Research on Multiple Products Aggregate Production Planning under Random Environment

Lanfen Liu, Xinfeng Yang, and Ke Yang

***Abstract*—Uncertain factors can significantly reduce the applicability of production plans and the production stability of enterprises. Therefore, considering the uncertain factors caused by external demands and internal capabilities, the impact of production in advance or backorder is analyzed, and the cost of backorder and loss of sales is determined. Moreover, a multi-objective stochastic programming model for aggregate production planning (APP) problem that minimizes the expected chance cost and employee instability is constructed considering external demand and internal capabilities as random variables. At a certain level of confidence, the opportunity constraints are transformed. In addition, combined with a local search strategy, two stochastic hybrid genetic algorithm-particle swarm optimization algorithms based on stage and multiple population strategies are designed to solve this model. Sensitivity analysis is performed on different confidence levels by using numerical examples, and numerical experiments show that the two hybrid algorithms proposed by this paper can effectively improve the solving efficiency.**

***Index Terms*—aggregate production planning, stochastic programming, multi objective, hybrid genetic algorithm**

I. Introduction

There are various uncertain factors in the environment faced by manufacturing enterprises, such as seasonal needs, price fluctuations, etc., which cause great uncertainty in demand level of products and lead to deviation from actual demand predictions of enterprises for products. In addition, the production capacity of manufacturing enterprises will also change due to equipment failures and personnel changes during the production process. These uncertain factors can significantly reduce the applicability of production plans, and frequent adjustments to production plans can disrupt the production arrangements of enterprises, and thus lead to a decrease in their service levels. In order to cope with these uncertainties, enterprises generally will increase their inventory levels. On the contrary, companies will try to reduce inventory in order to reduce costs. Therefore, it is necessary to consider the internal and external uncertain environment faced by manufacturing enterprises, and arrange production plans reasonably to improve the applicability of production plans.

In recent years, many scholars have adopted uncertain programming methods to solve aggregate production planning (APP) problems in uncertain environments, among which stochastic mathematical programming method and fuzzy programming method are two commonly used methods. Stochastic mathematical programming methods for APP mainly include stochastic linear programming, stochastic nonlinear programming, robust optimization and stochastic control [1]-[2]. We summarized the most important and related studies that consider random factors for APP problems in Table I.

These studies have made important contributions to solving the stability of production plans under random environments. Furthermore, from the review in Table I, it can be seen that APP problems mainly consider the uncertainty of product demand, and the chance constrained programming is a commonly used method to deal with the uncertainty APP problems.

Nowadays, decision makers pay great attention to the impact of labor changes, and the stability of workers has become more important than ever [19]. However, the market demand for products is difficult to determine due to the changes in market conditions. In addition, during the production process, the enterprise's equipment may require be repaired due to damage, which affects its production capacity. Production enterprises can produce in advance or be back ordered, both of which can affect the cost of the enterprise. Therefore, this paper mainly starts with the uncertain factors caused by external demands and internal capabilities. The impact of production in advance or backorder is analyzed, the cost of backorder and loss of sales is determined. Moreover, assuming that external demand and internal production capacity are random variables, a multi-objective stochastic programming model for APP problem that minimizes expected chance cost and employee instability is established to meet market demand and production capacity constraints at a certain level of confidence.

The remainder of this paper is organized as follows. Some assumptions and symbolic explanations are defined, expected losses of production in advance and backorder are analyzed, and the constraints and objective function are

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discussed in Section II, respectively. In addition, two stochastic hybrid genetic algorithm-particle swarm optimization algorithms based on stages (SHGA-PSO1) and multi population strategy (SHGA-PSO2) combined with the local search (LS) strategy are proposed in Section III. In section IV, the proposed algorithms are tested via some numerical examples. Finally, Section V provides concluding remarks.

TABLE I
THE RELATED STUDIES CONSIDERING THE BACKORDERING FOR APP

Article	Model category	Uncertain factors	Objective function	Solving approaches
Leung et al. [3]	A stochastic linear programming model	Demand	Minimize total costs consisting of the production cost, subcontracting cost, labour cost, inventory cost, hiring cost and lay-off cost, and penalty cost	--
Lababidi et al. [4]	The two-stage stochastic linear program with fixed recourse	Demand, market price, raw material cost and production yield	Minimize the total production costs, and raw material procurement, as well as lost demand, backlog, transportation and storage penalization	--
Demirel et al. [5]	A mixed-integer linear programming model	Demand	Minimize production-related costs	Solved by the CPLEX 12.0 solver
Jamalnia et al. [6]	A framework based on a set of stochastic, nonlinear, multi-objective optimization models	Market demand	Minimize total revenue; total production costs; utilization of production resources and capacity	The multiple criteria decision making methods
Ning et al. [7]	An uncertain expected value model	Market demand, deterioration rate, inventory cost, capital level, workforce production capacity, and machine production capacity	Maximize expected profits	Transformed into equivalent crisp form, used Genetic Algorithm and Direct Search Toolbox of MATLAB 8.5 to search for the optimal solutions
Alehashem et al.[8]	A nonlinear mixed integer programming model	Demand	Minimize the majority of supply chain cost and some more realistic cost	Model linearization and two-stage stochastic programming approach
Modarres et al.[9]	A multi-objective linear programming model	Operational cost, energy, carbon parameters, demand and maximum capacity	Minimize the operational cost, the energy cost and carbon emission	Solved as a single-objective model applying a goal attainment technique and robust optimization approach was applied
Pham *et* al. [10]	A joint production and microgrid planning model	Product demand and energy supply	Minimize the total cost comprised of production, logistics and energy generation	Solved as a two-stage decision-making process
Islam et al. [11]	A two-stage stochastic programming model	Product demand, labor and machine capacity, and power generation	Minimize the expected cost by considering all production and renewable energy expenses and revenues	Solved through CPLEX using the two-phase method
Hahn et al. [12]	Combines a deterministic linear programming model and an aggregate stochastic queuing network model	The key demand-side and operations parameters	Minimize WIP costs, and average weighted campaign lead time	A hierarchical decision support method
Entezaminia et al.[13]	A stochastic robust optimization approach lied on Mulvey's model	Demand and cost parameters	Minimize total losses of considered supply chain	The robust optimization approach
Sabah et al. [14]	A mixed integer linear programming model	Demand	Maximize the customer demand satisfaction level	Monte Carlo simulation
Li et al. [15]	A two-stage stochastic programming model	The sales volume and demand	Minimize the sum of distribution costs and the expected recourse costs	Transformed into an equivalent deterministic programming model and solved by CPLEX solver
Gozali et al. [16]	A linear programming model	Demand	Minimize total cost that consist of production cost, subcontracted cost and holding inventory cost	--
Ghaithan et al. [17]	A multi-objective stochastic optimization model	Demand and price	Minimization of total cost, maximization of total revenue, and maximization of service level.	The improved augmented ε-constraint algorithm
Tirkolaee *et* al. [18]	A multi-objective mixed-integer linear programming model	Demand, budgets	Total cost minimization, total resiliency-weighted purchasing maximization and total environmental pollution/footprint minimization	Weighted goal programming method and Robust optimization technique

II. MULTI-OBJECTIVE STOCHASTIC PROGRAMMING MODEL

In this section, the assumed conditions of the APP problem in a random environment are analyzed, and the relevant variables and parameter symbols are defined. On the basis of clarifying the expected losses of production in advance and backorder, some constraints such as supply and demand, inventory capacity, production capacity, and worker labor capacity are analyzed. Furthermore, a multi-objective stochastic optimization model is established for APP problem from two aspects: total production cost and employee stability.

A. Assumptions and Notations

(1) Assumptions

The following assumptions are made:

1) The failure rate of equipment in each period is a random variable and independent of each period.

2) The market demand for products in each period is a random variable, and the probability distribution of market demand is known.

3) Backorder and loss of sales are allowed in each period.

(2) Notations

1) Sets and indices

Let T be the set of periods in planning and t ($t \in T$) be the production planning period. Let I ($i \in I$) be the product category, which I is the set of product categories. Let j ($j \in J$) be the raw material category, which J is the set of raw material category.

2) Notations of product

$P\hat{D}_{it}$: Demand for product i in period t (units), discrete random variable;

PC_i: Unit production cost of product i;

$PB_{it}(t'-t,\theta)$: The maximum tolerant backorder quantity for product i in period t for the customer waiting time $t'-t$ under natural state θ ;

$cb_i(t'-t)$: The backorder cost for product i with waiting time $t'-t$;

$C1_t$: Total production cost of period t;

PR_{it} : The unit raw material cost of product i in period t;

$C2_t$: Total raw material cost in period t;

PW_i : Working hours required for a unit product i;

$P\hat{N}_{it}$: The production capacity for product i in period t, discrete random variable;

$\hat{C}5_{it}(\theta)$ Total backorder cost or lost sales cost in period t under natural state θ .

cl_i : The unit lost sales cost of product i.

2) Notations of inventory

$C\hat{I}_{it}$: The inventory of product i in period t, discrete random variable;

CK_i : Inventory cost of product i;

CN_i : The inventory capacity of product i;

$\hat{C}3_{it}(\theta)$: Total inventory cost of product i in period t under natural state θ .

