Trapezoidal Neutrosophic Program Evaluation and Review Technique Using Interval Arithmetic Operations

S. Sinika and G. Ramesh

Abstract—**In the discipline of project management, the occurence of ambiguity and vagueness are the major hurdles for its accomplishment. Neutrosophic sets can be a great solution to overcome such situations leading to improved decision-making, better communication and enhanced problem-solving in project management. By incorporating neutrosophic sets into the PERT, one can handle uncertainty in the project schedule and improve the accuracy of critical path analysis. This manuscript instigates an interval-based de-neutrosophication methodology and utilizes PERT problem in a neutrosophic environment. Trapezoidal neutrosophic forward and backward pass are employed to determine each activity's neutrosophic slack time and criticality. It uses the proposed interval-based ranking technique as a tool to ease the analysis. To validate the critical path, we develop a trapezoidal neutrosophic criticality degree for all the activities and further the efficiency of the proposed technique is analyzed using illustrations. Additionally, interval-valued trapezoidal neutrosophic fuzzy PERT is discussed by constructing the criticality degree using the interval-based de-neutrosophication technique and finally deliberated with examples.**

Index Terms—**Critical path, PERT, Trapezoidal neutrosophic fuzzy number, Interval-valued trapezoidal neutrosophic fuzzy number, Interval numbers, Interval arithmetic operations.**

I. Introduction

A structured approach is essential in the competitive business environment to achieve specific goals and objectives within a defined time frame. Project management plays an enormous role in planning, organizing and managing resources. The basic notion behind PERT is to divide a project into smaller, manageable tasks and to identify the dependencies between these tasks. Project managers can then use this information to build a schedule for the time needed to accomplish each task by providing the sequence for completion and the resources required to execute each work. In the late 1950s and early 1960s, the U.S. Navy's Special Projects Office developed project management methodologies such as PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) to manage complex defense projects. Since then, project management has evolved and expanded to other industries, including software development, healthcare, finance and marketing, particularly in the field of construction and engineering.

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Periodically, the project requirements and objectives may vary or remain unclear due to real-life situations and market unpredictability. These circumstances may lead to fuzziness in project management. Apparently, a proactive approach can define the objectives, requirements, etc. To address the incomplete or ambiguous information, Zadeh (1965) introduced the concept of Fuzzy set theory. Many researchers pay attention to CPM and PERT in a fuzzy environment to serve the project managers in executing their work efficiently. Chanas and Kamburowski (1981) [1] introduced PERT in a fuzzy environment and considered the activity duration of the project network as the undefined variable. Chanas and Zielinski (2001) [2] showed relations between fuzzy and interval criticality. They offered two approaches for calculating the path degree of criticality. Liang and Han (2004) [3] presented a paper in which they considered the parameters as trapezoidal fuzzy numbers to unravel problems in project networks. Chen and Huang (2007) [4] anticipated a novel method combining fuzzy triangular numbers with PERT. Also, they define the possibility index to recognize the necessary duration for the project network. Samman and Brahemi (2014) [5] explained fuzzy PERT as a case study in readymade factories. Yang et al. (2014) [6] apply time distribution in solving PERT problem in fuzzy environment. Mazlum and Guneri (2015) [7] projected a paper for improving the online internet using fuzzy management techniques with undefined triangular parameters. Elizabeth and Sujatha (2016) [8] developed a dynamic programming recursion formulation for identifying the critical path's fuzzy version and the parameters in fuzzy triangular numbers. Eventhough a fuzzy set can manage uncertain situations, handling them mathematically in unstabilized real-life applications is challenging.

Hence, Atanassov (1986) made further extensions, naming it an intuitionistic set. The fuzzy set can tackle a single membership grade with each element, whereas, intuitionistic set can handle membership and non-membership with each component. The concept of PERT in an intuitionistic environment with trapezoidal fuzzy numbers was proposed by Jayagowri and Geetharamani (2014) [9]. They defined the graded mean integration formula to convert trapezoidal intuitionistic fuzzy numbers to comparable crisp numbers. Later on, both [10] used the metric distance ranking approach to compute the total slack time for each path in the triangular intuitionistic fuzzy environment to analyze the criticality of the project network. To determine the critical path, Elizabeth and Sujatha (2015) [11] created two unique algorithms, and the time of each action is represented by triangular numbers in fuzzy and intuitionistic environments. Sudha et al. (2017)

[12] futured an algorithm for obtaining a critical path using triangular intuitionistic fuzzy numbers. Yogashanthi et al. (2019) [13] discussed an airfreight ground operating system application for finding critical paths in a trapezoidal intuitionistic environment. They applied a new centroid method for ranking trapezoidal intuitionistic numbers.

Giving credit to neutral thoughts makes decision-making even more comfortable and efficient. The concept of neutrosophic set which was proposed by Smarandache (1998) [14] defines that each element can belong to the set, not the set, or partially belong to the collection, with degrees of truth, falsity, and indeterminacy. Analyzing any problem using a neutrosophic environment makes one handle the situation in every possible way. Using neutrosophic set theory in PERT can improve project management's accuracy, flexibility, and effectiveness, particularly in complex projects with high levels of uncertainty and variability. Mai et al. (2017) [15] initiated the concept of PERT in a neutrosophic environment with three time estimates. Mullai and Surya (2019) [16] projected the idea of PERT in a neutrosophic field by analyzing the parameters in all possible ways, particularly truth, indeterminacy, and falsity. Avishek (2020) [17] focussed on the pentagonal neutrosophic environment for computing time in networking problems using the proposed score function. Vijaya et al. (2022) [19] handled neutrosophic fuzzy numbers to find the critical path in project management with an example.

The current research has focussed more on interval numbers than crisp ones because of their complication in real-life environments. The numbers are more convenient and flexible when we express them within the interval boundary. Several researchers have concentrated and dealt with the PERT problem on interval numbers [20], [21], [22]. We incorporate the interval concept for resolving the trapezoidal neutrosophic fuzzy PERT problem. In the literature survey, we have identified that most of the de-neutrosophication approaches for solving PERT problem in trapezoidal neutrosophic settings are in crisp numbers. In practical situations, considering problems in terms of intervals rather than precise values proves to be more cost-effective, especially when decision-makers require supplementary information. This circumstance serves as a driving force behind the creation of this article, aimed at tackling the PERT problem through the transformation of trapezoidal neutrosophic fuzzy numbers into intervals.

This paper marks the use of an interval-based de-neutrosophication technique for solving PERT problem in trapezoidal neutrosophic and interval-valued trapezoidal neutrosophic environment. The effectiveness of this technique is examined by solving two problems from [17] and [18] in a neutrosophic PERT environment with trapezoidal parameters. Additionally, the validity of interval-based de-neutrosophication techique is inspected by two examples in an interval-valued trapezoidal neutrosophic PERT environment and compared with [15] and [29]. This article provides a well-defined structure that consists of an introduction in section I, the necessary background of the article in section II, interval-based de-neutrosophication methodology of a trapezoidal neutrosophic number highlighted in section III, detailed explanation of the core concept trapezoidal neutrosophic fuzzy PERT and interval-valued trapezoidal neutrosophic PERT in sections IV and V, and demonstrated illustrations in sections VI and VII, strength and limitations of the proposed method in section VIII, finally section IX summarizes the keypoints and offer further exploration of the article as a conclusion.

II. Preliminaries

A. Neutrosophic number [23]

Trapezoidal neutrosophic fuzzy number (TrpNFN) is defined as $T_{\widetilde{Neu}} = \langle (t_{11}, t_{12}, t_{13}, t_{14}), (i_{11}, i_{12}, i_{13}, i_{14}), (f_{11}, f_{12}, f_{13}, f_{14}) \rangle$, where the parameters are from real numbers $\mathbf{R}$, and it satisfies $t_{11} \leq t_{12} \leq t_{13} \leq t_{14},\ i_{11} \leq i_{12} \leq i_{13} \leq i_{14}$ and $f_{11} \leq f_{12} \leq f_{13} \leq f_{14}$. The membership degrees of truth, indeterminacy and falsity are defined as

$$\sigma_{T_{\widetilde{Neu}}}(r) = \begin{cases} \frac{r-t_{11}}{t_{12}-t_{11}}, & t_{11} \leq r \leq t_{12}, \\ 1, & t_{12} \leq r \leq t_{13}, \\ \frac{t_{14}-r}{t_{14}-t_{13}}, & t_{13} \leq r \leq t_{14}, \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{T_{\widetilde{Neu}}}(r) = \begin{cases} \frac{i_{12}-r}{i_{12}-i_{11}}, & i_{11} \leq r \leq i_{12}, \\ 0, & i_{12} \leq r \leq i_{13}, \\ \frac{r-i_{13}}{i_{14}-i_{13}}, & i_{13} \leq r \leq i_{14}, \\ 1, & \text{otherwise} \end{cases}$$

$$\omega_{T_{\widetilde{Neu}}}(r) = \begin{cases} \frac{f_{12}-r}{f_{12}-f_{11}}, & f_{11} \leq r \leq f_{12}, \\ 0, & f_{12} \leq r \leq f_{13}, \\ \frac{r-f_{13}}{f_{14}-f_{13}}, & f_{13} \leq r \leq f_{14}, \\ 1, & \text{otherwise} \end{cases}$$

B. Interval-valued trapezoidal neutrosophic fuzzy number [28]

Let $a_1, a_2, a_3, a_4 \in \mathbf{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. An interval-valued trapezoidal neutrosophic fuzzy number can be expressed as $T_{\widetilde{IvNeu}} = \langle (a_1, a_2, a_3, a_4); [\rho^L, \rho^R], [\kappa^L, \kappa^R], [\nu^L, \nu^R] \rangle$, where $\rho^L : X \to [0, 1],\ \rho^R : X \to [0, 1]$ are the lower truth and upper truth degrees whose functions are defined as

$$\rho^L(x) = \begin{cases} \rho^L \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \rho^L, & a_2 \leq x \leq a_3 \\ \rho^L \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

$$\rho^R(x) = \begin{cases} \rho^R \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \rho^R, & a_2 \leq x \leq a_3 \\ \rho^R \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Also, $\kappa^L : X \to [0, 1],\ \kappa^R : X \to [0, 1]$ are the lower indeterminacy and upper indeterminacy degrees whose functions are defined as

$$\kappa^L(x) = \begin{cases} \frac{(a_2-x)+\kappa^L(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \kappa^L, & a_2 \leq x \leq a_3 \\ \frac{(x-a_3)+\kappa^L(a_4-x)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases}$$

$$\kappa^R(x) = \begin{cases} \frac{(a_2-x)+\kappa^R(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \kappa^R, & a_2 \le x \le a_3 \\ \frac{(x-a_3)+\kappa^R(a_4-x)}{a_4-a_3}, & a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

Similarly, $\nu^L : X \to [0,1]$, and $\nu^R : X \to [0,1]$ are the lower falsity and upper falsity degrees whose functions are defined as

$$\nu^L(x) = \begin{cases} \frac{(a_2-x)+\nu^L(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \nu^L, & a_2 \le x \le a_3 \\ \frac{(x-a_3)+\nu^L(a_4-x)}{a_4-a_3}, & a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

$$\nu^R(x) = \begin{cases} \frac{(a_2-x)+\nu^R(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ \nu^R, & a_2 \le x \le a_3 \\ \frac{(x-a_3)+\nu^R(a_4-x)}{a_4-a_3}, & a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

C. Interval number [20]

An interval number $\widetilde{k}$ on $\mathbf{R}$ is defined as $\widetilde{k} = [k^L, k^R] = \{k : k^L \le k \le k^R, k \in \mathbf{R}\}$, where k^L and k^R are the left and the right limits of $\widetilde{k}$ respectively. The mid-point and width (half-width) of an interval are defined as $\mathcal{C}(\widetilde{k}) = \frac{k^L+k^R}{2}$ and $\mathcal{W}(\widetilde{k}) = \frac{k^R-k^L}{2}$. And it can also be written in the form of mid-point and width (half-width) as

$$\widetilde{k} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle = \{k : \mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) \le k \le \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k}), k \in \mathbf{R}\}.$$

D. Interval arithmetic operations [24]

Below are the interval arithmetic operations.
If $\widetilde{k} = [k^L, k^R]$ and $\widetilde{l} = [l^L, l^R]$ and for $* \in \{+, -, \times, \div\}$, then $\widetilde{k} * \widetilde{l} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle * \langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle$
$= \langle \mathcal{C}(\widetilde{k}) * \mathcal{C}(\widetilde{l}), \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \rangle$.
In particular, we have
$\widetilde{k} + \widetilde{l} = \langle \mathcal{C}(\widetilde{k}) + \mathcal{C}(\widetilde{l}),\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \rangle$.
$\widetilde{k} - \widetilde{l} = \langle \mathcal{C}(\widetilde{k}) - \mathcal{C}(\widetilde{l}),\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \rangle$
$\widetilde{k} \times \widetilde{l} = \langle \mathcal{C}(\widetilde{k}) \times \mathcal{C}(\widetilde{l}),\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \rangle$.
$\widetilde{k} \div \widetilde{l} = \langle \mathcal{C}(\widetilde{k}) \div \mathcal{C}(\widetilde{l}),\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \rangle$,
provided that $\mathcal{C}(\widetilde{l}) \neq 0$.
For scalar multiplication,
$\widetilde{\alpha}\widetilde{k} = \langle \mathcal{C}(\widetilde{\alpha}) \times \mathcal{C}(\widetilde{k}),\ \max\{\mathcal{W}(\widetilde{\alpha}), \mathcal{W}(\widetilde{k})\} \rangle$

E. Power properties of interval numbers

Based on the definition for exponential of intervals provided by Hema Surya et al. [25], we define the power properties of interval numbers.

Property 1: For any three interval numbers $\widetilde{k}, \widetilde{m}$ & $\widetilde{n}$, then $\left(\widetilde{k}^{\widetilde{m}}\right)^{\widetilde{n}} = \widetilde{k}^{\widetilde{m}\widetilde{n}}$.

Proof: We have $\widetilde{k} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle$,
$\widetilde{m} = \langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle$ & $\widetilde{n} = \langle \mathcal{C}(\widetilde{n}), \mathcal{W}(\widetilde{n}) \rangle$,

Now, $\left(\widetilde{k}^{\widetilde{m}}\right)^{\widetilde{n}}$

$$= \left(\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle} \right)^{\langle \mathcal{C}(\widetilde{n}), \mathcal{W}(\widetilde{n}) \rangle}$$
$$= \left\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m})\} \right\rangle^{\langle \mathcal{C}(\widetilde{n}), \mathcal{W}(\widetilde{n}) \rangle}$$
$$= \left\langle \left(\mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})}\right)^{\mathcal{C}(\widetilde{n})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m}), \mathcal{W}(\widetilde{n})\} \right\rangle$$
$$= \left\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})\mathcal{C}(\widetilde{n})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m}), \mathcal{W}(\widetilde{n})\} \right\rangle \quad (1)$$

Also, $\widetilde{k}^{\widetilde{m}\widetilde{n}} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle \langle \mathcal{C}(\widetilde{n}), \mathcal{W}(\widetilde{n}) \rangle}$

$$= \left\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \right\rangle^{\left\langle \mathcal{C}(\widetilde{m})\mathcal{C}(\widetilde{n}),\ \max\{\mathcal{W}(\widetilde{m})\mathcal{W}(\widetilde{n})\} \right\rangle}$$
$$= \left\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})\mathcal{C}(\widetilde{n})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m}), \mathcal{W}(\widetilde{n})\} \right\rangle \quad (2)$$

From equations (1) & (2), the property (1) holds.

Property 2: For any three interval numbers $\widetilde{k}, \widetilde{l}$ & $\widetilde{m}$, then $\widetilde{k}^{\widetilde{m}} \times \widetilde{l}^{\widetilde{m}} = (\widetilde{k} \times \widetilde{l})^{\widetilde{m}}$.

Proof: We have $\widetilde{k} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle$,
$\widetilde{l} = \langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle$ & $\widetilde{m} = \langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle$

Now, $\widetilde{k}^{\widetilde{m}} \times \widetilde{l}^{\widetilde{m}}$

$$= \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle} \times \langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}$$
$$= \left\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m})\} \right\rangle \times \left\langle \mathcal{C}(\widetilde{l})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle$$
$$= \left\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})} \times \mathcal{C}(\widetilde{l})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle$$
$$= \left\langle \left(\mathcal{C}(\widetilde{k}) \times \mathcal{C}(\widetilde{l})\right)^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle \quad (3)$$

Also, $(\widetilde{k} \times \widetilde{l})^{\widetilde{m}} = \left(\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle \times \langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle \right)^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}$

$$= \left\langle \mathcal{C}(\widetilde{k}) \times \mathcal{C}(\widetilde{l}),\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \right\rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}$$
$$= \left\langle \left(\mathcal{C}(\widetilde{k}) \times \mathcal{C}(\widetilde{l})\right)^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle \quad (4)$$

From equations (3) & (4), the property (2) holds.

Property 3: For any three interval numbers $\widetilde{k}, \widetilde{l}$ & $\widetilde{m}$, then $\frac{\widetilde{k}^{\widetilde{m}}}{\widetilde{l}^{\widetilde{m}}} = \left(\frac{\widetilde{k}}{\widetilde{l}}\right)^{\widetilde{m}}$.

