzy sets. Chuang and Chu [5] proved that the traffic model has a unique optimal solution and then offered a formulated approximated solution. Roy et al. [6] studied a multiple choice stochastic transportation problem in which cost coefficients of the objective function are of multiple choices and the demands and supplies are follow an exponential random variables. Therefore, Roy et al. [6] developed a mathematical model for multiple choice stochastic transportation problem. In this study, we will present an improvement for Roy et al. [6] to help practitioners in the future when they deal with multiple choice objective functions.

Manuscript received April 5, 2024; revised October 9, 2024.

Yu-Lan Wang is a Professor of the School of Education, Qingdao Huanghai University, Shandong, China (e-mail: yulan.duker@gmail.com).

II. A Brief Review of Previous Results

Roy et al. [6] developed a multiple choice transportation problem where the coefficients containing exponential random variable in all constraints and cost coefficients of objective function are also satisfied the multiple choices as follows:

for $k = 1, 2, ..., K$,

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\left\{C_{ij}^{1}, C_{ij}^{2}, ..., C_{ij}^{k}\right\}x_{ij}, \tag{2.1}$$

subject to

$$\Pr\left(\sum_{j=1}^{n} x_{ij} \le a_i\right) \ge 1-\alpha_i, \tag{2.2}$$

for $i = 1, 2, ..., m$,

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \ge b_j\right) \ge 1-\beta_j, \tag{2.3}$$

for $j = 1, 2, ..., n$, $x_{ij} \ge 0$, with $0 < \alpha_i < 1$ and $0 < \beta_j < 1$ for every i, and j.

They assumed that a_i, for $i = 1, 2, ..., m$ and b_j, for $j = 1, 2, ..., n$ both are exponential random variables with mean $E(a_i) = \theta_i$ and variance $Var(a_i) = \theta_i^2$ and then they transformed the stochastic constraint (2.2) into deterministic constraints as follows:

$$\sum_{j=1}^{n} x_{ij} \le -\theta_i \ln(1-\alpha_i), \tag{2.4}$$

for $i = 1, 2, ..., m$.

Moreover, with $E(b_j) = \theta'_j$ and variance $Var(b_j) = \theta'^2_j$ and then they transformed the stochastic constraint (2.3) into deterministic constraints as follows:

$$\sum_{i=1}^{m} x_{ij} \geq -\theta_j' \ln\left(\beta_j\right), \quad (2.5)$$

for $j = 1, 2, \ldots, n$.

After they converted the constraints from probabilistic format to the equivalent deterministic representation, they tried to transform the multiple choice cost coefficient of the objective function into an equivalent model. To save the precious space of this journal, we directly quote the numerical example of Roy et al. [6] to illustrate their procedure.

The price rates of transportation costs in each route are listed below:

x_{11} routes either 10 or 11 or 12 required admissible costs in Rupees.

x_{12} routes either 15 or16 required admissible costs in Rupees.

x_{13} routes either 20 or 21 or 22 or 23 required admissible costs in Rupees.

x_{14} routes either 15 or 16 or 17 required admissible costs in Rupees.

x_{21} routes either 12 or 13 or 14 or 15 or 16 required admissible costs in Rupees.

x_{22} routes either 10 or 11 or 12 or 13 or 14 or 15 required admissible costs in Rupees.

x_{23} routes either 9 or 10 or 11 required admissible costs in Rupees.

x_{24} routes either 18 or 19 required admissible costs in Rupees.

x_{31} routes either 20 or 21 or 22 or 23 or 24 or 25 or 26 required admissible costs in Rupees.

x_{32} routes either 9 or 10 or 11 or 12 or 13 or 14 or 15 or 17 required admissible costs in Rupees.

x_{33} routes either 24 or 25 or 26 required admissible costs in Rupees.

x_{34} routes either 27 or 28 required admissible costs in Rupees.

For example, the cost coefficients of the objective function with x_{31} have seven choices as

$$\left\{20, 21, 22, 23, 34, 25, 26\right\}, \quad (2.6)$$

out of which one is to be selected. We may abstractly denote them as $\left\{C_{31}^1, C_{31}^2, \ldots, C_{31}^7\right\}$. Owing to $2^2 < 7 \leq 2^3$, Roy et al. [6] will use three binary variables z_{31}^1, z_{31}^2 and z_{31}^3 to represent the seven possible choice for x_{31} as follows:

$$\left\{C_{31}^1\left(1-z_{31}^1\right)\left(1-z_{31}^2\right)\left(1-z_{31}^3\right)+C_{31}^2 z_{31}^1\left(1-z_{31}^2\right)\left(1-z_{31}^3\right)\right.$$
$$+C_{31}^3\left(1-z_{31}^1\right)z_{31}^2\left(1-z_{31}^3\right)+C_{31}^4\left(1-z_{31}^1\right)\left(1-z_{31}^2\right)z_{31}^3$$
$$+C_{31}^5(1-z_{31}^1)z_{31}^2 z_{31}^3 + C_{31}^6 z_{31}^1(1-z_{31}^2)z_{31}^3$$
$$\left.+C_{31}^7 z_{31}^1 z_{31}^2(1-z_{31}^3)\right\}x_{31}, \quad (2.7)$$

with $z_{31}^t = 0/1$ for $t = 1, 2, 3$.

III. Our Improvement for Multiple Choice

If the decision maker has the freedom to select the possible cost in a minimum problem, then automatically he will select the minimum among those possible multiple choice costs. Owing to

$$\min\left\{20, 21, 22, 23, 34, 25, 26\right\} = 20, \quad (3.1)$$

we will directly assume the coefficient of x_{31} is 20. Consequently, the tedious procedure in Roy et al. [6] for multiple choice cost formulation becomes redundant.

Similarly, for other x_{ij}, we can directly take

$$\min\left\{C_{ij}^1, C_{ij}^2, \ldots, C_{ij}^k\right\} \quad (3.2)$$

as the cost for x_{ij}.

We will apply an intuitive approach to solve their minimum problem without referring the any computer programming package. To illustrate our approach, we reconsider the same numerical example in Roy et al. [6]. Based on our simplification of Equation (3.2), we will solve the following minimum problem for a stochastic transformation model:

$$\min: z = 10x_{11} + 15x_{12} + 20x_{13} + 15x_{14} + 12x_{21} + 10x_{22}$$
$$+9x_{23} + 18x_{24} + 20x_{31} + 9x_{32} + 24x_{33} + 27x_{34}, \quad (3.3)$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 4.040541464, \quad (3.4)$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 9.137762245, \quad (3.5)$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 16.32879781, \quad (3.6)$$

$$x_{11} + x_{21} + x_{31} \geq 11.25364287, \quad (3.7)$$

$$x_{12} + x_{22} + x_{32} \geq 7.977780111, \quad (3.8)$$

$$x_{13} + x_{23} + x_{33} \geq 5.051457289, \quad (3.9)$$

and

$$x_{14} + x_{24} + x_{34} \geq 2.40794509, \quad (3.10)$$

where the constraints of Equations (3.4-3.10) are quoted from Roy et al. [6].

