Electronic Circuit in the Two-Scroll Hyperchaotic System with a Unique Saddle-Point Rest Point and its Synchronization via Integral Sliding Mode Control

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Abstract—A new 4-D two-scroll hyperchaotic system with three quadratic nonlinearities is detailed in this research paper. It is shown that the new hyperchaotic system has a unique saddle-point rest point at the origin and it exhibits self-excited multistability. A detailed stability analysis is carried out for the new hyperchaotic system. It is also proved that the new hyperchaotic system has multistability with coexisting attractors. Next, we invoke integral sliding mode control for the global self-synchronization of the new hyperchaotic system taken as leader-follower systems. An electronic circuit design of the new two-scroll hyperchaotic system is developed in MultiSim, which is beneficial for engineering applications.

Index Terms—Hyperchaos; hyperchaotic systems; rest points; multi-stability; sliding mode control; circuit simulation.

I. INTRODUCTION

Hyperchaotic systems are defined as chaotic systems having two or more positive Lyapunov exponents [1]. Hyperchaotic systems exhibit more complexity than the chaotic systems and hence they have important engineering applications such as neural networks [2], chemical reactors [3], mechanical systems [4], wind turbine [5]-[6], robotics [7], electronic circuit [8]-[10], finance systems [11][12], etc.

In this research work, a new 4-D two-scroll hyperchaotic system with three quadratic nonlinearities is proposed and the properties of the new hyperchaotic system are analyzed in detail. It is established that the new two-scroll hyperchaotic system has a unique saddle-point rest point at the origin. Hence, it follows that the new hyperchaotic system has hidden attractor [13]-[15]. Multistability is a special property of chaotic and hyperchaotic systems which is the coexistence of attractors for same parameter set but different initial conditions [16]-[19]. In this work, it is also established that the new two-scroll hyperchaotic system has multistability with coexisting attractors.

Control and synchronization of chaotic and hyperchaotic systems are important research topics in the chaos literature [20]-[25]. Many control methods are used in control engineering for chaotic systems such as nonlinear control [20], adaptive control [21], backstepping control [22], sliding mode control [23]-[24], fuzzy logic control [25], etc. In this research work, we use integral sliding mode control for the global self-synchronization of the new hyperchaotic system. Sliding mode control has attractive properties such as finite-time convergence, robust to parameter variations, etc [26].

Section 2 describes the modelling of the new two-scroll hyperchaotic system with three quadratic nonlinear terms. Section 3 provides a dynamic analysis of the new two-scroll hyperchaotic system. Section 4 deals with the global self-synchronization of the new two-scroll hyperchaotic system considered as leader-follower systems using integral sliding mode control. Section 5 contains the circuit simulation of the new two-scroll hyperchaotic system using MultiSim.

II. A NEW TWO-SCROLL HYPERCHAOTIC SYSTEM

In this work, we consider a new 4-D system with three quadratic nonlinear terms:

\[
\begin{align*}
\dot{y}_1 &= a(y_2 - y_1) + py_2 y_3 - y_4 \\
\dot{y}_2 &= b y_1 - d y_1 y_3 - y_4 \\
\dot{y}_3 &= y_1 y_2 - c y_3 \\
\dot{y}_4 &= d y_2 
\end{align*}
\]  

In (1), \(Y = (y_1, y_2, y_3, y_4)\) is the state and \(a, b, c, d, p\) are positive constants. We show that the system (1) exhibits a
hyperchaotic attractor when the parameters are taken as
\[ a = 8, \ b = 32, \ c = 3, \ d = 3, \ p = 0.2 \]  \hspace{2cm} (2)

For MATLAB simulations, we take the initial values of the 4-D system (1) as
\[ y_1(0) = 0.2, \ y_2(0) = 0.3, \ y_3(0) = 0.3, \ y_4(0) = 0.2 \]  \hspace{2cm} (3)