Proof: We have $\widetilde{k} = \langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle$,
$\widetilde{l} = \langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle$ & $\widetilde{m} = \langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle$

$$\text{Now, } \frac{\widetilde{k}^{\widetilde{m}}}{\widetilde{l}^{\widetilde{m}}} = \frac{\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}}{\langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}}$$
$$= \frac{\langle \mathcal{C}(\widetilde{k})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{m})\} \rangle}{\langle \mathcal{C}(\widetilde{l})^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \rangle}$$
$$= \left\langle \left(\frac{\mathcal{C}(\widetilde{k})}{\mathcal{C}(\widetilde{l})}\right)^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle \quad (5)$$

$$\text{Also, } \left(\frac{\widetilde{k}}{\widetilde{l}}\right)^{\widetilde{m}} = \left(\frac{\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle}{\langle \mathcal{C}(\widetilde{l}), \mathcal{W}(\widetilde{l}) \rangle} \right)^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}$$
$$= \left\langle \frac{\mathcal{C}(\widetilde{k})}{\mathcal{C}(\widetilde{l})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l})\} \right\rangle^{\langle \mathcal{C}(\widetilde{m}), \mathcal{W}(\widetilde{m}) \rangle}$$
$$= \left\langle \left(\frac{\mathcal{C}(\widetilde{k})}{\mathcal{C}(\widetilde{l})}\right)^{\mathcal{C}(\widetilde{m})},\ \max\{\mathcal{W}(\widetilde{k}), \mathcal{W}(\widetilde{l}), \mathcal{W}(\widetilde{m})\} \right\rangle \quad (6)$$

From equations (5) & (6), the property (3) holds.

Property 4: For any interval number $\widetilde{k}$, then $\langle \mathcal{M}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^0 = 1$.

Proof: $\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle^{\langle 0,0 \rangle} = \langle \mathcal{C}(\widetilde{k})^0, \ \max\{\mathcal{W}(\widetilde{k}), 0\} \rangle = \langle 1, \ \max\{\mathcal{W}(\widetilde{k}), 0\} \rangle = 1.$

III. Interval-based De-neutrosophication Technique

This section explains a grading methodology based on the interval number using (α, β, γ) - cut of a trapezoidal neutrosophic fuzzy number (TrpNFN). According to the literature study, most of the de-neutrosophication technique occurs in a neutrosophic environment are mainly in crisp numbers. Based on the reference [26], we derive the interval-based de-neutrosophication approach for the trapezoidal neutrosophic fuzzy number (TrpNFN) of the form $T_{\widetilde{Neu}} = \langle (t_{11}, t_{12}, t_{13}, t_{14}), (i_{11}, i_{12}, i_{13}, i_{14}), (f_{11}, f_{12}, f_{13}, f_{14}) \rangle$. The graphical representation of trapezoidal neutrosophic fuzzy number is displayed in Fig. 1.

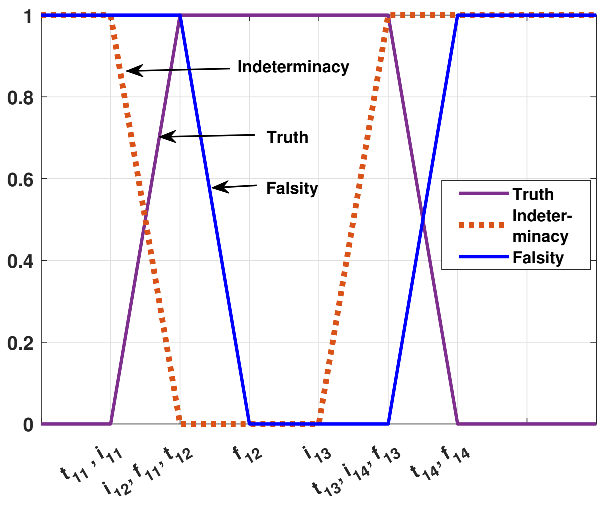

Fig. 1. Trapezoidal Neutrosophic Fuzzy Number

Let $\mathcal{C}(\alpha_{T_{\widetilde{Neu}}})$, $\mathcal{C}(\beta_{T_{\widetilde{Neu}}})$ and $\mathcal{C}(\gamma_{T_{\widetilde{Neu}}})$ are the mid-point & $\mathcal{W}(\alpha_{T_{\widetilde{Neu}}})$, $\mathcal{W}(\beta_{T_{\widetilde{Neu}}})$ and $\mathcal{W}(\gamma_{T_{\widetilde{Neu}}})$ are the width (half-width) of the (α, β, γ) - cut of a TrpNFN referring to truth, indeterminacy and falsity respectively. Then,

1. For $\alpha \in [0, 1]$, any α- cut of a TrpNFN $(T_{\widetilde{Neu}})$ can be expressed by a closed interval and is defined as

$$\alpha_{T_{\widetilde{Neu}}} = [\alpha^L_{T_{\widetilde{Neu}}}, \alpha^R_{T_{\widetilde{Neu}}}] = [t_{11} + \alpha(t_{12} - t_{11}), t_{14} - \alpha(t_{14} - t_{13})] \quad (7)$$

Based on equation (7), the interval form (mid-point and width (half-width)) of the α-cut of a TrpNFN is defined as

$$\langle \mathcal{C}(\alpha_{T_{\widetilde{Neu}}}), \mathcal{W}(\alpha_{T_{\widetilde{Neu}}}) \rangle = \Big\langle \frac{\alpha^L_{T_{\widetilde{Neu}}} + \alpha^R_{T_{\widetilde{Neu}}}}{2}, \frac{\alpha^R_{T_{\widetilde{Neu}}} - \alpha^L_{T_{\widetilde{Neu}}}}{2} \Big\rangle = \Big\langle \frac{(t_{14} + t_{11})(1 - \alpha) + \alpha(t_{13} + t_{12})}{2}, \frac{(t_{14} - t_{11})(1 - \alpha) + \alpha(t_{13} - t_{12})}{2} \Big\rangle \quad (8)$$

2. For $\beta \in [0, 1]$, any β- cut of a TrpNFN $(T_{\widetilde{Neu}})$ can be expressed by a closed interval and is denoted by $\beta_{T_{\widetilde{Neu}}}$ and expressed as

$$\beta_{T_{\widetilde{Neu}}} = [\beta^L_{T_{\widetilde{Neu}}}, \beta^R_{T_{\widetilde{Neu}}}] = [i_{12} + \beta(i_{12} - i_{11}), i_{13} + \beta(i_{14} - i_{13})] \quad (9)$$

Based on equation (9), the interval form (mid-point and width (half-width)) of the β-cut of a TrpNFN is expressed as

$$\langle \mathcal{C}(\beta_{T_{\widetilde{Neu}}}), \mathcal{W}(\beta_{T_{\widetilde{Neu}}}) \rangle = \Big\langle \frac{\beta^L_{T_{\widetilde{Neu}}} + \beta^R_{T_{\widetilde{Neu}}}}{2}, \frac{\beta^R_{T_{\widetilde{Neu}}} - \beta^L_{T_{\widetilde{Neu}}}}{2} \Big\rangle = \Big\langle \frac{\beta((i_{14} - i_{13}) + (i_{12} - i_{11})) + (i_{13} + i_{12})}{2}, \frac{\beta((i_{14} - i_{13}) - (i_{12} - i_{11})) + (i_{13} - i_{12})}{2} \Big\rangle \quad (10)$$

3. For $\gamma \in [0, 1]$, any γ- cut of a TrpNFN $(T_{\widetilde{Neu}})$ can be expressed by a closed interval and is denoted by $\gamma_{T_{\widetilde{Neu}}}$ and given as

$$\gamma_{T_{\widetilde{Neu}}} = [\gamma^L_{T_{\widetilde{Neu}}}, \gamma^R_{T_{\widetilde{Neu}}}] = [f_{12} + \gamma(f_{12} - f_{11}), f_{13} + \gamma(f_{14} - f_{13})] \quad (11)$$

Based on equation (11), the interval form (mid-point and width (half-width)) of the γ-cut of a TrpNFN is given as

$$\langle \mathcal{C}(\gamma_{T_{\widetilde{Neu}}}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}}) \rangle = \Big\langle \frac{\gamma^L_{T_{\widetilde{Neu}}} + \gamma^R_{T_{\widetilde{Neu}}}}{2}, \frac{\gamma^R_{T_{\widetilde{Neu}}} - \gamma^L_{T_{\widetilde{Neu}}}}{2} \Big\rangle = \Big\langle \frac{\gamma((f_{14} - f_{13}) + (f_{12} - f_{11})) + (f_{13} + f_{12})}{2}, \frac{\gamma((f_{14} - f_{13}) - (f_{12} - f_{11})) + (f_{13} - f_{12})}{2} \Big\rangle \quad (12)$$

A. Ranking function of TrpNFN using interval number

Define a ranking function that assigns each TrpNFN to an interval number. Based on the equations (8), (10) and (12) the ranking is expressed as

$$\begin{aligned}\mathcal{R}(T_{\widetilde{Neu}}) &= s.\langle \mathcal{C}(\alpha_{T_{\widetilde{Neu}}}), \mathcal{W}(\alpha_{T_{\widetilde{Neu}}}) \rangle + (1 - s). \{\langle \mathcal{C}(\beta_{T_{\widetilde{Neu}}}), \mathcal{W}(\beta_{T_{\widetilde{Neu}}}) \rangle + \langle \mathcal{C}(\gamma_{T_{\widetilde{Neu}}}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}}) \rangle\}, \\ &= s.\langle \mathcal{C}(\alpha_{T_{\widetilde{Neu}}}), \mathcal{W}(\alpha_{T_{\widetilde{Neu}}}) \rangle + (1 - s).\{\langle \mathcal{C}(\beta_{T_{\widetilde{Neu}}}) + \mathcal{C}(\gamma_{T_{\widetilde{Neu}}}), \ \max\{\mathcal{W}(\beta_{T_{\widetilde{Neu}}}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}})\}\} \\ &= \langle s.\mathcal{C}(\alpha_{T_{\widetilde{Neu}}}) + (1 - s).\mathcal{C}(\beta_{T_{\widetilde{Neu}}}) + (1 - s).\mathcal{C}(\gamma_{T_{\widetilde{Neu}}}), \max\{\mathcal{W}(\alpha_{T_{\widetilde{Neu}}}), \mathcal{W}(\beta_{T_{\widetilde{Neu}}}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}})\} \rangle \\ &= \langle \mathcal{C}(T_{\widetilde{Neu}}), \mathcal{W}(T_{\widetilde{Neu}}) \rangle \end{aligned} \quad (13)$$

where,
$\mathcal{C}(T_{\widetilde{Neu}}) = s.\mathcal{C}(\alpha_{T_{\widetilde{Neu}}}) + (1 - s).\mathcal{M}(\beta_{T_{\widetilde{Neu}}}) + (1 - s).\mathcal{C}(\gamma_{T_{\widetilde{Neu}}})$
and $\mathcal{W}(T_{\widetilde{Neu}}) = \max\{\mathcal{W}(\alpha_{T_{\widetilde{Neu}}}), \mathcal{W}(\beta_{T_{\widetilde{Neu}}}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}})\}$.

Here $s \in [0, 1]$ and it denotes the various choices for the mid-point and the width (half-width) version of (α, β, γ) - cuts of the truth, indeterminacy and falsity membership degrees. And the choices are preferably chosen by the decision maker that may be uncertainty, neutral or certainty.

B. Relation between trapezoidal neutrosophic fuzzy number

Sengupta and Pal [27] introduced ranking of interval numbers and our aim is to rank two trapezoidal neutrosophic fuzzy number. We introduce the following ranking methodology for trapezoidal neutrosophic fuzzy numbers. We apply it to relate two trapezoidal neutrosophic fuzzy numbers based on interval numbers.

Let $T_{\widetilde{Neu}}1 = \langle (t_{11}, t_{12}, t_{13}, t_{14}), (i_{11}, i_{12}, i_{13}, i_{14}), (f_{11}, f_{12}, f_{13}, f_{14}) \rangle$ & $T_{\widetilde{Neu}}2 = \langle (t_{21}, t_{22}, t_{23}, t_{24}), (i_{21}, i_{22}, i_{23}, i_{24}), (f_{21}, f_{22}, f_{23}, f_{24}) \rangle$ be any two trapezoidal neutrosophic fuzzy numbers.

We create an interval number from the trapezoidal neutrosophic fuzzy numbers using the sections III & III-A. Then the connection between trapezoidal neutrosophic fuzzy numbers corresponds to interval numbers are outlined as follows:

The interval form of the given two trapezoidal neutrosophic fuzzy numbers are represented as

$$\mathcal{R}(T_{\widetilde{Neu}}1) = \langle \mathcal{C}(T_{\widetilde{Neu}}1), \mathcal{W}(T_{\widetilde{Neu}}1) \rangle$$
$$\text{and } \mathcal{R}(T_{\widetilde{Neu}}2) = \langle \mathcal{C}(T_{\widetilde{Neu}}2), \mathcal{W}(T_{\widetilde{Neu}}2) \rangle.$$

Now, we can determine $\mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2)$ by interpreting it as the first interval's acceptability grade being inferior than that of the second interval and is given as

$$\mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2) = \frac{\mathcal{C}(T_{\widetilde{Neu}}2) - \mathcal{C}(T_{\widetilde{Neu}}1)}{\mathcal{W}(T_{\widetilde{Neu}}2) + \mathcal{W}(T_{\widetilde{Neu}}1)}, \quad (14)$$

where $\mathcal{W}(T_{\widetilde{Neu}}2) + \mathcal{W}(T_{\widetilde{Neu}}1) \neq 0$

And it is observed in the subsequent steps:

(i) If $\mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2) = 0$, then $T_{\widetilde{Neu}}1$ inferior to $T_{\widetilde{Neu}}2$ is not accepted.

(ii) If $0 < \mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2) < 1$ & $\mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2) \geq 1$, then $T_{\widetilde{Neu}}1$ inferior to $T_{\widetilde{Neu}}2$ is accepted.

1) Remark: Transforming trapezoidal neutrosophic fuzzy number as an interval number.

Let $T_{\widetilde{Neu}}1 = (1, 1.5, 2.5, 3; 0.5, 1.25, 1.75, 2.5; 1.2, 2.25, 3.15, 3.5)$.

Assume α, β, γ and s as the highest membership grade "1". Using equations (8), (10), (12),

$\langle \mathcal{C}(\alpha_{T_{\widetilde{Neu}}1}), \mathcal{W}(\alpha_{T_{\widetilde{Neu}}1}) \rangle = \langle 2, 0.5 \rangle$,

$\langle \mathcal{C}(\beta_{T_{\widetilde{Neu}}1}), \mathcal{W}(\beta_{T_{\widetilde{Neu}}1}) \rangle = \langle 2.25, 0.25 \rangle$,

$\langle \mathcal{C}(\gamma_{T_{\widetilde{Neu}}1}), \mathcal{W}(\gamma_{T_{\widetilde{Neu}}1}) \rangle = \langle 3.4, 0.10 \rangle$.

Using section III-A,

$$\mathcal{R}(T_{\widetilde{Neu}}1) = s\langle 2, 0.5 \rangle + (1-s)\{\langle 2.25, 0.25 \rangle + \langle 3.4, 0.10 \rangle\}$$
$$= \langle 2, 0.5 \rangle$$

2) Remark: Grading two trapezoidal neutrosophic fuzzy numbers utilizing interval numbers

Let $T_{\widetilde{Neu}}1 = (1, 1.5, 2.5, 3; 0.5, 1.25, 1.75, 2.5; 1.2, 2.25, 3.15, 3.5)$, then $\mathcal{R}(T_{\widetilde{Neu}}1) = \langle 2, 0.5 \rangle$.

& $T_{\widetilde{Neu}}2 = (1, 2, 8, 9; 1.5, 4, 5, 6; 4, 4.5, 9.5, 10)$, then $\mathcal{R}(T_{\widetilde{Neu}}2) = \langle 5, 3 \rangle$.

Using equation (14),

$\mathcal{A}(T_{\widetilde{Neu}}1 \otimes T_{\widetilde{Neu}}2) = \frac{\mathcal{C}(T_{\widetilde{Neu}}2) - \mathcal{C}(T_{\widetilde{Neu}}1)}{\mathcal{W}(T_{\widetilde{Neu}}2) + \mathcal{W}(T_{\widetilde{Neu}}1)}$,

$= \frac{5-2}{3+0.5} = \frac{3}{3.5} = 0.86 < 1$

Hence by III-B, $T_{\widetilde{Neu}}1$ is inferior to $T_{\widetilde{Neu}}2$.

Therefore, $\mathcal{R}(T_{\widetilde{Neu}}1) < \mathcal{R}(T_{\widetilde{Neu}}2)$.

IV. Trapezoidal Neutrosophic Fuzzy PERT

PERT, a statistical approach, plays a phenomenal role in project management that helps the project managers to schedule, coordinate and track the various tasks and activities involved in a project. This method usually consolidates expected time calculation, backward pass, forward pass, and slack time. The main focus of this study is to solve the trapezoidal neutrosophic fuzzy PERT problem by utilizing the proposed de-neutrosophication technique in the form of an interval number. Using neutrosophic set theory in PERT can help to improve project management's accuracy, flexibility, and effectiveness, particularly in complex projects with high uncertainty and variability. This section presents the definitions for Expected Duration, Forward Pass, Backward Pass, Slack Time, Critical Path and Critical Degree in terms of trapezoidal neutrosophic fuzzy numbers and its interval version.