3) Notations of raw material

R_{ij} : The demand of raw material j for producing unit product i;

RM_{jt} : Total demand of raw material j in period t;

RC_{jt} : The price of raw material j in period t.

4) Notations of worker

WH_t : Number of new employees in period t;

WL_t : Number of workers laid off in period t;

WHC : Training cost for one new employee;

WR : Maximum regular work hours in each period;

WO : Maximum overtime labor hours for in each period;

WRT_t : Total regular labor hours in period t;

WOT_t : Total overtime labor hours in period t;

WRC : Regular time labor cost per hour;

WOC : Overtime labor cost per hour;

$C4_t$ Total labor cost in period t.

5) Decision variables

PP_{it} : Production quantity of product i in period t;

W_t : Number of employees in period t.

B. Expected Loss Cost of Production in Advance and Stockout

(1) The inventory cost

Production enterprises can utilize the abundant production capacity in the early stage. Since the demand for products in each stage $P\hat{D}_{it}$ is a random variable, the initial inventory is also a random variable. According to the production and inventory balance conditions, the inventory at the beginning of the period can be expressed as:

$$C\hat{I}_{it} = C\hat{I}_{it-1} + PP_{it-1} - P\hat{D}_{it-1} . \quad (1)$$

For the beginning inventory of each planning period, if $C\hat{I}_{it}(\theta) \geq 0$, it means that there is no delay in the previous planning period and inventory costs will be incurred.

If the probability of each natural state occurring is known in advance, the inventory cost for each period under each natural state can be calculated using equation (2).

$$\hat{C}3_{it}(\theta) = C\hat{I}_{it}(\theta) \cdot CK_i , C\hat{I}_{it}(\theta) > 0 . \quad (2)$$

(2) The backorder cost

1) Maximum stockout quantity that customers can tolerate

Stockout will lead to a decline in customer satisfaction or loss of sales. Generally, the longer the replenishment time and the greater the backorder quantity are, the greater the possibility of loss of sales will be [19].

The customer's loss threshold is defined as Equation (3) according to the definition of exponential partial backlogging rate.

$$\beta(t'-t) = k_0 \cdot e^{-k_1(t'-t-1)} , \quad (3)$$

where k_0 is the backordering intensity coefficient, and k_1 is the waiting time resistance, and $0 < k_0 < 1,\ k_1 > 0$.

So, based on the demand $P\hat{D}_{it}(\theta)$ of the product i under a natural state θ during the period t, the maximum tolerant stockout quantity for customers in different delayed periods can be calculated as:

$$PB_{it}(t'-t,\theta) = P\hat{D}_{it}(\theta) \cdot \beta(t'-t) = P\hat{D}_{it}(\theta) \cdot k_0 \cdot e^{-k_1(t'-t-1)} . \quad (4)$$

2) Delayed delivery/loss of sales cost

The cost of delayed delivery per unit of out of stock quantity gradually increases with waiting time. The delayed delivery cost per unit of out of stock quantity is defined by the penalty fee for delayed delivery as follows:

$$cb_i(t'-t)=f_i+a_i\cdot(t'-t)+b_i\cdot(t'-t)^2\,, \tag{5}$$

where f_i is the fixed cost part of unit out of stock quantity, a_i and b_i are the cost rate and cost increasing rate of unit delivery cost, respectively.

If the ending inventory of a planning period $C\hat{I}_{it}(\theta)<0$, it indicates that there has been a delayed delivery situation in that planning period. Therefore, when the delayed delivery quantity in a natural state θ is less than or equal to the maximum tolerant stockout quantity, the backorder cost can be expressed as:

$$\hat{C}5_{it}(\theta)=-C\hat{I}_{it}(\theta)\cdot cb_i(t'-t),\; C\hat{I}_{it}(\theta)<0, -C\hat{I}_{it}(\theta)\le PB_{it}(t'-t)\;. \tag{6}$$

If the out of stock quantity exceeds the maximum tolerant stockout quantity, the enterprise will lose this part of the product. The cost of out of stock in a certain natural state θ can be expressed as:

$$\hat{C}5_{it}(\theta)=-C\hat{I}_{it}(\theta)\cdot cl_i\,, -C\hat{I}_{it}(\theta)>PB_{it}(t'-t)\,. \tag{7}$$

C. Objective Functions

The total production cost and the employee stability are mainly considered in APP problem. The total production cost mainly includes five aspects: product production cost, raw material cost, product inventory cost, employee cost and backorder cost. Due to the uncertainty of product inventory costs and delayed delivery/loss of sales costs, the expected value approach is used to handle the total cost. Then, the total production cost can be defined as

$$Z_1=\sum_{t=1}^{T}\left(C1_t+C2_t+E(\hat{C}3_{it})+C4_t+E(\hat{C}5_{it})\right), \tag{8}$$

where the total production cost $C1_t=\sum_{i=1}^{I}PP_{it}\cdot PC_i$, the total raw material cost $C2_t=\sum_{i=1}^{I}\sum_{j=1}^{J}PP_{it}\cdot R_{ij}\cdot RC_{jt}$, and $C4_t=WH_t\cdot WHC+W_t\cdot WI+WRT_t\cdot WRC+WOT_t\cdot WOC$.

$E(\hat{C}3_t)$ and $E(\hat{C}5_t)$ can be calculated by Equations (9) and (10).

$$E(\hat{C}3_{it})=\sum_{i=1}^{I}\sum_{\theta}\pi(C\hat{I}_{it},\theta)\cdot\hat{C}3_{it}(\theta)\,, \tag{9}$$

$$E(\hat{C}5_{it})=\sum_{i=1}^{I}\sum_{\theta}\pi(C\hat{I}_{it},\theta)\cdot\hat{C}5_{it}(\theta)\,, \tag{10}$$

where $E(\cdot)$ represents the expected value of a random variable, and $\pi(\cdot)$ expresses the prediction probability function of discrete random variables.

For the first planning period, if the initial inventory is a fixed value, then $\pi(C\hat{I}_{i1})=1$. The probability of occurrence in a certain natural state of other initial storage periods can be expressed as:

$$\pi(C\hat{I}_{it},\theta)=\sum_{t'=1}^{t-1}\pi(C\hat{I}_{it'},\theta)\cdot\pi(P\hat{D}_{it'},\theta)\,. \tag{11}$$

The sum of changes in employee numbers is used to measure its stability according to Equation (12).

$$Z_2=\sum_{t=1}^{T}\left(WH_t+WL_t\right), \tag{12}$$

where the hired worker number, $WH_t=\max\{W_t-W_{t-1},0\}$, and the number of laid off workers, $WL_t=\max\{W_{t-1}-W_t,0\}$.

D. Analysis of Constraints

APP problem generally requires consideration of constraints such as supply and demand, inventory capacity, production capacity, and worker labor capacity.

Production and inventory balance:

$$C\hat{I}_{it}=C\hat{I}_{it-1}+PP_{it-1}-P\hat{D}_{it-1}\,. \tag{13}$$

Workforce balance:

$$W_t=W_{t-1}+WH_t-WL_t\,. \tag{14}$$

Due to the fact that the initial inventory of each period is a random variable, we expect the warehouse capacity to meet the requirement at confidence level α_1. Then, the opportunity constraint of the inventory capacity can be expressed as:

$$\Pr\left\{C\hat{I}_{it}\le CN_i\right\}\ge\alpha_1\,. \tag{15}$$

During the production process, it is inevitable that the equipment will malfunction, be damaged or repaired, which will reduce the production capacity of the enterprise. Due to the above situation in the equipment, the production capacity of the enterprise changes randomly. Similarly, if it is expected that the production capacity meets the requirement at the confidence level α_2, the production capacity constraint can be expressed as:

$$\Pr\left\{PP_{it}\le P\hat{N}_{it}\right\}\ge\alpha_2\,. \tag{16}$$

If it is also expected to meet product requirement at a confidence level α_3, then the product demand constraints for each stage can be expressed as:

$$\Pr\left\{PP_{it}+C\hat{I}_{it}\ge P\hat{D}_{it}\right\}\ge\alpha_3\,. \tag{17}$$

The production time constraint of workers can be expressed as:

$$\sum_{i=1}^{I}PP_{it}\cdot PW_i\le W_t(WR+WO)\,. \tag{18}$$

E. The Optimization Model

Based on the above analysis, the multi-objective stochastic APP model in a random environment is constructed as follows:

$$\min\, Z_1=\sum_{t=1}^{T}\left(C1_t+C2_t+E(\hat{C}3_t)+C4_t+E(\hat{C}5_t)\right),$$

$$\min\, Z_2=\sum_{t=1}^{T}\left(WH_t+WL_t\right),$$

$$\text{s.t.}\begin{cases} C\hat{I}_{it}=C\hat{I}_{it-1}+PP_{it-1}-P\hat{D}_{it-1}, & \forall i,t,\\ W_t=W_{t-1}+WH_t-WL_t, & \forall t,\\ \Pr\left\{C\hat{I}_{it}\le CN_i\right\}\ge\alpha_1, & \forall i,t,\\ \Pr\left\{PP_{it}\le P\hat{N}_{it}\right\}\ge\alpha_2, & \forall i,t,\\ \sum_{i=1}^{I}PP_{it}\cdot PW_i\le W_t(WR+WO), & \forall t,\\ \Pr\left\{PP_{it}+C\hat{I}_{it}\ge P\hat{D}_{it}\right\}\ge\alpha_3, & \forall i,t,\\ W_t, PP_{it}\in N, C\hat{I}_{it}\in Z, & \forall i,t. \end{cases}$$

III. Design Of Solving Algorithms

According to the comparative analysis of several algorithms of [19], it can be seen that the quality of solutions obtained by the local search-based GA (LS-GA), hybrid genetic algorithm-particle swarm optimization based on stages (HGA-PSO1) and multi population strategy (HGA-PSO2) is relatively good, and HGA-PSO1 and HGA-PSO2 have significant advantages in computational time. Therefore, two stochastic hybrid genetic algorithm-particle swarm optimization algorithms based on stages (SHGA-PSO1) and multi population strategy (SHGA-PSO2) combined with the local search (LS) strategy are designed to solve -objective stochastic APP model.