We compare the coefficients of x_{ij} for $i = 1,2,3$ and $j = 1,2,3,4$ to find that x_{23} and x_{32} both have the lowest coefficient 9 so we will assume that

$$x_{23} = 5.051457289, \quad (3.11)$$

with $x_{13} = 0$, and $x_{33} = 0$.

On the other hand, we obtain that

$$x_{32} = 7.977780111, \quad (3.12)$$

with $x_{12} = 0$, and $x_{22} = 0$.

Up to now, we plug our results into constraints, Equations (3.4-3.7) and (3.10), to imply that

$$x_{11} + x_{14} \leq 4.040541464, \quad (3.13)$$

$$x_{21} + x_{24} \leq 4.086304956, \quad (3.14)$$

$$x_{31} + x_{34} \leq 8.351017699, \quad (3.15)$$

$$x_{11} + x_{21} + x_{31} \geq 11.25364287, \quad (3.16)$$

and

$$x_{14} + x_{24} + x_{34} \geq 2.40794509. \quad (3.17)$$

Next, we compare the coefficients of x_{ij} for $i = 1,2,3$ and $j = 1,4$ to find that x_{34} had the highest coefficient 27 and then we compare the coefficient of x_{14} and x_{24} to decide $x_{34} = 0$, $x_{24} = 0$ and $x_{14} = 2.40794509$.

Consequently, we can further simplify constraints as follows

$$x_{11} \leq 1.632596354, \quad (3.18)$$

$$x_{21} \leq 4.086304956, \quad (3.19)$$

$$x_{31} \leq 8.351017699, \quad (3.20)$$

and

$$x_{11} + x_{21} + x_{31} \geq 11.25364287, \quad (3.21)$$

Finally, we compare the coefficients of x_{11}, x_{21} and x_{31} to claim that

$$x_{11} = 1.632596354, \quad (3.22)$$

$$x_{21} = 4.086304956, \quad (3.23)$$

and

$$x_{31} = 5.53474156. \quad (3.24)$$

Based on our approach, without using any complex computer programming, we obtain the same findings as Roy et al. [6].

IV. Two Related Open Questions

After Tung [7] published a paper to discuss inventory models with ramp type demand under different probability distributions, there are six articles that had cited Tung [7] in their references. We provide a brief reviewing for these six articles. Chao et al. [8] solved the open question proposed by Tung [7] to show that the optimal solution is independent of the demand such that different demand can derive the same optimal solution. Pal et al. [9] studied inventory systems with crisp or fuzzy parameters, Weibull distribution of decay, and time value of money. Chuang et al. [10] provided a detailed numerical analysis to support Chao et al. [8] to show that some derivations of Tung [7] contained questionable findings. Luo [11] examined inventory models with ramp type demand and presented revisions for Mandal and Pal [12] and Wu and Ouyang [13]. Based on Chao et al. [8], Luo [14] improved Deng et al. [15] by two approaches: (a) Analytic method with calculus, and (B) Operational research point of view without deriving the explicit objective function. Lin and Schaeffer [16] studied similarity measures under intuitionistic fuzzy sets environment to amend Li and Cheng [17].

Based on our brief literature reviewing, we can assert that the open question proposed by Tung [7] is an interesting research topic that deserves further examinations.

On the other hand, we recall Acharya and Biswal [18] that considered to solve probabilistic programming problems under the restriction of multiple choice parameters. In the next section, we will present our improvement for Acharya and Biswal [18] that will help practitioners to realize those probabilistic programming problems with multiple choice parameters.

V. Our Examination

We examine the probabilistic programming problem proposed by Acharya and Biswal [18] as follows,

$$\min 24.83x_1 + 28.5x_2 + 43.5x_3 + 45.21x_4, \quad (5.1)$$

subject to

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 \geq 5, \quad (5.2)$$

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta} \geq 21, \quad (5.3)$$

with a abbreviation,

$$\Delta = (0.53x_1)^2 + (0.44x_2)^2 + (4.5x_3)^2 + (0.79x_4)^2, \quad (5.4)$$

and several restrictions,

$$x_1 + x_2 + x_3 + x_4 = 1, \quad (5.5)$$

and $x_1 \geq 0$, $x_2 \geq 0.0100$, $x_3 \geq 0$, with $x_4 \geq 0$.

First we recall that Acharya and Biswal [18] derived that

$$x_1 = 0.6127, \quad (5.6)$$

$$x_2 = 0.0100, \quad (5.7)$$

$$x_3 = 0.3106, \quad (5.8)$$

and

$$x_4 = 0.0667. \quad (5.9)$$

We examine to find thatat the optimal solution proposed by Acharya and Biswal [18] listed as Equations (5.6-5.9) to find out that

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 = 4.99958 < 5. \quad (5.10)$$

Based on our result of Equation (5.10), we know that the solution proposed by Acharya and Biswal [18] that violated the restriction of Equation (5.2) such that researchers should not accept the findings of Equations (5.6-5.9) as derived by Acharya and Biswal [18].

For completeness, we also compute that

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta} = 22.493649 > 21, \quad (5.11)$$

such that the condition of Equation (5.3) is satisfied.

Based on our above examination, we will provide our revision for Acharya and Biswal [18] in the nest section.

VI. Our Improvement

We begin to solve the probabilistic programming problem by an approximated method, such that we will locate an upper bound and a lower bound in the following.

We derive that

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta} \geq 12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\left(\sqrt{(0.53x_1)^2} + \sqrt{(0.44x_2)^2} \sqrt{(4.5x_3)^2} + \sqrt{(0.79x_4)^2}\right), \quad (6.1)$$

such that the solution of

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 = 5, \quad (6.2)$$

and

$$(12 - 1.645 \times 0.53)x_1 + (11.9 - 1.645 \times 0.44)x_2 + (41.8 - 1.645 \times 4.5)x_3 + (52.1 - 1.645 \times 0.79)x_4 = 21, \quad (6.3)$$

that will provide a lower bound for the optimal solution.
On the other hand, we consider the dominate coefficient in the square root as 4.5 and then we derive that

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{(4.5x_3)^2} \geq 12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta}, \quad (6.4)$$

such that

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 = 5, \quad (6.5)$$

and

$$12x_1 + 11.9x_2 + (41.8 - 1.645 \times 4.5)x_3 + 52.1x_4 = 21, \quad (6.6)$$

that will provide an approximated upper bound.