A. Trapezoidal Neutrosophic Fuzzy Expected Duration

Let $T_{\widetilde{Neu}}ED_i$ be the trapezoidal neutrosophic fuzzy expected duration (TrpNFED). The trapezoidal neutrosophic fuzzy PERT activity duration is expressed in three estimates: Trapezoidal neutrosophic fuzzy optimistic time ($T_{\widetilde{Neu}}OT_i$), Trapezoidal neutrosophic fuzzy pessimistic time ($T_{\widetilde{Neu}}PT_i$) and Trapezoidal neutrosophic fuzzy most likley time ($T_{\widetilde{Neu}}MT_i$). Then the formula to obtain the TrpNFED is

$$T_{\widetilde{Neu}}ED_i = \frac{T_{\widetilde{Neu}}OT_i + 4T_{\widetilde{Neu}}MT_i + T_{\widetilde{Neu}}PT_i}{6} \quad (15)$$

Now, using the proposed interval-based de-neutrosophication technique (section III-A), the TrpNFED in interval number is calculated using the following:

$$\mathcal{R}(T_{\widetilde{Neu}}ED_i) = \frac{\mathcal{R}(T_{\widetilde{Neu}}OT_i) + 4\mathcal{R}(T_{\widetilde{Neu}}MT_i) + \mathcal{R}(T_{\widetilde{Neu}}PT_i)}{6} \quad (16)$$

B. Trapezoidal Neutrosophic Fuzzy Forward Pass Calculations

Let $T_{\widetilde{Neu}}A_i$ be the trapezoidal neutrosophic fuzzy duration of the i^{th} activity, $T_{\widetilde{Neu}}A_j$ be the trapezoidal neutrosophic fuzzy duration of j^{th} activity, P_i be the predecessor of j^{th} activity and S_i be the successor of j^{th} activity. Trapezoidal neutrosophic fuzzy forward pass calculations (TrpNFFPC) is a technique that is used to move forward throughout a network diagram in order to estimate project duration and identify the critical path. It is performed by the trapezoidal neutrosophic fuzzy early Start (TrpNFES) and trapezoidal neutrosophic fuzzy early finish (TrpNFEF) and is defined as

$$T_{\widetilde{Neu}}ES_j = \max_{P_i} [T_{\widetilde{Neu}}ES_i + T_{\widetilde{Neu}}A_i], \quad (17)$$

$$T_{\widetilde{Neu}}EF_i = T_{\widetilde{Neu}}ES_i + T_{\widetilde{Neu}}A_i \quad (18)$$

where $T_{\widetilde{Neu}}ES_i$ represents the earliest possible start time for the i^{th} activity while maintaining the project's scheduling constraints. $T_{\widetilde{Neu}}EF_i$ represents the earliest time when the i^{th} activity can be finished.

By applying the proposed interval-based de-neutrosophication technique (section III-A), the TrpNFES & TrpNFEF based on interval number is calculated using the following:

$$\mathcal{R}(T_{\widetilde{Neu}}ES_i) = \max_{P_i} \left[\mathcal{R}(T_{\widetilde{Neu}}ES_j) + \mathcal{R}(T_{\widetilde{Neu}}A_j)\right], \tag{19}$$

$$\mathcal{R}(T_{\widetilde{Neu}}EF_i) = \mathcal{R}(T_{\widetilde{Neu}}ES_i) + \mathcal{R}(T_{\widetilde{Neu}}A_i) \tag{20}$$

C. Trapezoidal Neutrosophic Fuzzy Backward Pass Calculations

The trapezoidal neutrosophic fuzzy backward pass calculation (TrpNFBPC) is used to determine a late start or determine whether there is any slack in the action. It is performed by the trapezoidal neutrosophic fuzzy late start (TrpNFLS) and trapezoidal neutrosophic fuzzy late finish (TrpNFLF) and is defined as

$$T_{\widetilde{Neu}}LF_i = \min_{S_i} \left[T_{\widetilde{Neu}}LF_j - T_{\widetilde{Neu}}A_j\right], \tag{21}$$

$$T_{\widetilde{Neu}}LS_i = T_{\widetilde{Neu}}LF_i - T_{\widetilde{Neu}}A_i \tag{22}$$

where $T_{\widetilde{Neu}}LS_i$ is the latest dates for an activity's start in order to prevent project delays and it is the i^{th} latest start for computing backward pass. $T_{\widetilde{Neu}}LF_i$ is the latest dates that have been calculated for an activity to end, which is the i^{th} latest finish for the project network's backward pass computation.

The TrpNFLS and TrpNFLF based on interval number is calculated using the proposed interval-based de-neutrosophication technique (section III-A) and is defined as below.

$$\mathcal{R}(T_{\widetilde{Neu}}LF_i) = \min_{S_i} \left[\mathcal{R}(T_{\widetilde{Neu}}LF_j) - \mathcal{R}(T_{\widetilde{Neu}}A_j)\right], \tag{23}$$

$$\mathcal{R}(T_{\widetilde{Neu}}LS_i) = \mathcal{R}(T_{\widetilde{Neu}}LF_i) - \mathcal{R}(T_{\widetilde{Neu}}A_i) \tag{24}$$

D. Trapezoidal Neutrosophic Fuzzy Slack Time

Trapezoidal neutrosophic fuzzy slack time (TrpNFST) is the duration of time that a project's task or activity can be delayed without having an impact on the entire project schedule. The formula for calculating TrpNFST is provided in equation (25).

$$\begin{aligned} T_{\widetilde{Neu}}ST_i &= T_{\widetilde{Neu}}LS_i - T_{\widetilde{Neu}}ES_i \\ &= T_{\widetilde{Neu}}LF_i - T_{\widetilde{Neu}}EF_i \end{aligned} \tag{25}$$

Now the TrpNFST based on the proposed interval-based de-neutrosophic technique (section III-A) is obtained by the following formula.

$$\begin{aligned} \mathcal{R}(T_{\widetilde{Neu}}ST_i) &= \mathcal{R}(T_{\widetilde{Neu}}LS_i) - \mathcal{R}(T_{\widetilde{Neu}}ES_i) \\ &= \mathcal{R}(T_{\widetilde{Neu}}LF_i) - \mathcal{R}(T_{\widetilde{Neu}}EF_i) \end{aligned} \tag{26}$$

If the TrpNFST is 0 for each i^{th} activity, then it is called as critical activity. Otherwise, called as non-critical activity. The path formed by the critical activity is called as critical path.

E. Trapezoidal Neutrosophic Critical Degree of the Activity

Based on the definition given by Mukherjee and Basu [21], we define trapezoidal neutrosophic critical degree (TrpNFCD) of the activity using our proposed interval-based de-neutrosophication technique (section III-A) and it is described as follows.

If the interval slack time of the activity $i-j$ is $\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k}) \rangle$, then the critical degree (CD) of this activity is defined as $\mathcal{R}(T_{\widetilde{Neu}}CD_i) =$

$$\begin{cases} 1, & \text{if } \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) < 0 \\ \frac{\mathcal{W}(\widetilde{k}) - \mathcal{C}(\widetilde{k})}{\mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k})} & \text{if } \mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) < 0 < \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) \\ 0, & \text{if } \mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) \geq 0 \end{cases} \tag{27}$$

Equation (27) gives more validity for the proposed interval-based de-neutrosophication technique. i.e., If the critical degree takes the value 1 for all the critical activities, then we can say that the proposed method is more valid.

V. Interval-valued Trapezoidal Neutrosophic Fuzzy PERT

Let the interval-valued trapezoidal neutrosophic fuzzy number is in the form of $T_{\widetilde{IvNeu}} = \langle (a_1, a_2, a_3, a_4); [\rho^L, \rho^R], [\kappa^L, \kappa^R], [\nu^L, \nu^R] \rangle$. The graphical representation of interval-valued trapezoidal neutrosophic fuzzy number is displayed in Fig. 2. Based on the reference [28], we convert the interval-valued trapezoidal neutrosophic fuzzy number into an interval number (as same as section III). We consider this interval based de-neutrosophication technique to solve interval-valued trapezoidal neutrosophic fuzzy PERT problem. Using interval-valued trapezoidal neutrosophic set theory in PERT can help the project manager's to analyze the situations in interval truth, interval indeterminacy and interval falsity respectively. This section presents the definitions for Expected Duration, Forward Pass, Backward Pass, Slack Time, Critical Path and Critical Degree in the form of interval-valued trapezoidal neutrosophic fuzzy numbers and its interval version. It is similar as explained in section IV.

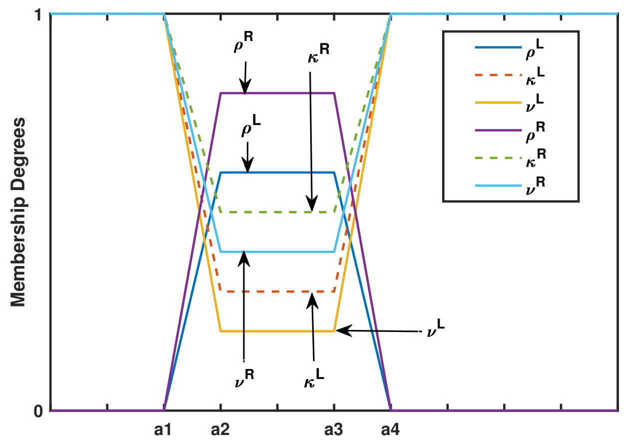

Fig. 2. Interval-valued Trapezoidal Neutrosophic Fuzzy Number

A. Interval-valued Trapezoidal Neutrosophic Fuzzy Expected Duration

Let $T_{\widetilde{IvNeu}}ED_i$ be the interval-valued trapezoidal neutrosophic fuzzy expected duration (IVTrpNFED). The interval-valued trapezoidal neutrosophic fuzzy PERT activity duration is expressed in three estimates: Interval-valued trapezoidal neutrosophic fuzzy optimistic time ($T_{\widetilde{IvNeu}}OT_i$), Interval-valued trapezoidal neutrosophic fuzzy pessimistic time ($T_{\widetilde{IvNeu}}PT_i$) and Interval-valued trapezoidal neutrosophic fuzzy most likley time ($T_{\widetilde{IvNeu}}MT_i$). Then the formula to obtain the IVTrpNFED is

$$T_{\widetilde{IvNeu}}ED_i = \frac{T_{\widetilde{IvNeu}}OT_i + 4T_{\widetilde{IvNeu}}MT_i + T_{\widetilde{IvNeu}}PT_i}{6} \tag{28}$$

Now, using the interval-based de-neutrosophication technique (provided by [28]), the IVTrpNFED in interval number is calculated using the following:

$$\mathcal{R}(T_{\widetilde{IvNeu}}ED_i) = \frac{\mathcal{R}(T_{\widetilde{IvNeu}}OT_i) + 4\mathcal{R}(T_{\widetilde{IvNeu}}MT_i) + \mathcal{R}(T_{\widetilde{IvNeu}}PT_i)}{6} \tag{29}$$

B. Interval-valued Trapezoidal Neutrosophic Fuzzy Forward Pass Calculations

Let $T_{\widetilde{IvNeu}}A_i$ be the i^{th} activity of the interval-valued trapezoidal neutrosophic fuzzy duration, $T_{\widetilde{IvNeu}}A_j$ be the interval-valued trapezoidal neutrosophic fuzzy duration of j^{th} activity, P_i be the predecessor and S_i be the successor of j^{th} activity. Interval-valued Trapezoidal neutrosophic fuzzy forward pass calculations (IVTrpNFFPC) is performed by the interval-valued trapezoidal neutrosophic fuzzy early start (IVTrpNFES) and interval-valued trapezoidal neutrosophic fuzzy early finish (IVTrpNFEF) and is defined as

$$T_{\widetilde{IvNeu}}ES_j = \max_{P_i}\left[T_{\widetilde{IvNeu}}ES_i + T_{\widetilde{IvNeu}}A_i\right], \tag{30}$$

$$T_{\widetilde{IvNeu}}EF_i = T_{\widetilde{IvNeu}}ES_i + T_{\widetilde{IvNeu}}A_i \tag{31}$$

where $T_{\widetilde{IvNeu}}ES_i$ ensures that the i^{th} activity doesnot start until all its predecessors are completed. $T_{\widetilde{IvNeu}}EF_i$ ensures that the i^{th} activity is completed as early as possible without violating any project constraints.

By applying the interval-based de-neutrosophication technique provided by [28], the IVTrpNFES & IVTrpNFEF based on interval number is calculated using the following:

$$\mathcal{R}(T_{\widetilde{IvNeu}}ES_i) = \max_{P_i}\left[\mathcal{R}(T_{\widetilde{IvNeu}}ES_j) + \mathcal{R}(T_{\widetilde{IvNeu}}A_j)\right], \tag{32}$$

$$\mathcal{R}(T_{\widetilde{IvNeu}}EF_i) = \mathcal{R}(T_{\widetilde{IvNeu}}ES_i) + \mathcal{R}(T_{\widetilde{IvNeu}}A_i) \tag{33}$$

C. Interval-valued Trapezoidal Neutrosophic Fuzzy Backward Pass Calculations

The interval-valued trapezoidal neutrosophic fuzzy backward pass calculation (IVTrpNFBPC) is used to determine the interval-valued trapezoidal neutrosophic fuzzy late start (IVTrpNFLS) and interval-valued trapezoidal neutrosophic fuzzy late finish (IVTrpNFLF) for each activity in a project schedule and is defined as

$$T_{\widetilde{IvNeu}}LF_i = \min_{S_i}\left[T_{\widetilde{IvNeu}}LF_j - T_{\widetilde{IvNeu}}A_j\right], \tag{34}$$

$$T_{\widetilde{IvNeu}}LS_i = T_{\widetilde{IvNeu}}LF_i - T_{\widetilde{IvNeu}}A_i \tag{35}$$

where $T_{\widetilde{IvNeu}}LS_i$ indicates the latest possible start time for the i^{th} activity. $T_{\widetilde{IvNeu}}LF_i$ represents the latest possible point in time when the i^{th} activity can be calculated without delaying the project completion.

The IVTrpNFLS & IVTrpNFLF based on interval number is calculated by using the interval-based de-neutrosophication technique (provided by [28]) and is defined below.

$$\mathcal{R}(T_{\widetilde{IvNeu}}LF_i) = \min_{S_i}\left[\mathcal{R}(T_{\widetilde{IvNeu}}LF_j) - \mathcal{R}(T_{\widetilde{IvNeu}}A_j)\right], \tag{36}$$

$$\mathcal{R}(T_{\widetilde{IvNeu}}LS_i) = \mathcal{R}(T_{\widetilde{IvNeu}}LF_i) - \mathcal{R}(T_{\widetilde{IvNeu}}A_i) \tag{37}$$

D. Interval-valued Trapezoidal Neutrosophic Fuzzy Slack Time

The formula for calculating interval-valued trapezoidal neutrosophic fuzzy slack time (IVTrpNFST) is provided in equation (38).

$$\begin{aligned} T_{\widetilde{IvNeu}}ST_i &= T_{\widetilde{IvNeu}}LS_i - T_{\widetilde{IvNeu}}ES_i \\ &= T_{\widetilde{IvNeu}}LF_i - T_{\widetilde{IvNeu}}EF_i \end{aligned} \tag{38}$$

Now the IVTrpNFST based on the interval-based de-neutrosophic technique (provided by [28]) is obtained by the following formula.

$$\begin{aligned} \mathcal{R}(T_{\widetilde{IvNeu}}ST_i) &= \mathcal{R}(T_{\widetilde{IvNeu}}LS_i) - \mathcal{R}(T_{\widetilde{IvNeu}}ES_i) \\ &= \mathcal{R}(T_{\widetilde{IvNeu}}LF_i) - \mathcal{R}(T_{\widetilde{IvNeu}}EF_i) \end{aligned} \tag{39}$$

If the IVTrpNFST is 0 for each i^{th} activity, then it is called as critical activity. Otherwise, called as non-critical activity. The path formed by the critical activity is called as critical path.

E. Interval-valued Trapezoidal Neutrosophic Critical degree of the Activity

The interval-valued trapezoidal neutrosophic critical degree (IVTrpNFCD) of the activity based on the interval-based de-neutrosophication technique (provided by [28]) is as same as described in section IV-E, equation (27).

VI. Illustration – Trapezoidal Neutrosophic Fuzzy PERT

To show the efficiency of the proposed interval-based de-neutrosophication technique (section III) in the proposed TrpNFPERT method (section IV), we solve two examples (VI-A & VI-B) based on trapezoidal neutrosophic fuzzy numbers. Here, we choose the choices of values for α, β, γ and s as the highest membership grade 1.

