Deposit Insurance Price Using Modified Option Approach Based on Variance-Gamma Process

Gunardi

Abstract—**This article proposes a new method to determine deposit insurance prices, specifically designed for financial institutions dealing with high-risk scenarios. The method is based on a modified option approach rooted in the Variance-Gamma process, which aims to provide a more accurate and refined means of assessing and pricing deposit insurance. The model assumes that the total deposit and asset price processes follow a Variance-Gamma process. The Monte Carlo simulation method is used to price the deposit insurance premium using the modified option method. The results show that kurtosis and skewness in the distribution of assets and deposits should be considered when calculating deposit insurance pricing.**

Index Terms—**Option valuation, deposit insurance, Variance-Gamma model, Monte Carlo simulation.**

I. Introduction

DEPOSIT insurance was established in various nations to mitigate bank losses. This legislative structure protects specific depositors' deposits in a bank up to a specified limit in case the bank cannot fulfill its payment obligations [1]. Studies conducted in the United States have demonstrated that implementing the Federal Deposit Insurance Corporation (FDIC) led to a decrease in bank failures compared to the previous period [2].

In contrast, deposit insurance has the potential to encourage moral hazard, whereby banks are more likely to engage in high-risk investments to maximize profits, as they know that customer deposits are protected by insurance [3], [4], [5]. Therefore, it is crucial to manage deposit insurance effectively to prevent bank defaults while minimizing moral hazards. One approach is introducing a risk-based insurance premium that is fair to all banks and mitigates moral hazards [6], [7].

II. Literature Review

The study conducted by Diamond and Dibvig [8] highlights the significance of deposit insurance in rebuilding confidence in the banking system. Banks face the potential threat of panic withdrawals in times of financial distress, which can adversely affect the entire banking sector. However, deposit insurance offers customers reassurance by ensuring the safety and protection of their savings, thus increasing their trust in the system.

According to a report in 2013, out of the 189 nations that had deposit insurance systems, only 112 had explicit deposit protection. Surprisingly, only 31 percent of these countries adjusted their insurance premiums based on risk, as indicated by Demirguc-Kunt [9]. For instance, Singapore Deposit Insurance Company (SDIC) employs a premium system that takes into account the asset maintenance ratio, as mentioned by Singapore Deposit Insurance Company (2022) and the Monetary Authority of Singapore (2018). Similarly, the Malaysian Deposit Insurance Corporation (PIDM) utilizes a differential premium method to categorize deposit insurance premiums based on the level of bank risk, as outlined by the Malaysia Deposit Insurance Corporation (2022).

The Deposit Insurance Corporation (DIC) and the Philippine Deposit Insurance Corporation (PDIC) continue to utilize the same interest rate for deposit insurance premiums. However, this standardized rate may result in inaccurate pricing, as financially stable institutions pay more than necessary despite the low likelihood of claims being filed. Consequently, these institutions are effectively subsidized by paying less than their fair share despite the higher probability of claims being made. The uniform premium rates fail to reflect appropriate actuarial calculations accurately [10]. Research conducted by the Federal Deposit Insurance Company (FDIC) has demonstrated that uniform interest rates are associated with inflated costs [11], [12], [13]. Buser et al. [14] also observed that deposit insurance premiums are priced lower than they should be.

Merton [15] introduced a technique for computing deposit insurance premiums and market information that are actuarially fair. This approach was the first to establish a link between deposit insurance and put options. Merton's model is a one-period system where the insured parties undergo an audit at the end of the period. If the insured party remains solvent, the insurance company will determine the policy for the next period. However, if the party becomes insolvent, the insurance company will exercise the put option and acquire the party's assets [16]. The premium equation can be calculated using the Black and Scholes method [17], [18], which considers the asset risk and the relative size of the insured party's total deposits and assets.

III. Materials and Methods

This paper proposes that the deposit and asset price processes follow the Variance-Gamma process. The deposit price process is denoted as $D = \{D(t) : t > 0\}$, and the bank asset price process is denoted as $V = \{V(t) : t > 0\}$. The dynamics of D and V under the risk-neutral measure are assumed to be described by the variability-gamma process [19], [20], [21]. The Monte Carlo simulation method determines the deposit insurance premium based on the Variance-Gamma model.

Manuscript received October 13, 2023; revised February 18, 2024. This work was supported in part by the laboratory mandate grant of the Department of Mathematics of Universitas Gadjah Mada 2023 (grant number: 135/UN1/FMIPA.1.3/MAT/PT.01.03/2023) provided by the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada 2023 towards the World Class University.

Gunardi is a lecturer of the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Unversitas Gadjah Mada, Yogyakarta, 55281, Indonesia (e-mail: gunardi@ugm.ac.id).

In finance, an option is a derivative financial instrument that grants the holder the privilege to purchase or sell an underlying asset at a predetermined price, known as the exercise price, before the expiration date. There are two distinct types of options: put options, which grant the holder the ability to sell a specific asset and call options, which grant the holder the ability to buy a specific asset.