A. Algorithm Preprocessing

(1) Chromosome encoding

The APP model in this paper uses the production quantity of product PP_{it} and the number of employees in each period W_t as the decision variables, and both are positive integers. Therefore, this paper still adopts the encoding method of positive integers, as detailed in [20]. Fig.1 shows a sub-chromosome P_{it} represents the production quantity with I product categories and a sub-chromosome W_t represents the number of workers with T periods.

	Period (t)	**1**	**2**	**...**	***T***
P	**Category 1**	PP_{11}	PP_{12}	...	PP_{1T}
	Category 2	PP_{21}	PP_{22}	...	PP_{2T}
	...	...			
	Category *I*	PP_{I1}	PP_{I2}	...	PP_{IT}
W	**Worker number**	W_1	W_2	...	W_T

Fig. 1. A simple example of a chromosome

(2) Handling of opportunity constraints

Since $C\hat{I}_{it} = C\hat{I}_{it-1} + PP_{it-1} - P\hat{D}_{it-1}$ and the production quantity is deterministic, the probability distribution of random inventory $C\hat{I}_{it}$ is the same as that of $P\hat{D}_{it-1} - C\hat{I}_{it-1}$. Let the probability distribution function of $P\hat{D}_{it-1} - C\hat{I}_{it-1}$ be $\phi(\cdot)$, and $\phi^{-1}(\cdot)$ be its inverse function. So, at a confidence level α_1, the opportunity constraint of inventory capacity can be transformed into:

$$\Pr\left\{PP_{it-1} - CN_i \le P\hat{D}_{it-1} - C\hat{I}_{it-1}\right\} \ge \alpha_1 . \tag{19}$$

Then, equation (14) can be equivalent to:

$$PP_{it-1} - \sup\left\{K \middle| K = \phi_{it-1}^{-1}(1-\alpha_1)\right\} \le CN_i . \tag{20}$$

Similarly, at a confidence level α_2, the production capacity opportunity constraint can be equivalent to:

$$PP_{it} \le \sup\left\{K \middle| K = \varphi_{it}^{-1}(1-\alpha_2)\right\}, \tag{21}$$

where $\varphi(\cdot)$ and $\varphi^{-1}(\cdot)$ are the probability distribution function and its inverse function of the production capacity $P\hat{N}_{it}$, respectively.

(3) Feasible range for production quantity

Due to the fact that the backorder cost is greater than the cost of on-time delivery, the on-time delivery strategy should be adopted when the production capacity is sufficient. If the production capacity exceeds the demand, the minimum production quantity can meet the minimum demand of the planned period. Otherwise, the production should be at its maximum production capacity. Therefore, the minimum production quantity of each product during the planning period can be expressed as:

$$\mathrm{MinPP}_{it} = \max\left\{0, \min\left\{P\hat{D}_{it} - C\hat{I}_{it}, \sup\left\{K \middle| K = \varphi_{it}^{-1}(1-\alpha_2)\right\}\right\}\right\}. \tag{22}$$

In addition, according to the equation (19) and equation (20), the maximum production quantity of each product should not exceed the production capacity and inventory capacity. Therefore, at the confidence level α_1 and α_2, the maximum production quantity of each product during the planning period can be expressed as:

$$\begin{aligned}\mathrm{MaxPP}_{it} = \min\Big\{&\sup\left\{K \middle| K = \phi_{it-1}^{-1}(1-\alpha_1)\right\} + CN_i, \\ &\sup\left\{K \middle| K = \varphi_{it}^{-1}(1-\alpha_2)\right\}\Big\}\end{aligned} . \tag{23}$$

(4) Feasible range of the number of employees

The total working hours required for the planning period t can be calculated based on the production quantity PP_{it} and the working hours required per unit product PW_i, so the minimum number and maximum number the of employees can be calculated using equation (24) and equation (25), respectively ($\lceil\ \rceil$ represents rounding up).

$$\mathrm{MinW}_t = \left\lceil \sum_{i=1}^{I} \mathrm{MinPP}_{it} \cdot PW_i \,/\, (WR + WO) \right\rceil . \tag{24}$$

$$\mathrm{MaxW}_t = \left\lceil \sum_{i=1}^{I} \mathrm{MaxPP}_{it} \,/\, WR \right\rceil . \tag{25}$$

B. Genetic Algorithm Design

(1) Initial population generation

In the process of initial population generation, the genes of sub-chromosome P and W are generated within the feasible range of product production and employee numbers. The specific method for generating a chromosome is as follows:

Step1. Initialize known parameters.

Step2. For t = 1 to T, repeat Step3 to Step4.

Step3. For i = 1 to I, calculate MinPP_{it} and MaxPP_{it}, and generate a random integer number $PP_{it} \in [\mathrm{MinPP}_{it}, \mathrm{MaxPP}_{it}]$ as a gene of sub-chromosome P. Calculate the ending inventory of products and its probability distribution $\phi(\cdot)$ according to equations (1) and (11).

Step4. Calculate MinW_t and MaxW_t, and generate a random integer number $W_t \in [\mathrm{MinW}_t, \mathrm{MaxW}_t]$ as a gene of sub-chromosome W.

Fig.2 is a schematic diagram of generating a sub-chromosome P.

The demands for this product in four planning periods are (80, 90, 100), (100, 110, 120), (100, 110, 120), and (130, 140, 150), respectively, with the corresponding probability distributions of (30%, 40%, 30%). The initial inventory of the first phase is 10, and the production capacity is (90, 100, 110), (120, 130, 140), (100, 110, 120), (130, 140, 150), respectively. The corresponding probability distributions are (10%, 20%, 70%), the confidence levels α_1 and α_2 both are 80%, and the inventory capacity is 50. According to equations (22) and (23), the minimum production capacity for the first phase is 70, and the maximum production capacity is $\min\{120, 100\} = 100$. Therefore, genes can be generated based on $PP_{it} \in [70,100]$. If the gene generated in the first planning period is 90, then update the inventory and probability distribution of the next period of the product according to equations (1) and (11) as (20,10,0) and (30%, 40%, 30%), respectively. Similarly, a feasible sub-chromosome of P can be obtained.

Period (t)	1	2	3	4
$P\hat{D}_{it}$	80,90,100	100,110,120	100,110,120	130,140,150
$P\hat{N}_{it}$	90,100,110	120,130,140	100,110,120	130,140,150
$C\hat{I}_{it}$	10	20,10,0	20,10,0, -10,-20	30,20,10,0, -10,-20,30
$\pi(C\hat{I}_{it},\theta)$	100	30,40,30	9,24,34,24,9	2.7,10.8,22.5, 28,22.5,10.8,2.7
$[\mathrm{MinPP}_{it}, \mathrm{MaxPP}_{it}]$	70 100	80 130	80 110	100 140
P	90	100	100	130

Fig. 2. Examples of the production process of chromosome

According to equations (22) and (23), the feasible range of yield sub-chromosomes is determined, and the genetic operators for gene crossover, mutation, chromosome repair and selection operations are designed as that of genetic algorithm in [20].

(2) Local search strategy

In order to improve the search depth of genetic algorithms, this paper designs a local search strategy. According to [20], the production quantity optimization problem is a special case of the minimum-cost flow (MCF) problem. Therefore, the LS strategy for production quantity can be designed based on the augmenting cycle.

For the sub-chromosome P, there are various situations in the initial inventory of the product during local search based on the augmented circle algorithm. When two different planning periods, $t_1, t_2 \in [1,T]$, are randomly selected to form a circle, the adjustable production will change accordingly. According to equations (22) and (23), the possible adjustment range of the augmenting circle can be obtained.

1) Anticlockwise cycle

If the formed circle is counterclockwise (as shown in Fig.3, θ_I is the adjustment flow), the maximum flow increase of the augmenting circle is limited by three factors: production quantity of t_1, inventory capacity from t_1 to t_2, and production quantity of t_2.

The production quantity of t_1: The maximum flow that can be increased during the t_1 planning period is limited by its maximum production quantity, which can be determined according to equation (23). The increase in maximum flow of the augmenting circle can be expressed as $\mathrm{MaxPP}_{it_1} - PP_{it_1}$.