A. Example 1

1) Description of the Problem and the Data: Consider a project management in farming Oyster mushroom and we choose the parameters as trapezoidal neutrosophic fuzzy numbers (TrpNFN). We design a production process for farming Oyster mushroom by the proposed trapezoidal neutrosophic fuzzy pert method (TrpNFPERT). In this example, triangular neutrosophic fuzzy values are adapted from [17] and we consider it as TrpNFN based on decision maker's perception. Considering TrpNFN makes one to

TABLE I
DURATIONS IN THE FORM OF TRAPEZOIDAL NEUTROSOPHIC FUZZY NUMBERS (EXAMPLE 1)

Nr	Durations (day)		
	Optimistic time ($T_{\widetilde{Neu}}OT_i$)	Pessimistic time ($T_{\widetilde{Neu}}PT_i$)	Most likely time ($T_{\widetilde{Neu}}MT_i$)
1	$(1, 1.5, 2.5, 3; 0.5, 1.25, 1.75, 2.5; 1.2, 2.25, 3.15, 3.5)$	$(1, 2, 8, 9; 1.5, 4, 5, 6; 4, 4.5, 9.5, 10)$	$(1.5, 3, 4, 5.5; 1, 1.5, 2.5, 3; 3, 4, 5, 6)$
2	$(1, 3, 7, 8; 1, 2, 4, 6; 4, 6, 8, 9)$	$(1, 1.5, 2.5, 3; 0.5, 1, 2, 2.5; 1.5, 2, 3, 3.5)$	$(1, 3, 7, 8; 1.5, 2.5, 3.5, 6.5; 4, 6.5, 7.5, 9)$
3	$(1, 2, 6, 7; 1, 2, 4, 5; 3.5, 5.5, 6.5, 7.5)$	$(1, 1.25, 1.75, 4; 0.5, 0.75, 1.25, 2.5; 1.25, 2.25, 3.75, 4.25)$	$(1, 3, 7, 9; 1.5, 3.5, 5.5, 6.5; 4, 6, 8, 10)$
4	$(1, 2, 4, 5; 0.5, 2, 3, 3.5; 2.5, 3, 5, 6)$	$(1.5, 2.5, 4.5, 5.5; 1, 1.5, 2.5, 3; 3, 3.5, 5.5, 6)$	$(0.5, 2, 3, 4.5; 1, 1.5, 2.5, 3; 1.5, 2.5, 4.5, 5.5)$
5	$(0.5, 2, 3, 4.5; 0.5, 1, 2, 3.5; 2, 3, 5, 6)$	$(1, 3, 7, 9; 1.5, 3.5, 5.5, 6.5; 4, 5.5, 8.5, 10.5)$	$(1.5, 2, 3, 3.5; 1, 1.25, 1.75, 3; 2, 2.75, 3.25, 4)$
6	$(2, 3, 5, 6; 1.5, 2, 3, 3.5; 3, 4, 6, 7)$	$(1, 1.5, 2.5, 3; 0.5, 1, 2, 2.5; 1.2, 2.4, 3, 3.5)$	$(1, 3, 5, 7; 1, 2, 4, 5; 3.5, 5.5, 6.5, 7.5)$
7	$(0.5, 2, 3, 4.5; 1, 1.5, 2.5, 3; 2, 3, 5, 6)$	$(1, 4, 6, 8; 1.5, 2.5, 3.5, 6.5; 4, 6, 8, 9)$	$(1, 2, 8, 9; 1.5, 3, 6, 6.5; 4, 4.5, 9.5, 10)$
8	$(1.5, 2.5, 4.5, 5.5; 1, 1.5, 2.5, 3; 3, 4, 5, 6)$	$(0.5, 2.5, 4.5, 6.5; 0.5, 1.5, 3.5, 4.5; 3, 4, 6, 7)$	$(1, 1.5, 2.5, 3; 0.5, 1, 2, 2.5; 1.5, 2, 3, 3.5)$
9	$(1, 4, 6, 9; 1.5, 3.5, 5.5, 6.5; 4, 6, 9, 10)$	$(0.5, 2, 3, 4.5; 1, 1.5, 2.5, 3; 1.5, 2.5, 4.5, 5.5)$	$(1, 4, 6, 8; 1, 2, 4, 6; 4, 6, 8, 9)$
10	$(0.5, 2.5, 4.5, 6.5; 0.5, 2, 3, 4.5; 3, 4, 6, 7)$	$(1.5, 2, 3, 3.5; 1, 1.25, 1.75, 3; 2, 2.5, 3.5, 4)$	$(1, 1.75, 2.25, 3; 0.5, 1, 2, 2.5; 1.2, 2.4, 3, 3.5)$
11	$(1, 4, 6, 8; 1.5, 2.5, 4.5, 6.5; 4, 5, 7, 8.5)$	$(1, 3, 5, 7; 1, 2.5, 3.5, 5; 3.5, 5.5, 6.5, 7.5)$	$(1, 1.25, 1.75, 4; 0.5, 0.75, 1.25, 2.5; 1.25, 2.25, 3.75, 4.25)$

analyze the problem in all the ways of possibility namely positive, negative and neutral. The activities of the problem with the description and the corresponding predecessor are given in Table II. Flow of the activities are represented in Fig. 3

TABLE II
DESCRIPTION OF THE ACTIVITY IN FARMING MUSHROOM (EXAMPLE 1)

Nr	Activity	Description	Predecessor
1	A	Choosing Mushroom type	-
2	B	Landscape Management	-
3	C	Preparing Compost	A
4	D	Natural Compost	B
5	E	Synthetic Compost	B
6	F	Filling the compost in the tray	A
7	G	Spawning	C
8	H	Casing	D
9	I	Harvesting	A
10	J	Storage	E,G,H
11	K	Selling	F,I,J

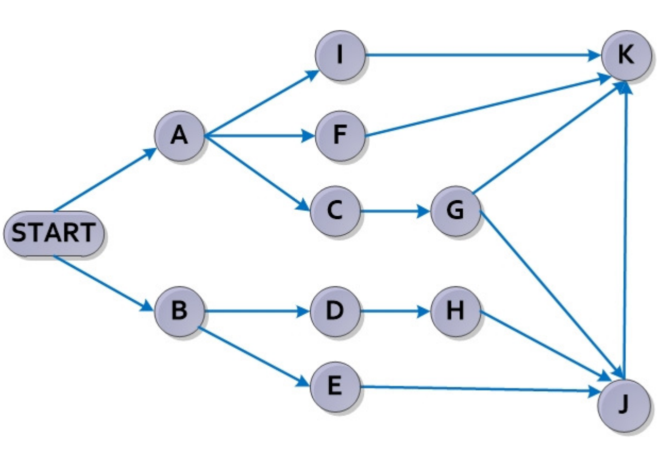

Fig. 3. Network diagram (Example 1)

2) *Calculation for TrpNFED:* The process of TrpNFED are defined based on proposed de-neutrosophication technique in interval form. The data given in Table I is in the form of TrpNFN. The interval version of trapezoidal neutrosophic fuzzy optimistic time, trapezoidal neutrosophic fuzzy pessimistic time and trapezoidal neutrosophic fuzzy most likely time are converted based on sections III, III-A. The interval form of trapezoidal neutrosophic fuzzy expected duration are now calculated using section IV-A and the equation (16) is used for computation. The i^{th} activity of the TrpNFED in the form of interval number is calculated as follows. For example, let us calculate for i=7.

$T_{\widetilde{Neu}}OT_7 = (0.5, 2, 3, 4.5; 1, 1.5, 2.5, 3; 2, 3, 5, 6)$
$T_{\widetilde{Neu}}PT_7 = (1, 4, 6, 8; 1.5, 2.5, 3.5, 6.5; 4, 6, 8, 9)$
$T_{\widetilde{Neu}}MT_7 = (1, 2, 8, 9; 1.5, 3, 6, 6.5; 4, 4.5, 9.5, 10)$

Then by applying equations (7) - (13), we get,
$\mathcal{R}(T_{\widetilde{Neu}}OT_7) = \langle 2.5, 1\rangle$, $\mathcal{R}(T_{\widetilde{Neu}}OT_7) = \langle 5, 1.5\rangle$,
$\mathcal{R}(T_{\widetilde{Neu}}OT_7) = \langle 4.58, 3\rangle$.

Using equation (16), we obtain the expected duration of the trapezoidal neutrosophic number for i=7 as $\langle 4.58, 3\rangle$.
The detailed expected duration for all activities in interval form are provided in Table III.

3) *Calculation for TrpNFFPC:* Table IV consists of the calculations to TrpNFFPC in detail and it is computed using equations (19) & (20). The forward pass in project calculations requires the completion of all j^{th} activities before starting the i^{th} activities, since specific activities are dependent on predecessor activities. To calculate the interval version of the i^{th} activity of TrpNFFPC is given as follows. Let us take i=10. Using equation (19), the interval version of TrpNFES is found as

$$\begin{aligned}\mathcal{R}(T_{\widetilde{Neu}}ES_{10}) &= \max_{5,7,8}\left[\mathcal{R}(T_{\widetilde{Neu}}ES_j) + \mathcal{R}(T_{\widetilde{Neu}}A_j)\right]\\ &= \max\left[\mathcal{R}(T_{\widetilde{Neu}}ES_5) + \mathcal{R}(T_{\widetilde{Neu}}A_5),\right.\\ &\qquad \mathcal{R}(T_{\widetilde{Neu}}ES_7) + \mathcal{R}(T_{\widetilde{Neu}}A_7),\\ &\qquad \left.\mathcal{R}(T_{\widetilde{Neu}}ES_8) + \mathcal{R}(T_{\widetilde{Neu}}A_8)\right]\\ &= \max\left[\langle 4.5, 2\rangle + \langle 2.9, 2\rangle,\ \langle 7.75, 3\rangle + \langle 4.58, 3\rangle,\right.\\ &\qquad \left.\langle 7.25, 2\rangle + \langle 2.5, 1\rangle\right]\end{aligned}$$

TABLE III
DURATIONS IN THE FORM OF INTERVAL NUMBER (EXAMPLE 1)

Nr	Activity	Predecessor	Durations (day)			
			Optimistic time	Pessimistic time	Most likely time	Expected days
1	A	-	$\langle 2, 0.5\rangle$	$\langle 5, 3\rangle$	$\langle 3.5, 0.5\rangle$	$\langle 3.5, 3\rangle$
2	B	-	$\langle 5, 2\rangle$	$\langle 2, 0.5\rangle$	$\langle 5, 2\rangle$	$\langle 4.5, 2\rangle$
3	C	A	$\langle 4, 2\rangle$	$\langle 1.5, 0.75\rangle$	$\langle 5, 2\rangle$	$\langle 4.25, 2\rangle$
4	D	B	$\langle 3, 1.25\rangle$	$\langle 3.5, 1\rangle$	$\langle 2.5, 1\rangle$	$\langle 2.75, 1.25\rangle$
5	E	B	$\langle 2.5, 1\rangle$	$\langle 5, 2\rangle$	$\langle 2.5, 0.75\rangle$	$\langle 2.9, 2\rangle$
6	F	A	$\langle 4, 1\rangle$	$\langle 2, 0.5\rangle$	$\langle 4, 1\rangle$	$\langle 3.7, 1\rangle$
7	G	C	$\langle 2.5, 1\rangle$	$\langle 5, 1.5\rangle$	$\langle 5, 3\rangle$	$\langle 4.58, 3\rangle$
8	H	D	$\langle 3.5, 1\rangle$	$\langle 3.5, 1\rangle$	$\langle 2, 0.5\rangle$	$\langle 2.5, 1\rangle$
9	I	A	$\langle 5, 1\rangle$	$\langle 2.5, 1\rangle$	$\langle 5, 1.5\rangle$	$\langle 4.58, 1.5\rangle$
10	J	E,G,H	$\langle 3.5, 1\rangle$	$\langle 2.5, 0.75\rangle$	$\langle 2, 0.5\rangle$	$\langle 2.3, 1\rangle$
11	K	F,I,J	$\langle 5, 1.5\rangle$	$\langle 4, 1\rangle$	$\langle 3.75, 1.5\rangle$	$\langle 2.5, 1.5\rangle$

$$= \max\left[\langle 7.4, 2\rangle,\ \langle 12.33, 3\rangle,\ \langle 9.75, 2\rangle\right] = \langle 12.33, 3\rangle.$$

Also, using equation (20), the TrpNFEF based on interval number is

$$\mathcal{R}(T_{\widetilde{Neu}}EF_{10}) = \mathcal{R}(T_{\widetilde{Neu}}ES_{10}) + \mathcal{R}(T_{\widetilde{Neu}}A_{10})$$
$$= \langle 12.33, 3\rangle + \langle 2.3, 1\rangle = \langle 14.63, 3\rangle.$$

4) Calculation for TrpNFBPC: The calculations to TrpNFBPC is computed using equations (23) & (24). When calculating the backward pass from the finish of the project, certain activities require successor activities, therefore, all the j^{th} activities must finish to start the i^{th} activities. To calculate the interval version of the i^{th} activity of TrpNFBPC is given as follows.
Let us take i=1. Using equation (23), the TrpNFLF is found as

$$\begin{aligned}\mathcal{R}(T_{\widetilde{Neu}}LF_1) &= \min_{3,6,9}\left[\mathcal{R}(T_{\widetilde{Neu}}LF_j) - \mathcal{R}(T_{\widetilde{Neu}}A_j)\right]\\ &= \min\left[\mathcal{R}(T_{\widetilde{Neu}}LF_3) - \mathcal{R}(T_{\widetilde{Neu}}A_3),\right.\\ &\quad \mathcal{R}(T_{\widetilde{Neu}}LF_6) - \mathcal{R}(T_{\widetilde{Neu}}A_6),\\ &\quad \left.\mathcal{R}(T_{\widetilde{Neu}}LF_9) - \mathcal{R}(T_{\widetilde{Neu}}A_9)\right]\\ &= \min\left[\langle 7.75, 3\rangle - \langle 4.25, 2\rangle,\ \langle 14.63, 3\rangle - \langle 3.7, 1\rangle,\right.\\ &\quad \left.\langle 14.63, 3\rangle - \langle 4.58, 1.5\rangle\right]\\ &= \min\left[\langle 3.5, 3\rangle,\ \langle 10.93, 3\rangle,\ \langle 10.05, 3\rangle\right] = \langle 3.5, 3\rangle.\end{aligned}$$

Also, using equation (24), the TrpNFEF based on interval number is $\mathcal{R}(T_{\widetilde{Neu}}LS_1) = \mathcal{R}(T_{\widetilde{Neu}}LF_1) - \mathcal{R}(T_{\widetilde{Neu}}A_1)$
$= \langle 3.5, 3\rangle - \langle 3.5, 3\rangle = \langle 0, 3\rangle.$

5) Calculation for TrpNFST: TrpNFST is used to identify the critical and non-critical activities by using equation (25). Let us calculate for the activity i=3,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{Neu}}ST_3) &= \mathcal{R}(T_{\widetilde{Neu}}LS_3) - \mathcal{R}(T_{\widetilde{Neu}}ES_3)\\ &= \mathcal{R}(T_{\widetilde{Neu}}LF_3) - \mathcal{R}(T_{\widetilde{Neu}}EF_3)\\ &= \langle 3.5, 3\rangle - \langle 3.5, 3\rangle = \langle 7.75, 3\rangle - \langle 7.75, 3\rangle\\ &= \langle 0, 3\rangle = \widetilde{0},\ \text{a critical activity}\end{aligned}$$

For example, let us calculate for the activity i=8,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{Neu}}ST_8) &= \mathcal{R}(T_{\widetilde{Neu}}LS_8) - \mathcal{R}(T_{\widetilde{Neu}}ES_8)\\ &= \mathcal{R}(T_{\widetilde{Neu}}LF_8) - \mathcal{R}(T_{\widetilde{Neu}}EF_8)\\ &= \langle 9.83, 3\rangle - \langle 7.25, 3\rangle = \langle 12.33, 3\rangle - \langle 9.75, 3\rangle\\ &= \langle 2.58, 3\rangle,\ \text{a non-critical activity}\end{aligned}$$

The activities that has the TrpNFST values as 0 is considered as the critical path.

6) Calculation for TrpNFCD: The critical degree for TrpNFN is calculated for showing the validity of the proposed grading technique. And it is calculated based on the TrpNFST for each activity. Let us calculate the TrpNFCD for i=3 and the TrpNFST is $\langle 0, 3\rangle$.
Therefore, based on section IV-E,
$\mathcal{M}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) < 0 < \mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) \implies -3 < 0 < 3$.

$$\text{Hence } \mathcal{R}(T_{\widetilde{Neu}}CD_3) = \frac{\mathcal{W}(\widetilde{k}) - \mathcal{M}(\widetilde{k})}{\mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k})} = \frac{3-0}{0+3} = 1,\ \text{is the critical activity.}$$

For i=4, the TrpNFST is $\langle 2.58, 3\rangle$. Therefore, based on section IV-E,
$\mathcal{M}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) < 0 < \mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) \implies -0.42 < 0 < 5.58.$

$$\text{Hence } \mathcal{R}(T_{\widetilde{Neu}}CD_4) = \frac{\mathcal{W}(\widetilde{k}) - \mathcal{M}(\widetilde{k})}{\mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k})} = \frac{3-2.58}{2.58+3} = 0.075,\ \text{is the non-critical activity.}$$

It is clearly visible that the TrpNFCD is 1 for all critical activities and the rest are 0.
The detailed values of TrpNFFPC, TrpNFBPC, TrpNFST and TrpNFCD are provided in Table IV.