Using Wolf algorithm [27], we calculate the Lyapunov characteristic exponents (LCE) for the system (1) for the parameter values (2) and the initial state (3) for \( T = 1E5 \) seconds as follows:
\[ \mu_1 = 0.7223, \ \mu_2 = 0.2449, \ \mu_3 = 0, \ \mu_4 = -11.9546 \]  \hspace{2cm} (4)

Fig. 1: Lyapunov exponents of the new 4-D system (1) for \((a, b, c, d, p) = (8,32,3,0.2)\)

Fig. 1 shows the Lyapunov exponents of the new hyperchaotic system (1) for the parameter values \((a, b, c, d, p) = (8,32,3,0.2)\). The system (1) is hyperchaotic since it possesses two positive Lyapunov characteristic exponents \(\mu_1\) and \(\mu_2\). As the sum of the LCE values in (4) is negative, the system (1) is also dissipative.

Fig. 2 shows the MATLAB plots of the new hyperchaotic system (1) in various coordinate planes for \((a, b, c, d, p) = (8,32,3,0.2)\) and \(Y(0) = (0.2,0.3,0.3,0.2)\).

From Fig. 2, it is clear that the hyperchaotic system (1) is equipped with a two-scroll hyperchaotic attractor.

III. DYNAMIC ANALYSIS OF THE NEW TWO-SCROLL HYPERCHAOTIC SYSTEM

A. Rest Points of the New Hyperchaotic System

The rest points of the new two-scroll hyperchaotic system are calculated by means of finding the roots of the following algebraic system of equations:
\[ a(y_2 - y_1) + py_2y_3 - y_4 = 0 \]  \hspace{2cm} (5a)
\[ by_1 - dy_1y_3 - y_4 = 0 \]  \hspace{2cm} (5b)
\[ y_1y_2 - cy_3 = 0 \]  \hspace{2cm} (5c)
\[ dy_2 = 0 \]  \hspace{2cm} (5d)

Solving (5d), we get \( y_2 = 0 \).

Substituting \( y_2 = 0 \) in (5c), we get \( y_3 = 0 \).

Next, we put \( y_2 = y_3 = 0 \) into equations (5a) and (5b) and obtain the following:
\[ ay_1 + y_4 = 0 \]  \hspace{2cm} (6a)
\[ by_1 - y_4 = 0 \]  \hspace{2cm} (6b)

Adding (6a) and (6b), we get \((a + b)y_1 = 0\).

Since \( a > 0 \) and \( b > 0 \), it follows that \( y_1 = 0 \).

Substituting \( y_1 = 0 \) into (6a), we get \( y_4 = 0 \). Hence, \( K_0 = (0,0,0,0) \) is the unique rest point of the two-scroll hyperchaotic system (1).

To investigate the stability of \( K_0 \), we determine the Jacobian matrix of the two-hyperchaotic system (1) at the rest point \( K_0 \) for the hyperchaotic case \((a, b, c, d, p) = (8,32,3,0.2)\) as follows:
\[ J_0 = \begin{bmatrix} -8 & 8 & 0 & -1 \\ 32 & 0 & 0 & -1 \\ 0 & 0 & -3 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix} \]  \hspace{2cm} (7)

The spectral values of the Jacobian matrix \( J_0 \) are numerically estimated in MATLAB as follows:
\[ \alpha_1 = -3, \ \alpha_2 = -20.5780, \ \alpha_3 = 0.4821, \ \alpha_4 = 12,0959 \]  \hspace{2cm} (8)

This pinpoints that \( K_0 \) is a saddle-point and unstable rest point for the two-scroll hyperchaotic system (1). Hence, we deduce that the system (1) has a self-excited two-scroll hyperchaotic attractor.

B. Bifurcation and Lyapunov Exponents

In this section, we delve into the dynamic characteristics of the proposed four-dimensional hyperchaotic system (1), examining how these dynamics evolve in relation to various parameters. Our exploration involves the utilization of tools such as the Lyapunov exponents spectrum, bifurcation diagram, and phase plots to gain a comprehensive understanding of the system’s behavior and the intricate interplay with its associated parameters.