The bank's receipt of funds from the deposit insurance manager is based on the disparity between the loan's face value and the collateral's market value. Merton [22] showed that the bank can be seen as possessing a put option, with the deposit insurance manager acting as the option seller. The payment received by the bank, or the put option's payoff, is determined by the difference between the loan's face value and the collateral's market value.

$$\Lambda(V(T)) = \max(D - V(T), 0). \tag{1}$$

The bank's asset value at time T is represented as $V(T)$, while the deposit made at the start of the period is denoted as D.

Nonetheless, this article suggests that the assessment of deposit insurance premiums can be likened to a call option, with the modified option incorporating systematic risk into the model. The deposit insurance agreement at time T is represented as the payoff of a call option, which can be expressed as follows:

$$\Lambda(D(T)) = \max(D(T) - V, 0) \tag{2}$$

where V is the bank asset at time 0, and $D(T)$ is the deposit at time T and the payoff of a modified option, that is

$$\Lambda(D(T), V(T)) = \max(D(T) - V(T), 0) \tag{3}$$

where $V(T)$ is the bank asset at time T and $D(T)$ is the deposit at time T. Hence, the deposit insurance premium could be determined similarly to the valuation of an option and a modified option.

A. Variance-Gamma Process

This paper proposes proposes a method to model Brownian motion as a random time change based on the gamma process. Specifically, it generates a variance gamma (VG) process using Brownian-Gamma Bridge Sampling (BGBS) introduced by Avramidis et al. [23]. The VG process is characterized by three parameters: the volatility of the Brownian motion, the rate of variance of the change in gamma time (which controls kurtosis), and the drift in the Brownian motion with drift (which controls skewness). To estimate these parameters, the authors employ the moment method proposed by Seneta [24].

B. Brownian-Gamma Bridge Sampling

The Variance-Gamma process can be characterized by applying the second algorithm, BGBS, which assumes that the processes G (gamma) and B (Brownian) are independent. This assumption implies that, given any set of gamma process (G) increments, the Brownian process (B) increments are independent and normally distributed. To achieve this, the increments are sampled alternately. The first step involves sampling the increments in the sample of G using the gamma bridge, then sampling the increments in the $B(G(t))$ using Brownian bridge sampling, conditioned on the corresponding G increments.

Algorithm to generate Variance Gamma process by BGBS

$G(0) = 0; X(0) = 0$
Generate $G(T) \sim \mathcal{G}(T/\nu, \nu)$;
Generate $X(T) \sim N(\theta G(T), \sigma^2 G(T))$;
For $l = 1$ to k
    For $m = 1$to 2^{l-1}
        $i = 2m - 1$;
        Generate $Y \sim \mathcal{B}(T/(\nu 2^l), T/(\nu 2^l))$;
        $G(iT/2^l) = G((i-1)T/2^l) + (G((i+1)T/2^l) - G((i-1)T/2^l))Y$;
        $b = G((i+1)T/2^l) - G(iT/2^l)$;
        Generate $Z \sim N(0, b\sigma^2 Y)$;
        $X(iT/2^l) = YX((i+1)T/2^l) + (1-Y)X((i-1)T/2^l) + Z$;
    Next m.
Next, l

C. VG Put Option Pricing by Brownian-Gamma Bridge Sampling

This study examines deposit insurance pricing using a put option contract. This contract grants the holder the option to pay the deposit amount, denoted as D, at the maturity date (T) if D is less than the value $V(T)$. Conversely, if D exceeds $V(T)$, the holder can pay $V(T)$.

Let $V_n = V(t_n)$ denote the bank asset at t_n, we have

$$V_n = V_{n-1} \exp((r + \omega)\Delta t + X_n). \tag{4}$$

We can estimate the price of deposit insurance based on the put option using the following algorithm:

Algorithm of VG-BGBS for the put option

For $i = 1, ..., M$
    For $n = 1, ..., N$
        Generate X_n by BGBS
        Set $\omega = \frac{1}{\nu}\ln(1 - \theta\nu - \sigma^2\nu)$
        Set $V_n = V_{n-1}\exp((r + \omega)\Delta t + X_n)$
        set $P_i = e^{-rT}\max(D - V_N, 0)$
end for.
Set $P = \frac{1}{M}\sum_{i=1}^{M} P_i$
Set $DIP_{vgp} = P$

D. VG Call Option Pricing by Brownian-Gamma Bridge Sampling

This study examines deposit insurance pricing using a call option contract. Under this contract, the policyholder has the option to pay the deposit amount, denoted as $D(T)$, at maturity date T when $D(T)$ is less than the value $V(T)$. Conversely, if $D(T)$ exceeds $V(T)$, the policyholder pays the value V.