7) Comparison of Results:

- Using our proposed interval-based de-neutrosophication methodology for trapezoidal neutrosophic fuzzy number, the critical path of the project in the TrpNFPERT is A $\to$ C $\to$ G $\to$ J $\to$ K and it is obtained as same as the path in the existing method [17].
- The project completion days we obtained as $\langle 3.5, 3\rangle + \langle 4.25, 2\rangle + \langle 4.58, 3\rangle + \langle 2.3, 1\rangle + \langle 2.5, 1.5\rangle = \langle 17.13, 3\rangle = [14.13, 20.13]$ days.

TABLE IV
DURATIONS CALCULATIONS BASED ON FORWARD AND BACKWARD PASS (EXAMPLE 1)

Nr	Activity	Early start	Early finish	Late finish	Late start	Slack time	Description	Critical degree
1	A	$\langle 0,0\rangle$	$\langle 3.5,3\rangle$	$\langle 3.5,3\rangle$	$\langle 0,3\rangle$	$\langle 0,3\rangle$	Critical	1
2	B	$\langle 0,0\rangle$	$\langle 4.5,2\rangle$	$\langle 7.08,3\rangle$	$\langle 2.58,3\rangle$	$\langle 2.58,3\rangle$	Non-Critical	0.075
3	C	$\langle 3.5,3\rangle$	$\langle 7.75,3\rangle$	$\langle 7.75,3\rangle$	$\langle 3.5,3\rangle$	$\langle 0,3\rangle$	Critical	1
4	D	$\langle 4.5,2\rangle$	$\langle 7.25,3\rangle$	$\langle 9.83,3\rangle$	$\langle 7.08,3\rangle$	$\langle 2.58,3\rangle$	Non-Critical	0.075
5	E	$\langle 4.5,2\rangle$	$\langle 7.4,2\rangle$	$\langle 12.33,3\rangle$	$\langle 9.43,3\rangle$	$\langle 4.93,3\rangle$	Non-Critical	0
6	F	$\langle 3.5,3\rangle$	$\langle 7.2,3\rangle$	$\langle 14.63,3\rangle$	$\langle 10.93,3\rangle$	$\langle 7.43,3\rangle$	Non-Critical	0
7	G	$\langle 7.75,3\rangle$	$\langle 12.33,3\rangle$	$\langle 12.33,3\rangle$	$\langle 7.75,3\rangle$	$\langle 0,3\rangle$	Critical	1
8	H	$\langle 7.25,2\rangle$	$\langle 9.75,2\rangle$	$\langle 12.33,3\rangle$	$\langle 9.83,3\rangle$	$\langle 2.58,3\rangle$	Non-Critical	0.075
9	I	$\langle 3.5,3\rangle$	$\langle 8.08,3\rangle$	$\langle 14.63,3\rangle$	$\langle 10.05,3\rangle$	$\langle 6.55,3\rangle$	Non-Critical	0
10	J	$\langle 12.33,3\rangle$	$\langle 14.63,3\rangle$	$\langle 14.63,3\rangle$	$\langle 12.33,3\rangle$	$\langle 0,3\rangle$	Critical	1
11	K	$\langle 14.63,3\rangle$	$\langle 17.13,3\rangle$	$\langle 17.13,3\rangle$	$\langle 14.63,3\rangle$	$\langle 0,3\rangle$	Critical	1

TABLE V
PROJECT COMPLETION TIME FOR VARIOUS PARAMETERS (EXAMPLE 1)

$\alpha,\ \beta$ & γ	Project completion time (Interval form)					
	s=0		s=0.5		s=1	
	$\langle \mathcal{C}(\tilde{k}),\mathcal{W}(\tilde{k})\rangle$	$[\mathcal{C}(\tilde{k})-\mathcal{W}(\tilde{k}),\ \mathcal{C}(\tilde{k})+\mathcal{W}(\tilde{k})]$	$\langle \mathcal{C}(\tilde{k}),\mathcal{W}(\tilde{k})\rangle$	$[\mathcal{C}(\tilde{k})-\mathcal{W}(\tilde{k}),\ \mathcal{C}(\tilde{k})+\mathcal{W}(\tilde{k})]$	$\langle \mathcal{C}(\tilde{k}),\mathcal{W}(\tilde{k})\rangle$	$[\mathcal{C}(\tilde{k})-\mathcal{W}(\tilde{k}),\ \mathcal{C}(\tilde{k})+\mathcal{W}(\tilde{k})]$
0	$\langle 37.67,4\rangle$	$[33.64,41.64]$	$\langle 27.75,4\rangle$	$[23.75,31.75]$	$\langle 17.83,4\rangle$	$[13.83,21.83]$
0.1	$\langle 38.66,3.9\rangle$	$[34.76,42.56]$	$\langle 28.21,3.9\rangle$	$[24.31,32.11]$	$\langle 17.77,3.9\rangle$	$[13.87,21.67]$
0.2	$\langle 39.65,3.8\rangle$	$[35.85,43.45]$	$\langle 28.68,3.8\rangle$	$[24.88,32.48]$	$\langle 17.70,3.8\rangle$	$[13.90,21.50]$
0.3	$\langle 40.65,3.7\rangle$	$[36.95,44.35]$	$\langle 29.14,3.7\rangle$	$[25.44,32.84]$	$\langle 17.63,3.7\rangle$	$[13.93,21.33]$
0.4	$\langle 41.64,3.6\rangle$	$[38.04,45.24]$	$\langle 29.60,3.6\rangle$	$[26.00,33.2]$	$\langle 17.57,3.6\rangle$	$[13.97,21.17]$
0.5	$\langle 42.63,3.5\rangle$	$[39.13,46.13]$	$\langle 30.07,3.5\rangle$	$[26.57,33.57]$	$\langle 17.50,3.5\rangle$	$[14.00,21.00]$
0.6	$\langle 43.63,3.4\rangle$	$[40.23,47.03]$	$\langle 30.53,3.4\rangle$	$[27.13,33.93]$	$\langle 17.43,3.4\rangle$	$[14.03,20.83]$
0.7	$\langle 44.62,3.3\rangle$	$[41.32,47.92]$	$\langle 30.99,3.3\rangle$	$[27.69,34.29]$	$\langle 17.37,3.3\rangle$	$[14.07,20.67]$
0.8	$\langle 45.61,3.2\rangle$	$[42.41,48.81]$	$\langle 31.46,3.2\rangle$	$[28.26,34.66]$	$\langle 17.30,3.2\rangle$	$[14.10,20.50]$
0.9	$\langle 46.61,3.1\rangle$	$[43.51,49.71]$	$\langle 31.92,3.1\rangle$	$[28.82,35.02]$	$\langle 17.23,3.1\rangle$	$[14.13,20.33]$
1.0	$\langle 47.60,3\rangle$	$[44.6,50.6]$	$\langle 32.38,3\rangle$	$[29.38,38.38]$	$\langle 17.13,3\rangle$	$[14.13,20.13]$

- But the project completion days acquired by [17] is 15.9.
- It shows that our method is efficient than the existing and our result is more flexible that suits better in current real-life scenarios.

TABLE VI
POSSIBLE PATHS OF THE PROJECT

Path	Possible path	Project completion days	Rank
1	A→I→K	$\langle 10.58,3\rangle$	5
2	A→F→K	$\langle 9.7,3\rangle$	6
3	A→C→G→K	$\langle 14.83,3\rangle$	2
4	A→C→G→J→K	$\langle 17.13,3\rangle$	1
5	B→D→H→J→K	$\langle 14.55,2\rangle$	3
6	B→E→J→K	$\langle 12,2\rangle$	4

8) Sensitivity Analysis: In Table V, the trapezoidal neutrosophic fuzzy PERT problem is analyzed for various parameters of α,β and γ between [0,1]. Also, the problem is discussed for various values of s between [0,1]. Fig. 4 represents the lower limit values for s=0, s=0.5 and s=1. From the analysis, we observe that the project completion days are more efficient for s=1, i.e., the highest membership grade. In Table VI, the validity of our proposed method are checked based on the possible path of the project.

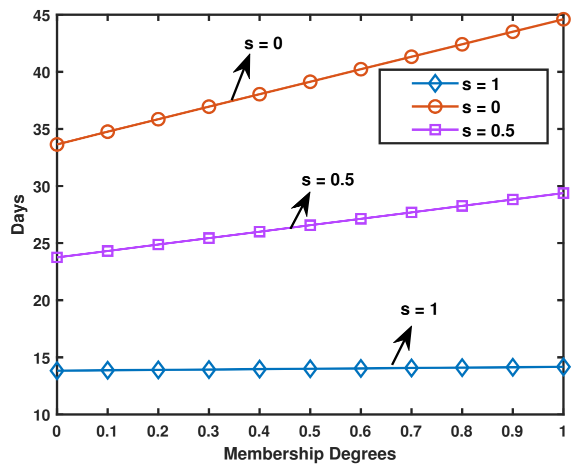

Fig. 4. Lower limit for various values of “s” (Example 1)

TABLE VII
DURATIONS IN THE FORM OF TRAPEZOIDAL NEUTROSOPHIC FUZZY NUMBERS AND INTERVAL NUMBERS (EXAMPLE 2)

Acti -vity	Durations (day)		Early start	Early finish	Late finish	Late start	Slack time	Descr -iption	Critical degree
	Trapezoidal neutrosophic fuzzy form	Interval form							
A	$(7,8,9,12;3.5,4,4.5,6;1.4,1.6,1.8,2.4)$	$\langle 8.5,0.75\rangle$	$\langle 0,0\rangle$	$\langle 8.5,0.75\rangle$	$\langle 8.5,1.5\rangle$	$\langle 0,1.5\rangle$	$\langle 0,1.5\rangle$	Critical	1
B	$(17,19,21,22;8.5,9.5,10.5,11;3.4,3.8,4.2,4.4)$	$\langle 20,1\rangle$	$\langle 0,0\rangle$	$\langle 20,1\rangle$	$\langle 29,1.5\rangle$	$\langle 9,1.5\rangle$	$\langle 9,1.5\rangle$	Non-critical	0
C	$(39,42,45,50;19.5,21,22.5,25;7.8,8.4,9,10)$	$\langle 43.5,1.5\rangle$	$\langle 8.5,0.75\rangle$	$\langle 52,1.5\rangle$	$\langle 52,1.5\rangle$	$\langle 8.5,1.5\rangle$	$\langle 0,1.5\rangle$	Critical	1
D	$(14,15,17,18;7,7.5,8.5,9;2.8,3,3.4,3.6)$	$\langle 16,1\rangle$	$\langle 8.5,0.75\rangle$	$\langle 24.5,1\rangle$	$\langle 94,1.5\rangle$	$\langle 78,1.5\rangle$	$\langle 69.5,1.5\rangle$	Non-critical	0
E	$(14,15,17,18;7,7.5,8.5,9;2.8,3,3.4,3.6)$	$\langle 16,1\rangle$	$\langle 20,1\rangle$	$\langle 36,1\rangle$	$\langle 45,1.5\rangle$	$\langle 29,1.5\rangle$	$\langle 9,1.5\rangle$	Non-critical	0
F	$(13,14,15,16;6.5,7,7.5,8;2.6,2.8,3,3.2)$	$\langle 14.5,0.75\rangle$	$\langle 36,1\rangle$	$\langle 50.5,1\rangle$	$\langle 59.5,1.5\rangle$	$\langle 45,1.5\rangle$	$\langle 9,1.5\rangle$	Non-critical	0
G	$(16,17,18,21;8,8.5,9,10.5;3.2,3.4,3.6,4.2)$	$\langle 17.5,0.75\rangle$	$\langle 50.5,1\rangle$	$\langle 68,1\rangle$	$\langle 77,1.5\rangle$	$\langle 59.5,1.5\rangle$	$\langle 9,1.5\rangle$	Non-critical	0
H	$(44,45,48,52;22,22.5,24,26;8.8,9,9.6,10.4)$	$\langle 46.5,1.5\rangle$	$\langle 52,1.5\rangle$	$\langle 98.5,1.5\rangle$	$\langle 98.5,1.5\rangle$	$\langle 52,1.5\rangle$	$\langle 0,1.5\rangle$	Critical	1
J	$(3,4,5,6;1.5,2,2.5,3;0.6,0.8,1,1.2)$	$\langle 4.5,0.5\rangle$	$\langle 24.5,1\rangle$	$\langle 29,1\rangle$	$\langle 98.5,1.5\rangle$	$\langle 94.5,1.5\rangle$	$\langle 69.5,1.5\rangle$	Non-critical	0
L	$(20,21,22,25;10,10.5,11,12.5;28.5,30;4,4.2,4.4,5)$	$\langle 21.5,0.75\rangle$	$\langle 68,1\rangle$	$\langle 89.5,1\rangle$	$\langle 98.5,1.5\rangle$	$\langle 77,1.5\rangle$	$\langle 9,1.5\rangle$	Non-critical	0
N	$(54,55,57,60;27,27.5,28.5,30;10.8,11,11.4,12)$	$\langle 56,1\rangle$	$\langle 98.5,1.5\rangle$	$\langle 154.5,1.5\rangle$	$\langle 154.5,1.5\rangle$	$\langle 98.5,1.5\rangle$	$\langle 0,1.5\rangle$	Critical	1

B. Example 2

1) Description of the Problem and the Data: In this example, we design a problem to compute TrpNFPERT method in Diary farming. The process of the activities are given in Table VIII. The flow of the activities with the network diagram is shown in Fig. 5. The durations of the activities are defined based on the fuzzy parameters adapted from [18]. We consider into TrpNFN based on decision maker's perception with indeterminacy as 50 % and falsity as 20 % of the fuzzy membership values. Considering the trapezoidal fuzzy numbers in terms of TrpNFN makes the decision maker to analyze each duration in all possible thoughts, that includes true, neutral and falsity respectively. The durations in TrpNFN and its interval based de-neutrosophication form (mid-point and width(half-width)) are provided in Table VII. Also, the calculations to TrpNFFPC (TrpNFES & TrpNFEF), TrpNFBPC (TrpNFLS & TrpNFLF), TrpNFST, TrpNFCD and the description (critical/non-critical) are given in Table VII.

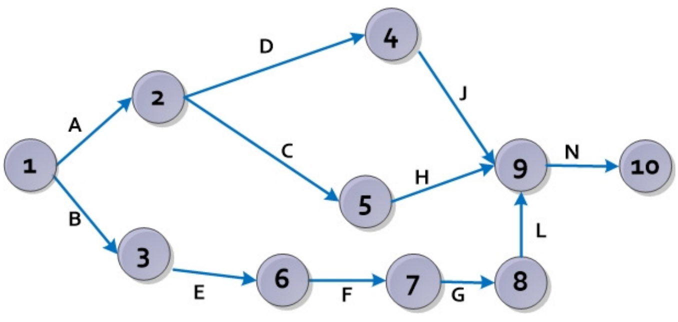

Fig. 5. Network diagram (Example 2)

TABLE VIII
DESCRIPTION OF THE ACTIVITY IN DIARY FARMING (EXAMPLE 2)

Activity	Description
A (1–2)	Aware about all species and breed
B (1–3)	Recognize each individual animal
C (2–5)	Layout / Outline a breeding plan
D (2–4)	Supply of necessary nutrition
E (3–6)	Learn the practice of farming
F (6–7)	Awareness of animal disease
G (7–8)	Amenities for animals
H (5–9)	Plot the local milk market
J (4–9)	Reach the government milk market
L (8–9)	Market with neighbours/ public
N (9–10)	Structure a business plan

2) Comparison of Results:

- Using our proposed interval-based de-neutrosophication methodology for trapezoidal neutrosophic fuzzy number, the critical path of the project in the TrpNFPERT is A $\to$ C $\to$ H $\to$ N and it is obtained as same as the path in the existing method [18].
- The project completion days we obtained as $\langle 8.5,0.75\rangle + \langle 43.5,1.5\rangle + \langle 46.5,1.5\rangle + \langle 56,1\rangle = \langle 154.5,1.5\rangle = [153,156]$ days.
- But the project completion days acquired by [18] is (144,150,159,174).
- It shows that our method is efficient than the existing and our result is more flexible that suits better in real-life scenarios.
- Interval numbers offer a convenient representation in situations where uncertainty or imprecision exists in the data or decision making process.