We believe that with our lower bound and upper bound, researchers can shrink the search domain for the optimal solution for the probabilistic programming problem of Equation (5.1) under the conditions of Equations (5.2-5.5).

Moreover, we point out the following analytic approach with Lagrange multiple,

$$\min f(x_1, x_2, x_3, x_4, \lambda_1, \lambda_2) = 24.83 + 3.67x_2 + 18.67x_3 + 20.38x_4 + \lambda_1(x_1 + 4.3x_2 + 9.8x_3 - 3.7), + \lambda_2(0.1x_1 + 29.9x_3 + 40.2x_4 - 9.1 - 1.645\sqrt{\Delta}), \quad (6.7)$$

that will be an interesting research approach for future study.

VII. Numerical Example

In this section, we run an example for our studied programming problem proposed by Acharya and Biswal [18]. We apply Mathcad program to solve the problem, and then we find that

$$x_1 = 0.600000, \quad (7.1)$$

$$x_2 = 0.040000, \quad (7.2)$$

$$x_3 = 0.300000, \quad (7.3)$$

and

$$x_4 = 0.060000. \quad (7.4)$$

Next, we begin to check the restrictions of Equations (5.2-5.5) in the following.
We examine that

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 = 5.0120, \quad (7.5)$$

which satisfies the condition of Equation (5.2).
We derive that

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta} = 21.0590, \quad (7.6)$$

where Δ is defined by Equation (5.4), that satisfies the condition of Equation (5.3).
We find that

$$x_1 + x_2 + x_3 + x_4 = 1, \quad (7.7)$$

which satisfies the condition of Equation (5.5).
At last, we evaluate that

$$24.83x_1 + 28.5x_2 + 43.5x_3 + 45.21x_4 = 31.800600. \quad (7.8)$$

We cannot compare our findings with that of Acharya and Biswal [18], because the results of Acharya and Biswal [18] as we cite as Equations (5.6-5.9) which violate the restriction of Equation (5.2).

VIII. A Related Problem

We study the inventory system proposed by Arcelus and Srinivasan [19] with a temporary price reduction. Arcelus and Srinivasan [19] modified the inventory model developed by Martin [20] such that the ordering cost is computed actually for the last partial order cycle. The purpose of our examination is twofold. First, we prove that there exist closed-form solutions for the supreme gain of the inventory system constructed by Arcelus and Srinivasan [19]. Second, sometimes this inventory system does not have the optimal solution for the special order quantity. Hence, this inventory system sometimes cannot help decision makers to decide the optimal special order quantity. We will advise decision makers more carefully to adopt the inventory system proposed by Arcelus and Srinivasan [19] as an alternative for the inventory model developed by Martin [20]. Numerical examples illustrate our findings.

For the inventory model with a temporary price reduction, Tersine [21] maximized the benefit between a special order under reduced purchase price and the normal EOQ ordering policy where the holding cost is proportional to time. Martin [20] revised Tersine's model to compute the actual holding cost for the last partial order cycle. Arcelus and Srinivasan [19] extended the inventory models of Tersine [21] and Martin [20] to five models such that (1) the AVCT model that is the inventory system of Tersine [21], where the inventory level is assumed as a constant of one half the economic ordering quantity, and the ordering cost is proportional to time, (2) the AVCM model that is the inventory model of Martin [20] where the holding cost is computed actually and the ordering cost is proportional to time, (3) the AVCI model that is a version of the AVCM model such that the ordering cost is revised to compute actually, (4) the AVCII model that is another revision of the AVCM model where the ordering cost is assumed to be equal to the holding cost, and (5) the NPV model that considered the present value for the ordering cost and the purchasing cost for the infinite time horizon. Since the objective function of AVCM, AVCI and AVCII models containing the greatest integer function, Arcelus and Srinivasan [19] claimed that "no closed-form solution for the resulting optimal order quantity, Q_s^*, $s = 2$, 3, 4. exists and thus its optimal value must be obtained iteratively".

In this study, first we will show that the closed-form solution for the objective function of the AVCI model exists. Second, we will demonstrate that sometimes the maximum solution for the special order quantity does not exist. Consequently, for the AVCI model, sometimes decision makers cannot find the optimal special order quantity from the AVCI model. Therefore, the AVCI model seems not a very applicable model to replace for the AVCM model which is the Martin [20]. In the first numerical example, we will find the maximum gain and the optimal special order quantity, then it shows that the iterative solution of Arcelus and Srinivasan [19] is not accurate. In the second numerical example, we will demonstrate that the supreme gain still exists but the optimal special order quantity does not exist. We use the same notation as Martin [20] and Arcelus and Srinivasan [19] with some terminologies to specify our expressions as follows:

$g_{\text{AVCI}}(\$)$ is the maximum value for $g_{\text{AVCI}}(Q')$.

Q'^* is the maximum point for $g_{\text{AVCI}}(Q')$.

B is the set for those m where $Q_{\text{AVCI}}(m)$ is in the interval $(mQ^*,(m+1)Q^*)$.

$Q_{\text{AVCI}}(m)$ is the solution for $\frac{d}{dQ'}g_{\text{AVCI}}(Q')=0$, when m is given.

Q^* is the optimal solution for regular economic ordering quantity.

C is the ordering cost per order.

R is the annual demand.

F is the annual holding cost fraction.

Q' denotes the special order quantity.

d is the unit price discount.

P is the regular price.

We will use $[\cdot]$ to indicate (i) the closed interval, and (ii) the greatest integer function, interchangeably. From the content of related descriptions, no vagueness will happen.