Let $D_n = D(t_n)$ denote the bank deposit at t_n, we have

$$D_n = D_{n-1} \exp((r + \omega)\Delta t + X_n). \tag{5}$$

We can estimate the price of deposit insurance based on the call option by the following algorithm:

Algorithm of VG-BGBS for Call Option

For $i = 1, ..., M$
  For $n = 1, ..., N$
    Generate X_n by BGBS
    Set $\omega = \frac{1}{\nu} \ln(1 - \theta\nu - \sigma^2\nu)$
    Set $D_n = D_{n-1} \exp((r + \omega)\Delta t + X_n)$
    set $P_i = e^{-rT} \max(D_N - V, 0)$
end for.
Set $P = \frac{1}{M} \sum_{i=1}^{M} P_i$
Set $DIP_{vgc} = P$

E. VG Modified Option Pricing by Brownian-Gamma Bridge Sampling

This study examines a pricing deposit insurance scheme that utilizes a modified option contract. Under this scheme, the insurance contract holder is granted the option to pay the deposit at maturity date T for a predetermined value $D(T)$ if $D(T)$ is less than $V(T)$. Conversely, if $D(T)$ exceeds $V(T)$, the contract holder has the option to pay $V(T)$.

Let $V_n = V(t_n)$ denote the bank asset and $D_n = D(t_n)$ denote the deposit at t_n, we have

$$D_n = D_{n-1} \exp((r + \omega_D)\Delta_D t + X_n) \quad (6)$$

$$V_n = V_{n-1} \exp((r + \omega_V)\Delta_V t + X_n) \quad (7)$$

We can estimate the price of deposit insurance based on the modified option by the following algorithm:

Algorithm of VG-BGBS for Modified Option

For $i = 1, ..., M$
  For $n = 1, ..., N$
    Generate X_n by BGBS
    Set $\omega_D = \frac{1}{\nu_D} \ln(1 - \theta_D\nu_D - \sigma_D^2\nu_D)$
    Set $\omega_V = \frac{1}{\nu_V} \ln(1 - \theta_V\nu_V - \sigma_V^2\nu_V)$
    Set $D_n = D_{n-1} \exp((r + \omega_D)\Delta_D t + X_n)$
    Set $V_n = V_{n-1} \exp((r + \omega_V)\Delta_V t + X_n)$
    set $P_i = e^{-rT} \max(D_N - V_N, 0)$
end for.
Set $P = \frac{1}{M} \sum_{i=1}^{M} P_i$
Set $DIP_{vgm} = P$

IV. Result and Discussions

The Monte Carlo simulation can use the Variance-Gamma method to determine the deposit insurance premium. By utilizing the BGBS formula discussed in Section III, we can individually assess the impact of parameter modifications.

The suitability of the Variance-Gamma (VG) process for non-normally distributed total deposits and total assets can be observed from Figures 1, 2, and 3. A comparison of the graphics in Figure 1 reveals the influence of the parameter σ on the variance. In contrast, Figure 3 demonstrates the relationship between the parameter ν and the distribution's kurtosis. Figure2 indicates that the parameter θ is associated with the skewness of the distribution. By carefully estimating the values of each parameter using a moment estimator, it is possible to obtain them based on the total deposit or total return data.