TABLE IX
PROJECT COMPLETION TIME FOR VARIOUS PARAMETERS (EXAMPLE 2)

α, β & γ	Project completion time (Interval form)					
	s=0		s=0.5		s=1	
	$\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k})]$	$\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k})]$	$\langle \mathcal{C}(\widetilde{k}), \mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k}) - \mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k}) + \mathcal{W}(\widetilde{k})]$
0	$\langle 108.10, 5.50\rangle$	$[102.60, 113.60]$	$\langle 133.55, 5.50\rangle$	$[128.05, 139.05]$	$\langle 159, 5.50\rangle$	$[153.5, 164.5]$
0.1	$\langle 108.87, 5.10\rangle$	$[103.77, 113.97]$	$\langle 133.71, 5.10\rangle$	$[128.61, 138.81]$	$\langle 158.55, 5.1\rangle$	$[153.45, 163.65]$
0.2	$\langle 109.64, 4.70\rangle$	$[104.94, 114.34]$	$\langle 133.87, 4.7\rangle$	$[129.17, 138.57]$	$\langle 158.10, 4.7\rangle$	$[153.4, 162.8]$
0.3	$\langle 110.41, 4.30\rangle$	$[106.11, 114.71]$	$\langle 134.03, 4.30\rangle$	$[129.73, 138.33]$	$\langle 157.65, 4.3\rangle$	$[153.35, 161.95]$
0.4	$\langle 111.18, 3.90\rangle$	$[107.28, 115.08]$	$\langle 134.19, 3.90\rangle$	$[130.29, 138.09]$	$\langle 157.20, 3.9\rangle$	$[153.30, 161.10]$
0.5	$\langle 111.95, 3.50\rangle$	$[108.45, 115.45]$	$\langle 134.35, 3.5\rangle$	$[130.85, 137.85]$	$\langle 156.75, 3.5\rangle$	$[153.25, 160.25]$
0.6	$\langle 112.72, 3.10\rangle$	$[109.62, 115.82]$	$\langle 134.51, 3.1\rangle$	$[131.41, 137.61]$	$\langle 156.30, 3.10\rangle$	$[153.20, 159.40]$
0.7	$\langle 113.49, 2.70\rangle$	$[110.79, 116.19]$	$\langle 134.67, 2.7\rangle$	$[131.97, 137.37]$	$\langle 155.85, 2.7\rangle$	$[153.15, 158.55]$
0.8	$\langle 114.26, 2.30\rangle$	$[111.96, 116.56]$	$\langle 134.83, 2.3\rangle$	$[132.33, 137.13]$	$\langle 155.4, 2.30\rangle$	$[153.10, 157.70]$
0.9	$\langle 115.03, 1.90\rangle$	$[113.13, 116.93]$	$\langle 134.99, 1.5\rangle$	$[133.09, 136.89]$	$\langle 154.95, 1.90\rangle$	$[153.05, 156.85]$
1.0	$\langle 115.8, 1.50\rangle$	$[114.30, 117.30]$	$\langle 135.15, 1.5\rangle$	$[133.65, 136.65]$	$\langle 154.50, 1.50\rangle$	$[153, 156]$

TABLE X
POSSIBLE PATHS OF THE PROJECT (EXAMPLE 2)

Path	Possible path	Project completion days	Rank
1	A→D→J→N	$\langle 85, 1\rangle$	3
2	A→C→H→N	$\langle 154.5, 1.5\rangle$	1
3	B→E→F→G→L→N	$\langle 145.5, 1$	2

3) Sensitivity Analysis: The trapezoidal neutrosophic fuzzy PERT problem is analyzed for various parameters of α, β and γ between [0,1]. It is analyzed for various values of s between [0,1] and mentioned in Table IX. Also, Fig. 6 and 7 represents the lower and upper limit values for s=0, s=0.5 and s=1 respectively. The efficiency of the project completion days can be perceived in two ways.
(i) all upper limit values of $s \in [0, 1]$.
(ii) all lower limit values of $s \in [0, 0.7]$.
In Table X, the validity of our proposed method are checked based on the alternative or possible path together with its project completion days and ranking order.

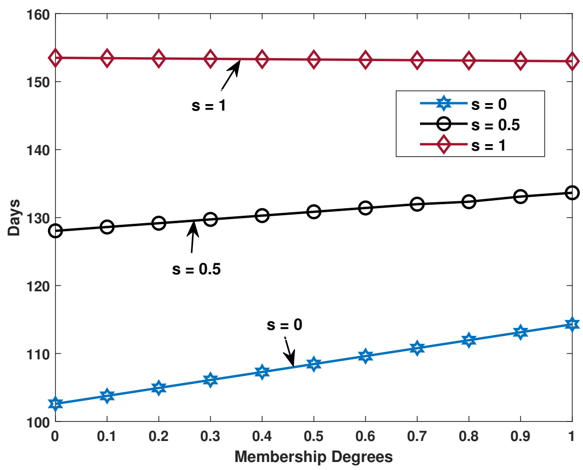

Fig. 6. Lower limit for various values of "s" (Example 2)

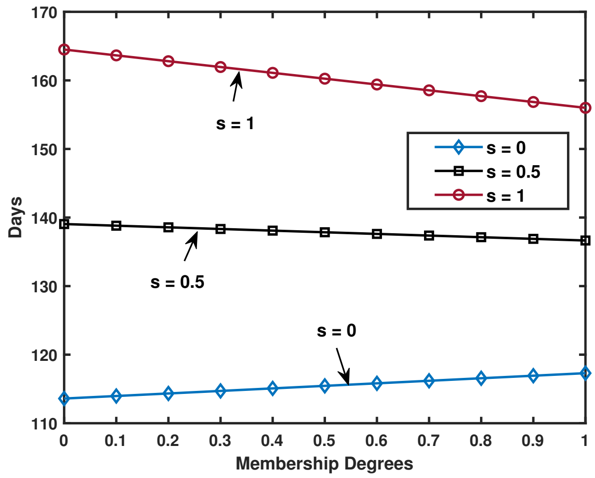

Fig. 7. Upper limit for various values of "s" (Example 2)

VII. ILLUSTRATION – INTERVAL-VALUEDTRAPEZOIDAL NEUTROSOPHIC FUZZY PERT

A. Example 3

1) Description of the Problem and the Data: A project management in Goat farming is considered in this example and the interval-valued trapezoidal neutrosophic fuzzy numbers (IVTrpNFN) are chosen as the parameters. The activities of the problem with the description and the corresponding predecessor are given in Table XIII. The flow of activities are graphically presented in Fig. 8. The problem is adapted from [15] and we consider it as IVTrpNFN based on the decision maker's perception. A production process is designed for Goat farming with the proposed interval - valued trapezoidal neutrosophic fuzzy pert method (IVTrpNFPERT). Considering IVTrpNFN makes one to analyze the problem in all the 3 known possibilites in certain limits.

2) Calculation for IVTrpNFED: The data in the form of interval-valued trapezoidal neutrosophic fuzzy number in

TABLE XI
DURATIONS IN THE FORM OF INTERVAL-VALUED TRAPEZOIDAL NEUTROSOPHIC FUZZY NUMBERS (EXAMPLE 3)

Nr	Durations (day)		
	Optimistic time $(IVT_{\widetilde{Neu}}OT_i)$	Most likely time $(IVT_{\widetilde{Neu}}MT_i)$	Pessimistic time $(IVT_{\widetilde{Neu}}PT_i)$
1	$(0, .5, .75, 2); [.7, 0.9]; [.5, 0.7]; [.3, 0.5]$	$(0, 1, 4, 5); [0.1, 0.3]; [0.4, 0.6]; [0.5, 0.7]$	$(2, 3, 5, 6); [0.7, 0.9]; [0.1, 0.3]; [0.3, 0.5]$
2	$(0, 1, 3, 4); [0.1, 0.3]; [0.2, 0.4]; [0.4, 0.6]$	$(1, 2, 6, 7); [0.4, 0.6]; [0.3, 0.5]; [0.8, 0.9]$	$(3, 5, 7, 8); [0.6; 0.8]; [0.1, 0.3]; [0.4, 0.6]$
3	$(5, 6, 8, 9); [0.3, 0.5]; [0.5, 0.7]; [0.7, 0.9]$	$(1, 3, 10, 11); [0.5, 0.7]; [0.3, 0.5]; [0.6, 0.8]$	$(1, 5, 10, 16); [0.4, 0.6]; [0.1, 0.3]; [0.3, 0.5]$
4	$(1, 3, 9, 10); [0.8, 0.9]; [0, 0.2]; [0.2, 0.4]$	$(4, 6, 9, 11); [0.6, 0.8]; [0.5, 0.7]; [0.2, 0.4]$	$(1, 6, 10, 17); [0.6, 0.8]; [0.4, 0.6]; [0.5, 0.7]$
5	$(3, 5, 7, 8); [0.6; 0.8]; [0.1, 0.3]; [0.4, 0.6]$	$(1, 3, 10, 11); [0.5, 0.7]; [0.3, 0.5]; [0.6, 0.8]$	$(4, 6, 8, 15); [0.7, 0.9]; [0.3, 0.5]; [0.6, 0.8]$
6	$(2, 4, 7, 12); [.7, 0.9]; [0.1, 0.3]; [0.4, 0.6]$	$(1, 5, 10, 16); [0.4, 0.6]; [0.1, 0.3]; [0.3, 0.5]$	$(1, 6, 18, 22); [0.1, 0.3]; [0.3, 0.5]; [0.5, 0.7]$
7	$(3, 5, 7, 8); [0.6; 0.8]; [0.1, 0.3]; [0.4, 0.6]$	$(2, 4, 7, 12); [0.7, 0.9]; [0.1, 0.3]; [0.4, 0.6]$	$(1, 6, 13, 20); [0.8, 0.9]; [0.6, 0.8]; [0.7, 0.9]$
8	$(1, 3, 9, 10); [0.8, 0.9]; [0, 0.2]; [0.2, 0.4]$	$(3, 5, 8, 13); [0.2, 0.4]; [0.5, 0.7]; [0.3, 0.5]$	$(8, 10, 18, 21); [.1, 0.3]; [0.2, 0.4]; [0.4, 0.6]$

TABLE XII
DURATIONS IN THE FORM OF INTERVAL NUMBER (EXAMPLE 3)

Nr	Activity	Predecessor	Durations (day)			
			Optimistic time	Most likely time	Pessimistic time	Expected days
1	A	-	$\langle 0.46, 0.03\rangle$	$\langle 2.50, 0.83\rangle$	$\langle 4, 0.89\rangle$	$\langle 2.41, 0.89\rangle$
2	B	-	$\langle 2, 0.75\rangle$	$\langle 4, 1.57\rangle$	$\langle 6.33, 0.83\rangle$	$\langle 4.06, 1.57\rangle$
3	C	A	$\langle 7, 0\rangle$	$\langle 7, 2.86\rangle$	$\langle 6, 1.94\rangle$	$\langle 6.83, 2.86\rangle$
4	D	A	$\langle 6.13, 3\rangle$	$\langle 7.50, 1\rangle$	$\langle 7.33, 0.50\rangle$	$\langle 7.24, 3\rangle$
5	E	B	$\langle 6.33, 0.83\rangle$	$\langle 7, 2.86\rangle$	$\langle 5.93, 0.50\rangle$	$\langle 6.71, 2.86\rangle$
6	F	C, D	$\langle 4.86, 1.11\rangle$	$\langle 6, 1.94\rangle$	$\langle 16.50, 4.07\rangle$	$\langle 7.56, 4.07\rangle$
7	G	D, E	$\langle 6.33, 0.83\rangle$	$\langle 4.86, 1.11\rangle$	$\langle 9.25, 2.83\rangle$	$\langle 5.84, 2.83\rangle$
8	H	F, G	$\langle 6.13, 3\rangle$	$\langle 0.50, 0\rangle$	$\langle 9.50, 3.38\rangle$	$\langle 2.94, 3.38\rangle$

TABLE XIII
DESCRIPTION OF THE ACTIVITY IN GOAT FARMING (EXAMPLE 3)

Nr	Activity	Description	Predecessor
1	A	Decide the type of Goat farm	-
2	B	Select a breed	-
3	C	Advantages of Goat farming	A
4	D	Purchase Goats	A
5	E	Choosing proper land	B
6	F	Gather experienced farmers opinion	C, D
7	G	Proper housing, equipment and food	D, E
8	H	Marketing the farm	F, G

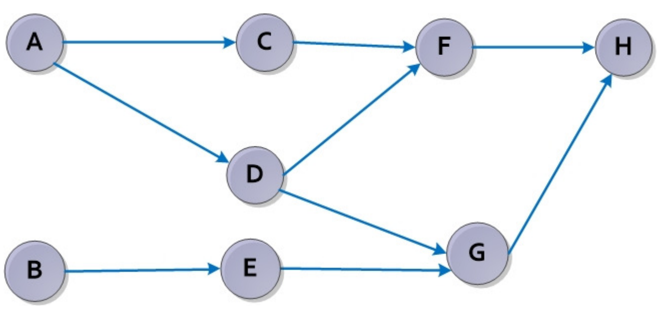

Fig. 8. Network diagram (Example 3)

three forms namely, optimistic, most-likely and pessimistic are given in Table XI and its interval version is mentioned in Table XII. The IVTrpNFED are calculated using section V-A. For example, let us calculate for the activity i=6.

$T_{\widetilde{IvNeu}}OT_6 = (2, 4, 7, 12); [0.7, 0.9]; [0.1, 0.3]; [0.4, 0.6]$
$T_{\widetilde{IvNeu}}PT_6 = (1, 6, 18, 22); [0.1, 0.3]; [0.3, 0.5]; [0.5, 0.7]$
$T_{\widetilde{IvNeu}}MT_6 = (1, 5, 10, 16); [0.4, 0.6]; [0.1, 0.3]; [0.3, 0.5]$

Then, using the interval-based de-neutrosophication technique provided by [28],

$$\mathcal{R}(T_{\widetilde{IvNeu}}OT_6) = \langle 4.86, 1.11\rangle,$$
$$\mathcal{R}(T_{\widetilde{IvNeu}}PT_6) = \langle 16.50, 4.07\rangle$$
$$\text{and } \mathcal{R}(T_{\widetilde{IvNeu}}MT_6) = \langle 6, 1.94\rangle$$

Then, using equation (29), the expected duration of interval-valued trapezoidal neutrosophic fuzzy number is $\langle 7.56, 4.07\rangle$.
The IVTrpNFED for all activities in interval form are detailed in Table XII.

3) Calculation for IVTrpNFFPC: The early start of interval-valued trapezoidal neutrosophic fuzzy number can be calculated using the equation (32) and for i=6,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}ES_6) &= \max_{3,4}\left[\mathcal{R}(T_{\widetilde{IvNeu}}ES_j) + \mathcal{R}(T_{\widetilde{IvNeu}}A_j)\right]\\ &= \max\left[\mathcal{R}(T_{\widetilde{IvNeu}}ES_3) + \mathcal{R}(T_{\widetilde{IvNeu}}A_3),\right.\\ &\quad \left.\mathcal{R}(T_{\widetilde{IvNeu}}ES_4) + \mathcal{R}(T_{\widetilde{IvNeu}}A_4)\right]\\ &= \max\left[\langle 9.24, 2.86\rangle, \langle 9.65, 3\rangle\right]\\ &= \langle 9.65, 3\rangle.\end{aligned}$$

Also, the early finish of interval-valued trapezoidal neutrosophic fuzzy number can be calculated using the equation

TABLE XIV
DURATIONS CALCULATIONS BASED ON FORWARD AND BACKWARD PASS (EXAMPLE 3)

Nr	Activity	Early start	Early finish	Late finish	Late start	Slack time	Description	Critical degree
1	A	$\langle 0,0\rangle$	$\langle 2.41,0.89\rangle$	$\langle 2.41,4.07\rangle$	$\langle 0,4.07\rangle$	$\langle 0,4.07\rangle$	Critical	1
2	B	$\langle 0,0\rangle$	$\langle 4.06,1.57\rangle$	$\langle 4.06,3.38\rangle$	$\langle 0.61,4.07\rangle$	$\langle 0.61,4.07\rangle$	Non-Critical	0.74
3	C	$\langle 2.41,0.89\rangle$	$\langle 9.24,2.86\rangle$	$\langle 9.65,4.07\rangle$	$\langle 2.82,4.07\rangle$	$\langle 0.41,4.07\rangle$	Non-Critical	0.82
4	D	$\langle 2.41,0.89\rangle$	$\langle 9.65,3\rangle$	$\langle 9.65,4.07\rangle$	$\langle 2.41,4.07\rangle$	$\langle 0,4.07\rangle$	Critical	1
5	E	$\langle 4.06,1.57\rangle$	$\langle 10.77,2.86\rangle$	$\langle 11.38,4.07\rangle$	$\langle 4.67,4.07\rangle$	$\langle 0.61,4.07\rangle$	Non-Critical	0.74
6	F	$\langle 9.65,3\rangle$	$\langle 17.21,4.07\rangle$	$\langle 17.21,4.07\rangle$	$\langle 9.65,4.07\rangle$	$\langle 0,4.07\rangle$	Critical	1
7	G	$\langle 10.77,3\rangle$	$\langle 16.60,3\rangle$	$\langle 17.21,4.07\rangle$	$\langle 11.38,4.07\rangle$	$\langle 0.61,4.07\rangle$	Non-Critical	0.74
8	H	$\langle 17.21,4.07\rangle$	$\langle 20.15,4.07\rangle$	$\langle 20.15,4.07\rangle$	$\langle 17.21,4.07\rangle$	$\langle 0,4.07\rangle$	Critical	1

(33) and for i=6,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}EF_6) =&\mathcal{R}(T_{\widetilde{IvNeu}}ES_6) + \mathcal{R}(T_{\widetilde{IvNeu}}A_6)\\ =&\langle 9.65,3\rangle + \langle 7.56,4.07\rangle\\ =&\langle 17.21,4.07\rangle.\end{aligned}$$

The detailed values of IVTrpNFES and IVTrpNFEF are mentioned in Table XIV.

