Inventory Models of Decaying Items with Shortages and Salvage Value

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Abstract—**Recently, Annadurai published an important paper in International journal of Management Science and Engineering Management for the optimal replenishment policy for inventory models of decaying items with shortages and salvage value. However, his solution procedure contained some questionable results. The purpose of this note is fourfold. First, we recap the inventory model proposed by Annadurai. Second, we provide our improvement. Third, we point out questionable results of Annadurai and then offer our revisions. Furth, we consider two boundary minimums that will be an improvement for the no shortage case and no stock case, respectively.**

Index Terms—**Optimization, Salvage value, Shortage, Deterioration, Inventory**

I. Introduction

THERE is a trend of several papers that were published by Yugoslav Journal of Operations Research to apply an analytical method to revise previous findings. We just name few of them. Wu et al. [1] examined the Newton method to find an optimal replenishment strategy for inventory models of Dohi et al. [2] and Chung et al. [3] so that the convergence of their sequence is better than the existing bisection approach. Hung [4] developed inventory models with crashable lead and present value to generalize the inventory model of Ouyang et al. [5]. He compared the boundary minimum with the interior minimum to verify that the interior minimum is the global minimum. Lin et al. [6] studied inventory systems to generalize Deng et al. [7] constructed a new inventory models from a ramp type demand to a more practical condition and then the optimal solution for the replenishment time is not related to demand. Their results dramatically simplified the solution procedure. Lin [8] published an interesting article with the original paper of fuzzy set theorem to provide revisions for some analytical works for Zadeh [9]. Chuang and Chu [10] proved that the traffic model has a unique optimal solution and then offered a formulated approximated solution that was superior to the results of Hendrickson [11] for headways. Tung et al. [12] revised some questionable results in Chang et al. [13] to help researchers apply their important finding of smoothly connected property. We follow this trend to study Annadurai [14] to point out that his two theorems contained questionable results and then we provide our improvements. Moreover, we examine two boundary minimum to provide a complete solution procedure for inventory model with decaying items, shortages and salvage value.

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II. Assumptions and Notation

To be compatible with Annadurai [14], we adopt the same assumptions and notation as him. The assumptions and notation is summarized as follows.

(1) The inventory carrying cost and deterioration cost are assumed to be proportional to the inventory level and are incurred instantaneously.

(2) There is no repair or replacement of deteriorated units during the planning horizon. Items are withdrawn from warehouse immediately they become deteriorated.

(3) The salvage value χC where $0 \le \chi < 1$ is associated with deteriorated units during the cycle time.

(4) Shortages are allowed to occur. It is assumed that only a fraction of demand is backlogged. Additionally, the longer the waiting time, the smaller the backlogging rate will be. Let $B(t)$ denote this fraction where t is the waiting time up to the next replenishment. Following Abad [15, 16] and Chang and Dye [17], we take

$$B(t) = 1/(1+\delta t), \tag{2.1}$$

where the backlogging parameter δ is a positive constant.

(5) The demand D units per unit time is deterministic and constant.

(6) Replenishment occurs instantaneously at an infinite rate and the lead time is negligible.

(7) The system involves single items.

We list notation as follows.

TC total relevant cost for the retailer per unit time.

$I(t)$ inventory level at time t.

h holding cost per unit per unit time.

C_2 unit cost of lost sales.

C_1 shortage cost for backlogged item.

C unit purchase cost of the item.

χ salvage value parameter, where $0 \le \chi < 1$, associated with deteriorated units during the cycle time.

δ backlogging parameter, with $0 \le \delta \le 1$.

θ deterioration rate, with $0 \le \theta < 1$.

D annual demand rate.

Q ordering quantity.

A ordering cost (set up cost) per order.

III. RECAP THE MODEL PROPOSED BY ANNADURAI

To save the precious space of this journal, we directly quote his average cost per unit time,

$$TC(t_1,T)=\frac{D}{T}\left(\frac{A}{D}+\frac{h+C(1-\chi)\theta}{\theta^2}\left(e^{\theta t_1}-1-\theta t_1\right)\right.$$
$$\left.+\frac{C_1+\delta C_2}{\delta^2}\left[\delta(T-t_1)-\ln\left(1+\delta(T-t_1)\right)\right]\right). \quad (3.1)$$

Remark. We must point out that the salvage value χC was proposed by Annadurai [14]. However, in his derivation, he used χ to denote the salvage value. To be consistent with the notation, we revise the repression from χ to χC. Consequently, all expressions of $C-\chi$ are improved to $C(1-\chi)$.