Inventory surplus capacity: The inventory that can be increased is $CN_i - C\hat{I}_{it}$. Since $C\hat{I}_{it} = C\hat{I}_{it-1} + PP_{it-1} - P\hat{D}_{it-1}$, the minimum inventory surplus capacity from t_1 to t_2 can be inferred from equation (20) at the confidence level α_1, which can be expressed as:

$$\min\left\{CN_i - PP_{it-1} + \sup\left\{K \middle| K = \phi_{it-1}^{-1}(1-\alpha_1)\right\} \middle| t_1 < t \le t_2\right\}. \quad (26)$$

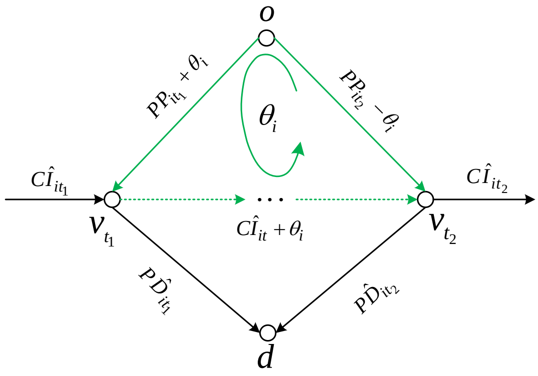


Fig. 3. Examples of flow increase in anticlockwise circle

The production quantity of t_2: The adjustment amount for the production limit during t_2 is $PP_{it_2} - \mathrm{MinPP}_{it_2}$.

Then, the maximum adjustment amount of anticlockwise cycle for product i can be obtained by the following equation:

$$\begin{aligned} FP_i = \min\Big\{&\mathrm{MaxPP}_{it_1} - PP_{it_1}, PP_{it_2} - \mathrm{MinPP}_{it_2}, \\ &CN_i - PP_{it-1} + \sup\left\{K \middle| K = \phi_{it-1}^{-1}(1-\alpha_1)\right\} \Big| t_1 < t \le t_2\Big\}. \end{aligned} \quad (27)$$

Taking the chromosome in Fig.2 as an example, the two randomly selected planning periods are t_1=1 and t_2=3, and according to equation (27), the maximum adjustment can be obtained as $FP_i = \min\{10, 40, 20\}$=10. As shown in Fig.4, there is a counterclockwise flow increasing chain, with a maximum adjustment of 10.

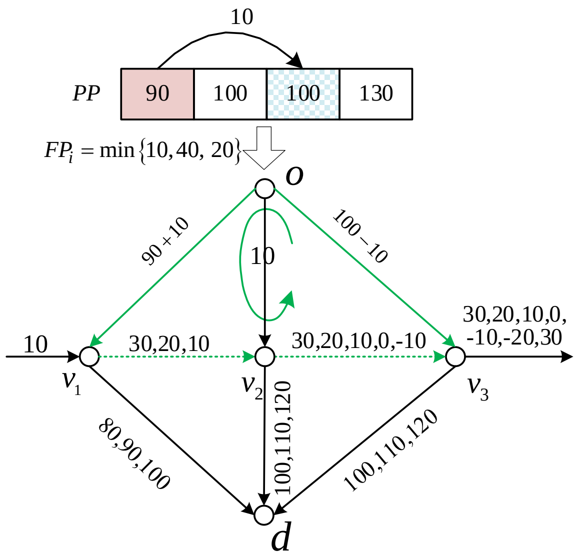


Fig. 4. Illustration of local search for sub-chromosomes pp in anticlockwise circle

2) Clockwise cycle

If the formed circle is in a clockwise direction (as shown in Fig.5), the maximum flow increase is also limited by the production quantity of t_1, inventory from t_1 to t_2, and production quantity of t_2.

The production quantity of t_1: The adjustment amount for the production limit during t_1 is $PP_{it_1} - \mathrm{MinPP}_{it_1}$.

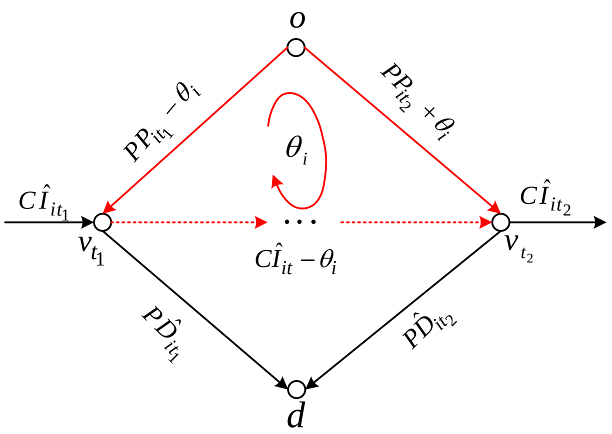


Fig. 5. Example of flow increase in clockwise circle

Inventory from t_1 to t_2. The minimum inventory quantity from t_1 to t_2 is $\min\left\{\max(C\hat{I}_{it})|t_1 < t \le t_2\right\}$.

The production quantity of t_2: The adjustment amount for the maximum production quantity limit during t_2 is $\mathrm{MaxPP}_{it_2} - PP_{it_2}$.

Then, when the circle is clockwise, the maximum adjustment amount of product i can be obtained by the following equation:

$$\begin{aligned} FP_i = \min\{ & PP_{it_1} - \mathrm{MinPP}_{it_1}, \max(C\hat{I}_{it}), \\ & \mathrm{MaxPP}_{it_2} - PP_{it_2} \mid t_1 < t \le t_2 \} \end{aligned} . \quad (28)$$

C. Particle Swarm Optimization Algorithm Design

Since *Gbest* is the best position ever found by all the particles that can guide particles towards the global optimal position. Selecting a suitable global optimal position *Gbest* to guide each particle will greatly improve the quality of the obtained Pareto solutions and maintain the diversity of non-dominated solutions. An external file is established, mainly used to record the global Pareto solution set and Pareto frontiers of the population. Furthermore, as the Particle Swarm Optimization (PSO) algorithm evolves, external file is updated. The global optimal guide selection mechanism, as well as the individual global optimal guide allocation and selection mechanism, is detailed in [19].

During each iteration, each particle updates its speed and position according to the following strategy based on the global and individual optimal guides selected for allocation [19], [21]:

$$\begin{aligned} v_k^1(g+1) = \chi\big[& \omega_k v_k^1(g) + c_1 r_1 (pb_k^1(g) - x_k^1(g)) \\ & + c_2 r_2 (gb_k^1(g) - x_k^1(g)) \big] \end{aligned} , \quad (29)$$

$$\begin{aligned} v_k^2(g+1) = \chi\big[& \omega_k v_k^2(g) + c_1 r_1 (pb_k^2(g) - x_k^2(g)) \\ & + c_2 r_2 (gb_k^2(g) - x_k^2(g)) \big] \end{aligned} , \quad (30)$$

$$x_k^1(g+1) = x_k^1(g) + \left\lceil v_k^1(g+1) \right\rceil, \quad (31)$$

$$x_k^2(g+1) = x_k^2(g) + \left\lceil v_k^2(g+1) \right\rceil, \quad (32)$$

where k is *a* particle, χ is the constriction factor, and ω_k is the inertia weight. c_1, c_2 are respectively learning factors. r_1, r_2 are two random parameters and g is the number of iterations. $v_k^1(g)$ and $v_k^2(g)$ represent the velocity of particle k in P layer and W layer of iteration g, respectively. $pb_k^1(g)$, $pb_k^2(g)$, $gb_k^1(g)$ and $gb_k^2(g)$ represent the local optimal guide and global optimal guide, respectively. $x_k^1(g)$ and $x_k^2(g)$ are the position of particle *k*.

After updating the particle speed and position, the *P* and *W* layers of the particles may not be feasible and need to be repaired.

Repair method of *P* layer: Firstly, calculate the feasible range of product production, $[\mathrm{MinPP}_{it}, \mathrm{MaxPP}_{it}]$,based on equations (22) and (23). Secondly, check whether the position of a new particle of *P* layer meets $\mathrm{MinPP}_{it} \le PP_{it} \le \mathrm{MaxPP}_{it}$. If it is not satisfied, the position of the unsatisfied particle is regenerated, that is, a $PP'_{it} \in [\mathrm{MinPP}_{it}, \mathrm{MaxPP}_{it}]$ is randomly generated as a position of the particle.

Repair method of *W* layer: Calculate the feasible range of worker number based on equations (24) and (25), $[\mathrm{MinW}_t, \mathrm{MaxW}_t]$, and check whether the position of a new particle of *W* layer meets $\mathrm{MinW}_t \le W_t \le \mathrm{MaxW}_t$. If it is not satisfied, a $W_t \in [\mathrm{MinW}_t, \mathrm{MaxW}_t]$ is randomly generated as a position of *W*.

D. Hybrid Strategies

The idea of the SHGA-PSO1 strategy is: Firstly, the PSO algorithm is used for global search. When it falls into a local optimal solution, a genetic algorithm based on LS is used to perform local search in the neighborhood of the global optimal solution, in order to improve the quality of the Pareto solution set.

