Optimal Control and Analysis of the SEIRS Model on the Problem of Online Game Addiction: A Case Study Among Class VIII Students of the State Junior High Schools in Makassar City

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Abstract—This study aimed to develop a Suspected-Exposed-Infected-Recovered (SEIRS) mathematical model to solve addiction to online games. The model would use optimal control to analyze and predict addiction cases among the State Junior High School students. The model was analyzed through determination of the equilibrium point, stability, and basic reproduction number ($R_0$). The optimal control problem was solved using the Pontryagin principle to minimize the number of addicted groups. The analysis without control gave an initial value of $R_0 = 1.94 > 1$, which indicated that online game addiction in students at a worrying level. The value $R_0$ with 1% control was $R_0 = 1.892$, which showed that the problem is still worrying. Meanwhile, 50% and 90% controls gave $R_0 = 0.814$ and $R_0 = 0.556$, respectively, which indicated that the problems can be overcome because the values are less than one. This shows that the greater the control, the lower the transmission rate for online game addiction.

Index Terms—SEIRS Model, Online Game Addiction, Optimal Control, Basic Reproductive Numbers,

I. INTRODUCTION

In the health sector, mathematical modeling is employed to determine whether a disease is epidemic, communicable, or non-communicable [1]. It can also be applied to model online games that have a negative impact on users when played excessively, leading to behaviors like laziness to engage in other activities and reduced social interaction within the community [2].

In 2017, the Indonesian Internet Service Providers Association (APJII) reported that the country had 143.26 million internet users (50% of the 262 million population) [3]. According to a marketing research institute in Amsterdam called Newzoo, the country had 43.7 million gamers in 2017, with 56% being male and 44% female. Indonesia ranked 17th globally for having the highest number of active game players and the most in Southeast Asia. These games are played on mobile phones, personal computers, and laptops.

A theoretical study was carried out on the SEIRS mathematical modeling using a qualitative approach in the case of online game addiction with optimal control. The SEIR model was built by adding control parameters such as parental supervision and analyzed using a generation matrix to obtain basic reproduction numbers and model stability [20]. The numerical simulation used primary data obtained from class VIII students of the State Junior High School in Makassar City and analyzed with Matlab software.

III. RESULT AND DISCUSSION

A. SEIRS Mathematical Model on the Problem of Online Game Addiction

The SEIRS model scheme in problems of online game addiction with optimal control is shown in Figure 1 below:

The variables in Figure 1 consist of a total population of $N$, which is divided into four compartments, namely Susceptible (S) class, which is a group of students who are prone to online game addiction. Exposed (E), those have started playing but have not become addicted, Infected (I), the addicted group, and Recovered (R), students who are free from addiction.

http://www.iaeng.org/IJAM/v54n2p232.pdf
from online game addiction. The definition of the SEIRS model of online game addiction is shown in Table 1.

The formulation of the SEIRS model on the problem of online game addiction based on Figure 1 is a non-linear system of differential equations as expressed in Equations 1 below:

\[
\begin{align*}
\frac{dS}{dt} &= \mu N - \theta R - \alpha SI - \mu S \\
\frac{dE}{dt} &= \alpha SI - \beta E - \mu E - \tau(1-u)E \\
\frac{dI}{dt} &= \beta E + \mu I - \delta I \\
\frac{dR}{dt} &= \delta I + \tau(1-u)E - \mu R - \theta R
\end{align*}
\]

(1)

1) Basic Reproduction Number: The value of basic reproduction number \( (R_0) \) obtained using the Next Generation Matrix method from Equations (1) is:

\[
F = \begin{pmatrix}
\beta SI \\
0
\end{pmatrix}
\]

\[
F' = \begin{pmatrix}
0 & \alpha \delta \\
0 & 0
\end{pmatrix}
\]