IX. Our Mathematical Results

In Arcelus and Srinivasan [19], they only wrote down the holding cost for the last partial cycle. Hence, we recall the objective function of Martin [20], then we have the gain of the AVCM model, say $g_{\text{AVCM}}(Q')$ as follows,

$$g_{\text{AVCM}}(Q')=d(Q'-Q^*)+C(Q'/Q^*-1)+\frac{(p-d)F}{2R}(Q^{*2}-Q'^2)+\frac{Fp}{2R}\left(\left[\frac{Q'}{Q^*}\right]Q^{*2}-\left(Q'-\left(\left[\frac{Q'}{Q^*}\right]+1\right)Q^*\right)^2\right), \quad (9.1)$$

where the ordering cost is computed proportional to time. If we consider the AVCI model of Arcelus and Srinivasan [19] then the ordering cost is computed actually so we will change the ordering cost from $C((Q'/Q^*)-1)$ to (a) $C([Q'/Q^*]-1)$ for Q'/Q^* is an integer, or (b) $C[Q'/Q^*]$ for Q'/Q^* is not an integer. Hence, we write down the gain for AVCI model as follows,

$$g_{\text{AVCI}}(Q')=d(Q'-Q^*)+C([Q'/Q^*]-1)+\frac{(p-d)F}{2R}(Q^{*2}-Q'^2)+\frac{Fp}{2R}\left(\left[\frac{Q'}{Q^*}\right]Q^{*2}-\left(Q'-\left(\left[\frac{Q'}{Q^*}\right]+1\right)Q^*\right)^2\right), \quad (9.2)$$

when Q'/Q^* is an integer, or

$$g_{\text{AVCI}}(Q')=d(Q'-Q^*)+C[Q'/Q^*]+\frac{(p-d)F}{2R}(Q^{*2}-Q'^2)+\frac{Fp}{2R}\left(\left[\frac{Q'}{Q^*}\right]Q^{*2}-\left(Q'-\left(\left[\frac{Q'}{Q^*}\right]+1\right)Q^*\right)^2\right), \quad (9.3)$$

when Q'/Q^* is not an integer.

From Equation (9.3), we know that $g_{\text{AVCI}}(Q')$ is a continuous function in the open interval $(mQ^*,(m+1)Q^*)$ for every non-negative integer value of m. Next, we will show that $g_{\text{AVCI}}(Q')$ is discontinuous at mQ^* such that the left hand limit and the right hand limit are different as

$$\lim_{Q'\to mQ^{*-}} g_{\text{AVCI}}(Q')=g_{\text{AVCI}}(mQ^*), \quad (9.4)$$

and

$$\lim_{Q'\to mQ^{*+}} g_{\text{AVCI}}(Q')=g_{\text{AVCI}}(mQ^*)+C. \quad (9.5)$$

Based on the above discussion, we will divide the domain of $g_{\text{AVCI}}(Q')$ into sub-intervals as $(mQ^*,(m+1)Q^*]$ for $m=0,1,2\cdots$ such that for each subinterval we will try to find the criterion to determine the local supreme value in each subinterval then compare those local supreme value to locate the global supreme value.

For $mQ^*<Q'<(m+1)Q^*$, we rewrite Equation (9.3) as

$$g_{\text{AVCI}}(Q')=-\frac{(2p-d)F}{2R}Q'^2-\frac{Cd}{p}-Cm^2+\left(\frac{pF}{R}(m+1)Q^*+d\right)Q'-dQ^*. \quad (9.6)$$

It yields that

$$\frac{d}{dQ'}g_{\text{AVCI}}(Q')=-\frac{(2p-d)F}{R}Q'.+\left(\frac{pF}{R}(m+1)Q^*+d\right). \quad (9.7)$$

If we assume that the solution for $\frac{d}{dQ'}g_{\text{AVCI}}(Q')=0$, say $Q_{\text{AVCI}}(m)$, then

$$Q_{\text{AVCI}}(m)=\frac{p}{2p-d}(m+1)Q^*+\frac{dR}{(2p-d)F}. \quad (9.8)$$

In the following, we will derive the criterion for $Q_{\text{AVCI}}(m)$ being in the interior point of the truncated sub-domain, $(mQ^*,(m+1)Q^*)$. From Equation (9.8), it is a direct calculation for inequalities, so we omit its proof.

Lemma 1. For each m, the sufficient and necessary condition for $Q_{\text{AVCI}}(m)$ being in the truncated sub-domain $(mQ^*,(m+1)Q^*)$ is

$$E-1<m<\frac{p}{p-d}+E, \quad (9.9)$$

with

$$E=\frac{dR}{(p-d)FQ^*}. \quad (9.10)$$

According to Lemma 1, we assume that m_S (starting) is the least integer and m_e (ending) is the last integer in $(E-1,(p/(p-d))+E)$ such that m_s, $1+m_s,\cdots$, m_e-1, m_e are in $(E-1,(p/(p-d))+E)$. To simplify the expression, we assume that

$$B_{\mathrm{AVCI}} = \{m : m_s \le m \le m_e\}. \quad (9.11)$$

For each *m* in B_{AVCI}, we find that

$$g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(m)) = \frac{dQ^*(p(m-1)+d)}{2p-d} + \frac{d^2R}{2(2p-d)F} + C\left(\frac{p(m+1)^2}{2p-d} - m^2 - \frac{d}{p}\right). \quad (9.12)$$

For each $m > m_e$, we have

$$g_{\mathrm{AVCI}}(mQ^*) + C = d(m-1)Q^* + C\left(\frac{d}{p}(m^2-1) - m^2 + 2m\right). \quad (9.13)$$

Now, we will derive the main theorem for this note.

Theorem 1. The supreme value for $g_{\mathrm{AVCI}}(Q')$ equals to the superior value of

$$\{g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(m)) : m \in B_{\mathrm{AVCI}}\} \cup \{g_{\mathrm{AVCI}}(mQ^*) + C : m > m_e\}. \quad (9.14)$$

Proof of Theorem 1. Recalled Equations (9.4) and (9.5) that we already show that when $m \notin B$, the maximum point for $g(Q')$ on the sub-interval $(mQ^*, (m+1)Q^*]$ will be equal to (a) $g((m+1)Q^*)$ when $g(Q')$ is on the left slope of a concave down function or (b) $g(mQ^*) + C$ when $g(Q')$ is on the right slope of a concave down function, for each *m*, respectively. Therefore, we need to compare the value of $g(mQ^*) + C$ for $m > m_e$ with those local maximum values of $g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(m))$ for $m \in B_{\mathrm{AVCI}}$.

Theorem 1 point out that $g(Q')$ may have the supreme value but may not have maximum value or maximum point. We will demonstrate this phenomenon in the second numerical example.

X. Numerical Examples

We will first consider the same numerical example in Arcelus and Srinivasan [19] with the following data: p = \$10 per unit, d = \$1 per unit, R = 8000 units per year, C=\$30 per order, F = 0.3 per year, to locate the optimal special order quantity, Q'^* and the maximum value of $g_{\mathrm{AVCI}}(Q')$, say $g_{\mathrm{AVCI}}(\$)$. We find that

$$E = \frac{dR}{(p-d)FQ^*} = 7.4, \quad (10.1)$$

such that $E - 1 = 6.4$ and

$$\frac{p}{p-d} + E = 8.5, \quad (10.2)$$

Based on our results of Equations (10.1) and (10.2), we derive that $m_S = 7$, $m_e = 8$, and $B_{\mathrm{AVCI}} = \{7,8\}$.