1) Fix \( b = 32, \ c = 3, \ d = 3, \ p = 0.2, \) and vary \( a \): In the context of a fixed parameter set where \( b = 32, \ c = 3, \ d = 3, \) and \( p = 0.2 \), and the control parameter \( a \) is systematically adjusted within the range \([0.5, 8]\), the bifurcation diagram and Lyapunov exponents spectrum are presented in Fig. 3 and Fig. 4, respectively.

For values of \( a \) in the interval \([0.5, 1.2]\), System (1) exhibits a periodic nature, characterized by one zero and three negative Lyapunov exponents, as depicted in Fig. 5(a). Specifically, at \( a = 0.5 \), the corresponding Lyapunov exponents are detailed as follows: \( LE1 = 0, \ LE2 = -0.407, \ LE3 = -1.249, \) and \( LE4 = -1.9546 \).

Upon entering the range \( a \in [1.2, 6] \), System (1) undergoes a transition to chaotic behavior, with a single positive Lyapunov exponent, showcased in Fig. 5(b). Notably, at \( a = 3 \), the specific Lyapunov exponents are outlined as follows: \( LE1 = 0.632, \ LE2 = 0, \ LE3 = -0.836 \) and \( LE4 = -5.795 \). The associated Kaplan-Yorke dimension maintains a fractional value of 2.756 during this chaotic regime.
Extending further into the range $a \in [6, 8]$, System (1) reveals hyperchaotic behavior, marked by the presence of two positive Lyapunov exponents, as illustrated in Fig. 5(c). Specifically, at $a = 7$, the corresponding Lyapunov exponents are noted as follows: $LE1 = 0.589$, $LE2 = 0.197$, $LE3 = 0$, and $LE4 = -10.787$. The Kaplan-Yorke dimension maintains a fractional value of 3.073 during this chaotic regime.

This detailed examination provides a coherent narrative of the dynamic transformations experienced by System (1) across varying values of the control parameter $a$, shedding light on its periodic, chaotic, and hyperchaotic behaviors.

2) Fix $a = 8$, $c = 3$, $d = 3$, $p = 0.2$, and vary $b$:
When the parameters $a$, $c$, and $d$ are held constant at specific values ($a = 8$, $c = 3$, $d = 3$, and $p = 0.2$), the control
a fractional value of 3.007 during this chaotic phase.

As $b$ continues into the range $[12, 32]$, the system further evolves into a state of hyperchaos, characterized by two positive Lyapunov exponents, as shown in Fig. 8(b)(c). For instance, at $b = 20$, the corresponding Lyapunov exponents are $LE1 = 0.415$, $LE2 = 0.356$, $LE3 = 0$, and $LE4 = -11.773$. Importantly, the Kaplan-Yorke dimension assumes a fractional value of 3.066 during this intriguing hyperchaotic phase.

This comprehensive analysis offers a cohesive account of the dynamic changes observed in System (1) as the control parameter "$b$" varies. It provides insights into the system's periodic, chaotic, and hyperchaotic behaviors, offering a clearer understanding of its intricate dynamics across different values of the control parameter.

3) Fix $a = 8$, $b = 32$, $d = 3$, $p = 0.2$ and vary $c$: When the values of parameters $a$, $b$, and $d$ are kept constant at specific settings ($a = 8$, $b = 32$, $d = 3$, and $p = 0.2$), the control parameter $c$ experiences variation within the interval $[0, 3]$. Fig. 9 and Fig. 10 visually represent the Lyapunov exponents spectrum and bifurcation diagram for the new system.