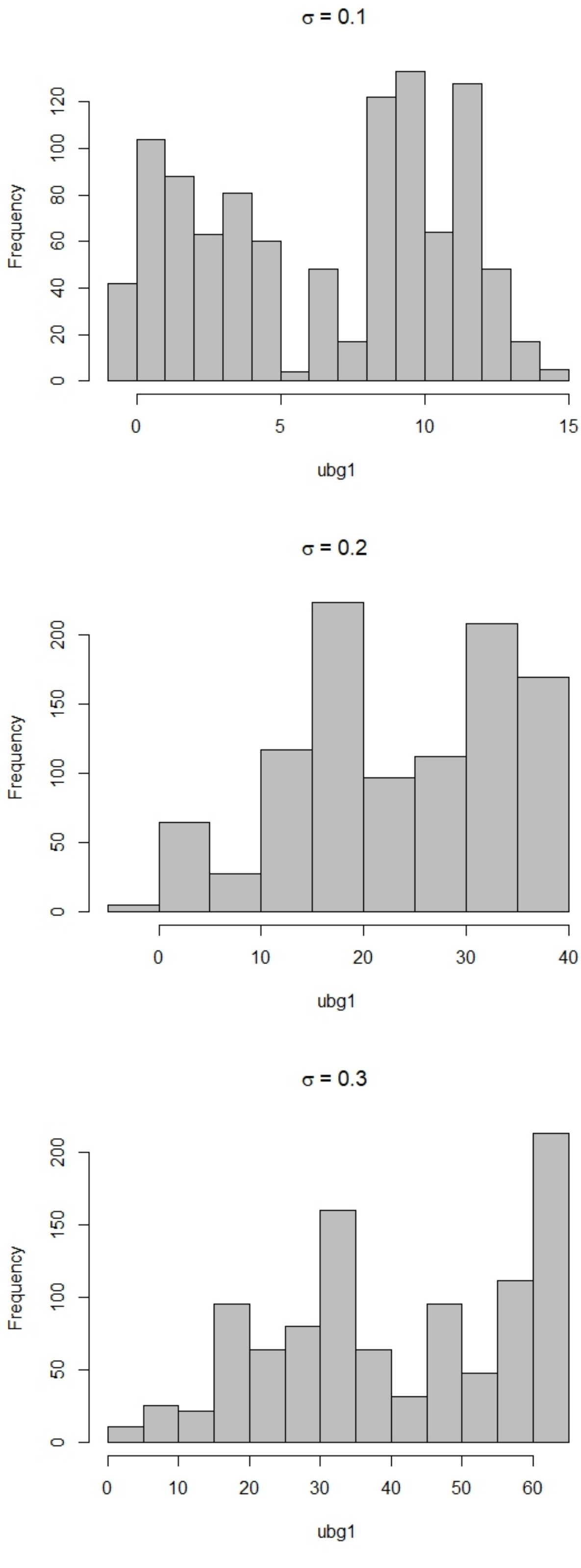

Fig. 1. Effects of parameter σ in Variance Gamma Process

Given that the precise formula is not accessible, the pricing of deposit insurance can be carried out using the Monte Carlo simulation method with the utilization of the Variance-Gamma process. This study examines three approaches: the