We compare $g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(m))$ for $m = 7$, and $m = 8$. We also evaluate $g(mQ^*) + C$ for $m > 8$ then list the results in the above Table 1. Since in Arcelus and Srinivasan [19], they claimed that ${Q'_{\mathrm{AVCI}}}^* = 3289$ so we also compute the value of $g_{\mathrm{AVCI}}(3289)$ to put it inside Table 1.

Based on Table 1, it yields that the maximum solution of ordering quantity, ${Q'_{\mathrm{AVCI}}}^* = 3298.25$, and the maximum value, $g_{\mathrm{AVCI}}(\$) = 1552.4386$.

Moreover, for this numerical example, we find the formulated solutions for the special order quantity and the maximum gain as

$${Q'_{\mathrm{AVCI}}}^* = Q_{\mathrm{AVCI}}(8) = \frac{9p}{2p-d}Q^* + \frac{dR}{(2p-d)F}, \quad (10.3)$$

and

$$g_{\mathrm{AVCI}}(\$) = g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(8)) = \frac{7p+d}{2p-d}dQ^* + \frac{d^2R}{2(2p-d)F} + C\left(\frac{81p}{2p-d} - \frac{d}{p} - 64\right). \quad (10.4)$$

Arcelus and Srinivasan [19] mentioned that

$${Q'_{\mathrm{AVCI}}}^* = 3289, \quad (10.5)$$

and

$$g_{\mathrm{AVCI}}(\$) = 1944.91. \quad (10.6)$$

Our results indicate that their iterative solutions of Arcelus and Srinivasan [19] are not as accurate as our analytical solutions.

We point out that (a) there are local maximum point in $(mQ^*, (m+1)Q^*)$ for $m = 7$, and $m = 8$, and (b) at mQ^* for $m \ge 9$, or $m \le 7$, there are jump.

To indicate that the AVCI model sometimes does not have maximum solution for the special order quantity, we change the value of ordering cost from $C = 30$ to $C = 10$, then we have $A_3 = \frac{dR}{(2p-d)FQ^*} = 12.83$, to imply that

$$A_3 - 1 = 11.83, \quad (10.7)$$

and

$$\frac{p}{p-d} + A_3 = 13.94, \quad (10.8)$$

so $m_S = 12$, $m_e = 13$ and $B_{\mathrm{AVCI}} = \{12,13\}$. We compare $g_{\mathrm{AVCI}}(Q_{\mathrm{AVCI}}(m))$ for $m = 12$, and $m = 13$, and we also evaluate $g(mQ^*) + C$ for $m > 13$ then list the results in the above Table 2.

Based on Table 2, we know that the supreme gain equals to

$$g_{\mathrm{AVCI}}(\$) = C + g_{\mathrm{AVCI}}(14Q^*) = 1517.22. \quad (10.9)$$

However, we do not have the optimal solution for the special order quantity, ${Q'_{\mathrm{AVCI}}}^*$ such that can attain the supreme value. The theoretical solution for ${Q'_{\mathrm{AVCI}}}^*$ is

$${Q'_{\mathrm{AVCI}}}^* = (14Q^*)^+, \quad (10.10)$$

Table 1. Comparison among $g_{\text{AVCI}}(Q_{\text{AVCI}}(m))$ for $m \in B_{\text{AVCI}}$ and $g_{\text{AVCI}}(mQ^*)+C$ for $m > m_e$ with $C = 30$.

	$Q_{\text{AVCI}}(m)$	$g_{\text{AVCI}}(Q_{\text{AVCI}}(m))$		$C + g_{\text{AVCI}}(mQ^*)$
$m = 7$	3087.72	1523.4912	$m = 9$	1550
$m = 8$	3298.25	1552.4386	$m = 10$	1497
			$m = 11$	1390
$g_{\text{AVCI}}(3289) = 1552.4081$			$m = 12$	1229

Table 2. Comparison among $g_{\text{AVCI}}(Q_{\text{AVCI}}(m))$ for $m \in B_{\text{AVCI}}$ and $g_{\text{AVCI}}(mQ^*)+C$ for $m > m_e$ with $C = 10$.

	$Q_{\text{AVCI}}(m)$	$g_{\text{AVCI}}(Q_{\text{AVCI}}(m))$		$C + g_{\text{AVCI}}(mQ^*)$
$m = 12$	2983.63	1499.40	$m = 14$	1517.22
$m = 13$	3105.17	1513.06	$m = 15$	1507.16
			$m = 16$	1479.10

where $(14Q^*)^+$ means the left hand limit point for $14Q^*$, but in the real word situation, we cannot choice a point to represent $(14Q^*)^+$. Hence, The AVCI model of Arcelus and Srinivasan [19], sometimes does not have optimal solution for Q'^*_{AVCI}, even we can find the closed-form solution for the supreme gain. Consequently, by the numerical method mentioned in Arcelus and Srinivasan [19] to derive the maximum gain iteratively will not attain this supreme value. By our analytical method, we still can derive the closed-form solution for the supreme gain as

$$g_{\text{AVCI}}(\$) = C + g_{\text{AVCI}}(14Q^*) = 13dQ^* + C((195d/p) - 168) = 1517.22. \quad (10.11)$$

This phenomenon indicates that the AVCI model of Arcelus and Srinivasan [19], does not serve as a good revision for the AVCM model which is the inventory model of Martin [20].

XI. Second numerical Test

With the help of a colleague, we rerun the numerical example that was proposed in section VII by Maple computer programming. We obtain that

$$x_1 = 0.603000, \quad (11.1)$$

$$x_2 = 0.037000, \quad (11.2)$$

$$x_3 = 0.300000, \quad (11.3)$$

and

$$x_4 = 0.060000. \quad (11.4)$$

Next, we begin to check the restrictions of Equations (5.2-5.5) in the following.
We examine that

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 = 5.0021, \quad (11.5)$$

which satisfies the condition of Equation (5.2).
We derive that

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645\sqrt{\Delta} = 21.0587, \quad (11.6)$$

where Δ is defined by Equation (5.4), that satisfies the condition of Equation (5.3).
We find that

$$x_1 + x_2 + x_3 + x_4 = 1, \quad (11.7)$$

which satisfies the condition of Equation (5.5).
At last, we evaluate that

$$24.83x_1 + 28.5x_2 + 43.5x_3 + 45.21x_4 = 31.7896. \quad (11.8)$$

Next, we compare our findings of Equations (7.8) and (11.8) to find the Maple programming is superior to that of Mathcad programming.