4) Calculation for IVTrpNFBPC: The latest finish of interval-valued trapezoidal neutrosophic fuzzy number can be calculated using the equation (36) and for i=4,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}LF_4) =& \min_{6,7}\left[\mathcal{R}(T_{\widetilde{IvNeu}}LF_j) - \mathcal{R}(T_{\widetilde{IvNeu}}A_j)\right]\\ =& \min\left[\mathcal{R}(T_{\widetilde{IvNeu}}LF_6) - \mathcal{R}(T_{\widetilde{IvNeu}}A_6),\right.\\ &\left.\mathcal{R}(T_{\widetilde{IvNeu}}LF_7) - \mathcal{R}(T_{\widetilde{IvNeu}}A_7)\right]\\ =& \min\left[\langle 9.65,4.07\rangle, \langle 11.38,4.07\rangle\right]\\ =&\langle 9.65,4.07\rangle.\end{aligned}$$

Also, the latest start of interval-valued trapezoidal neutrosophic fuzzy number can be calculated using the equation (37) and for i=4,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}LS_i) =&\mathcal{R}(T_{\widetilde{IvNeu}}LF_4) - \mathcal{R}(T_{\widetilde{IvNeu}}A_4)\\ =&\langle 9.65,4.07\rangle - \langle 7.24,3\rangle\\ =&\langle 2.41,4.07\rangle.\end{aligned}$$

The detailed values of IVTrpNFLF and IVTrpNFLS are mentioned in Table XIV.

5) Calculation for IVTrpNFST: Using equation (39), let us calculate for i=4.

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}ST_4) =&\mathcal{R}(T_{\widetilde{IvNeu}}LS_4) - \mathcal{R}(T_{\widetilde{IvNeu}}ES_4)\\ =&\langle 2.41,4.07\rangle - \langle 2.41,0.89\rangle\\ =&\langle 0,4.07\rangle\\ \text{Also, } \mathcal{R}(T_{\widetilde{IvNeu}}ST_4) =&\mathcal{R}(T_{\widetilde{IvNeu}}LF_4) - \mathcal{R}(T_{\widetilde{IvNeu}}EF_4)\\ =&\langle 9.65,4.07\rangle - \langle 9.65,3.00\rangle\\ =&\langle 0,4.07\rangle, \text{a critical activity}\end{aligned}$$

Also, for i=5,

$$\begin{aligned}\mathcal{R}(T_{\widetilde{IvNeu}}ST_5) =&\mathcal{R}(T_{\widetilde{IvNeu}}LS_5) - \mathcal{R}(T_{\widetilde{IvNeu}}ES_5)\\ =&\langle 4.67,4.07\rangle - \langle 4.06,1.57\rangle\\ =&\langle 0.61,4.07\rangle\\ \text{Also, } \mathcal{R}(T_{\widetilde{IvNeu}}ST_5) =&\mathcal{R}(T_{\widetilde{IvNeu}}LF_5) - \mathcal{R}(T_{\widetilde{IvNeu}}EF_5)\\ =&\langle 11.38,4.07\rangle - \langle 10.77,2.86\rangle\\ =&\langle 0.61,4.07\rangle, \text{a non-critical activity}\end{aligned}$$

The detailed values of IVTrpNFST are mentioned in Table XIV.

6) Calculation for IVTrpNFCD: The critical degree of interval-valued trapezoidal neutrosophic fuzzy number is calculated based on section V-E. For example, we calculate for i=2 and the IVTrpNFST is $\langle 0.61,4.07\rangle$.

$$\mathcal{M}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) < 0 < \mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) \implies -3.46 < 0 < 4.68 \quad .$$

$$\begin{aligned}\text{Hence } \mathcal{R}(T_{\widetilde{IvNeu}}CD_2) =& \frac{\mathcal{W}(\widetilde{k}) - \mathcal{M}(\widetilde{k})}{\mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k})}\\ =& \frac{4.07-0.61}{4.07+0.61} = 0.74,\end{aligned}$$

is the non - critical activity.

For i=4, the IVTrpNFST is $\langle 0,4.07\rangle$.

Therefore, based on section V-E,

$$\mathcal{M}(\widetilde{k}) - \mathcal{W}(\widetilde{k}) < 0 < \mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k}) \implies -4.07 < 0 < 4.07.$$

$$\begin{aligned}\text{Hence } \mathcal{R}(T_{\widetilde{IvNeu}}CD_4) =& \frac{\mathcal{W}(\widetilde{k}) - \mathcal{M}(\widetilde{k})}{\mathcal{M}(\widetilde{k}) + \mathcal{W}(\widetilde{k})}\\ =& \frac{4.07-0}{0+4.07}\\ =& 1, \text{is the critical activity.}\end{aligned}$$

The detailed values of IVTrpNFCD are mentioned in Table XIV. It is clearly visible that the IVTrpNFCD is 1 for all critical activities and the rest are 0.

7) Comparison of Results:

- Using interval-based de-neutrosophication methodology [28] for interval-valued trapezoidal neutrosophic fuzzy number, the critical path of the project in the IVTrpNF-PERT is A $\rightarrow$ D $\rightarrow$ F $\rightarrow$ H and it is obtained as same as the path in the existing method [15].
- The project completion days we obtained as $\langle 2.41,0.89\rangle$ + $\langle 7.24,3\rangle$ + $\langle 7.56,4.07\rangle$ + $\langle 2.94,3.38\rangle$ = $\langle 20.15,4.07\rangle = [16.08, 24.22]$ days.
- But the project completion days acquired by [15] is 18.
- It shows that our method is efficient than the existing. Representing the days in interval numbers is more flexible and convenient in current scenarios.

8) Sensitivity Analysis: The interval-valued trapezoidal neutrosophic fuzzy PERT problem is examined for various parameters of α, β and γ between [0,1]. The problem is studied for various values of s between [0,1], which is mentioned in Table XV. Also, Fig. 9 represents the lower limit values for s=0, s=0.5 and s=1. From the analysis, we

TABLE XV
PROJECT COMPLETION TIME FOR VARIOUS PARAMETERS (EXAMPLE 3)

α, β & γ	Project completion time (Interval form)					
	s=0		s=0.5		s=1	
	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$
0	$\langle 54.67, 10.50\rangle$	$[44.17, 65.17]$	$\langle 41, 10.50\rangle$	$[30.5, 51.50]$	$\langle 27.33, 10.50\rangle$	$[16.83, 37.83]$
0.1	$\langle 54.04, 9.86\rangle$	$[44.18, 63.9]$	$\langle 40.33, 9.86\rangle$	$[30.47, 50.19]$	$\langle 26.62, 9.86\rangle$	$[16.76, 36.48]$
0.2	$\langle 53.42, 9.21\rangle$	$[44.21, 62.63]$	$\langle 39.66, 9.21\rangle$	$[30.45, 48.87]$	$\langle 25.90, 9.21\rangle$	$[16.69, 35.11]$
0.3	$\langle 52.80, 8.57\rangle$	$[44.23, 61.37]$	$\langle 38.99, 8.57\rangle$	$[30.42, 47.56]$	$\langle 25.18, 8.57\rangle$	$[16.61, 33.75]$
0.4	$\langle 52.18, 7.93\rangle$	$[44.25, 60.11]$	$\langle 38.32, 7.93\rangle$	$[30.39, 46.25]$	$\langle 24.46, 7.93\rangle$	$[16.53, 32.39]$
0.5	$\langle 51.55, 7.29\rangle$	$[44.26, 58.84]$	$\langle 37.65, 7.29\rangle$	$[30.36, 44.94]$	$\langle 23.74, 7.29\rangle$	$[16.45, 31.03]$
0.6	$\langle 50.93, 6.64\rangle$	$[44.29, 57.57]$	$\langle 36.98, 6.64\rangle$	$[30.34, 43.62]$	$\langle 23.02, 6.64\rangle$	$[16.38, 29.66]$
0.7	$\langle 50.31, 6\rangle$	$[44.31, 56.31]$	$\langle 36.31, 6\rangle$	$[30.31, 42.31]$	$\langle 22.31, 6\rangle$	$[16.31, 28.31]$
0.8	$\langle 49.69, 5.36\rangle$	$[44.33, 55.05]$	$\langle 35.64, 5.36\rangle$	$[30.28, 41]$	$\langle 21.59, 5.36\rangle$	$[16.23, 26.95]$
0.9	$\langle 49.07, 4.71\rangle$	$[44.36, 53.78]$	$\langle 34.97, 4.71\rangle$	$[30.26, 39.68]$	$\langle 20.87, 4.71\rangle$	$[16.16, 25.58]$
1.0	$\langle 48.44, 4.07\rangle$	$[44.37, 52.51]$	$\langle 34.30, 4.07\rangle$	$[30.23, 38.37]$	$\langle 20.15, 4.07\rangle$	$[16.08, 24.22]$

TABLE XVI
POSSIBLE PATHS OF THE PROJECT (EXAMPLE 3)

Path	Possible path	Project completion days	Rank
1	A→C→F→H	$\langle 19.74, 4.07\rangle$	2
2	A→D→F→H	$\langle 20.15, 4.07\rangle$	1
3	A→D→G→H	$\langle 18.43, 3.38\rangle$	4
4	B→E→D→H	$\langle 19.55, 3.38\rangle$	3

observed that the project completion days are more efficient for s=1, i.e., the highest membership grade. In Table XVI, the validity of interval-based de-neutrosophication technique in example VII-A is checked based on the possible paths of the project together with its project completion days and ranking order.

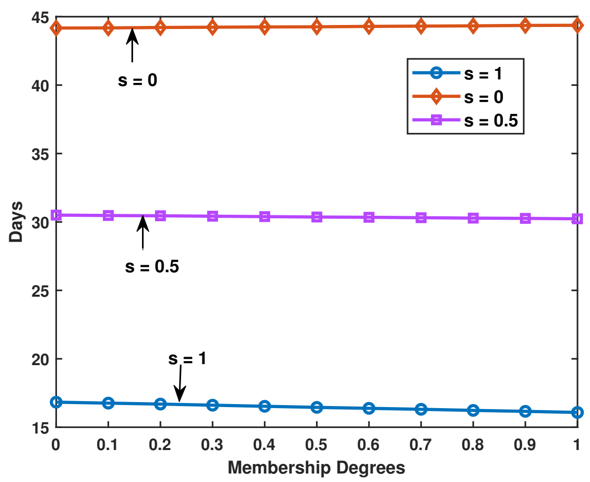

Fig. 9. Lower limit for various values of "s" (Example 3)

B. Example 4

1) Description of the Problem and the Data: In this example, we design a project to compute IVTrpNFPERT method to start restaurant business. The process of the activities are given in Table XVII. The flow of the activities with network diagram is shown in Fig. 10. The duration of the activities is defined based on the decision maker's perception which is adapted from [29]. The durations in IVTrpNFN and its interval based de-neutrosophication form (mid-point and width (half-width)) are provided in Table XVIII and XIX. Also, the calculations to IVTrpNFFPC (IVTrpNFES & IVTrpNFEF), IVTrpNFBPC (IVTrpNFLS & IVTrpNFLF), IVTrpNFST, IVTrpNFCD(CD) and the critical/non-critical (C/N-C) are given in Table XIX.

TABLE XVII
DESCRIPTION OF THE ACTIVITY IN DIARY FARMING (EXAMPLE 4)

Activity	Description
1	Beginning of the project / Select the restaurant's model
2	Choose the perfect site
3	Evaluate the budget
4	Arrange funds and finance
5	Get the license
6	Make contact with suppliers (Kitchenware and ingredients)
7	Get the assistance of Chef's and advisor's about the meal
8	Employ talented workers
9	In connect with food delivery partner
10	Market your restaurant
11	Completion of project

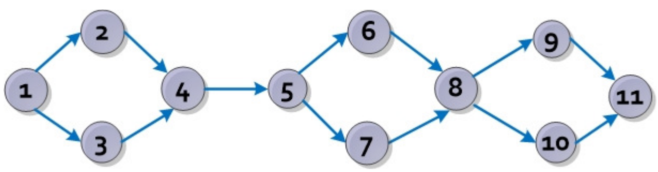

Fig. 10. Network diagram (Example 4)

TABLE XVIII
DURATIONS IN THE FORM OF INTERVAL-VALUED TRAPEZOIDAL NEUTROSOPHIC FUZZY NUMBERS (EXAMPLE 4)

Nr	Durations (day)		
	Optimistic time ($IVT_{\widetilde{Neu}}OT_i$)	Most likely time ($IVT_{\widetilde{Neu}}MT_i$)	Pessimistic time ($IVT_{\widetilde{Neu}}PT_i$)
1	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$
2	$(0.,0.5,0.75,2)$; $[0.7,0.9]$; $[0.5,0.7]$; $[0.3,0.5]$	$(0,1,3,4)$; $[0.1,0.3]$; $[0.2,0.4]$; $[0.4,0.6]$	$(0,1,4,5)$; $[0.1,0.3]$; $[0.4,0.6]$; $[0.5,0.7]$
3	$(0,1,4,5)$; $[0.1,0.3]$, $[0.4,0.6]$; $[0.5,0.7]$	$(1,2,6,9)$; $[0.4,0.6]$; $[0.3,0.5]$; $[0.8,0.9]$	$(1,3,6,12)$; $[0.7,0.9]$; $[0.1,0.3]$; $[0.3,0.5]$
4	$(0,1,4,5)$; $[0.1,0.3]$; $[0.4,0.6]$; $[0.5,0.7]$	$(1,3,6,12)$; $[0.7,0.9]$; $[0.1,0.3]$; $[0.3,0.5]$	$(4,5,7,15)$; $[0.6,0.8]$; $[0.1,0.3]$; $[0.4,0.6]$
5	$(1,3,9,15)$; $[0.8,0.9]$; $[0,0.2]$; $[0.2,0.4]$	$(3,5,10,18)$; $[0.5,0.7]$; $[0.3,0.5]$; $[0.6,0.8]$	$(3,5,8,13)$; $[0.2,0.4]$; $[0.5,0.7]$; $[0.3,0.5]$
6	$(1,2,6,9)$; $[0.4,0.6]$; $[0.3,0.5]$; $[0.8,0.9]$	$(1,3,8,11)$; $[0.3,0.5]$, $[0.5,0.7]$; $[0.7,0.9]$	$(3,5,10,18)$; $[0.5,0.7]$; $[0.3,0.5]$; $[0.6,0.8]$
7	$(0,1,3,4)$; $[0.1,0.3]$; $[0.2,0.4]$; $[0.4,0.6]$	$(1,2,6,9)$; $[0.4,0.6]$; $[0.3,0.5]$; $[0.8,0.9]$	$(1,3,6,12)$; $[0.7,0.9]$; $[0.1,0.3]$; $[0.3,0.5]$
8	$(0,1,4,5)$; $[0.1,0.3]$; $[0.4,0.6]$; $[0.5,0.7]$	$(1,2,6,9)$; $[0.4,0.6]$; $[0.3,0.5]$; $[0.8,0.9]$	$(4,5,7,15)$; $[0.6,0.8]$; $[0.1,0.3]$; $[0.4,0.6]$
9	$(0.,0.5,0.75,2)$; $[0.7,0.9]$; $[0.5,0.7]$; $[0.3,0.5]$	$(0,1,3,4)$; $[0.1,0.3]$; $[0.2,0.4]$; $[0.4,0.6]$	$(0,1,4,5)$; $[0.1,0.3]$; $[0.4,0.6]$; $[0.5,0.7]$
10	$(0,1,3,4)$; $[0.1,0.3]$; $[0.2,0.4]$; $[0.4,0.6]$	$(0,1,4,5)$; $[0.1,0.3]$; $[0.4,0.6]$; $[0.5,0.7]$	$(1,3,6,12)$; $[0.7,0.9]$; $[0.1,0.3]$; $[0.3,0.5]$
11	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$	$(0,0,0,0)$; $[1,1]$; $[0,0]$; $[0,0]$

TABLE XIX
DURATIONS IN THE FORM OF INTERVAL NUMBER AND CALCULATIONS BASED ON FORWARD AND BACKWARD PASS (EXAMPLE 4)