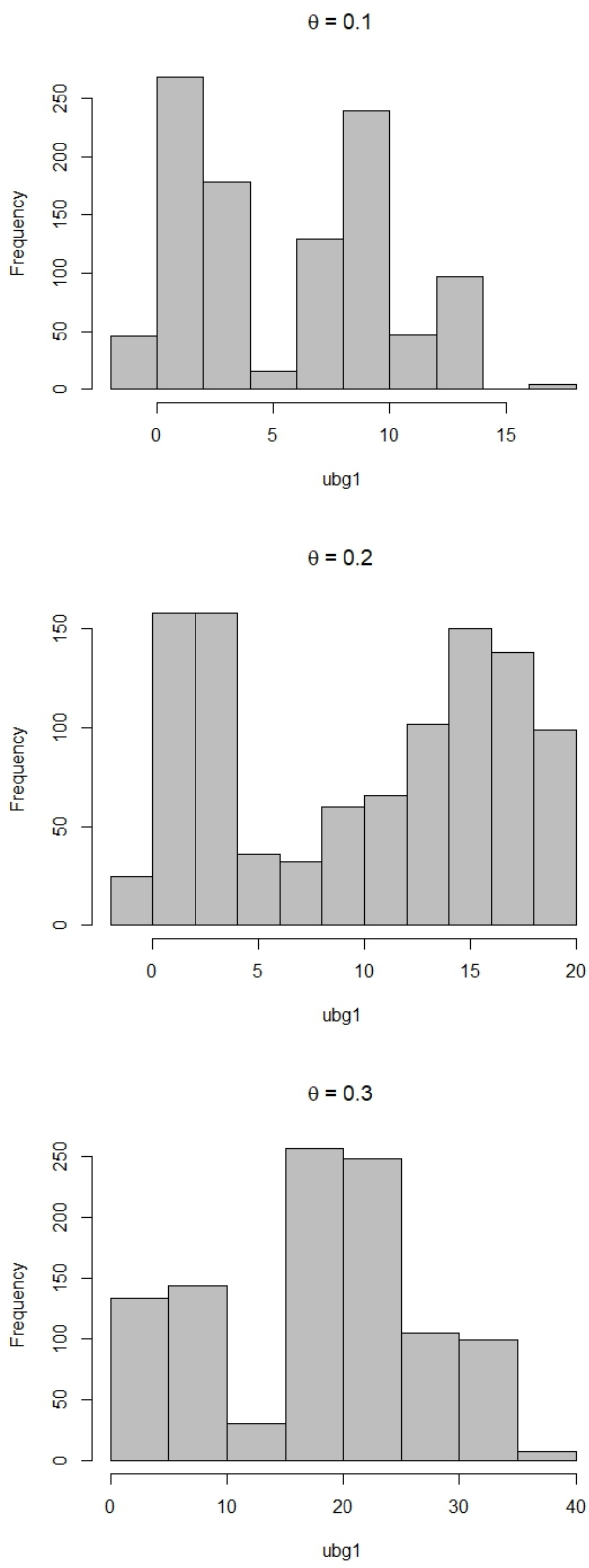

Fig. 2. Effects of parameter θ in Variance Gamma Process

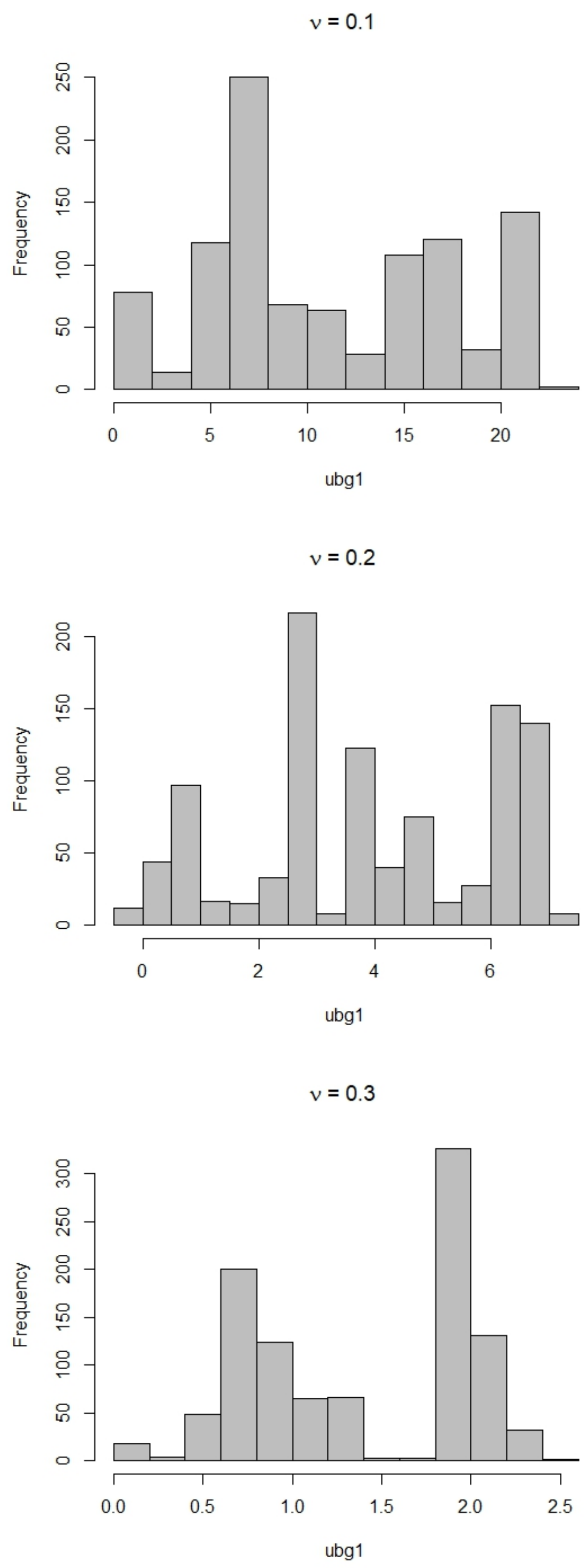

Fig. 3. Effects of parameter ν in Variance Gamma Process

put option, the call option, and the modified option. The outcomes of our simulation, conducted 100,000 times, are displayed in Figures 4-6.

Figures 4-6 demonstrate the consistent relationship between the deposit insurance premium and the (D/V) ratio. When comparing Figures 1 and Figure 4, a higher value of σ is associated with a greater premium for the deposit insurance holder. However, it is important to note that to achieve similar premium values, the σ of the deposit (in the call option approach) must be at least one-hundredth of the σ of the asset in the put option approach. The deposit insurance premium's excessively high and unreasonable value results

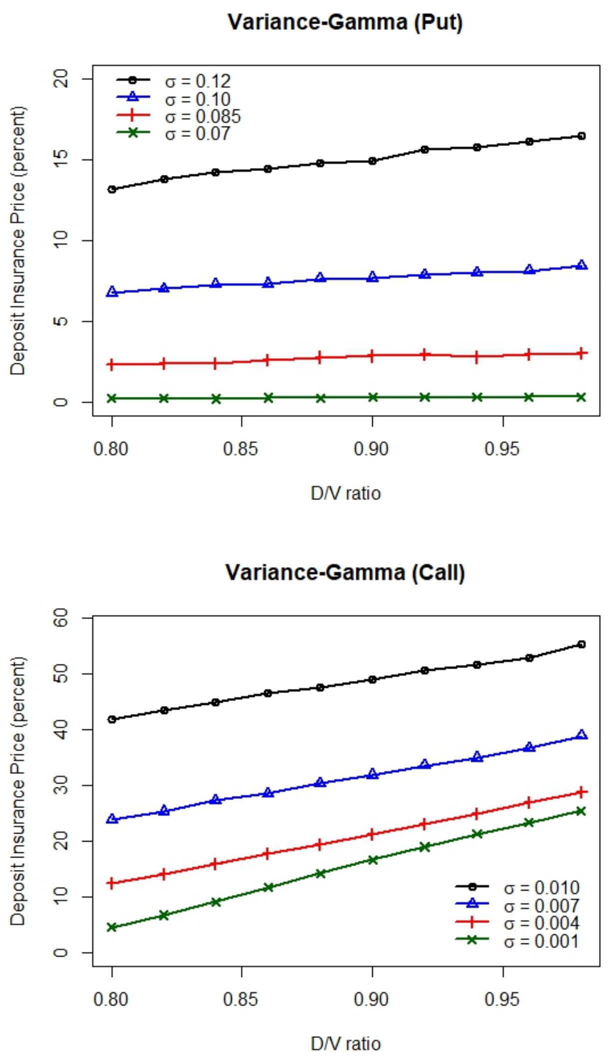

Fig. 4. Deposit insurance pricing using an option approach based on Variance-Gamma process at several σ values