Annadurai [14] derived the partial derivatives with respect to t_1 and T, then to simplify the expression, he assumed

$$U=(C_1/\delta)+C_2, \quad (3.2)$$

and

$$V=(h/\theta)+C(1-\chi). \quad (3.3)$$

From $\frac{\partial}{\partial t_1}TC(t_1,T)=0$, he found that

$$T-t_1=\frac{V\left(e^{\theta t_1}-1\right)}{\delta\left[U-V\left(e^{\theta t_1}-1\right)\right]}. \quad (3.4)$$

From $\frac{\partial}{\partial T}TC(t_1,T)=0$, he derived that

$$U\left[\frac{(T-t_1)(\delta t_1-1)}{1+\delta(T-t_1)}+\frac{\ln\left[1+\delta(T-t_1)\right]}{\delta}\right]$$
$$-\frac{V}{\theta}\left(e^{\theta t_1}-1-\theta t_1\right)-\frac{A}{D}=0. \quad (3.5)$$

He plugged Equation (3.4) to derive a function only in variable t_1 to imply that

$$V\left(e^{\theta t_1}-1\right)\frac{1-\delta t_1}{\delta}+\frac{U}{\delta}\ln\left(\frac{U}{U-V\left(e^{\theta t_1}-1\right)}\right)$$
$$-\frac{V}{\theta}\left(e^{\theta t_1}-1-\theta t_1\right)-\frac{A}{D}=0. \quad (3.6)$$

Motivated by Equation (3.6), he assumed an auxiliary function, $F(t_1)$, as

$$F(t_1)=V\left(e^{\theta t_1}-1\right)\frac{1-\delta t_1}{\delta}+\frac{U}{\delta}\ln\left(\frac{U}{U-V\left(e^{\theta t_1}-1\right)}\right)$$
$$-\frac{V}{\theta}\left(e^{\theta t_1}-1-\theta t_1\right)-\frac{A}{D}. \quad (3.7)$$

Annadurai [14] raised the following two theorems.

Theorem 4.1 of Annadurai [14].

If $F(t_1)>0$, then the solution of (t_1,T) which satisfied Equations (3.5) and (3.6) not only exists but is also unique.

Outline of his proof. Annadurai [14] mentioned that $dF(t_1)/dt_1>0$.

Theorem 4.2 of Annadurai [14].

If $F(t_1)>0$, then $TC(t_1,T)$ is convex and reaches its global minimum at (t_1^*,T^*) which satisfies Equations (3.4) and (3.5).

Outline of his proof. Annadurai [14] showed that at the stationary point (that is the point satisfying Equations (3.4) and (3.5)), the Hessian matrix is positive definite.

IV. OUR APPROACH FOR THE FIRST PARTIAL DERIVATIVE SYSTEM

From Equation (3.4) and $T\ge t_1$, we know that there is a restriction

$$U-V\left(e^{\theta t_1}-1\right)>0, \quad (4.1)$$

that will imply an upper bound of t_1 as

$$\frac{1}{\theta}\ln\left(1+\frac{U}{V}\right)>t_1. \quad (4.2)$$

For later discussion, we will denote the upper bound

$$\frac{1}{\theta}\ln\left(1+\frac{U}{V}\right)=t_1^{\&}. \quad (4.3)$$

We must point out that the finding of Equation (3.6) proposed by Annadurai [14] is questionable. We find the right expression should be revised as

$$V\left(e^{\theta t_1}-1\right)\frac{\delta t_1-1}{\delta}+\frac{U}{\delta}\ln\left(\frac{U}{U-V\left(e^{\theta t_1}-1\right)}\right)$$
$$-\frac{V}{\theta}\left(e^{\theta t_1}-1-\theta t_1\right)-\frac{A}{D}=0. \quad (4.4)$$

Consequently, we will assume a new auxiliary function, say $G(t_1)$, as

$$G(t_1)=V\left(e^{\theta t_1}-1\right)\frac{\delta t_1-1}{\delta}+\frac{U}{\delta}\ln\left(\frac{U}{U-V\left(e^{\theta t_1}-1\right)}\right)$$
$$-\frac{V}{\theta}\left(e^{\theta t_1}-1-\theta t_1\right)-\frac{A}{D}. \quad (4.5)$$

We derive that

$$G'(t_1)=\frac{V\theta}{\delta}e^{\theta t_1}\left(\delta t_1+\frac{V\left(e^{\theta t_1}-1\right)}{U-V\left(e^{\theta t_1}-1\right)}\right)>0, \quad (4.6)$$

to imply that $G(t_1)$ increases function from

$$G(0) = -\frac{A}{D} < 0, \tag{4.7}$$

and

$$\lim_{t_1 \to t_1^{\&}} G(t_1) = \infty, \tag{4.8}$$

since we derive that

$$\lim_{t_1 \to t_1^{\&}} \ln\left(\frac{U}{U - V\left(e^{\theta t_1} - 1\right)}\right) = \ln\frac{U}{0^+} = \infty. \tag{4.9}$$

Hence, there is a unique point, say t_1^*, with

$$G(t_1^*) = 0, \tag{4.10}$$

such that t_1^* also satisfies the restriction of $t_1^* < t_1^{\&}$. Based on Equation (3.4), we will denote T^* as

$$T^* = t_1^* + \frac{V\left(e^{\theta t_1^*} - 1\right)}{\delta\left[U - V\left(e^{\theta t_1^*} - 1\right)\right]}. \tag{4.11}$$

We prove that (t_1^*, T^*) is the unique solution that satisfies the first partial derivative system of Equations (3.4) and (3.5).

V. Discussion of results of Annadurai

We point out the condition in Theorems 4.1 and 4.2 of Annadurai [14] to require that $F(t_1) > 0$ must be revised. Based on $F(t_1)$ containing a term with opposite sign, his assertion of $F'(t_1) > 0$ that cannot be derived. Moreover, his proof of Theorem 4.2 shows the Hessian matrix is positive definite at the stationary point that will imply the stationary point is a local minimum point. It can not imply $TC(t_1, T)$ is convex.

