An Inventory Model for Deteriorating Goods with Exponential Demand, Variable Holding Costs, and Partial Backlogging Across Two-Warehouses in an Inflationary Environment

Karan Pathak, Ajay Singh Yadav* and Priyanka Agarwal

***Abstract*—Establishing warehouses is necessary in societies where commercial activities have grown to the extent that efficient storage becomes a prerequisite for seamless exchange. Variable holding cost is a pivotal factor influencing warehouse costs, as is the dynamic nature of holding expenses, which tend to increase over time. This study focuses on an inventory model designed for goods subject to degradation, stored across two distinct warehouses (rented and owned) within a partial backlogging and inflationary economic environment. The cost associated with holding these goods is linearly dependent on time. The demand pattern in the inventory model for goods follows an exponential trajectory over time. The primary objective of the proposed model is to forecast the optimal quantity and the corresponding timeframe, thereby minimizing the overall cost. Rigorous validation of the model's outcomes is undertaken through sensitivity analysis utilizing MATLAB R2017b software, ensuring the robustness and reliability of the findings.**

***Index Terms*—Two-warehouse; Variable holding cost; Exponential demand; Inflation; Partially backlogging**

I. Introduction

WAREHOUSING a vital element of both logistics and supply chain management. It functions as a vital intermediary between the production and consumption processes, facilitating the storage and distribution of items. Hartley [1] proposed the foundational two-warehouse inventory model. However, supervising a warehouse presents difficulties, particularly when it pertains to the preservation of degraded goods. Ghare and Schrader [2] proposed an EOQ model for items that undergo exponential deterioration over time. Deterioration may arise from various conditions, such as exposure to humidity, temperature, or simply over some time. The cost of storing deteriorated things can vary significantly depending on storage duration, extent of deterioration, and other relevant variables.

Moreover, warehouses are vulnerable to scarcity and price increases, which can complicate storing and distributing goods. For instance, supply shortages that result in stockouts may affect a warehouse's capacity to meet client demand. Nevertheless, as time progresses, inflation can increase the expenses associated with maintaining inventory, posing an incredible difficulty in managing a warehouse.

This research study addresses the challenges of shortages and inflation by examining the impact of varying holding costs on rented and owned warehouses for degrading goods. The impact of scarcity, inflation, and holding costs are significant factors when considering the total storage cost. The findings of this study will offer significant perspectives on the administration of warehouses for deteriorating commodities, enabling warehouse managers to make better-informed decisions in response to these difficulties.

II. Literature Review

A key component of supply chain management is inventory management, which can significantly impact an organisation's operational and financial performance. One of the most significant issues with inventory control is maintaining optimal inventory levels while ensuring efficient use of resources. A two-warehouse inventory system is a widely adopted approach to address this challenge. This paper reviews recent literature on two warehouse inventory models and critically analyses the proposed models and their contributions. Furthermore, it discusses the relevance and significance of the article titled "A Two-Warehouse Inventory Model with Degrading Items that Exhibit Exponential Demand during Shortages under Inflation" according to the available literature, as shown in Table I.

Jaggi CK and Verma P. [3] presented a two-warehouse inventory model for non-instantaneous degrading items. To determine whether renting a warehouse is feasible, the model offers an approach. A two-warehouse inventory model formulated by Verma et al. [4] with a linear demand under inflation conditions and various deterioration rates. A three-echelon supply chain model with two storage facilities was evaluated by Kumari et al. [5] under various partial supply chain delay conditions. Goswami et al. [6] developed a two-warehouse inventory model with the assumption of quadratic

Manuscript received October 3, 2023; revised April 12, 2024.

Karan Pathak is a PhD scholar in the Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad 201204, India. (Email: karanpathak18@gmail.com)

Ajay Singh Yadav is an Associate Professor in the Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad-201204, India. (Email: ajay29011984@gmail.com).

Priyanka Agarwal is an Assistant Professor in the Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Ghaziabad-201204, India. (Email: priyankv@srmist.edu.in).

demand, which is useful for commodities whose demand is rising quickly.

Dash et al. [7] suggested a model of inventory that examines the best-order quantity in the face of exponentially declining demand. In a situation of inflation and partially backlogged shortages, Jaggi CK et al. [8] developed a model of inventory involving non-instantaneous degradation of products. Dutta and Kumar [9] provided an inventory model of the partial demand backlog and time-dependent holding costs for deteriorating assets. The demand rate is a function of both time and selling price in a Sen and Saha model [10].