XII. Some Related Problems

Wang et al. [22] studied Ardajan [23], Aull-Hyde [24], and Chu et al. [25] to present further revisions. Hence, Wu and Hung [26] examined another related paper of Aull-Hyde et al. [27] to consider consistent test in analytic hierarchy process. In this section, we follow Wu and Hung [26] to provide a further examination for Aull-Hyde et al. [27].
If we consider an analytic hierarchy process problem with three alternatives, B_1, B_2 and B_3, and there are many decision makers, E_j, with $j \in \{1,2,\ldots,m\}$. In Saaty [28], each decision maker, say E_j, by pairwise comparison to decide the relative weight for the 1-9 scale then the individual comparison matrix, denoted as $M_j = [m_{stj}]_{3\times 3}$, that satisfies

$$m_{stj} \in \{1/9, 1/8, \ldots, 1, 2, \ldots, 9\}, \quad (12.1)$$

and the reciprocal property,

$$m_{stj} m_{tsj} = 1, \quad (12.2)$$

for $s, t \in \{1,2,3\}$.
On the other hand, in Aull-Hyde [27], she arbitrarily developed a group of ninety persons, and then randomly assign a reciprocal comparison matrix, say

$$N_k = [n_{stk}]_{3\times 3}, \quad (12.3)$$

with $k \in \{1,2,\ldots,90\}$ such that n_{stk} is randomly picked up from the permissible set, $\{1/9, 1/8, \ldots, 1, 2, \ldots, 9\}$.
The group comparison matrix, say

$$G = [g_{st}]_{3\times 3}, \quad (12.4)$$

such that

$$g_{st} = \left(\prod_{k=1}^{90} n_{stk}\right)^{1/90}. \quad (12.5)$$

Based on the simulation test of Aull-Hyde [27], the group comparison matrix of Equation (12.4) will pass the consistent examination. If we follow this trend to further extend the group from ninety peoples to nine hundred peoples, and then further extend to nine thousand peoples such that our simulation results will all pass the consistency test as proposed by Aull-Hyde [27]. From our three proposed tests, it demonstrates that if we (a) extend the individual group to a sufficient large group, and (b) randomly assign the individual comparison matrix, then the normalized group priority vector will converge to $(1/3, 1/3, 1/3)^T$, where T indicates the transpose operator. Since the group comparison matrix passes the consistency test, then the equally weight results will be

derived. It reveals that by the approach proposed by Aull-Hyde [29], then we will derive that every alternative has the same weight. It is not a reasonable conclusion for group decision environment. Hence, we can conclude that something must be wrong in the geometric mean average algorithm proposed by Aull-Hyde [29].
Owing to

$$\lim_{k\to\infty}(n_{stk})^{1/k} = 1, \tag{12.6}$$

if n_{stk} is randomly selected from the set of Equation (12.1). There is a threshold, say δ, from the arithmetic mean point of view, or denoted as W, from the geometric mean point of view, and a natural number, expressed as P, such that when $k \geq P$, then

$$1+\delta \geq (n_{stk})^{1/k} \geq 1-\delta, \tag{12.7}$$

or

$$W \geq (n_{stk})^{1/k} \geq (1/W). \tag{12.8}$$

Based on our above discussion of Equations (12.7) and (12.8), researchers will develop new estimation theorem for consistent test.

XIII. A Related Examination

We recall that in Chang et al. [29] and Gallego and Moon [30], $\pi^G(Q)$ has a lower bound as

$$m\mu - \sigma\sqrt{md} = m\mu\left(1-\frac{\sigma}{\mu}\sqrt{\frac{d}{m}}\right),$$

$$= \pi^G(Q^S) \leq \pi^G(Q). \tag{13.1}$$

If the lower bound is negative. It is too underestimated then Gallego and Moon [15] recalled that

$$\pi^G(Q=0)=0, \tag{13.2}$$

so the minimum problem $\max \pi^G(Q)$ cannot be estimated by the lower bound,

$$m\mu - \sigma\sqrt{md} = m\mu\left(1-\frac{\sigma}{\mu}\sqrt{\frac{d}{m}}\right),$$

$$= \pi^G(Q^S) < 0. \tag{13.3}$$

Gallego and Moon [30] claimed that

(i) if $\frac{m}{d} \geq \frac{\sigma^2}{\mu^2}$, and then $\hat{Q}^S = Q^S$,

(ii) if $\frac{m}{d} < \frac{\sigma^2}{\mu^2}$, and then $\hat{Q}^S = 0$.

We observe the solution procedure of Gallego and Moon [30]: if the lower bound is underestimated to imply a negative lower bound, then Gallego give up the lower bound, directly take $Q^* = 0$ with $\pi^G(Q^*=0)=0$.

We raise the following problems,

(1) Why the relation between $g(0)$ and $g(Q^S)$ can be decided as follows,

$$g(0) = \frac{m+d}{2}\left(\left(\sigma^2+\mu^2\right)^{1/2}+\mu\right),$$

$$\geq g(Q^S) = d\mu + \sigma\sqrt{md}\ ? \tag{13.4}$$

(2) if $\frac{m}{d} < \frac{\sigma^2}{\mu^2}$, then $Q^S < 0$

(3) if $\frac{m}{d} < \frac{\sigma^2}{\mu^2}$, then there should be a point, say $\widetilde{Q}$, that satisfies

$$(m+d)\mu - g(\widetilde{Q}) = 0, \tag{13.5}$$

where the objective function, $g(Q)$ is denoted as follows,

$$g(Q) = dQ + \left(\frac{m+d}{2}\right)\times,$$

$$\left(\left(\sigma^2+(Q-\mu)^2\right)^{1/2} - (Q-\mu)\right). \tag{13.6}$$

XIV. Direction for Future Study

We provide a brief literature reviewing for related problems. Under the restriction of demand expressed in a linear formation, Ouyang and Rau [31] constructed a simple procedure for production strategies. Referring to nearest feature character, Chen et al. [32] studied machine intelligence and pattern analysis for facial cognition. With two competing retailers, Li and Mao [33] developed inventory systems with respect to decay product. To establish a component patterns database, Hsiao et al. [34] examined a new procedure to execute character recognition under an internet environment. With a fuzzy and changeable conditions, Yan [35] generated inventory models under arbitrary supply chain to decide the optimal price and replenishment policy. On the other hand, some recently published articles aroused our attention. With respect to fifteen major crowded United States city region, Matloub, and Kostanic [36] examined individual travel freedom variables through proportional analysis. Referring to exponential essence, Yuan, and Du [37] studied Hermite and Hadamard inequality by partial integration. To trace breast cancer, Ashilla et al. [38] applied several variable control diagram. To learn automobile fire in subway, Bai et al. [39] considered serious speed and film span for battery. Related to complex coefficient, Al-Shorman et al. [40] derived algebraic formulas for Newton-Raphson approach. To realize bug killers in rice field with Tungro infection, Amelia et al. [41] constructed organized system. Based on distributed message, Li, and Zhang [42] analyzed presentation. According to chaos method, Song et al. [43] developed Cauchy transformation. Under time and space web structure, Zhang et al. [44] acquired fast shipment under fewer carbon transfer design. With respect to various forms of smash wave, Unyapoti and Pochai [45] obtained two types of peak wave systems. For tour question, Xu and Zhang [46] employed a novel grey approach. Considering incident judgment diagram, Li et al. [47] gained evolutionary judgment for municipal crisis. Through interaction with individual, Kusuma, and Prasasti [48] found out a heuristic procedure to merge huge information. Owing to novel intellectual systems, Zihan et al. [49] got forecast for earthquake. Owing to involvement regulations, Arboleda et al. [50] took plans for superstore assignment. Researchers can find interesting study topic and hot academic current through our literature reviewing.