Nr	Durations (day)				Early	Early	Late	Late	Slack	C	CD
	Optimistic time	Most likely time	Pessimistic time	Expected days	start	finish	finish	start	time	/NC	
1	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,3\rangle$	$\langle 0,3\rangle$	$\langle 0,3\rangle$	C	1
2	$\langle 0.46,0.03\rangle$	$\langle 2,0.75\rangle$	$\langle 2.5,0.83\rangle$	$\langle 1.83,0.83\rangle$	$\langle 0,0\rangle$	$\langle 1.83,0.83\rangle$	$\langle 2.69,3\rangle$	$\langle 0.86,3\rangle$	$\langle 0.86,3\rangle$	N-C	0.55
3	$\langle 2.50,0.83\rangle$	$\langle 2.50,1.14\rangle$	$\langle 3.64,1.06\rangle$	$\langle 2.69,1.14\rangle$	$\langle 0,0\rangle$	$\langle 2.69,1.14\rangle$	$\langle 2.69,3\rangle$	$\langle 0,3\rangle$	$\langle 0,3\rangle$	C	1
4	$\langle 2.50,0.83\rangle$	$\langle 3.64,1.06\rangle$	$\langle 3.67,0.50\rangle$	$\langle 3.46,1.06\rangle$	$\langle 2.69,1.14\rangle$	$\langle 6.15,1.14\rangle$	$\langle 6.15,3\rangle$	$\langle 2.69,3\rangle$	$\langle 0,3\rangle$	C	1
5	$\langle 5.5,3\rangle$	$\langle 4.50,0.36\rangle$	$\langle 0.50,0\rangle$	$\langle 4,3\rangle$	$\langle 6.15,1.14\rangle$	$\langle 10.15,3\rangle$	$\langle 10.15,3\rangle$	$\langle 6.15,3\rangle$	$\langle 0,3\rangle$	C	1
6	$\langle 3.64,1.06\rangle$	$\langle 4.33,0\rangle$	$\langle 4.50,0.36\rangle$	$\langle 4.25,1.06\rangle$	$\langle 10.15,3\rangle$	$\langle 14.39,3\rangle$	$\langle 14.39,3\rangle$	$\langle 10.15,3\rangle$	$\langle 0,3\rangle$	C	1
7	$\langle 2,0.75\rangle$	$\langle 2.50,1.14\rangle$	$\langle 3.64,1.06\rangle$	$\langle 2.61,1.14\rangle$	$\langle 10.15,3\rangle$	$\langle 12.75,3\rangle$	$\langle 14.39,3\rangle$	$\langle 11.79,3\rangle$	$\langle 1.64,3\rangle$	N-C	0.29
8	$\langle 2.50,0.83\rangle$	$\langle 2.50,1.14\rangle$	$\langle 3.64,1.06\rangle$	$\langle 2.69,1.14\rangle$	$\langle 14.39,3\rangle$	$\langle 17.08,3\rangle$	$\langle 17.08,3\rangle$	$\langle 14.39,3\rangle$	$\langle 0,3\rangle$	C	1
9	$\langle 0.46,0.03\rangle$	$\langle 2,0.75\rangle$	$\langle 2.50,0.83\rangle$	$\langle 1.83,0.83\rangle$	$\langle 17.08,3\rangle$	$\langle 18.91,3\rangle$	$\langle 19.69,3\rangle$	$\langle 17.86,3\rangle$	$\langle 0.78,3\rangle$	N-C	0.58
10	$\langle 2,0.75\rangle$	$\langle 2.50,0.83\rangle$	$\langle 3.64,1.06\rangle$	$\langle 2.61,1.06\rangle$	$\langle 17.08,3\rangle$	$\langle 19.69,3\rangle$	$\langle 19.69,3\rangle$	$\langle 17.08,3\rangle$	$\langle 0,3\rangle$	C	1
11	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 0,0\rangle$	$\langle 19.69,3\rangle$	$\langle 19.69,3\rangle$	$\langle 19.69,3\rangle$	$\langle 19.69,3\rangle$	$\langle 0,3\rangle$	C	1

TABLE XX
POSSIBLE PATHS OF THE PROJECT (EXAMPLE 4)

Path	Possible path	Project completion days	Rank
1	1→2→4→5→6→8→9→11	$\langle 18.06,3\rangle$	4
2	1→2→4→5→6→8→10→11	$\langle 18.84,3\rangle$	3
3	1→2→4→5→7→8→9→11	$\langle 16.42,3\rangle$	8
4	1→2→4→5→7→8→10→11	$\langle 17.2,3\rangle$	7
5	1→3→4→5→6→8→9→11	$\langle 18.92,3\rangle$	2
6	1→3→4→5→6→8→10→11	$\langle 19.69,3\rangle$	1
7	1→3→4→5→7→8→9→11	$\langle 17.28,3\rangle$	6
8	1→3→4→5→7→8→10→11	$\langle 18.06,3\rangle$	4

2) Comparison of Results:

- Using the interval-based de-neutrosophication methodology [28] for interval-valued trapezoidal neutrosophic fuzzy number, the critical path of the project in the IVTrpNFPERT is 1→3→4→5→6→8→10→11 and it is obtained as same as the path in the existing method [29].
- The project completion days we obtained as $\langle 2.69,1.14\rangle$ + $\langle 3.46,1.06\rangle$ + $\langle 4,3\rangle$ + $\langle 4.25,1.06\rangle$ + $\langle 2.69,1.14\rangle$ + $\langle 2.61,1.06\rangle = \langle 19.69,3\rangle = [16.69,22.69]$ days.
- But the project completion days acquired by [29] is 19.86.
- It shows that our method is efficient than the existing and our result is more flexible that suits better in current real-life scenarios.
- Interval numbers is a convenient representation in situations where uncertainty or imprecision exists in the data or decision making process.

3) Sensitivity Analysis: The trapezoidal neutrosophic fuzzy PERT problem is analyzed for various parameters of α, β and γ between [0,1]. It is analyzed for various values of s between [0,1] and mentioned in Table XXI. Also, Fig. 11 represents the lower limit values for s=0, s=0.5 and s=1 respectively. In Table XX, the validity of section VII-B (Example 4) are checked based on the possible path together with its project completion days and ranking order.

TABLE XXI
PROJECT COMPLETION TIME FOR VARIOUS PARAMETERS (EXAMPLE 4)

α, β & γ	Project completion time (Interval form)					
	s=0		s=0.5		s=1	
	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$	$\langle\mathcal{C}(\widetilde{k}),\mathcal{W}(\widetilde{k})\rangle$	$[\mathcal{C}(\widetilde{k})-\mathcal{W}(\widetilde{k}), \mathcal{C}(\widetilde{k})+\mathcal{W}(\widetilde{k})]$
0	$\langle 71.17, 7.50\rangle$	$[63.67, 78.67]$	$\langle 53.38, 7.50\rangle$	$[45.88, 60.88]$	$\langle 35.58, 7.50\rangle$	$[16.83, 37.83]$
0.1	$\langle 66, 6.79\rangle$	$[59.21, 72.79]$	$\langle 50.65, 6.79\rangle$	$[43.86, 57.44]$	$\langle 33.99, 6.79\rangle$	$[16.76, 36.48]$
0.2	$\langle 60.83, 6.20\rangle$	$[54.63, 67.03]$	$\langle 47.93, 6.2\rangle$	$[41.73, 54.13]$	$\langle 32.40, 6.2\rangle$	$[16.69, 35.11]$
0.3	$\langle 55.66, 5.8\rangle$	[49.86,61.46]	$\langle 45.21, 5.8\rangle$	[39.41,51.01]	$\langle 30.82, 5.8\rangle$	$[16.61, 33.75]$
0.4	$\langle 50.49, 5.40\rangle$	$[45.09, 55.89]$	$\langle 42.49, 5.4\rangle$	$[37.09, 47.89]$	$\langle 29.23, 5.4\rangle$	$[16.53, 32.39]$
0.5	$\langle 45.32, 5\rangle$	$[40.32, 50.32]$	$\langle 39.76, 5\rangle$	$[34.76, 44.76]$	$\langle 27.64, 5\rangle$	$[16.45, 31.03]$
0.6	$\langle 40.16, 4.60\rangle$	$[35.56, 44.76]$	$\langle 37.04, 4.60\rangle$	$[32.44, 41.64]$	$\langle 26.05, 4.6\rangle$	$[16.38, 29.66]$
0.7	$\langle 35.32, 4.20\rangle$	$[31.12, 39.52]$	$\langle 34.32, 4.20\rangle$	$[30.12, 38.52]$	$\langle 24.46, 4.2\rangle$	$[16.31, 28.31]$
0.8	$\langle 31.03, 3.80\rangle$	$[27.23, 34.83]$	$\langle 31.60, 3.80\rangle$	$[27.8, 35.4]$	$\langle 22.87, 3.8\rangle$	$[16.23, 26.95]$
0.9	$\langle 26.75, 3.40\rangle$	$[23.35, 30.15]$	$\langle 28.87, 3.40\rangle$	$[25.47, 32.27]$	$\langle 21.28, 3.4\rangle$	$[16.16, 25.58]$
1.0	$\langle 22.46, 3\rangle$	$[19.46, 25.46]$	$\langle 26.15, 3\rangle$	$[23.15, 29.15]$	$\langle 19.69, 3\rangle$	$[16.08, 24.22]$

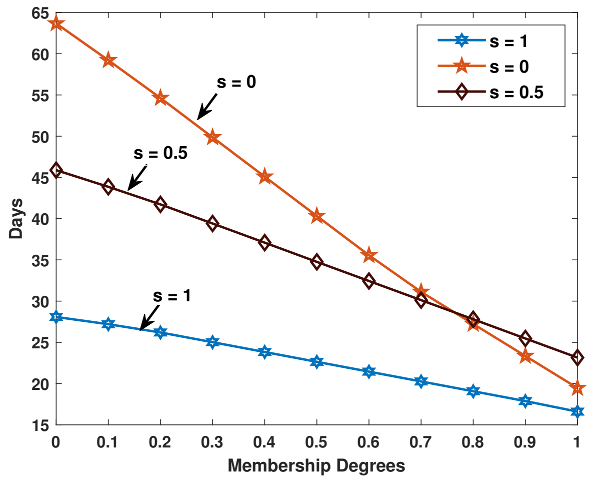

Fig. 11. Lower limit for various values of "s" (Example 4)

VIII. STRENGTH AND LIMITATION OF THE STUDY

Trapezoidal neutrosophic fuzzy numbers and interval-valued trapezoidal neutrosophic fuzzy numbers are more flexible and expressive. It helps the decision makers in the decision making process to model and analyze the imprecise data and preferences, providing more comprehensive approach to handle uncertainty. In addition, analyzing such data in a neutrosophic way helps the decision makers to handle it neutrally. While performing calculations with a large number of trapezoidal neutrosophic fuzzy number makes difficult than the traditional crisp or fuzzy methods. Moreover, interval number is more beneficial when the exact value is not known. Hence we convert the trapezoidal neutrosophic number into an interval number using the α, β & γ- cut. While solving numerically, we obtain an optimal project completion days for section VI-A, VII-A, VII-B (Example 1, 3 & 4) for a fixed α, β, γ & $s = 1$, i.e., the highest membership grade. Section VI-B (Example 2) is optimal for all α, β, γ & s. In most of the cases, if "s" decreases, the project completion time reached the maximum for various membership grades and this is considered as a limitation of the study. Further research and standardization may help to overcome some of the current limitations as the field of neutrosophic computing and fuzzy system evolves.

IX. CONCLUSION

In this article, we address two methods of PERT problem (Trapezoidal neutrosophic and Interval-valued trapezoidal neutrosophic environment) by employing a novel interval de-neutrosophication technique. To evaluate the efficacy of the proposed approach, numerical examples are conducted (two from each method), yielding superior results compared to the existing method referenced as [17], [18], [15] and [29]. The outcomes obtained in various stages of trapezoidal neutrosophic PERT problem and interval-valued trapezoidal neutrosophic PERT problem are thoroughly discussed in sections VI and VII. Sensitivity analysis is done for all the examples using various parameters and mentioned in Table V, IX, XV and XXI. It is graphically shown in Fig. 4, 6, 7, 9 and 11. Additionally, Table VI, X, XVI and XX clearly highlighted the possible paths of the project for all the examples in section VI and VII. Furthermore, the advantages and limitations of the developed approach is detailed in section VIII. Future investigation may include the development of advanced neutrosophic PERT models, hybrid approaches and utilization of the proposed techniques in real-world applications. Comparative studies shall be conducted against other uncertainty modeling techniques that helps accessing the model's strength and weakness. Furthermore, developing decision support systems or software tools that integrate neutrosophic PERT can facilitate the former's practical implementation.

REFERENCES

[1] S. Chanas and J. Kamburowski, "The use of fuzzy variables in PERT," *Fuzzy sets and systems*, vol. 5, no. 1, pp. 11-19, 1981.
[2] S. Chanas and P. Zieliński, "Critical path analysis in the network with fuzzy activity times," *Fuzzy sets and systems*, vol. 122, no. 2, pp. 195-204, 2001.

[3] G. S. Liang and T. C. Han, “The fuzzy critical path for project network,” *Information and Management Sciences*, vol. 15, no. 4, pp. 29-40, 2004.
[4] C. T. Chen and S. F. Huang, “Applying fuzzy method for measuring criticality in project network,” *Information sciences*, vol. 177, no. 12, pp. 2448-2458, 2007.
[5] T. A. S. Al Samman and R. M. R. Al Brahemi, “Fuzzy PERT for project management,” *International Journal of Advances in Engineering and Technology*, vol. 7, no. 4, pp. 1150, 2014.
[6] M.F. Yang, Y.T. Chou, M.C. Lo and W.C. Tseng, “Applying Fuzzy Time Distribution in PERT Model,” *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2014*, 12-14 March, 2014, Hong Kong, pp1084-1087 .
[7] M. Mazlum and A. F. Guneri, “CPM, PERT and project management with fuzzy logic technique and implementation on a business,” *Procedia-Social and Behavioral Sciences*, vol. 210, pp. 348-357, 2015.
[8] S. Elizabeth and L. Sujatha, “Finding critical path in a project network under fuzzy environment,” *Math. Sci. Int. Res. J*, vol. 5, pp. 31-36, 2016.
[9] P. Jayagowri and G. Geetharamani, “A critical path problem using intuitionistic trapezoidal fuzzy number,” *Applied Mathematical Sciences*, vol. 8, no. 52, pp. 2555-2562, 2014.
[10] P. Jayagowri and G. Geetharamani, “A critical path problem using intuitionistic triangular fuzzy number,” *Int. Journal of Engineering Research and Applications*, vol. 4, no. 5, pp. 55-60, May 2014.
[11] S. Elizabeth and L. Sujatha, “Project scheduling method using triangular intuitionistic fuzzy numbers and triangular fuzzy numbers,” *Applied Mathematical Sciences*, vol. 9, no. 4, pp. 185-198, 2015.
[12] G. Sudha, R. Porchelvi and K. Gnanaselvi, “Solving critical path problem using triangular intuitionistic fuzzy number,” *International Journal of Fuzzy Mathematical Archive*, vol. 14, no. 1, pp. 1-8, 2017.
[13] T. Yogashanthi, S. Mohanaselvi and K. Ganesan, “A new approach for solving flow shop scheduling problems with generalized intuitionistic fuzzy numbers,” *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 3, pp. 4287-4297, 2019.
[14] F. Smarandache, “Neutrosophy: neutrosophic probability, set, and logic,” *Rehoboth: American Research Press*, 1998.
[15] M. Mohamed, M. Abdel-Basset, A. N. Hussien and F. Smarandache, “Using neutrosophic sets to obtain PERT three-times estimates in project management,” *Infinite Study*, 2017.
[16] M. Mullai and R. Surya, “Neutrosophic project evaluation and review techniques,” *Infinite Study*, 2019.
[17] A. Chakraborty, S. P. Mondal, A. Ahmadian, N. Senu, S. Alam and S. Salahshour, “Different forms of triangular neutrosophic numbers, de-neutrosophication techniques and their applications,” *Symmetry*, vol. 10, no. 8, pp. 327, 2018.
[18] A. H. Mahmoud, “Critical Paths in a fuzzy construction project network,” *International Journal of Civil Engineering and Technology (IJCIET)*, vol. 10, no. 1, pp. 1313-1321, 2019.
[19] V. Vijaya, D. Rajalaxmi and H. Manikandan, “Finding critical path in a fuzzy project network using neutrosophic fuzzy number,” *Advances and Applications in Mathematical Sciences*, vol. 21, no. 10, pp. 5743-5753, 2022.
[20] G. Ramesh, G. Sudha and K. Ganesan, “A study on interval PERT/CPM network problems,” in *AIP Conference Proceedings*, AIP Publishing LLC, vol. 2112, no. 1, pp. 020123, June 2019.
[21] S. Mukherjee and K. Basu, “Solution of interval PERT/CPM network problems by a simplified tabular method,” *Opsearch*, vol. 48, pp. 355-370, 2011.
[22] S. Aramesh, S. M. Mousavi, V. Mohagheghi, E. K. Zavadskas, and J. Antucheviciene, “A soft computing approach based on critical chain for project planning and control in real-world applications with interval data,” *Applied Soft Computing*, vol. 98, 106915, 2021.
[23] P. Biswas, S. Pramanik, and B. C. Giri, Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making, *Neutrosophic Sets and Systems*, vol. 12(July), pp. 127-138, 2016.
[24] Ming Ma, M. Friedman and A. Kandel, “A new fuzzy arithmetic,” *Fuzzy Sets and Systems*, vol. 108, pp. 83-90, 1999.
[25] S. H. Surya, T. Nirmala and K. Ganesan, “Interval linear algebra–A new perspective,” *Journal of King Saud University-Science*, vol. 35, no. 2, 102502, 2023.
[26] S. Sinika and G. Ramesh, “Trapezoidal neutrosophic assignment problem with new interval arithmetic costs,” *Journal of Intelligent & Fuzzy Systems*, vol. 45, no. 2, pp. 2179-2191, 2023
[27] A. Sengupta, T. K. Pal and D. Chakraborty, “Interpretation of inequality constraints involving interval coefficients and a solution to interval linear programming,” *Fuzzy Sets and Systems*, vol. 119, no. 1, pp. 129-138, 2001.
[28] S. Sinika and G. Ramesh, “To enhance new interval arithmetic operations in solving linear programming problem using interval-valued trapezoidal neutrosophic numbers,” *Mathematics and Statistics*, vol. 11, no. 4, pp. 710-725, 2023.
[29] A. R. Fernandez, L. V. M. Rosales, O. G. A. Paspuel, W. B. J. Lopez and A. R. S. Leon, “Neutrosophic Statistics for Project Management. Application to a Computer System Project,” *Neutrosophic Sets and Systems*, vol. 44, pp. 308-314, 2021.