The values of \( V^{\prime-1} \) is determined using the equation below:

\[
V = \begin{pmatrix}
(\beta + \mu + \tau)E \\
(\mu I + \delta + \beta)E
\end{pmatrix}
\]

\[
V^{\prime-1} = \begin{pmatrix}
\frac{1}{\mu + \beta + \tau} & 0 \\
\frac{1}{\mu + \beta + \tau}
\end{pmatrix}
\]

therefore, the basic reproduction number is obtained as in Equation 4 below:

\[
F'V^{\prime-1} = \begin{pmatrix}
\frac{\alpha \delta \beta}{\mu + \beta + \tau} \\
\mu + \beta + \tau
\end{pmatrix}
\]

\[
R_0 = \frac{\alpha \beta}{(\beta + \mu + \tau)(\delta \mu)}
\]

(4)

2) Solution of SEIRS Model Optimal Control on Online Game Addiction: The SEIRS model on the problem of online game addiction in Equations 1 with the addition of control is expressed in Equations 5 below:

\[
\begin{align*}
\frac{dS}{dt} &= \mu N - \theta R - \alpha SI - \mu S \\
\frac{dE}{dt} &= \alpha SI - \beta E - \mu E - \tau(1-u)E \\
\frac{dI}{dt} &= \beta E + \mu I - \delta I \\
\frac{dR}{dt} &= \tau \mu E - \mu R - \theta R
\end{align*}
\]

(5)

Optimal control in form of parental supervision added to the SEIRS model of online game addiction is an effort to reduce the number of addicted individuals. Basic Reproductive Numbers \( R_0 \) with optimal control using the next-generation matrix method based on Equations 5 is stated in Equation 6 below:

\[
R_0 u = \frac{\alpha \beta \delta}{(\beta + \mu + (\tau + u)))(\mu + \delta)}
\]

(6)

Based on the Pontryagin principle, the first step taken to obtain optimal control is to form a Hamilton function, according to the objective function obtained:

\[
H(S, E, I, R, u, \sigma) = I(t) + \frac{e}{2} u^2 t + \sum_{i=1}^{\sigma f}(\sigma f)
\]

(7)

Where \( \sigma \) and \( \epsilon \) represent the costate variables and the right-hand side of the system of equations, respectively. Equation 7 can be rewritten as:

\[
H(S, E, I, R, u, \sigma) = I(t) + \frac{C}{2} u^2 t + \sum_{i=1}^{\sigma f}(\sigma f)
\]

(8)

According to Pontryagin’s principle, the Hamilton function reaches an optimal solution when the state and costate variables are stationary, as well as stationary conditions, apply:

**State Equation**

\[
\frac{\partial H}{\partial S} = \mu N - \theta R - \alpha SI - \mu S \\
\frac{\partial H}{\partial E} = \alpha SI - \beta E - \mu E - \tau(1-u)E \\
\frac{\partial H}{\partial I} = \beta E + \mu I - \delta I \\
\frac{\partial H}{\partial R} = \tau \mu E - \mu R - \theta R
\]

(9)

**Co-State Equation**

\[
-\frac{\partial H}{\partial S} = -[\sigma_1(t)\alpha I - \sigma_1(t)S + \sigma_2(t)\alpha I] \\
-\frac{\partial H}{\partial E} = -[\sigma_2(t)(\beta + \mu) - \sigma_2(t)(\tau - u)] \\
-\frac{\partial H}{\partial I} = -[\sigma_3(t)(\beta + \mu) + \sigma_4(t)(\mu + \tau)] \\
-\frac{\partial H}{\partial R} = -[\sigma_4(t)(\beta + \mu) + \sigma_4(t)(\mu + \tau)]
\]

(10)

**Stationary Condition**

\[
\frac{\partial H}{\partial \epsilon} = 0 \\
C\sigma_2(t)E(t) - \sigma_3(t)E(t) = 0
\]