In the segments where $c$ lies within $[0, 0.2]$, $[1.5, 2.3]$, System (1) demonstrates periodic behavior, characterized by one zero and three negative Lyapunov exponents, as depicted in Fig. 11(a). For example, at $c = 1.7$, the corresponding Lyapunov exponents are $LE1 = 0$, $LE2 = -1.531$, $LE3 = -4.028$, and $LE4 = -4.146$.

Moving into the range $c \in [2.3, 2.4]$, System (1) undergoes a transition towards chaotic dynamics, characterized by the
Fig. 8: Phase plots of system (1) for specifics values of the control parameter $b$: (a). $b = 2$, (b). $b = 11$ and (c). $b = 20$

emergence of one positive Lyapunov exponent, as illustrated in Fig. 11(b). Specifically, at $c = 2.34$, the specific Lyapunov exponents are detailed as follows: $\text{LE}_1 = 0.015$, $\text{LE}_2 = 0$, $\text{LE}_3 = -0.004$, and $\text{LE}_4 = -10.348$. Notably, the Kaplan-Yorke dimension maintains a fractional value of 3.001 during this chaotic phase.

In the intervals ($[0.2, 1.5]$, $[2.4, 3]$), the system evolves further into a state of hyperchaos, marked by two positive Lyapunov exponents, as depicted in Fig. 11(c). For instance, at $d=0.11$, the corresponding Lyapunov exponents are detailed as follows: $\text{LE}_1 = 0$, $\text{LE}_2 = -0.003$, $\text{LE}_3 = -5.496$, and $\text{LE}_4 = -5.509$. Notably, the Kaplan-Yorke dimension maintains a fractional value of 2.060, and the overall Kaplan-Yorke dimension is 3.016.

As the parameter $d$ extends into the range $[0.7, 3]$, System (1) reveals a hyperchaotic state with two positive Lyapunov exponents, as depicted in Fig. 14(c). At $d = 2.5$, the corresponding Lyapunov exponents are $\text{LE}_1 = 0.736$, $\text{LE}_2 = 0.192$, $\text{LE}_3 = 0$, and $\text{LE}_3 = -11932$. Additionally, the Kaplan-Yorke dimension assumes a value of 3.078 during this hyperchaotic phase.

4) Fix $a = 8$, $b = 32$, $c = 3$, $p = 0.2$ and vary $d$: In the scenario where parameters $a$, $b$, $c$, and $p$ are held constant at specific values ($a = 8$, $b = 32$, $c = 3$, and $p = 0.2$), the control parameter $d$ is subjected to variation within the range $[0.1, 3]$. The Lyapunov exponents spectrum and bifurcation diagram for the novel system are visually represented in 12 and Fig. 13, respectively.

Within the interval where $d \in [0.1, 0.15]$, System (1) exhibits periodic behavior with two zero Lyapunov exponents, as illustrated in Fig. 14(a). For instance, at $d=0.11$, the corresponding Lyapunov exponents are detailed as follows: $\text{LE}_1 = 0$, $\text{LE}_2 = -0.003$, $\text{LE}_3 = -5.496$, and $\text{LE}_4 = -5.509$.

Continuing within the interval $[0.15, 0.70]$, the system undergoes a transition into chaotic behavior, maintaining one positive Lyapunov exponent, as showcased in Fig. 14(b). For example, at $d = 0.3$, the specific Lyapunov exponents are outlined as follows: $\text{LE}_1 = 0.503$, $\text{LE}_2 = 0$, $\text{LE}_3 = -0.319$ and $\text{LE}_4 = -11.186$. Notably, the Kaplan-Yorke dimension maintains a fractional value of 2.060, and the overall Kaplan-Yorke dimension is 3.016.