The idea of the SHGA-PSO2 strategy is: Divide the PSO population into two subgroups (GA and PSO), with sizes set as *POPSIZE*/4 and *POPSIZE*×3/4, respectively, and use a selection method based on non-dominated sorting and crowding distance sorting to complete the clustering operation. The GA subgroup uses selection operations to select the initial population from external files, while the PSO subgroup selects individual particles from the overall parent population. The GA subgroup utilizes a local search based genetic algorithm for optimization, while the PSO subgroup utilizes the PSO algorithm for global search.

IV. Numerical Experiments

In order to verify the effectiveness of SHGA-PSO1 and SHGA-PSO2 and analyze the impact of uncertain conditions on the calculation results, this section uses these two algorithms to perform 10 calculations on 4 test cases. In addition, the sensitivity of product demand and production capacity under different confidence levels is analyzed using numerical examples.

A. Algorithm Performance Analysis

The average objectives value ($\mathrm{avg}(\cdot)$) [22], the runtime of algorithm (Runtime (S)), the number of nondominated set (M_1), set coverage measure (M_2) [23]-[24] and mean ideal distance (*MID*), are used to analyze the performance of two algorithms.

The maximum iteration number $G = 1000$, the number of chromosomes $popsize = 30$. Moreover, the constriction factor $\chi = 0.73$, the maximum and minimum inertia weight $\omega_{\max}$=0.8 , $\omega_{\min}$=0.4 , and learning factors c_1=2.0 , c_2=2.1 , respectively. The backordering intensity coefficient k_0=0.25 , the waiting time resistance k_1=0.3 , the fixed cost part f_i=0.05PC_i , the cost rate a_i=0.5f_i and cost increasing rate b_i=0.05f_i . Confidence levels α_1 , α_2 and α_3 are all 80%. The unit lost sales cost cl_i=0.5PC_i .

The initial inventory information and raw material related parameters are shown in Table II, the unit production cost, working hours, and raw material cost are shown in Table III, the product demand and its probability are shown in Table IV, and the production capacity and probability of each period are shown in Table V.

TABLE II
INITIAL INVENTORY INFORMATION AND RAW MATERIAL COMPOSITION PARAMETERS

i	CI_{i1}	CK_i	CN_i	R_{ij}		
				1	2	3
1	60	1.0	100	0.8	0.5	0
2	30	1.0	50	0.3	0.5	0.4
3	10	0.5	70	0.2	0.3	0.3
4	20	1.0	100	0.5	0.2	0.6

TABLE III
THE UNIT PRODUCTION COST, WORKING HOURS AND RAW MATERIAL COST

Parameter	i, j	t					
		1	2	3	4	5	6
PC_i /(Yuan)	1	500	500	500	500	500	500
	2	450	450	450	450	450	450
	3	200	200	200	200	200	200
	4	300	300	300	300	300	300
PW_i /(h)	1	1.0	1.0	1.0	1.0	1.0	1.0
	2	1.5	1.5	1.5	1.5	1.5	1.5
	3	2.0	2.0	2.0	2.0	2.0	2.0
	4	0.8	0.8	0.8	0.8	0.8	0.8
RC_{jt} /(Yuan)	1	2.0	2.0	3.0	1.0	2.0	2.0
	2	3.0	2.0	3.0	3.0	2.0	2.0
	3	3.0	3.5	3.0	2.8	3.0	4.0

TABLE IV
THE PRODUCT DEMAND AND ITS PROBABILITY DISTRIBUTION

Experiment No.	i		t					
			1	2	3	4	5	6
S1	1	$P\hat{D}_{it}$	(70,80,90)	(90,100,110)	(150,160,170)	(200,220,250)	--	--
		$\pi(P\hat{D}_{it})$	(0.3,0.4,0.3)	(0.2, 0.6, 0.2)	(0.3, 0.5, 0.2)	(0.2,0.6,0.2)	--	--
	2	$P\hat{D}_{it}$	(20,30,40)	(40,50,60)	(40,50,60)	(50,60,70)	--	--
		$\pi(P\hat{D}_{it})$	(0.2,0.5,0.3)	(0.2,0.6,0.2)	(0.3,0.5,0.2)	(0.3,0.4,0.3)	--	--
S2	1	$P\hat{D}_{it}$	(70,80,90)	(110,120,130)	(160,170,180)	(200,210,220)	(180,200,220)	(140,150,160)
		$\pi(P\hat{D}_{it})$	(0.3,0.4,0.3)	(0.3,0.4,0.3)	(0.3,0.5,0.2)	(0.3,0.4,0.3)	(0.3,0.5,0.2)	(0.2,0.6,0.2)
	2	$P\hat{D}_{it}$	(20,30,40)	(40,50,60)	(50,60,70)	(50,60,70)	(70,80,90)	(50,60,70)
		$\pi(P\hat{D}_{it})$	(0.3,0.5,0.2)	(0.2,0.6,0.2)	(0.3,0.5,0.2)	(0.3,0.4,0.3)	(0.2,0.6,0.2)	(0.2,0.6,0.2)
S3	1	$P\hat{D}_{it}$	(80,90,100)	(110,120,130)	(170,180,190)	(190,200,210)	--	--
		$\pi(P\hat{D}_{it})$	(0.3,0.4,0.3)	(0.3, 0.5, 0.2)	(0.3, 0.5, 0.2)	(0.2,0.6,0.2)	--	--
	2	$P\hat{D}_{it}$	(20,30,40)	(40,45,50)	(40,50,60)	(70,80,90)	--	--
		$\pi(P\hat{D}_{it})$	(0.2,0.5,0.3)	(0.3,0.4,0.3)	(0.2,0.6,0.2)	(0.2,0.5,0.3)	--	--
	3	$P\hat{D}_{it}$	(40,50,60)	(50,60,70)	(50,60,70)	(60,70,80)	--	--
		$\pi(P\hat{D}_{it})$	(0.2,0.6,0.2)	(0.3,0.4,0.3)	(0.2,0.5,0.3)	(0.2,0.5,0.3)	--	--
	4	$P\hat{D}_{it}$	(90,100,110)	(110,120,130)	(80,90,100)	(90,100,110)	--	--
		$\pi(P\hat{D}_{it})$	(0.3,0.4,0.3)	(0.3,0.4,0.3)	(0.2,0.5,0.3)	(0.3,0.4,0.3)	--	--
S4	1	$P\hat{D}_{it}$	(80,85,90)	(120,130,140)	(170,180,190)	(200,210,220)	(190,200,210)	(170,180,190)
		$\pi(P\hat{D}_{it})$	(0.3,0.4,0.3)	(0.3, 0.5, 0.2)	(0.2, 0.6, 0.2)	(0.2,0.6,0.2)	(0.3,0.5,0.2)	(0.3, 0.4, 0.3)
	2	$P\hat{D}_{it}$	(20,30,40)	(40,50,60)	(40,50,60)	(70,80,90)	(60,70,80)	(60,65,70)
		$\pi(P\hat{D}_{it})$	(0.2,0.5,0.3)	(0.2,0.5,0.3)	(0.2,0.6,0.2)	(0.2,0.5,0.3)	(0.3,0.4,0.3)	(0.3,0.4,0.3)
	3	$P\hat{D}_{it}$	(40,50,60)	(50,60,70)	(50,60,70)	(60,70,80)	(50,55,60)	(70,75,80)
		$\pi(P\hat{D}_{it})$	(0.2,0.6,0.2)	(0.3,0.4,0.3)	(0.2,0.5,0.3)	(0.2,0.5,0.3)	(0.2,0.5,0.3)	(0.3,0.4,0.3)
	4	$P\hat{D}_{it}$	(90,100,110)	(110,120,130)	(80,90,100)	(90,100,110)	(90,100,110)	(110,120,130)
		$\pi(P\hat{D}_{it})$	(0.2,0.5,0.3)	(0.3,0.4,0.3)	(0.2,0.5,0.3)	(0.2,0.4,0.4)	(0.3,0.4,0.3)	(0.2,0.5,0.3)