His proof did not show the existence for the stationary point.

Owing to above discussion, our derivation in Section 4 provides an improvement for Annadurai [14] for the interior minimum. In the following Sections 6 and 7, we will further revise Annadurai [14] for the two boundary minimums.

VI. The Boundary Minimum Along $T = t_1$

We point that the solution for the first partial derivative system is only considered for the interior points such that the suitable domain is shifted from the original one,

$$\{(t_1, T): 0 \le t_1 \le T, 0 < T\}, \tag{6.1}$$

to its interior

$$\{(t_1, T): 0 < t_1 < T, 0 < T\}. \tag{6.2}$$

Therefore, two boundaries,

$$\{(t_1, T = t_1): 0 < t_1\}, \tag{6.3}$$

and

$$\{(t_1 = 0, T): 0 < T\}, \tag{6.4}$$

will be discussed in Sections 6 and 7, respectively.

Along the boundary, $\{(t_1, T = t_1): 0 < t_1\}$, which corresponds to the no shortage inventory models, from Equation (3.1), we derive the average cost as follows

$$TC(t_1, T = t_1) = \frac{A}{t_1} + \frac{D}{t_1}\frac{h + C(1-\chi)\theta}{\theta^2}\left(e^{\theta t_1} - 1 - \theta t_1\right). \tag{6.5}$$

We find that

$$\frac{d}{dt_1}TC(t_1, T = t_1) = \frac{D\left(h + C(1-\chi)\theta\right)}{\theta^2 t_1^2} g(t_1), \tag{6.6}$$

where $g(t_1)$ ia an auxiliary function, with

$$g(t_1) = t_1\theta e^{\theta t_1} - e^{\theta t_1} + 1 - \frac{A\theta^2}{D\left(h + C(1-\chi)\theta\right)}. \tag{6.7}$$

We know that

$$g'(t_1) = t_1\theta^2 e^{\theta t_1} > 0, \tag{6.8}$$

so $g(t_1)$ is an increasing function from

$$g(0) = \frac{-A\theta^2/D}{h + C(1-\chi)\theta} < 0, \tag{6.9}$$

to

$$\lim_{t_1 \to \infty} g(t_1) = (t_1\theta - 1)e^{\theta t_1} + 1 - \frac{A\theta^2/D}{h + C(1-\chi)\theta} = \infty. \tag{6.10}$$

Hence, there is a point, say $t_1^{\#}$, satisfying $g(t_1^{\#}) = 0$.

Moreover, for $0 \le t_1 < t_1^{\#}$, $g(t_1) < 0$ that is $\frac{d}{dt_1}TC(t_1, T = t_1) < 0$ so $TC(t_1, T = t_1)$ decreases for $0 \le t_1 < t_1^{\#}$. On the other hand, if $t_1^{\#} < t_1$, $g(t_1) > 0$ that is $\frac{d}{dt_1}TC(t_1, T = t_1) > 0$ so $TC(t_1, T = t_1)$ increases for $t_1^{\#} < t_1$. Consequently, $t_1^{\#}$ is the global minimum for $TC(t_1, T = t_1)$.

VII. The Boundary Minimum Along $t_1 = 0$

We consider along the boundary $\{(t_1 = 0, T): 0 < T\}$ that is corresponding to no stock inventory models. From Equation (3.1), we derive the average cost as follows

$$TC(t_1 = 0, T) = \frac{A}{T} + \frac{D(C_1 + \delta C_2)}{\delta^2 T}\left[\delta T - \ln(1 + \delta T)\right]. \tag{7.1}$$

We find that

$$\frac{d}{dT}TC(0,T)=\frac{D(C_1+\delta C_2)}{\delta^2 T^2}h(T), \quad (7.2)$$

where h(T) is an auxiliary function, with

$$h(T)=\frac{(1+\delta T)\ln(1+\delta T)-\delta T}{1+\delta T} -\frac{A\delta^2}{D(C_1+\delta C_2)}. \quad (7.3)$$

Owing to $0 \le \delta \le 1$, we derive that

$$h'(T)=\frac{1-\delta+\delta T}{(1+\delta T)^2}>0, \quad (7.4)$$

so $h(T)$ increases from

$$h(0)=-\frac{A\delta^2}{D(C_1+\delta C_2)}<0, \quad (7.5)$$

to

$$\lim_{T\to\infty} h(T)=\infty, \quad (7.6)$$

where

$$h(T)=\ln(1+\delta T)-\frac{\delta T}{1+\delta T}-\frac{A\delta^2}{D(C_1+\delta C_2)}. \quad (7.7)$$

Therefore, there is a point, say $T^{\%}$, satisfying $h(T^{\%})=0$.

Moreover, for $0 \le T < T^{\%}$, $h(T)<0$ to imply that $\frac{d}{dT}TC(0,T)<0$ and then $TC(0,T)$ is a decreasing function for $0 \le T < T^{\%}$.

Similarly, for $T^{\%}<T$, $h(T)>0$ that yields $\frac{d}{dT}TC(0,T)>0$ and then $TC(0,T)$ is a increasing function for $T^{\%}<T$.