As the parameter $d$ extends into the range $[0.7, 3]$, System (1) reveals a hyperchaotic state with two positive Lyapunov exponents, as depicted in Fig. 14(c). At $d = 2.5$, the corresponding Lyapunov exponents are $\text{LE}_1 = 0.736$, $\text{LE}_2 = 0.192$, $\text{LE}_3 = 0$, and $\text{LE}_3 = -11932$. Additionally, the Kaplan-Yorke dimension assumes a value of 3.078 during this hyperchaotic phase.
This thorough examination offers a clear representation of the dynamic changes in System (1) as the control parameter $d$ is altered. It provides valuable insights into the system’s periodic, chaotic, and hyperchaotic behaviors, contributing to an enhanced comprehension of its intricate dynamics across varying values of the control parameter.

5) Fix $a = 3.2$, $b = 1.7$, $c = 2.34$ and vary $p$: Here, the parameters $a$, $b$, $c$, and $d$ are fixed at particular values ($a = 3.2$, $b = 1.7$, $c = 2.34$, and $d = 3$), while the control parameter $p$ undergoes variation within the interval $[-1, 1]$. The Lyapunov exponents spectrum and bifurcation diagram for the novel system are graphically depicted in Fig. 15 and Fig. 16, respectively.

In the range where $p \in [-1, -0.4]$, System (1) displays periodic behavior, as depicted in Fig. 17(a). Specifically, at $p = -0.6$, the associated Lyapunov exponents are detailed as follows: $LE_1 = 0$, $LE_2 = -2.164$, $LE_3 = -2.167$, and $LE_4 = -6.670$.

Advancing within the interval $[-0.4, -0.3]$, the system undergoes a shift towards chaotic dynamics, maintaining one positive Lyapunov exponent, as illustrated in Fig. 17(b). For instance, at $p = -0.35$, the specific Lyapunov exponents are outlined as follows: $LE_1 = 0.268$, $LE_2 = 0$, $LE_3 = -0.412$, and $LE_4 = -10.856$. Notably, the Kaplan-Yorke dimension retains a fractional value of 2.651.

As the parameter $"p"$ varies within the interval $[-0.3, 1]$, System (1) exhibits a hyperchaotic state characterized by two positive Lyapunov exponents, as illustrated in Fig. 17(c). When $p$ is set to 4, the corresponding Lyapunov exponents are $LE_1 = 0.732$, $LE_2 = 0.168$, $LE_3 = 0$, and $LE_4 = -11.901$. Additionally, the Kaplan-Yorke dimension attains a value of 3.076 during this hyperchaotic phase.

This in-depth analysis provides a vivid depiction of the dynamic transformations in System (1) with changes in the control parameter $"p"$.

C. Multistability of the New 4D Hyperchaotic System

The hyperchaotic system (1) demonstrates the existence of various attractors even when the parameters remain constant. Take into account two separate starting points for the new 4D hyperchaotic system (1): $X_{01} = (3, 3, 3, 3)$ and $X_{02} = (3, 0, 0, 3)$. The state trajectory linked to $X_{01}$ is depicted in blue, while the state trajectory associated with $X_{02}$ is illustrated in red.

Set the parameters to $a = 8$, $b = 8.7$, $c = 3$, $d = 0$, and $p = 0.2$. In this configuration, the new 4D Hyperchaotic
Fig. 14: Phase plots of system (1) for specifics values of the control parameter $d$: (a) $d = 0.11$, (b) $d = 0.3$ and (c) $d = 2.5$