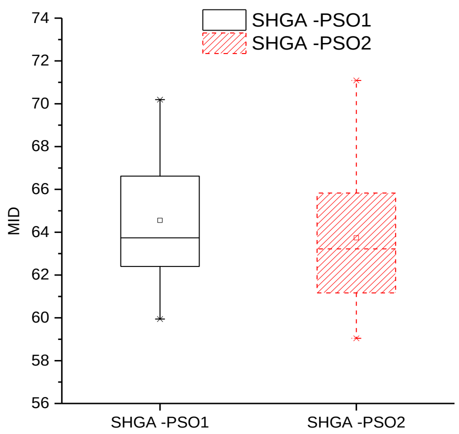
TABLE V
PRODUCT PRODUCTION CAPACITY AND ITS PROBABILITY DISTRIBUTION

Experiment No.	i		t					
			1	2	3	4	5	6
S1	1	$P\hat{N}_{it}$	(80,90,100)	(120,150,170)	(180,190,200)	(240,250,260)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1, 0.2, 0.7)	(0.1, 0.1, 0.8)	(0.1,0.2,0.7)	--	--
	2	$P\hat{N}_{it}$	(70,80,90)	(70,80,90)	(100,110,120)	(100,110,120)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	--	--
S2	1	$P\hat{N}_{it}$	(80,90,100)	(140,160,170)	(180,190,200)	(240,250,260)	(210,220,230)	(180,190,200)
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1, 0.2, 0.7)	(0.1, 0.2, 0.7)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1, 0.2, 0.7)
	2	$P\hat{N}_{it}$	(60,70,80)	(70,80,90)	(120,130,140)	(100,110,120)	(80,90,100)	(80,90,100)
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.3,0.6)	(0.1,0.2,0.7)
S3	1	$P\hat{N}_{it}$	(100,110,120)	(120,140,150)	(180,190,200)	(230,240,250)	--	--
		$\pi(P\hat{N}_{it})$	(0.1, 0.2, 0.7)	(0.2, 0.2, 0.6)	(0.1, 0.2, 0.7)	(0.1,0.2,0.7)	--	--
	2	$P\hat{N}_{it}$	(60,70,80)	(70,80,90)	(130,140,150)	(90,100,110)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	--	--
	3	$P\hat{N}_{it}$	(60,70,75)	(50,60,70)	(60,70,80)	(60,70,80)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	--	--
	4	$P\hat{N}_{it}$	(130,140,150)	(130,140,150)	(130,140,150)	(130,140,150)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	--	--
S4	1	$P\hat{N}_{it}$	(100,110,120)	(150,160,170)	(180,190,200)	(240,250,260)	(200,210,220)	(200,210,220)
		$\pi(P\hat{N}_{it})$	(0.2, 0.2, 0.6)	(0.1, 0.2, 0.7)	(0.1, 0.1, 0.8)	(0.1,0.2,0.7)	(0.1, 0.1, 0.8)	(0.1, 0.2, 0.7)
	2	$P\hat{N}_{it}$	(60,70,80)	(70,80,90)	(130,140,150)	(100,110,120)	(80,90,100)	(80,90,100)
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.2,0.7)
	3	$P\hat{N}_{it}$	(60,70,75)	(50,60,70)	(60,70,80)	(60,70,80)	(60,70,75)	(60,70,80)
		$\pi(P\hat{N}_{it})$	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.8)
	4	$P\hat{N}_{it}$	(130,140,150)	(130,140,150)	(140,150,160)	(130,140,150)	(130,140,150)	(120,130,140)
		$\pi(P\hat{N}_{it})$	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.8)

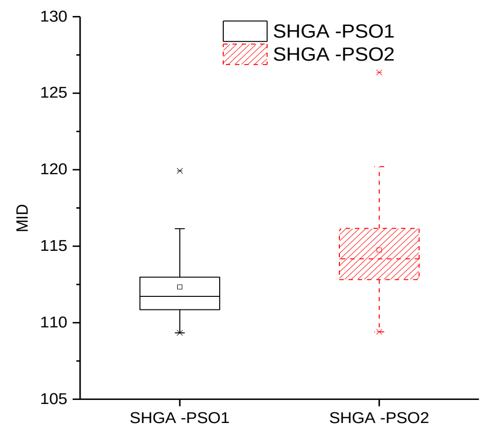
The results obtained after 10 calculations are shown in Fig.6. From the comparison results of evaluation indicators avg(Z_1), avg(Z_2), M_1 and MID, there is no significant difference in the performance of SHGA-PSO1 and SHGA-PSO2. From the perspective of algorithm runtime, the SHGA-PSO1 algorithm has a slightly shorter runtime than the SHGA-PSO2 algorithm (as shown in Fig.7).

Comparing the running time of deterministic and stochastic APP examples of the same scale, the results are shown in Fig.8. It can be seen that the running time of stochastic APP examples is significantly longer than that of deterministic APP examples of the same scale. Due to the fact that the number of natural states of random variables in stochastic APP problems increases with the increase of planning period, the calculation time of stochastic APP problems increases sharply with the increase of problem size. For example, for 2 products with 4 planning periods, it takes about 70 seconds for deterministic APP, while for stochastic APP, it takes about 190 seconds. The calculation of 6 planning periods for 4 products takes approximately 120 seconds for deterministic APP, while the calculation for stochastic APP takes approximately 450 seconds.

The obtained Pareto solution sets of two algorithms are compared in Fig.9. The results show that there is no significant difference in the number of nondominated sets obtained by two algorithms. However, in terms of the set coverage measure, the overall performance of SHGA-PSO2 algorithm is slightly better than that of SHGA-PSO1 algorithm.

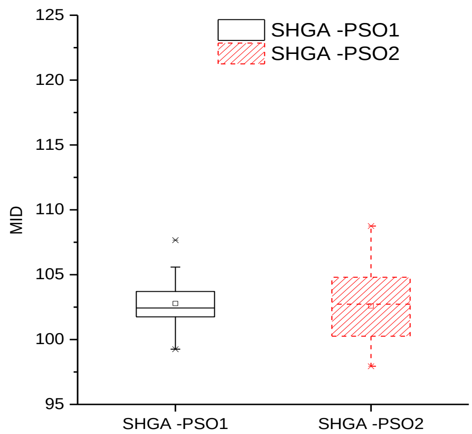


(a) Experiment

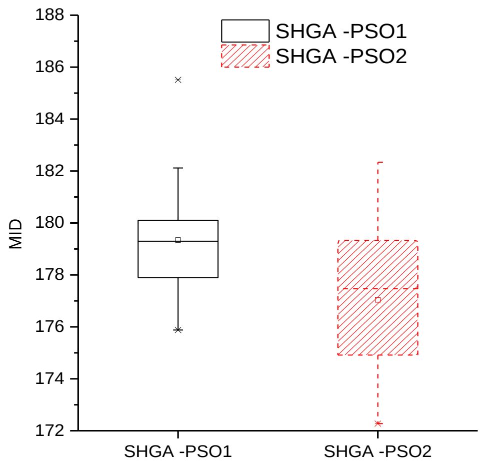


S1

(b) Experiment S2

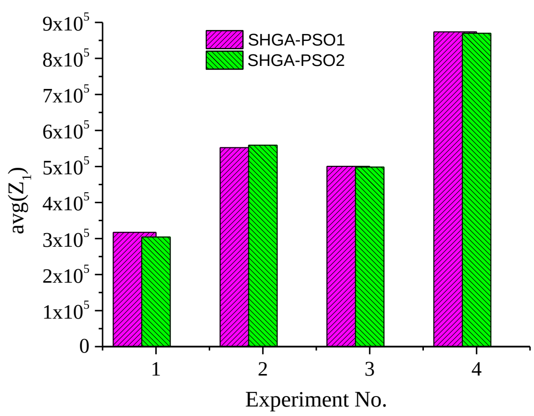


(c) Experiment S3

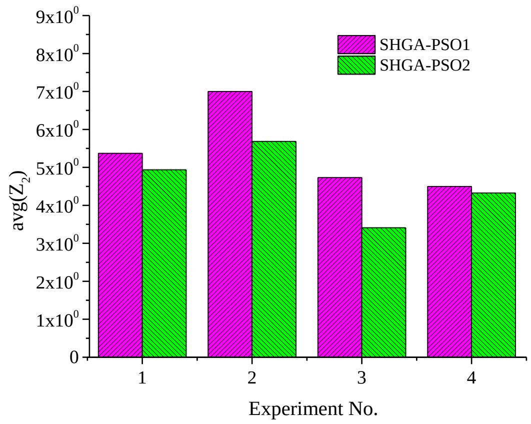


(d) Experiment S4

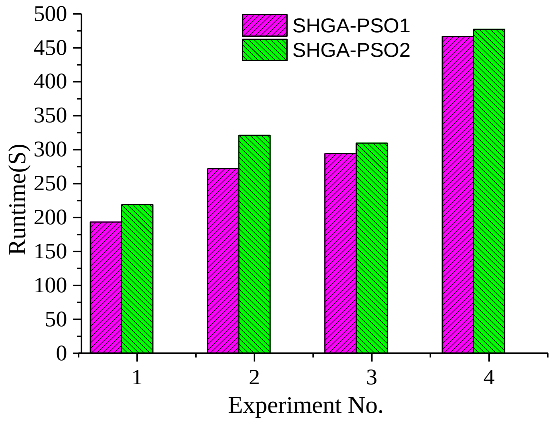
Fig. 6. Comparison of calculation results between two strategies.