Consequently, $T^{\%}$ is the global minimum for $TC(0,T)$.

VIII. Numerical Example

To be compatible with Annadurai [14], we consider the same numerical example as his proposed one with the following data: $A=250$, $D=1000$, $h=15$, $C_1=30$, $C_2=25$, $\theta=0.08$, $\chi=0.1$, and $\delta=0.56$.

However, Annadurai [14] did not inform us the purchasing cost, C. We refer to Dye and Ouyang [18] to know that the purchasing cost $C=5$, the shortage cost $C_1=3$, and the holding cost $h=0.35(5)$.

We approximate the purchasing cost by the ratio between purchasing cast and holding cost, to derive $\frac{C}{15}=\frac{5}{0.35(5)}$ to find $C=\frac{300}{7}\approx 43$.

On the other hand, we estimate the purchasing cost by the ratio between purchasing cast and shortage cost, to yield $\frac{C}{30}=\frac{5}{3}$ to obtain $C=50$. Hence, in our paper, we assume that $C=45$.

We find the interior minimum point and minimum value,

$$t_1^*=0.138174, \quad (8.1)$$

$$T^*=0.197691, \quad (8.2)$$

and

$$TC(t_1^*,T^*)=2148.6983. \quad (8.3)$$

The local minimum along the boundary $t_1=T$ is obtained as

$$t_1^{\#}=T=0.164840, \quad (8.4)$$

and

$$TC(t_1^*,t_1^*)=3026.5931, \quad (8.5)$$

for the special case of no shortage inventory model.

The local minimum along the boundary $t_1=0$ is obtained

$$T^{\%}=0.110983, \quad (8.6)$$

and

$$TC(0,T^{\%})=4597.5504, \quad (8.7)$$

for the special case of no stock inventory model.

From Equations (8.3), (8.5) and (8.7) we know that the global minimum point is (t_1^*,T^*) and the absolute minimum value is $TC(t_1^*,T^*)$.

IX. A Related Multi-choice Stochastic Transportation Problem

The multi-choice stochastic transportation problem proposed by Mahapatra et al. [19] has been a significant contribution to the field. However, there are a few problems in the proposed solution, which will be addressed in the following sections. Their complicated yet incomplete transformation of the objective function of seven cases can be simplified. Also, the multi-choice structure for the transportation cost is unnecessary. An analytical and simplified solution approach will be provide to replace their non-linear mixed integer programming problem solved by Lingo 10 package. Our findings will help the future development of transportation models.

Mahapatra et al. [19] published a paper for a multi-choice stochastic transportation problem. The supplies and demands follow the extreme value distribution, where the probability constraints are converted into deterministic constraints. Interested readers should refer to Equations (2.6) to (2.18) of [19] for more details. Of the three different models mentioned, only the most complicate model, where both supply and demand follows an extreme value distribution is considered. Our findings can be easily applied to their other two simpler models where either supply or demand follows the extreme value distribution.

Mahapatra et al. [19] tried to solve the following minimum transportation problem:

for $k=1,2,...,K$,

$$\min : z = \sum_{s=1}^{n}\sum_{t=1}^{m} x_{ts}\left\{C_{ts}^{1}, C_{ts}^{2}, ..., C_{ts}^{k}\right\}, \quad (9.1)$$

subject to for $t = 1,2,...,m$,

$$\alpha_s' - \beta_s'\left(\ln\left[-\ln\left(1-\delta_s\right)\right]\right) \le \sum_{i=1}^{m} x_{ts}, \quad (9.2)$$

$$\alpha_t - \beta_t\left(\ln\left[-\ln\left(\gamma_t\right)\right]\right) \le \sum_{j=1}^{n} x_{ts}, \quad (9.3)$$

$$\alpha_s' - \beta_s'\left(\ln\left[-\ln\left(1-\delta_s\right)\right]\right) \le \alpha_t - \beta_t\left(\ln\left[-\ln\left(\gamma_t\right)\right]\right), \quad (9.4)$$

and for $t = 1,2,...,m$, and $s = 1,2,...,n$,

$$0 \le x_{ts}, \quad (9.5)$$

where two random variables: supply, a_t and demand b_s satisfy an extreme value distribution, with location factors α_s', α_t and scale factors β_s', β_t with

$$1-\gamma_t \le \Pr\left(\sum_{j=1}^{n} x_{ts} \le a_t\right), \quad (9.6)$$

and for $s = 1,2,...,n$, and $t = 1,2,...,m$,

$$1-\delta_s \le \Pr\left(\sum_{i=1}^{m} x_{ts} \ge b_j\right). \quad (9.7)$$

X. Review of Their Transformation of the Objective Function Involving Multi-choice Cost Parameter

Mahapatra et al. [19] constructed a long procedure to convert their objective function of Equation (9.1) to an equivalent form. For example, when $K = 7$, for Case 6, they tried to convert the objective function,

$$\min : z = \sum_{s=1}^{n}\sum_{t=1}^{m} x_{ts}\left\{C_{ts}^{1}, C_{ts}^{2}, ..., C_{ts}^{7}\right\}, \quad (10.1)$$

to the following two different models,
Model 6(a):