XV. Conclusion

We examine stochastic transportation models with multiple choice costs to point out that for a minimum problem, and then we can directly use the minimum among the possible choice costs to dramatically simplify the transportation model. Moreover, we propose an intuitive approach to observe the global minimum or maximum in constraints and then examine the maximum and minimum in each constraint to simplify the model iteratively to obtain the optimal solution. Our proposed approach will be useful to solve transportation models in the future. We studied the AVCI model of Arcelus and Srinivasan [19] that is a modification of the AVCM model of Martin [20]. We found the criterion to derive those local maximum points inside the truncated sub-intervals and then compared with the gain value at those jumps. We can derive the exact formulated solution of supreme gain for each numerical example. However, sometimes, we pointed out that the optimal special order quantity did not exist. It indicates that we may advise decision makers not to treat the AVCI model as a revision of the AVCM model of Martin [20].

REFERENCES

[1] J. K. J. Wu, H. L. Lei, S. T. Jung, P. Chu, “Newton method for determining the optimal replenishment policy for EPQ model with present value,” *Yugoslav Journal of Operations Research*, vol. 18, pp. 53-61, 2008.

[2] K. C. Hung, “Continuous review inventory models under time value of money and crashable lead time consideration,” *Yugoslav Journal of Operations Research*, vol. 21, no. 2, pp. 293-306, 2011.

[3] J. Lin, H. Chao, P. Julian, “A demand independent inventory model,” *Yugoslav Journal of Operations Research*, vol. 23, no. 1. pp. 129-135, 2013.

[4] R. Lin, “Note on fuzzy sets,” *Yugoslav Journal of Operations Research*, vol. 24, no. 2, pp. 299-303, 2013.

[5] J. P. C. Chuang, P. Chu, “Improving the public transit system for routes with scheduled headways,” *Yugoslav Journal of Operations Research*, vol. 24, no. 2, pp. 237-248, 2014.

[6] S. K. Roy, D. R. Mahapatra, M. P. Bistwal, “Multi-choice Stochastic Transportation Problem with Exponential Distribution,” *Journal of Uncertain Systems*, vol. 6, no. 3, pp. 200-213, 2012.

[7] C. T. Tung, "A note on the inventory models with ramp type demand of negative exponentially distributed changing point," *Journal of Industrial and Production Engineering*, vol. 30, no. 1, 2013, pp. 15-19.

[8] H. C. J. Chao, S. C. Lin, J. Lin. "Reply to a note on the inventory models with ramp-type demand of negative exponentially distributed changing point," *Journal of Industrial and Production Engineering*, vol. 31, no. 6, pp. 359-362, 2014.

[9] S. Pal, G. S. Mahapatra, G. P. Samanta, "An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment," *International Journal of Production Economics*, vol. 156, pp. 159-166, 2014.

[10] J. P. C. Chuang, W. S. Chou, P. Julianne, "Inventory models with stochastic ramp type demand," *Journal of Interdisciplinary Mathematics*, vol. 19, no. 1, pp. 55-64, 2016.

[11] X. R. Luo, "Analysis of inventory models with ramp type demand," Journal of Interdisciplinary Mathematics, vol. 20, no. 2, pp. 543-554, 2017.

[12] B. Mandal. A. K. Pal, "Order level inventory system with ramp type demand rate for deteriorating items," *Journal of Interdisciplinary Mathematics,* vol. 1, pp. 49-66, 1998.

[13] K. S. Wu, L. Y. Ouyang, "A replenishment policy for deteriorating items with ramp type demand rate," *Proceeding of National Science Council ROC (A)*, vol. 24, pp. 279-286, 2000.

[14] X. R. Luo, "Inventory models solved with hybrid approaches," *International Journal of Computer Science and Engineering*, vol. 7, no. 4, pp. 1-10, 2018.

[15] P. S. Deng, R. Lin, P. Chu, "A note on the inventory models for deteriorating items with ramp type demand rate," *European Journal of Operational Research*, vol. 178, pp. 112-120, 2007.

[16] E. Lin, D. Schaeffer, "Discussion on Weighted Similarity Measure under Intuitionistic Fuzzy Sets Environment," *International Journal of Engineering Research & Science*, vol. 5, no. 9, pp. 48-56, 2019.

[17] D. Li, C. Cheng, “New similarity measure of intuitionistic fuzzy sets and application to pattern recognitions," *Pattern Recognition Letters*, vol. 23, pp. 221-225, 2002.

[18] S. Acharya, M. P. Biswal, "Solving probabilistic programming problems involving multi-choice parameters," *Opsearch*, vol. 48, no. 3, pp. 217-235, 2011.

[19] F. J. Arcelus, G. Srinivasan, “Costing partial order cycles in the temporary sale price problem,” *International Journal of Production Economics*, vol. 56-57, pp. 21-27, 1998.

[20] G. E. Martin, , “Note on an EOQ model with a temporary sale price,” *International Journal of Production Economics*, vol. 37, pp. 241-243, 1994.

[21] R. J. Tersine, , *Principles of Inventory and Materials Management*, 4th ed., Prentice-Hall, Englewood Cliffs, N.J., 1994.

[22] C. M. Wang, Y. L. Wang, M. L. Chen, "Discussion for Consistency Test in Analytic Hierarchy Process," *IAENG International Journal of Applied Mathematics*, vol. 53, no.3, pp. 1123-1131, 2023.

[23] A. Ardajan, "Optimal ordering policies in response to a sale," *IIE Transactions*, vol. 20, pp. 292-294, 1988.

[24] R. L. Aull-Hyde, "Evaluation of supplier-restricted purchasing options under temporary price discounts," *IIE Transactions*, vol. 24, pp. 184-186, 1992.