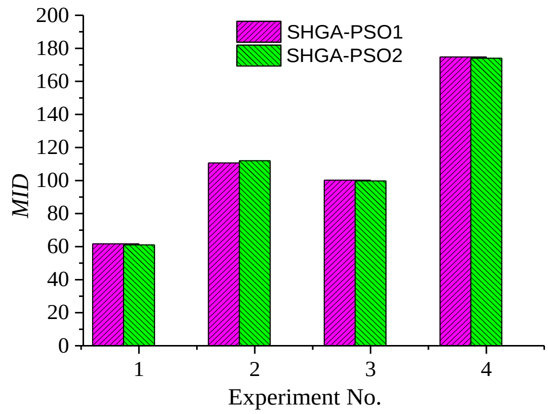
(a) avg(Z_1)



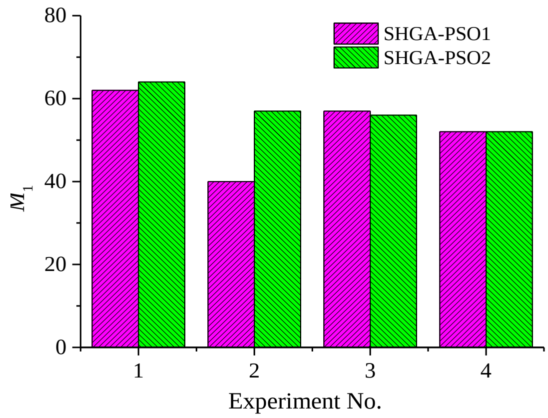
(b) avg(Z_2)



(c) Runtime



(d) *MID*



(e) M_1

Fig.7. Comparative analysis of performance measures

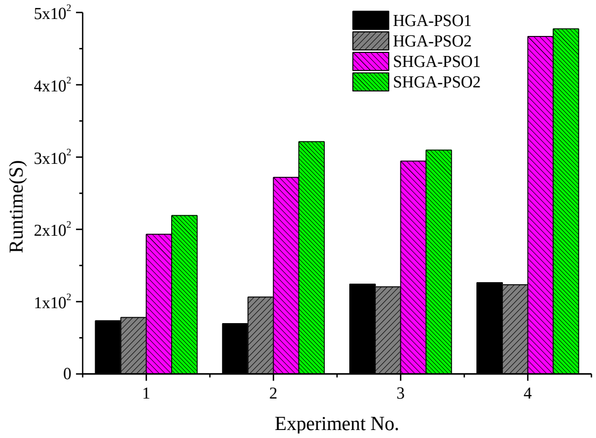
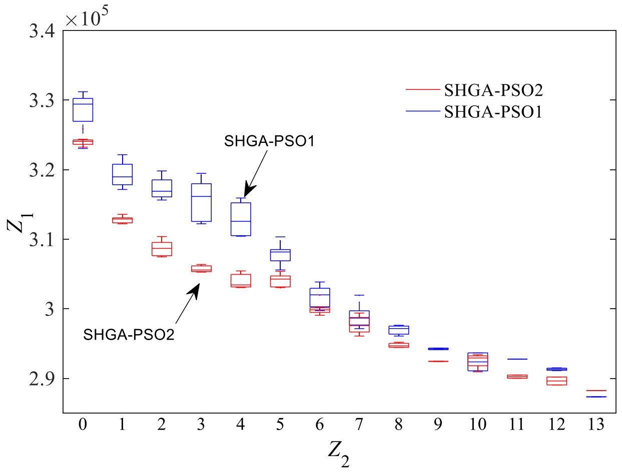
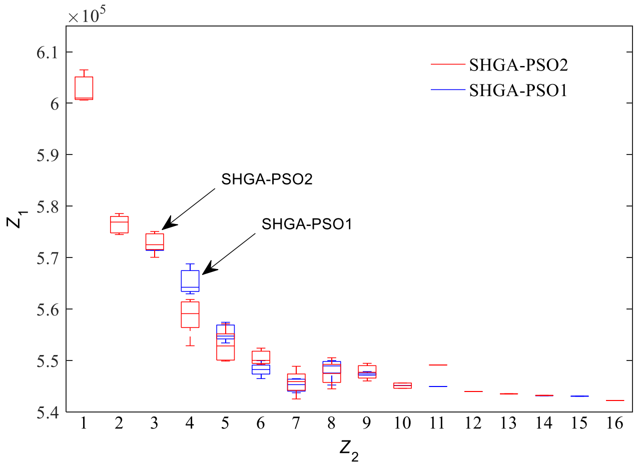


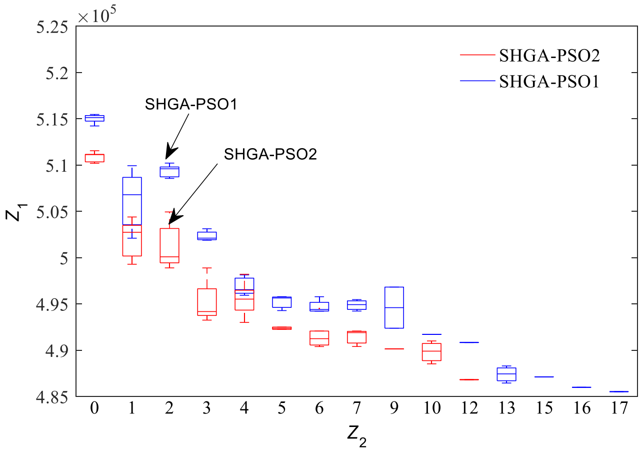
Fig.8. Comparative analysis of running time between deterministic and stochastic examples



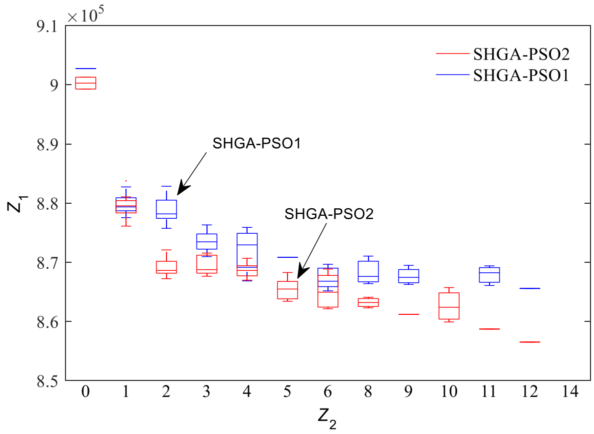
(a) Experiment S1



(b) Experiment S2



(c) Experiment S3



(d) Experiment S4

Fig.9. Comparison of the Pareto solution sets obtained by two algorithms

B. Sensitivity Analysis

Due to the setting of product demand $P\hat{D}_{it}$ and production capacity $P\hat{N}_{it}$ as random variables in this paper, the credibility of market forecasts by enterprises in uncertain environments can directly affect their production plans. Overoptimistic factors can lead to product backlog, while pessimistic factors can lead to loss of sales. Therefore, this section conducts sensitivity analysis on different confidence levels based on numerical examples above.

Based on the above data, 4 calculation examples are used for comparative analysis of product demand and production capacity at different confidence levels (calculated 5 times for each example at different confidence levels). As it is a multi-objective problem, MID parameters and average total production cost $\mathrm{avg}(Z_1)$ are used to measure the impact of confidence level changes.

(1) Sensitivity analysis of confidence level for production capacity

The impact of the change in confidence level of production capacity α_2 is shown in Fig.10 (α_1=0.8 , α_3=0.8). It can be seen that as the confidence level of production capacity continues to increase, the results of MID indicator remain basically unchanged. The main reason for this result is that the production capacity in the example is sufficient, and the impact on production is relatively small.

Adjust the production capacity and probability of each stage to Table VI. After calculation, the results of total production cost and MID index are shown in Fig.11 and Fig.12 It can be seen that when production capacity is limited, the total production cost and MID index show an upward trend as the confidence level of production capacity increases.

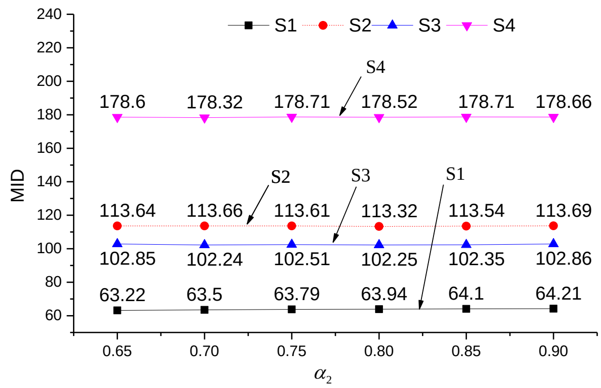
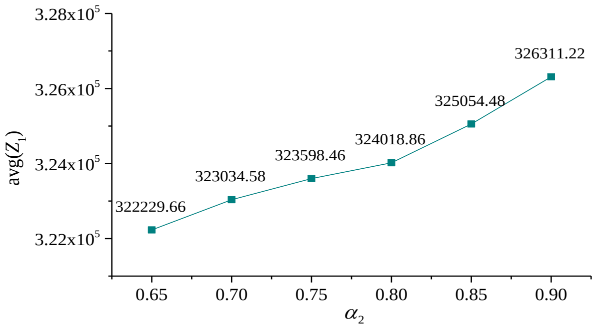


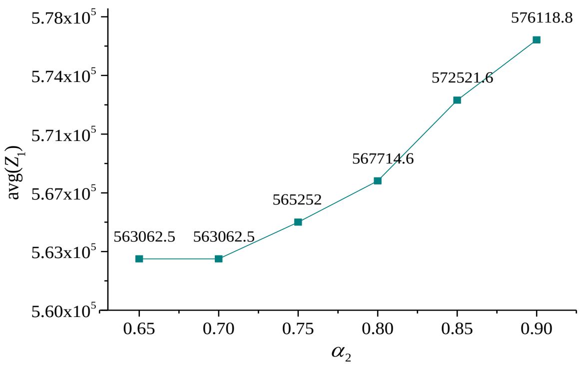
Fig.10. Sensitivity analysis of production capacity confidence level