Fig. 15: Bifurcation Diagram for varying parameter $p$

Fig. 16: Lyapunov exponents for varying parameter $p$

Fig. 17: Phase plots of system (1) for specifics values of the control parameter $p$: (a) $p = -0.6$, (b) $p = -0.35$ and (c) $p = 0.4$
System (1) converges to two distinct coexisting periodic orbits based on its initial conditions, as depicted in Fig. 18(a). Adjust the parameters to \(a = 8, b = 9.1, c = 3, d = 3\), and \(p = 0.2\). Under these settings, the new 4D Hyperchaotic System (1) exhibits two varied behaviors depending on its starting points, as illustrated in Fig. 18(b). When initiated from \(X_{01}\), the system generates a chaotic attractor, whereas it demonstrates periodic behavior when initiated from \(X_{02}\). Maintain the parameters at \(a = 8, b = 9.5, c = 3, d = 3\), and \(p = 0.2\). The 4D system (1) showcases two different behaviors based on its starting conditions, as shown in Fig. 18(c). Initiating from \(X_{01}\) results in periodic behavior, while starting from \(X_{02}\) leads to the generation of a chaotic attractor. Set the parameters to \(a = 8, b = 22, c = 3, d = 3\), and \(p = 0.2\). Under these conditions, the 4D system (1) displays the coexistence of two distinct hyperchaotic attractors, as depicted in Fig. 18(d).

IV. GLOBAL SYNCHRONIZATION VIA ISMC

As a control application, we invoke integral sliding mode control (ISMC) for the global synchronization between the states of the new two-scroll hyperchaotic systems taken as leader-follower systems.

As the leader system, we take the two-scroll hyperchaotic system

\[
\begin{align*}
\dot{y}_1 &= a(y_2 - y_1)p_1y_3 - y_4 \\
\dot{y}_2 &= by_1 - d_1y_3 - y_4 \\
\dot{y}_3 &= y_1y_2 - c_3y_3 \\
\dot{y}_4 &= dy_2
\end{align*}
\]  

(9)

We denote the state of the leader system (9) as \(Y = (y_1, y_2, y_3, y_4)\). As the follower system, we take the controlled two-scroll hyperchaotic system

\[
\begin{align*}
\dot{z}_1 &= a(z_2 - z_1)p_2z_3 - z_4 + v_1 \\
\dot{z}_2 &= b_1z_1 - d_2z_3 - z_4 + v_2 \\
\dot{z}_3 &= z_1z_2 - c_2z_3 + v_3 \\
\dot{z}_4 &= dz_2 + v_4
\end{align*}
\]  

(10)

We denote the state of the follower system (10) as \(Z = (z_1, z_2, z_3, z_4)\). The synchronization errors are determined as follows:

\[
\begin{align*}
\epsilon_1 &= z_1 - y_1 \\
\epsilon_2 &= z_2 - y_2 \\
\epsilon_3 &= z_3 - y_3 \\
\epsilon_4 &= z_4 - y_4
\end{align*}
\]  

(11)

We calculate the dynamics followed by the synchronization error variables.

We obtain the following system for the error dynamics:

\[
\begin{align*}
\dot{\epsilon}_1 &= a(\epsilon_2 - \epsilon_1) - \epsilon_4 - p(z_2y_3 - y_2y_3) + v_1 \\
\dot{\epsilon}_2 &= b\epsilon_1 - \epsilon_4 - d(z_1y_3 - y_1y_3) + v_2 \\
\dot{\epsilon}_3 &= z_1z_2 - y_1y_2 - c\epsilon_3 + v_3 \\
\dot{\epsilon}_4 &= d\epsilon_2 + v_4
\end{align*}
\]  

(12)

In the ISMC design, an integral sliding manifold is defined for each error variable as follows:

\[
\begin{align*}
\rho_1 &= \epsilon_1 + \alpha_1 \int_0^t \epsilon_1(\theta)d\theta \\
\rho_2 &= \epsilon_2 + \alpha_2 \int_0^t \epsilon_2(\theta)d\theta \\
\rho_3 &= \epsilon_3 + \alpha_3 \int_0^t \epsilon_3(\theta)d\theta \\
\rho_4 &= \epsilon_4 + \alpha_4 \int_0^t \epsilon_4(\theta)d\theta
\end{align*}
\]  

(13)

From (13), we deduce the following:

\[
\begin{align*}
\dot{\rho}_1 &= \dot{\epsilon}_1 + \alpha_1\epsilon_1 \\
\dot{\rho}_2 &= \dot{\epsilon}_2 + \alpha_2\epsilon_2 \\
\dot{\rho}_3 &= \dot{\epsilon}_3 + \alpha_3\epsilon_3 \\
\dot{\rho}_4 &= \dot{\epsilon}_4 + \alpha_4\epsilon_4
\end{align*}
\]  

(14)

In the ISMC design, we assume that \(\alpha_i > 0\) for \(i = 1, 2, 3, 4\).