$$\begin{aligned}\min : z = \sum_{t=1}^{m}\sum_{s=1}^{n} \Big\{ &\left(1-z_{ts}^{3}\right)\left(1-z_{ts}^{2}\right)\left(1-z_{ts}^{1}\right)C_{ts}^{1} + \\ &\left(1-z_{ts}^{3}\right)\left(1-z_{ts}^{2}\right)z_{ts}^{1}C_{ts}^{2} + \left(1-z_{ts}^{3}\right)z_{ts}^{2}\left(1-z_{ts}^{1}\right)C_{ts}^{3} \\ &+ z_{ts}^{3}\left(1-z_{ts}^{2}\right)\left(1-z_{ts}^{1}\right)C_{ts}^{4} + \left(1-z_{ts}^{3}\right)z_{ts}^{2}z_{ts}^{1}C_{ts}^{5} \\ &+ z_{ts}^{3}\left(1-z_{ts}^{2}\right)z_{ts}^{1}C_{ts}^{6} + z_{ts}^{3}z_{ts}^{2}\left(1-z_{ts}^{1}\right)C_{ts}^{7}, \quad (10.2)\end{aligned}$$

with

$$2 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1}, \quad (10.3)$$

and for all t and s, and $u = 1,2,3$,

$$z_{ts}^{u} = 0/1. \quad (10.4)$$

Model 6(b):

$$\begin{aligned}\min : z = \sum_{t=1}^{m}\sum_{s=1}^{n} \Big\{ &\left(1-z_{ts}^{3}\right)\left(1-z_{ts}^{2}\right)z_{ts}^{1}C_{ts}^{1} + \\ &\left(1-z_{ts}^{3}\right)z_{ts}^{2}\left(1-z_{ts}^{1}\right)C_{ts}^{2} + z_{ts}^{3}\left(1-z_{ts}^{2}\right)\left(1-z_{ts}^{1}\right)C_{ts}^{3} \\ &+ \left(1-z_{ts}^{3}\right)z_{ts}^{2}z_{ts}^{1}C_{ts}^{4} + z_{ts}^{3}\left(1-z_{ts}^{2}\right)z_{ts}^{1}C_{ts}^{5} \\ &+ z_{ts}^{3}z_{ts}^{2}\left(1-z_{ts}^{1}\right)C_{ts}^{6} + z_{ts}^{3}z_{ts}^{2}z_{ts}^{1}C_{ts}^{7}, \quad (10.5)\end{aligned}$$

with

$$1 \le z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1}, \quad (10.6)$$

and for all t and s, and $u = 1,2,3$,

$$z_{ts}^{u} = 0/1. \quad (10.7)$$

Their long process was presented from Equations (3.37) to (3.66) of Mahapatra et al. [41]. To save the precious space of this journal, we will not refer to those lengthy computation procedure. Instead, we will present our simplified version in the next section.

XI. Our Generalization of Their Transformation

For Model 6(a), a dummy cost, C_{ts}^{8}, was added to their objective function, making the coefficient,

$$z_{ts}^{3}z_{ts}^{2}z_{ts}^{1}C_{ts}^{8}. \quad (11.1)$$

For the transformation to work, the coefficient $z_{ts}^{3}z_{ts}^{2}z_{ts}^{1}$ of Equation (11.1) needs to always be zero, which is to say that

$$z_{ts}^{3} = 1 = z_{ts}^{2} = z_{ts}^{1}, \quad (11.2)$$

must not occur, and so

$$z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} = 3, \quad (11.3)$$

must be ruled out. Consequently, they improved the original condition of

$$3 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0, \quad (11.4)$$

to the desired restriction

$$2 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0, \quad (11.5)$$

to derive that

$$2 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1}. \quad (11.6)$$

Similarly, for Model 6(b), the dummy cost, C_{ts}^{8}, was added making the coefficient

$$\left(1-z_{ts}^{3}\right)\left(1-z_{ts}^{2}\right)\left(1-z_{ts}^{1}\right)C_{ts}^{8}. \quad (11.7)$$

The coefficient $\left(1-z_{ij}^{1}\right)\left(1-z_{ij}^{2}\right)\left(1-z_{ij}^{3}\right)$ of Equation (11.6) needs to always be zero, and to prevent

$$z_{ij}^{1} = 0 = z_{ij}^{2} = z_{ij}^{3}, \quad (11.8)$$

from happening, they ruled out

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} = 0. \quad (11.9)$$

Hence, they revised the original condition from $3 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0$ of Equation (11.4) to the desire restriction,

$$1 \le z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \le 3, \quad (11.10)$$

to derive that

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \ge 1. \quad (11.11)$$

After providing a reasonable explanation for their findings in Case 6, we will present our generalization for their objective function of Cases 1-7. We assume that

$$\min : z = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(b_{ij}^{1}C_{ij}^{1} + b_{ij}^{2}C_{ij}^{2} + ... + b_{ij}^{K}C_{ij}^{K}\right)x_{ij} \quad (11.12)$$

for $p = 1,2,...,K$, and $\sum_{p=1}^{K} b_{ij}^{p} = 1$,

$$\text{with } b_{ij}^{p} = 0/1. \quad (11.13)$$

Our approach is a generalization for a special case when $K \le 8$ to any finite range of p.