TABLE VI
THE PRODUCTION CAPACITY AND ITS PROBABILITY DISTRIBUTION

Experiment No.	i		t					
			1	2	3	4	5	6
S1-1	1	$P\hat{N}_{it}$	(80,90,100)	(70,80,90)	(180,190,200)	(240,250,260)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.2, 0.2, 0.6)	(0.1, 0.1, 0.8)	(0.1,0.2,0.7)	--	--
	2	$P\hat{N}_{it}$	(70,80,90)	(70,80,90)	(30,40,50)	(100,110,120)	--	--
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	--	--
S2-1	1	$P\hat{N}_{it}$	(80,90,100)	(140,160,170)	(140,150,160)	(240,250,260)	(210,220,230)	(180,190,200)
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1, 0.2, 0.7)	(0.1, 0.2, 0.7)	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1, 0.2, 0.7)
	2	$P\hat{N}_{it}$	(60,70,80)	(70,80,90)	(40,50,60)	(100,110,120)	(80,90,100)	(80,90,100)
		$\pi(P\hat{N}_{it})$	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0.1,0.2,0.7)	(0.1,0.2,0.7)	(0.1,0.3,0.6)	(0.1,0.2,0.7)



(a) Experiment S1-1



(b) Experiment S2-1

Fig.11. Changes in total production cost under different confidence levels of production capacity

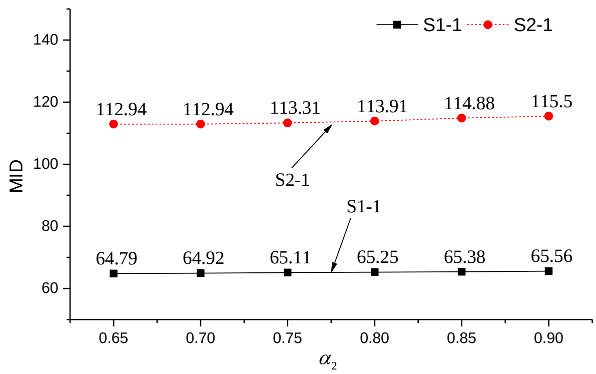
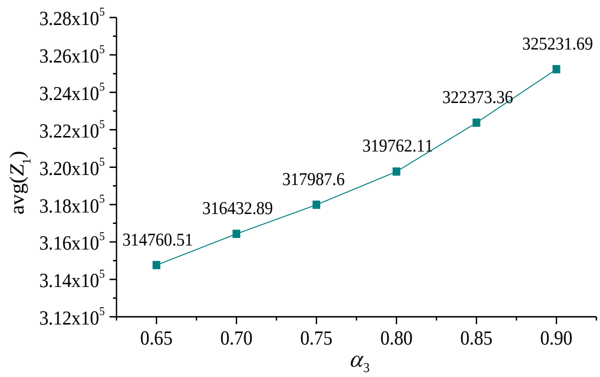


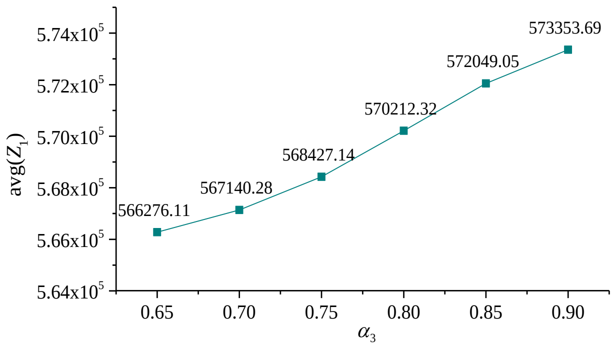
Fig.12. Changes in MID under different confidence levels of production capacity

(2) Sensitivity analysis of confidence level for product demand

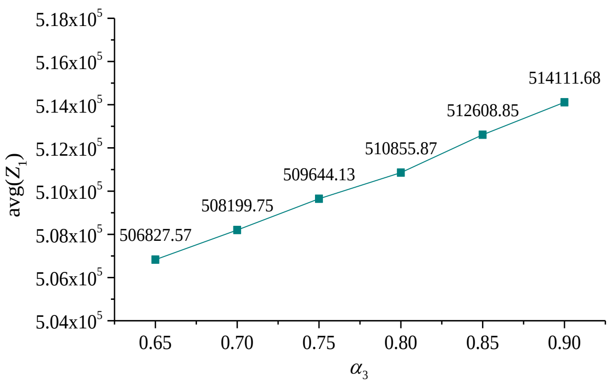
Similarly, the impact of changes in the confidence level of product demand is analyzed (α_1=0.8 , α_3=0.8). The results of the total production cost and MID indicators are shown in Fig.13 and Fig.14. From the figures, it can be seen that as the confidence level of product demand increases, both the total production cost and MID indicators show an upward trend. This indicates that the higher the confidence level is, the higher the cost that the enterprise will have to pay, and the greater the difficulty in obtaining a better plan.



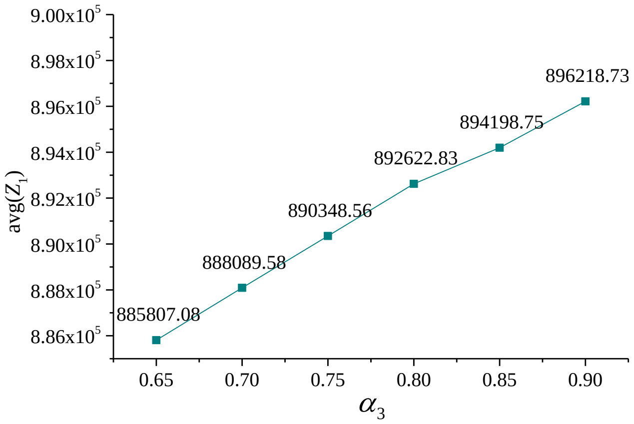
(a) Experiment S1



(b) Experiment s2



(c) Experiment S3



(d) Experiment S4

Fig.13. Changes in total production cost under different confidence levels of product demand

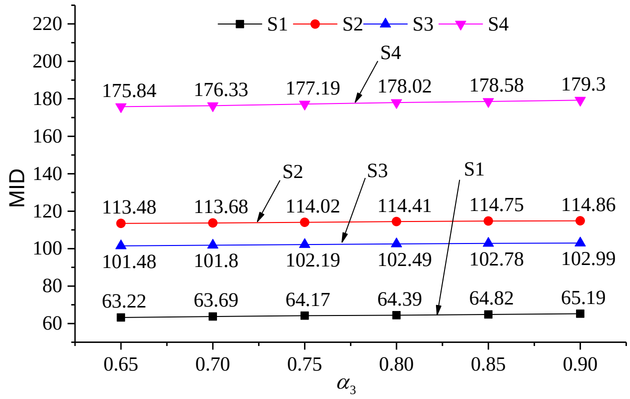


Fig.14. Changes in MID under different confidence levels of product demand

V. Conclusion

A multi-objective stochastic programming APP model is constructed considering external demand and internal capabilities as random variables, with minimal expected opportunity cost and employee instability. At a certain level of confidence, the opportunity constraint is transformed and processed. In addition, based on the determined feasible range of product output and employee number, SHGA-PSO1 and SHGA-PSO2 algorithms based on a local search strategy are designed to solve the model. Two algorithms are compared and analyzed from the evaluation indicators avg(·), Runtime, M_1, M_2 and MID by using numerical examples. Moreover, the sensitivity of product demand and production capacity under different confidence levels is analyzed. The following conclusions have been drawn:

(1) From the comparison results of evaluation indicators $\text{avg}(Z_1)$, $\text{avg}(Z_2)$, M_1 and MID, there is no significant difference in the performance of SHGA-PSO1 and SHGA-PSO2 algorithms. Both algorithms are suitable for solving the multi-objective stochastic programming APP problem.

(2) From the perspective of algorithm runtime, the SHGA-PSO1 algorithm has a slightly shorter runtime than the SHGA-PSO2 algorithm.

(3) By comparing the running time of deterministic and stochastic APP examples of the same scale, it is found that the calculation time of stochastic APP problems is significantly longer than that of deterministic APP problems, and the larger the problem size, the greater the difference in calculation time.

(4) By analyzing the sensitivity of product demand and production capacity under different confidence levels, it is known that when the production capacity is sufficient, the change in production capacity confidence level has a little impact on production. Otherwise, as the confidence level of production capacity increases, the total production cost and MID indicators show an upward trend. In addition, as the confidence level of product demand increases, the total production cost and MID indicators also show an upward trend. To increase confidence, enterprises must pay higher costs.

Two hybrid SHGA-PSO algorithms based on the global search ability of PSO algorithm and the local search ability of GA proposed by this paper can effectively improve the solving efficiency of the algorithm. With the rapid development of the Internet, changes in market demand will become more frequent. In the future, we will consider dynamically integrating dynamic demand information into the market and studying dynamic production planning problems for orders.

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