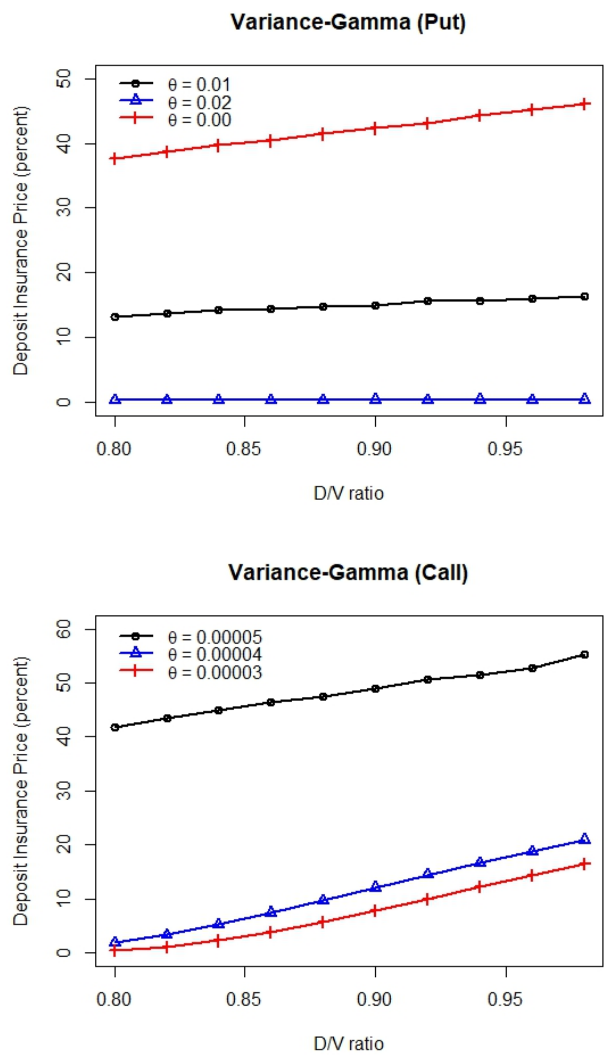

Fig. 5. Deposit insurance pricing using an option approach based on Variance-Gamma process at several θ values

from the high σ in the call option method.

The distribution's skewness, denoted by the parameter θ, exhibits a similar trend as σ. When using the call option approach, a higher θ value is linked to a higher deposit insurance premium, while the put option approach results in a lower premium (Figure 5). However, an excessively high θ value leads to an unreasonable premium. To achieve comparable premiums, the θ value for the deposit (in the call option approach) must be at least one-hundredth of the θ value for the asset when utilizing the put option approach.

The kurtosis parameter, denoted as ν, exhibits a distinct pattern. The valuation of deposit insurance based on the Variance-Gamma process at several ν values is illustrated in Figure 6. The highest deposit insurance premium is observed when $\nu = 0.01$, compared to smaller values ($\nu = 0.008$) and larger values ($\nu = 0.012$). This consistent pattern is observed in both the put option and call option approaches, suggesting that the kurtosis of the asset or deposit's distribution should be considered in calculating the deposit insurance premium rather than solely focusing on variance or volatility. Notably, a higher value of ν indicates a higher kurtosis or a distribution with fat tails.

The deposit insurance premium under the scenario where the deposit and asset follow an independent and identically distributed (i.i.d.) Variance-Gamma process is illustrated in Figure 7. There is no significant effect of the σ value on the pricing of deposit insurance using modified option approach for each D/V ratio. In the modified option approach, the higher the θ value, the higher the deposit insurance value. In contrast, the deposit insurance value decreases as the ν parameter increases.

V. Conclusion

Besides the use of put options, there are two alternative approaches to calculating the deposit insurance premium. One option is to assume that the total deposit follows a stochastic process and utilize the call option approach. Another option is to generalize the call-and-put option and adopt the modified option approach, which assumes that the total deposit and asset follow a stochastic process.

- The second row indicates that the parameter θ is associated with the skewness of the distribution. The parameter σ acts as the variance, while the third row demonstrates that the parameter ν is linked to the