Although it may be fascinating to use the minimum number for the possible minimum numbers of parameter for the coefficient for the derivation of Mahapatra et al. [41], since a characteristic of the multi-choice problem is that only one cost will be adopted for the objective function, making derivation of a compact expression with minimum number of parameters inconsequential. Moreover, their long and complicated transformation will detour the attention of practitioners who will not focus on the fact that only one cost will be adopted by the decision maker.

XII. Our Generalization of Their Transformation

For Model 6(a), a dummy cost, C_{ts}^{8}, was added to their objective function, making the coefficient,

$$z_{ts}^{3} z_{ts}^{2} z_{ts}^{1} C_{ts}^{8}. \quad (12.1)$$

For the transformation to work, the coefficient $z_{ts}^{3} z_{ts}^{2} z_{ts}^{1}$ of Equation (12.1) needs to always be zero, which is to say that

$$z_{ts}^{3} = 1 = z_{ts}^{2} = z_{ts}^{1}, \quad (12.2)$$

must not occur, and so

$$z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} = 3, \quad (12.3)$$

must be ruled out. Consequently, they improved the original condition of

$$3 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0, \quad (12.4)$$

to the desired restriction

$$2 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0, \quad (12.5)$$

to derive that

$$2 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1}. \quad (12.6)$$

Similarly, for Model 6(b), the dummy cost, C_{ts}^{8}, was added making the coefficient

$$\left(1 - z_{ts}^{3}\right)\left(1 - z_{ts}^{2}\right)\left(1 - z_{ts}^{1}\right)C_{ts}^{8}. \quad (12.7)$$

The coefficient $\left(1 - z_{ij}^{1}\right)\left(1 - z_{ij}^{2}\right)\left(1 - z_{ij}^{3}\right)$ of Equation (12.6) needs to always be zero, and to prevent

$$z_{ij}^{1} = 0 = z_{ij}^{2} = z_{ij}^{3}, \quad (12.8)$$

from happening, they ruled out

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} = 0. \quad (12.9)$$

Hence, they revised the original condition from $3 \ge z_{ts}^{3} + z_{ts}^{2} + z_{ts}^{1} \ge 0$ of Equation (12.4) to the desire restriction,

$$1 \le z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \le 3, \quad (12.10)$$

to derive that

$$z_{ij}^{1} + z_{ij}^{2} + z_{ij}^{3} \ge 1. \quad (12.11)$$

After providing a reasonable explanation for their findings in Case 6, we will present our generalization for their objective function of Cases 1-7. We assume that

$$\min : z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(b_{ij}^{1} C_{ij}^{1} + b_{ij}^{2} C_{ij}^{2} + ... + b_{ij}^{K} C_{ij}^{K}\right) x_{ij} \quad (12.12)$$

for $p = 1,2,...,K$, and $\sum_{p=1}^{K} b_{ij}^{p} = 1$,

$$\text{with } b_{ij}^{p} = 0/1. \quad (12.13)$$

Our approach is a generalization for a special case when $K \le 8$ to any finite range of p.

Although it may be fascinating to use the minimum number for the possible minimum numbers of parameter for the coefficient for the derivation of Mahapatra et al. [19], since a characteristic of the multi-choice problem is that only one cost will be adopted for the objective function, making derivation of a compact expression with minimum number of parameters inconsequential. Moreover, their long and complicated transformation will detour the attention of practitioners who will not focus on the fact that only one cost will be adopted by the decision maker.

XIII. The Multi-choice of Transportation Cost

In the section it will be proven that their multi-choice of transportation cost of Mahapatra et al. [19] is redundant, through the use of numerical examples provided in their paper. We have reproduced Table 1 from their paper for the transportation cost from three supplies to four demands. For further analysis, we have also listed their findings of C_{st}^{k} and x_{st} in our table 1.

From their findings of $C_{14} = 23$, $C_{31} = 22$ and $C_{33} = 22$, these three values are not the lowest cost of C_{ij}^{k}, since $\min C_{14}^{k} = 21$, $\min C_{31}^{k} = 20$ and $\min C_{33}^{k} = 20$. Even though, their approach is a reasonable method for finding the optimal solution under a multi-choice transportation cost environment, we will point out that it is rather redundant.

In the following we provide our comments for their approach with multi-choice transportation cost. From their approach as Equations (10.2) or (10.5) for $K = 7$, or our generalization of Equation (12.12), where z_{st}^{p}, $p = 1,2,3$ or b_{st}^{k}, $k = 1,2,...,K$ are decision variables such that for the minimum problem, we can directly take the smallest value among C_{st}^{k}. For example, in Table 1, for x_{32}, we should directly take

$$\min C_{32}^{k} = \min\{10,11,12,13,14,15,16,17\} = 10. \quad (13.1)$$

This assertion is violated by the optimal solution of [19] with $C_{14} = 23$, $C_{31} = 22$ and $C_{33} = 22$. However, we must point out that in Mahapatra et al. [19],

Table 1 Reproduction of multi-choice transportation cost for route x_{st} of Mahapatra et al. [19] and their findings of optimal C_{ij}^k and x_{ij}.