(11)
Where \(0 \leq u \leq 1\), obtain:

\[
U^* = \begin{cases} 
0, & \text{if } u'(t) \leq 0 \\
u'(t), & \text{if } 0 \leq u'(t) \leq 1 \\
1, & \text{if } u'(t) \geq 1 
\end{cases}
\] (11)

Therefore, the control form \(u(t)\) is obtained which is presented in Equation 12 below:

\[
u(t) = \min(1, \max(0, u'(t)))
\]

\[
u(t) = \min(1, \left(\max\left(0, \frac{\sigma_2(t)rE(t) + \sigma_4(t)E(t)}{C}\right)\right))
\] (12)

B. SEIRS Model Simulation on the Problem of Online Game Addiction in Makassar City

The SEIRS model's numerical simulation was conducted using Matlab by substituting parameters and initial values from the primary data. This encompasses the count of students from State Junior High School in Makassar City who initiate playing online games, are susceptible to addiction, and engage in activities unrelated to online games. Meanwhile, the parameter values employed in the simulation were derived from questionnaire responses. The initial values for the model’s variables and parameters are presented in Table 2 below.

<table>
<thead>
<tr>
<th>Variable &amp; Parameters</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_0)</td>
<td>0.27</td>
</tr>
<tr>
<td>(E_0)</td>
<td>0.57</td>
</tr>
<tr>
<td>(I_0)</td>
<td>0.12</td>
</tr>
<tr>
<td>(R_0)</td>
<td>0.04</td>
</tr>
<tr>
<td>(N)</td>
<td>364</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.03</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.70</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.3</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.27</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.03</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.16</td>
</tr>
</tbody>
</table>

When the parameter values are substituted into Equation (4), the basic reproduction number value is \(R_0 = 1.944 > 1\). This showed that online game addiction in students is a concern. The results of the SEIRS model simulation on the problem of online game addiction for grade VIII students are shown in Figures 3 and 4 below.
Figures 2 and 3 show that the suspected class or students who are prone to online game addiction continued to increase from the 1th month, but remained stable in the 50th month. In the exposed class, students who started playing online games continued to decline and stabilize in the 70th month. The infected class experienced a significant increase in the first month and stabilized in the 50th month. Meanwhile, the recovered class increased at the beginning, but decreased in the 15th month and stabilized in the 50th month.

1) SEIRS Model Simulation on Online Game Addiction Problems with Optimal Control: The SEIRS model simulation of online game addiction problem with optimal control aims to determine the maximum effort of supervision or control from parents on addicted students. The numerical simulation with optimal control was solved using the Runge Kutta back and forth scheme of order 4. The state equations were solved using the forward scheme, while the costate equations were estimated using the backward scheme. The state equation is defined as $S_0 = y(1)$, $E_0 = y(2)$, $I_0 = y(3)$, and $R_0 = y(4)$, while the costate equation is defined as $\sigma(1) = y(1)$, $\sigma(2) = y(2)$, $\sigma(3) = y(3)$, and $\sigma(4) = y(4)$. The model simulation used the initial value of $S=99, E=206, I=45, R=14$, and $N=364$, with the given weight value of $C=0.75$, while the parameter values were used to refer to Table 2. This study contains only one control variable, therefore, the simulation was carried out in one scenario when there is parental supervision on children playing online games.

Comparison of simulation results on students who are addicted to online games without and with parental supervision are shown in Figures 8 below:
in the present study, where the count of addicted students in the state junior high school initially continued to rise, but subsequently decreased following the implementation of optimal control measures in the form of parental supervision.

V. CONCLUSION

The findings demonstrate the feasibility of characterizing online game addiction using the SEIRS model. Through analysis and simulation of the model, it is evident that the incidence of cases among students diminishes when incorporating parameters, such as parental supervision, as an optimal control measure. Consequently, the SEIRS mathematical model, when employed with optimal control strategies, presents a viable solution to address the issue of online game addiction among State Junior High School students in Makassar City.

REFERENCES