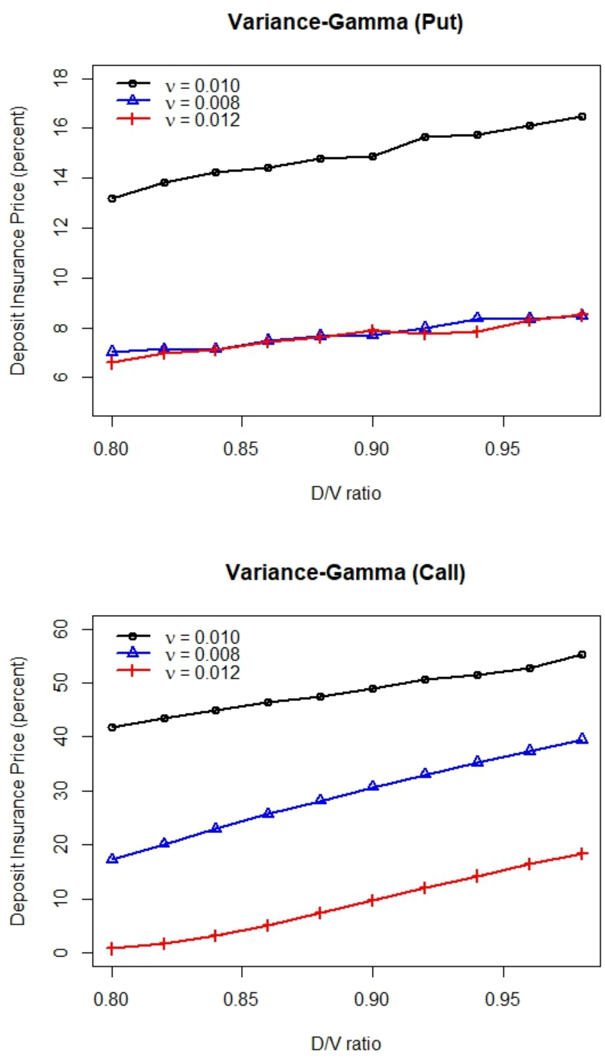

Fig. 6. Deposit insurance pricing using an option approach based on Variance-Gamma process at several ν values

- distribution's kurtosis. Through a meticulous estimation procedure, such as using a moment estimator, we can determine the value of each parameter using the total deposit or total return data.
- The increased value of σ is associated with an increased premium for the deposit insurance holder. The (D/V) ratio and the deposit insurance premium consistently exhibit the same trend. However, it is important to highlight that to achieve comparable deposit insurance premium values, the σ of the deposit (in the call option approach) must be at least one-hundredth of the σ of the asset in the put option approach. The elevated σ value in the call option method leads to an excessively high and unreasonable deposit insurance premium.
- The ν, or kurtosis parameter, exhibits different patterns. A higher value of $\nu = 0.01$ results in a higher deposit insurance premium than smaller values of $\nu(0.008)$ and higher values of $\nu(0.012)$. This pattern is observed in both the put option and call option approaches, indicating that the calculation of deposit insurance premiums should consider the kurtosis of the asset or deposit's distribution rather than the variance or volatility. It is important to note that a higher value of ν indicates a higher kurtosis or fat-tailed distribution in the data.

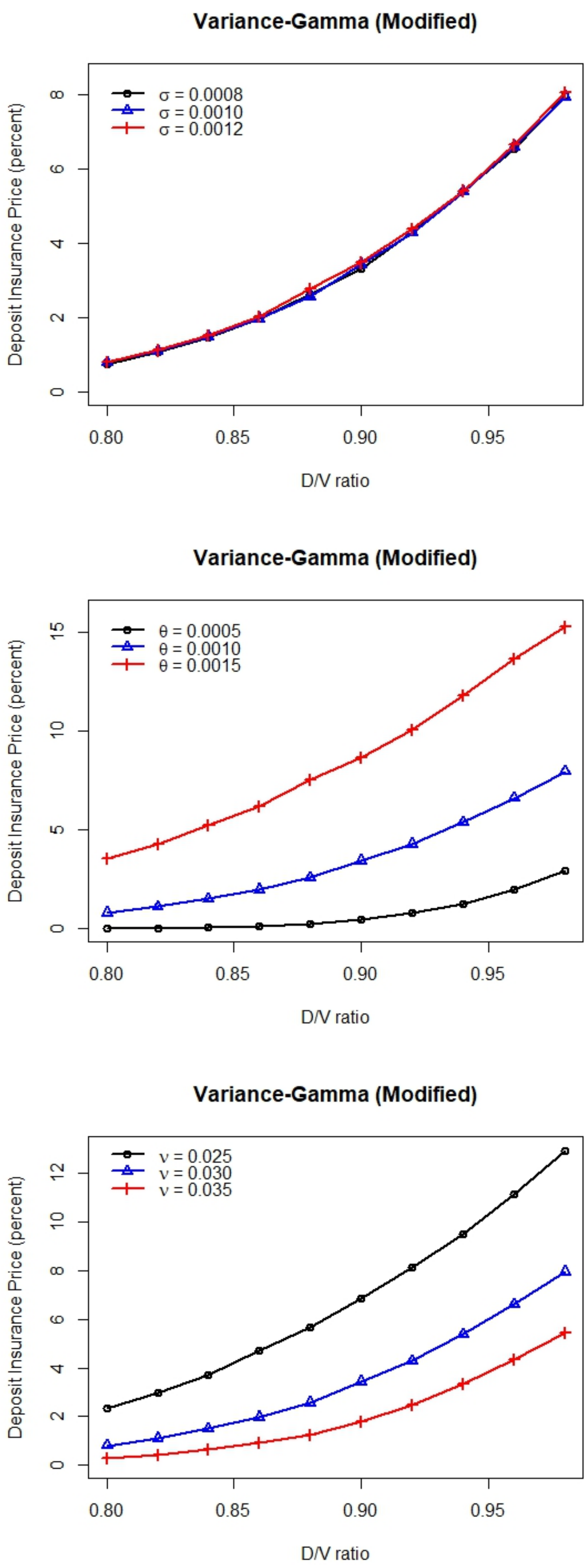

Fig. 7. Deposit insurance pricing using modified option approach based on the Variance-Gamma process

- The Variance-Gamma distribution experiment suggests that the kurtosis and skewness of the distribution of assets and deposits should be considered when determining deposit insurance price.