Sl. no.	Route: x_{st}	Transportation cost C_{st}^k	Optimal solution by Mahapatra et al. [19]	
			C_{st}^k	x_{st}
1	(1,1): x_{11}	10 or 11 or 12	10	615.8808
2	(1,2): x_{12}	15 or 16	15	0
3	(1,3): x_{13}	21 or 22 or 23 or 24	21	0
4	(1,4): x_{14}	21 or 23 or 25	23	0
5	(2,1): x_{21}	15 or 17 or 19 or 21 or 23 or 25	15	0
6	(2,2): x_{22}	10 or 12 or 14 or 16 or 18 or 20	10	382.1037
7	(2,3): x_{23}	9 or 10 or 11	9	408.3479
8	(2,4): x_{24}	18 or 19	18	0
9	(3,1): x_{31}	20 or 21 or 22 or 23 or 24 or 25 or 26	22	0
10	(3,2): x_{32}	10 or 11 or 12 or 13 or 14 or 15 or 16 or 17	10	129.7771
11	(3,3): x_{33}	20 or 22 or 25	22	0
12	(3,4): x_{34}	15 or 20	15	305.2464

$$x_{14} = x_{31} = x_{33} = 0. \qquad (13.2)$$

Therefore, outcomes of C_{14}, C_{31} and C_{33} will not influence their minimum solution. Hence, by directly taking the smallest value for the transportation cost, their multi-cost structure can be simplified dramatically. In the next section, we will present our simplified version for these kind of transportation problems.

We will demonstrate our approach by using the same problem of Mahapatra et al. [19], and so we are solving the following minimum problem:

$$\min\ 10x_{11} + 15x_{12} + 21x_{13} + 21x_{14} + 15x_{21} + 10x_{22} + 9x_{23} + 18x_{24} + 20x_{31} + 10x_{32} + 20x_{33} + 15x_{34}, \qquad (13.3)$$

under constraints

$$x_{11} + x_{12} + x_{13} + x_{14} \le 987.782536, \qquad (13.4)$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le 790.4516176, \qquad (13.5)$$

$$x_{31} + x_{32} + x_{33} + x_{34} \le 692.4721905, \qquad (13.6)$$

$$x_{11} + x_{21} + x_{31} \ge 651.880781, \qquad (13.7)$$

$$x_{12} + x_{22} + x_{32} \ge 511.880781, \qquad (13.8)$$

$$x_{13} + x_{23} + x_{33} \ge 408.3478976, \qquad (13.9)$$

and

$$x_{14} + x_{24} + x_{34} \ge 305.2463882, \qquad (13.10)$$

where upper bounds for supplies and lower bounds for demands are directly quoted from Mahapatra et al. [19] of their Equations (4.72) to (4.78).

The cost to the same destinations is compared to rule out expanse suppliers.

The cost is compared to find that the maximum is 21. To achieve the optimal solution for the minimum problem, we will assume that $x_{13} = 0$ and $x_{14} = 0$.

Based on Equation (13.9), when $x_{13} = 0$, the unit cost for x_{23} and x_{33} is compared to find that $9 < 20$ such that $x_{33} = 0$, and $x_{23} = 408.3478976$.

Similarly, based on Equation (13.10) and unit costs of x_{24} and x_{34} with $15 < 18$, we find that $x_{24} = 0$, and $x_{34} = 305.2463882$.

Based on Equation (13.7), and unit costs of x_{11}, x_{21} and x_{31} with $10 < 15 < 20$, we find that $x_{21} = 0$, $x_{31} = 0$ and $x_{11} = 651.880781$.

These findings are plugged into Equations (13.5) and (13.6) and converted to

$$x_{22} \le 382.1037200, \qquad (13.11)$$

and

$$x_{32} \le 387.2258023. \qquad (13.12)$$

Based on Equation (13.8), and unit costs of x_{12}, x_{22} and x_{32} with $10 = 10 < 15$, $x_{12} = 0$ and

$$x_{22} + x_{32} = 511.880781. \qquad (13.13)$$

Without referring to Lingo 10 package, the optimal solution was directly obtained. Moreover, it have been demonstrated that the finding of Mahapatra et al. [19] with $x_{22} = 382.1037$ and $x_{32} = 129.7771$ is a special result of our results of Equations (13.11), (13.12) and (13.13).

We provide a generalization for the transformation in Mahapatra et al. [19] for an objective function with multi-choice cost parameters was provided. Also, complex approach for the multi-choice cost was shown to be unnecessary, since it could be directly simplified to the smallest related cost. Finally, it was shown that a strict solution method could be derived by comparing the coefficient of the minimum problem without referring to any complex non-linear mixed integer programming, making it more efficient.

XIV. An Inventory System with Random Defective Rate

Huang [20] tried to introduce inventory system with random defective rate to those practitioners that are not familiar to calculus. Hence, he used algebraic method to reconsider the EPQ models of Chiu [21], where the original source is the paper of Chang et al. [22]. This section is a response of Huang [20] to point out that there are two questionable results in his paper. Our findings are only provided a patchwork for Huang's algebraic work, but also examine the sufficient conditions for the EPQ model proposed by Chiu [21].