Based on the exponential reaching law [28], we set the following:

\[
\begin{align*}
\dot{\rho}_1 &= -\beta_1\text{sgn}(\rho_1) - K_1\rho_1 \\
\dot{\rho}_2 &= -\beta_2\text{sgn}(\rho_2) - K_2\rho_2 \\
\dot{\rho}_3 &= -\beta_3\text{sgn}(\rho_3) - K_3\rho_3 \\
\dot{\rho}_4 &= -\beta_4\text{sgn}(\rho_4) - K_4\rho_4
\end{align*}
\]  

(15)

By comparing the equations (14) and (15), we get the following:

\[
\begin{align*}
\dot{\epsilon}_1 + \alpha_1\epsilon_1 &= -\beta_1\text{sgn}(\rho_1) - K_1\rho_1 \\
\dot{\epsilon}_2 + \alpha_2\epsilon_2 &= -\beta_2\text{sgn}(\rho_2) - K_2\rho_2 \\
\dot{\epsilon}_3 + \alpha_3\epsilon_3 &= -\beta_3\text{sgn}(\rho_3) - K_3\rho_3 \\
\dot{\epsilon}_4 + \alpha_4\epsilon_4 &= -\beta_4\text{sgn}(\rho_4) - K_4\rho_4
\end{align*}
\]  

(16)

We combine the equations (12) and (16) to obtain the following:

\[
\begin{align*}
(a\epsilon_2 - \epsilon_1) - \epsilon_4 - p(z_2y_3 - y_2y_3) + v_1 + \alpha_1\epsilon_1 &= -\beta_1\text{sgn}(\rho_1) - K_1\rho_1 \\
b\epsilon_1 - \epsilon_4 - d(z_1y_3 - y_1y_3) + v_2 + \alpha_2\epsilon_2 &= -\beta_2\text{sgn}(\rho_2) - K_2\rho_2 \\
z_1z_2 - y_1y_2 - c\epsilon_3 + v_3 + \alpha_3\epsilon_3 &= -\beta_3\text{sgn}(\rho_3) - K_3\rho_3 \\
d\epsilon_2 + v_4 + \alpha_4\epsilon_4 &= -\beta_4\text{sgn}(\rho_4) - K_4\rho_4
\end{align*}
\]  

(17)

Theorem 1. The new two-scroll hyperchaotic systems (9) and (10) are globally and asymptotically synchronized for all initial conditions \(Y(0), Z(0) \in R^4\) by the integral sliding mode controller (18), where the constants \(\alpha_i, \beta_i, K_i, (i = 1, 2, 3, 4)\) are all positive.

Proof. We establish this theorem using Lyapunov stability theory [29]. First, we consider the quadratic and positive definite Lyapunov function defined by

\[V(\rho_1, \rho_2, \rho_3, \rho_4) = \frac{1}{2}(\rho_1^2 + \rho_2^2 + \rho_3^2 + \rho_4^2)\]

(19)

We determine the time-derivative of \(V\) as follows:

\[\dot{V} = \sum_{i=1}^{4} \rho_i [-\beta_i\text{sgn}(\rho_i) - K_i\rho_i] = \sum_{i=1}^{4} \left[-\beta_i\rho_i - K_i\rho_i^2\right]\]

(20)
From (20), we see that $\dot{V}$ is negative definite at all points of $\mathbb{R}^4$. Using Lyapunov stability theory [29], we conclude that $\rho_1(t) \to 0$ as $t \to \infty$ for each $i = 1, 2, 3, 4$.

Hence, it follows that $\varepsilon_i(t) \to 0$ as $t \to \infty$ for each $i = 1, 2, 3, 4$.