[25] P. Chu, P. S. Chen, T. Niu, "Note on supplier-restricted order quantity under temporary price discounts," *Mathematical Methods of Operations Research*, vol. 58, pp. 141-147, 2003.

[26] L. C. Wu, C. F. Hung, "Further study for individual consistency control in consensus building," *IAENG International Journal of Computer Science*, vol. 51, no. 7, pp. 868-875, 2024.

[27] R. Aull-Hyde, S. Erdogan, J. M. Duke, "An experiment on the consistency of aggregated comparison matrices in AHP," European Journal of Operational Research, vol. 171, pp. 290–295, 2006.

[28] T. L. Saaty, *The analytic hierarchy process,* New York: McGraw-Hill, 1980.

[29] S. K. J. Chang, J. P. C. Chuang, H. J. Chen, Short comments on technical note—The EOQ and EPQ models with shortages derived without derivatives, *International Journal of Production Economics,* vol. 97, pp. 241–243, 2005.

[30] G. Gallego, I. Moon, "The Distribution-free newsboy problem- review and extensions," *Journal of the Operational Research Society*, vol. 44, pp. 825-834, 1993.

[31] B. Ouyang, H. Rau, "A simple algorithm for production policies with a linear trend in demand," *Journal of the Chinese Institute of Industrial Engineers*, vol. 25, no. 3, pp. 229-236, 2008.

[32] Y. Chen, C. Han, C. Wang, K. Fan, "Face recognition using nearest feature space embedding," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 6, pp. 1073-1086, 2011.

[33] J. Li, J. Mao, "An inventory model of perishable item with two types of retailers," *Journal of the Chinese Institute of Industrial Engineers*, vol. 26, no. 3, pp. 176-183, 2009.

[34] S. Hsiao, K. Fan, W. Sung, S. Ou, "Innovative algorithms for running a web-based pattern recognition search system for a component patterns database," *Malaysian Journal of Computer Science*, vol. 15, no. 2, pp. 78-93, 2002.

[35] X. Yan, "Replenishment and pricing decisions for inventory systems with random supply in a fluctuated environment," *Journal of the Chinese Institute of Industrial Engineers*, vol. 28, no. 1, pp. 45-54, 2010.

[36] Zaid Matloub, and Ivica Kostanic, "Comparative Analysis of Spatial Human Mobility Parameters in 15 Most Populous U.S. Metropolitan Statistical Areas," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 6, pp. 997-1012, 2024.

[37] Zhengrong Yuan, and Tingsong Du, "The Multipoint-based Hermite-Hadamard Inequalities for Fractional Integrals with Exponential Kernels," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 6, pp1013-1025, 2024.

[38] AurellFaza Ashilla, Awang Putra Sembada R, I Melda Puspita Loka, Sukma Adi Perdana, and Muhammad Ahsan, "Kernel PCA based on Hotelling Multivariate Control Chart for Monitoring Breast Cancer Diagnostic," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 6, pp. 1026-1032, 2024.

[39] Zhenpeng Bai, Xiaohan Zhao, Huaitao Song, Yang Zhang, Haowei Yao, and Jin Zhang, "Prediction of Back-layering Length and Critical Velocity with Lithium-ion Battery Car Fires in Tunnel," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 6, pp. 1033-1037, 2024.

[40] Ammar Al-Shorman, Mahmood Shareef Ajeel, and Kamel Al-Khaled, "Analyzing Newton's Method for Solving Algebraic Equations with Complex Variables: Theory and Computational Analysis," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 6, pp. 1038-1047, 2024.

[41] Rika Amelia, Nursanti Anggriani, Asep K. Supriatna, and Noor Istifadah, "Optimal Control Model for The Spread of Tungro Disease in Rice Plants by Controlling Using Pesticides," *Engineering Letters*, vol. 32, no. 6, pp. 1097-1106, 2024.

[42] Xinying Li, and Lijun Zhang, "Performance Analysis of NOMA-Based Symbiotic Ambient Backscatter Communication," *Engineering Letters*, vol. 32, no. 6, pp. 1107-1113, 2024.
[43] Yu-Wei Song, Wei-Zhong Sun, Jie-Sheng Wang, Yu-Liang Qi, Xun Liu, and Yuan-Zheng Gao, "Cauchy Mutation Chaotic Coati Optimization Algorithm," *Engineering Letters*, vol. 32, no. 6, pp. 1114-1131, 2024.
[44] Yuzhao Zhang, Zhenjiang Zhang, Zhimo Jiang, Xiaorong Wang, and Muchen Ye, "Study on Low-carbon Transport Scheme of Express Freight Based on Space-time Network," *Engineering Letters*, vol. 32, no. 6, pp. 1132-1145, 2024.
[45] Pidok Unyapoti, and Nopparat Pochai, "A Shoreline Evolution Model with Wave Crest Model on I-Head and T-Head Groin Structures with Different Types of Breaking Wave," *Engineering Letters*, vol. 32, no. 6, pp. 1146-1162, 2024.
[46] Zhinan Xu, and Xiaoxia Zhang, "An Improve Grey Wolf Optimizer Algorithm for Traveling Salesman Problems," *IAENG International Journal of Computer Science,* vol. 51, no. 6, pp. 602-612, 2024.
[47] Shiyong Li, Ruijun Wang, and Wei Sun, "Evolutionary Logic of Public Emergencies Based on Event Logic Graph," *IAENG International Journal of Computer Science*, vol. 51, no. 6, pp. 613-625, 2024.
[48] Purba Daru Kusuma, and Anggunmeka Luhur Prasasti, "Best-Other Algorithm: A Metaheuristic Combining Best Member with Other Entities," *IAENG International Journal of Computer Science*, vol. 51, no. 6, pp. 626-633, 2024.
[49] Zhang Zihan, Liu Xiaosheng, Wu Lijun, and Hu Guangqiu, "Prediction of Blast-induced Ground Vibration using Eight New Intelligent Models," *IAENG International Journal of Computer Science*, vol. 51, no. 6, pp. 642-649, 2024.
[50] Francisco Javier Moreno Arboleda, Georgia Garani, and Andres Felipe Arboleda Correa, "Supermarket Product Placement Strategies Based on Association Rules," *IAENG International Journal of Computer Science*, vol. 51, no. 6, pp. 650-662, 2024.

Yu-Lan Wang received her Ph.D. degree from Tianjin Nankai University in 2014 and is currently a Professor at the School of Education, Qingdao Huanghai University, Shandong, China. The main research directions are Preschool Education, Creative Flipped Education, Educational Management, Fuzzy Set Theorem, and Educational Psychology.