Before his algebraic operation, Huang [20] should check the sign of those terms in his formula such that we have to check the following two inequalities:

$$D > 0, \tag{13.1}$$

and

$$D - \frac{G^2}{4F} > 0. \tag{13.2}$$

We recall that

$$\mathrm{D} = \frac{\mathrm{h}\,\Omega}{2(1-\mathrm{E}[\mathrm{x}])}, \tag{13.3}$$

with an abbreviation, denoted as Ω, that was assumed by us,

$$\Omega = \left(\left(1-\frac{\lambda}{\mathrm{P}}\right) + \mathrm{E}[\mathrm{x}^2]\right) - 2\left(1-\frac{\lambda}{\mathrm{P}}\right)\mathrm{E}[\mathrm{x}]. \tag{13.4}$$

To prove $D > 0$ that is to verify the following inequality:

$$\left(1-\frac{\lambda}{P}\right) + E\left[x^2\right] - 2\left(1-\frac{\lambda}{P}\right)E\left[x\right] > 0. \tag{13.5}$$

We recall that the variance,

$$\sigma^2 = E\left[x^2\right] - \left(E\left[x\right]\right)^2 \ge 0, \tag{13.6}$$

to imply that

$$E\left[x^2\right] \ge \left(E\left[x\right]\right)^2. \tag{13.7}$$

On the other hand, from $0 < 1 - \frac{\lambda}{P} < 1$, it yields that

$$\left(1-\frac{\lambda}{P}\right)^2 < 1 - \frac{\lambda}{P}. \tag{13.8}$$

Hence, we compute that

$$\begin{aligned}&\left(1-\frac{\lambda}{P}\right) + E\left[x^2\right] - 2\left(1-\frac{\lambda}{P}\right)E\left[x\right] >\\ &\left(1-\frac{\lambda}{P}\right)^2 + \left(E\left[x\right]\right)^2 - 2\left(1-\frac{\lambda}{P}\right)E\left[x\right]\\ &= \left(1-\frac{\lambda}{P}-E\left[x\right]\right)^2 \ge 0.\end{aligned} \tag{13.9}$$

Based on the findings of Equation (13.9), we prove that

$$\Omega > 0. \tag{13.10}$$

We recall Equation (13.3), it follows that the inequality of Equation (13.1) is verified.

We obtain that

$$4\mathrm{DF} - \mathrm{G}^2 = \frac{\mathrm{h}(\mathrm{h}+\mathrm{b})}{(1-\mathrm{E}[\mathrm{x}])^2}\mathrm{E}\left(\frac{1-\mathrm{x}}{1-\mathrm{x}-(\lambda/\mathrm{P})}\right)\Omega - \mathrm{h}^2, \tag{13.11}$$

where Ω is defined in Equation (13.4).

If we want that $4FD - G^2 > 0$ of Equation (13.2), for all cases that is independent the magnitude of b and h, then we need to show the following inequality:

$$\mathrm{E}\left(\frac{1-\mathrm{x}}{1-\mathrm{x}-(\lambda/\mathrm{P})}\right)\Omega > (1-\mathrm{E}[\mathrm{x}])^2. \tag{13.12}$$

Based on our above discussion, we provide a possible direction for future research to present a patchwork for Huang [20] for the inequality of Equation (13.2).

XV. Direction for Further Study

We present several research hot spots in this section to help practitioners locate possible future research trends. With conditional value at risk, and copula-based value at risk, Ismail et al. [23] estimated currency exchange portfolio risk. Wang et al. [24] studied intermodal transport for travelers to optimize route with bounded rationality. With Holt-Winters approaches and Kalman filter, Al-Gounmeein et al. [25] examined ARFIMA models to revise models accuracy. For exogenous factors model on a two-sided extended EWMA control chart, Areepong and Sukparungsee [26] constructed capability process to move average. According to variable order transfer functions, Hou et al. [27] examined high dimensional feature selection question through binary equalization optimizer. Concerning unconstrained optimization problems, Semiu et al. [28] developed a novel Dai-Liao-type conjugate gradient method. Referring to multivariate adaptive generalized Poisson regression spline, Yasmirullah et al. [29] constructed parameter estimations to analysis spatial error model. Based on target-oriented opinion words extraction, Li et al. [30] derived graph convolutional network related to attention feature fusion. Referring to Spotted Hyena optimizer, for energy-efficient routing in MANET, Venkatasubramanian et al. [31] developed cluster head selection. Considering the spread of infectious diseases, Zuhairoh et al. [32] constructed continuous-time hybrid semi-Markov and Markov Models under Sojourn time approach. Using optimal adjustment of the control points, Li and Liu [33] tried to obtain smoothing connected Bezier surfaces and curves. Through P4-Enabled SDN Switch and gRPC API with cloud computing platform, Krishna and Sharma [34] studied an open Stack case to revise orchestration service. Based on our above literature reviewing for recently published papers, researchers can find suitable and hot topic for their future studies.

XVI. Conclusion

We examine the solution procedure of inventory model for decaying items with shortages and salvage value proposed by Annadurai [14] to provide a patch work. We not only point out his questionable results but also offer a revision. Moreover, we consider two boundary minimums for the two generated cases with (a) no shortage case and (b) no stock case such that there will be three minimums for the inventory model. Our amendment will be a useful improvement for the future development of more practical inventory models.

We also examined Mahapatra et al. [19] to reveal that thier complicated solution process is redadant and then we offered our revised solution approach.

At last, we provide a patchwork for Huang [20] for his first inequality. On the other hand, for his second inequality, we present a possible direction to verify it.

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