More research should be done to include the systematic and other risks in the formula.

REFERENCES

[1] D. Ognjenovic, *Deposit insurance schemes: funding, policy, and operational challenges*. Switzerland: Palgrave Macmillan, 2017.
[2] A. Saunders. and M. M. Cornett, *Financial institution management: a risk management approach*, 6th ed. New York: McGraw Hill, 2008.
[3] R. Cull, L. Senbet, and M. Sorge, "Deposit insurance and financial development," *Journal of Money, Credit and Banking*, vol. 37, pp. 43–82, 2005.
[4] H. Liu, R. Li, and J. Yuan, "Deposit insurance pricing under GARCH," *Finance Research Letters*, vol. 26, pp. 242–249, 2018.
[5] T. Mondschean and T. Opiela, "Bank time deposit rates and market discipline in Poland: The impact of state ownership and deposit insurance reform," *Journal of Financial Services Research*, vol. 15, pp. 179–196, 1999.
[6] E. S. Prescott, "Can risk-based deposit insurance premiums control moral hazard?" *Federal Reserve Bank of Richmond Economic Quarterly*, vol. 88, no. 2, 2002.
[7] S. C. Lee, T. C. Lin, and M. S. Tsai, "The pricing of deposit insurance in the presence of systematic risk," *Journal of Banking and Finance*, vol. 51, pp. 1–15, 2015.
[8] D. W. Diamond and P. H. Dybvig, "Bank runs, deposit insurance, and liqudity," *Journal of Political Economy*, vol. 91, no. 3, pp. 401–419, 1983.
[9] A. Demirguc-Kunt, L. Klapper, D. Singer, and P. van Oudheusden, "The global findex database 2014: measuring financial inclusion around the world," *World Bank Group Policy Research Working Paper*, vol. 7255, 2015.
[10] A. Camara, T. Davidson, and A. Fodor, "Bank asset structure and deposit insurance pricing," *Journal of Banking and Finance*, vol. 114, 2020.
[11] K. E. Scott and T. Mayer, "Risk and regulation in banking: some proposals for federal deposit insurance reform," *Stanford Law Rev*, vol. 23, pp. 857–902, 1971.
[12] D. B. Humphrey, "100 percent deposit insurance: what would it cost?" *J. Bank Res*, vol. 23, pp. 192–198, 1976.
[13] A. J. Marcus and I. Shaked, "The valuation of FDIC deposit insurance using option pricing estimates," *J. Money Credit Bank*, vol. 16, pp. 446–460, 1984.
[14] S. A. Buser, A. H. Chen, and E. J. Kane, "Federal deposit insurance, regulatory policy, and optimal bank capital," *J. Finance*, vol. 35, pp. 51–60, 1981.
[15] R. C. Merton, "An analytic derivation of the cost of deposit insurance loan guarantess," *J. Bank Finance*, vol. 1, pp. 3–11, 1977.
[16] A. Q. M. Khaliq, B. A. Wade, M. Yousuf, and J. Vigo-Aguiar, "High order smoothing schemes for in homogeneous parabolic problems with applications in option pricing," *Numerical Methods for Partial Differential Equations*, vol. 23, no. 5, pp. 1249–1276, 2007.
[17] F. Black and M. Scholes, "The pricing of options and corporate liabilities," *J. Polit. Econ*, vol. 81, pp. 637–659, 1973.
[18] H. Song and H. Jia, "Fast method for black-scholes equation of european options," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 1, pp. 25–30, 2023.
[19] D. B. Madan and E. Seneta, "The variance gamma (VG) model for share market returns," *Journal of Business*, vol. 63, pp. 511–524, 1990.
[20] D. B. Madan and F. Milne, "Option pricing with variance gamma martingale components," *Mathematical Finance*, vol. 1, pp. 39–55, 1991.
[21] D. B. Madan, P. P. Carr, and C. E. Chang, "The variance gamma process and option pricing," *European Finance Review*, vol. 2, pp. 79–105, 1998.
[22] R. C. Merton, "On the cost of deposit insurance when there are surveillance cost," *Journal of Business*, vol. 51, no. 3, pp. 439–452, 1978.
[23] A. N. Avramidis, P. L'Ecuyer, and P. A. Tremblay, "Efficient simulation of gamma and variance gamma processes," in *Proceedings of 2003 Winter Simulation Conference*, 2003, pp. 319–326.
[24] E. Seneta, "Fitting the variance-gamma model to financial data," *Journal of Applied Probability*, vol. 41A, pp. 177–187, 2004.