This completes the proof. ■

For MATLAB simulations, we take $(a, b, c, d, p) = (8, 32, 3, 3, 0.2)$. We take the sliding constants as $\alpha_i = \beta_i = 0.2$ and $K_i = 20$ for each $i = 1, 2, 3, 4$.

We take the initial state of the new two-scroll hyperchaotic systems (9) and (10) as $Y(0) = (12.3, 4.2, 16.7, 5.4)$ and $Z(0) = (5.8, 24.9, 3.1, 14.5)$.

Fig. 19 shows the complete synchronization of the new hyperchaotic systems (9) and (10). Fig. 20 shows the time-history of the synchronization errors.

V. CIRCUIT DESIGN

The implementation of the chaotic circuit based on the new hyperchaotic system is shown in Fig. 21. This circuit was designed with the MultiSIM platform. The circuit consists of four integrators made using the operational amplifiers U1A, U2A, U3A and U4, each corresponding to the line of the differential equation (1).

By using Kirchhoff’s circuit laws, the circuital equations of the designed circuit in Fig. 21 are derived as follows:

$$
\begin{align*}
\dot{y}_1 &= \frac{1}{c_1 R_5} y_2 - \frac{1}{c_5 R_5} y_1 + \frac{1}{10 c_5 R_5} y_2 y_3 - \frac{1}{c_4 R_4} y_4 \\
\dot{y}_2 &= \frac{1}{c_2 R_5} y_1 - \frac{1}{10 c_5 R_5} y_1 y_3 - \frac{1}{c_2 R_2} y_4 \\
\dot{y}_3 &= -\frac{1}{c_3 R_6} y_3 + \frac{1}{10 c_3 R_6} y_1 y_2 \\
\dot{y}_4 &= \frac{1}{c_4 R_4} y_2
\end{align*}
$$

(21)

Here, $y_1, y_2, y_3, y_4$ are the voltages across the capacitors $C_1, C_2, C_3$ and $C_4$, respectively. We choose the values of the circuital elements as $R_1 = R_2 = 50 \, \text{k}\Omega$, $R_3 = 200 \, \text{k}\Omega$, $R_4 = R_7 = 400 \, \text{k}\Omega$, $R_5 = 12.5 \, \text{k}\Omega$, $R_6 = 13.33 \, \text{k}\Omega$, $R_8 = R_{10} = 133.33 \, \text{k}\Omega$, $R_9 = 40\, \text{k}\Omega$, $R_{11} = R_{12} = R_{13} = R_{14} = 100 \, \text{k}\Omega$, $C_1 = C_2 = C_3 = C_4 = 1 \, \text{nF}$. The corresponding phase portraits on the oscilloscope are shown in Fig. 21. Multisim simulation have been performed.
VI. CONCLUSION

There is great interest in the modelling and applications of hyperchaotic systems in the chaos literature. A new 4-D two-scroll hyperchaotic system with three quadratic nonlinearities was described in this work. We showed that the new hyperchaotic system is dissipative and it has a unique saddle-point rest point at the origin. Thus, the new hyperchaotic system exhibits a self-excited hyperchaotic attractor. We also proved that the new hyperchaotic system has multistability with coexisting attractors. Next, we made use of integral sliding mode control to achieve global self-synchronization of the new hyperchaotic system taken as leader-follower systems. For real-world applications, we also designed an electronic circuit design of the new two-scroll hyperchaotic system using MultiSim.

in order to validate the numerical simulation results. A good agreement has been revealed between the results obtained from Multisim software and Matlab software.
Fig. 21: Circuit Design of the new Hyperchaotic system

Fig. 22: 2-D oscilloscope outputs of the new Hyperchaotic system (a) $y_1 - y_2$ plane, (b) $y_2 - y_3$ plane, (c) $y_3 - y_4$ plane and (d) $y_1 - y_4$ plane
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