Swami and Yadav [11] presented a stock model with two warehouses for items that do not spoil immediately, assuming that better detention facilities in the rented warehouse result in higher inventory and deterioration costs. Khan et al. [12] considered that the degradation of their warehouse starts sooner than in the scenario of a rented warehouse because the latter always provides better inventory and facility management. Sahoo et al. [13] developed a stock model with two warehouses and an exponentially declining order rate with a limited suspension price, including buybacks. Das [14] developed a production inventory model in which customer demand depends on three variables: proportional to the level of inventories, proportional to the replacement period, and harmful to the prices of the items. Chakraborty [15] looked at a multi-item inventory model for non-snapshot items that lose value over time in an uncertain, unstructured setting with multiple storage facilities.

Suman [16], and Karan Pathak et al. [17] studied a mathematical deterministic stock model for deteriorating items with a biquadratic demand function that enables shortages. Garg and Malik [18] analysed the system in the current model using linear assumptions of demand functions, considering two warehouses and avoiding shortages. Kumar et al. [19] used a deterministic inventory model with a set retention period to look at items that were going bad in a system with two warehouses that had some backlog.

The studies show that two warehouse inventory models efficiently address various inventory problems, including depreciation, inflation, and shortages. As the literature reveals, these models consider various demand functions, cost structures, and assumptions, reflecting the complex nature of inventory management problems. The work proposed in the paper focuses on the need for an ideal model that can address the problem of degrading goods in circumstances where there is a partial backlog, inflationary deficits, and holding costs that vary linearly with time.

The model also considers how shortages could affect the overall inventory system and increase costs. Understanding this is important since shortages can significantly impact an enterprise's ability to meet consumer demand, which could lead to lost revenue or lower customer satisfaction.

The proposed model offers a novel viewpoint on inventory management by including important components not previously considered in earlier models mentioned in Table I. This unique model integrates deteriorating goods with exponential demand, holding costs that vary linearly over time during partial backlogs and inflationary deficits. It provides a more accurate and realistic depiction of enterprises' complex issues with advanced inventory management. Thus, the suggested approach represents a significant addition to the literature on inventory management, especially for businesses looking to maximise their systems amid complicated challenges.

III. Notations And Assumptions

The following section articulates the notations and assumptions integral to the proposed model. This articulation aims to establish a consistent and unambiguous understanding of the concepts presented.

A. Notations

TABLE II
Notations for the Proposed Model

Notations	Description
RW	Rented warehouse
OW	Owned warehouse
σ	Coefficient parameter of demand
υ	Coefficient parameter of demand
Ω	Deterioration rate in RW
λ	Deterioration rate in OW
A	Ordering cost
ω_1	Holding cost's coefficient parameter of RW
υ_1	Holding cost's coefficient parameter of RW
ω_2	Holding cost's coefficient parameter of OW
υ_2	Holding cost's coefficient parameter of OW
s_1	Cost of shortage per unit time
s_2	Lost sale cost per unit time
φ	Rate of inflation
d	Deterioration cost per unit item per unit time
η	Shortage rate per unit time
G	Maximum inventory level in OW
H	Total quantity at initial time
S	Maximum backlogging quantity
$I_{rw}(t)$	The level of inventory in RW at time t
$I_{ow}(t)$	The level of inventory in OW at time t
$I_{sw}(t)$	Inventory level of backlogging at time t
TQR	Total quantity to be replenished in next cycle
$TCU(t_1,t_2,T)$	Average of total cost per unit time
Decision Variables	
t_1	Time at which RW becomes zero
t_2	Time at which OW becomes zero
T	Length of the replenishment cycle

B. Assumptions

The analysis is grounded in specific assumptions, forming the foundational framework for subsequent discussions:

Assumption 1

The rate of demand increases exponentially over time.

i.e. $Demand = e^{(\upsilon+\sigma t)}$, where $\upsilon,\sigma > 0$

Assumption 2

Deterioration occurs in both warehouses (RW & OW) at constant rate α & β such that $\alpha < \beta$ at the time $t \in [0,t_1]$ &

$t \in [t_1, t_2]$ respectively.

Assumption 3

In both warehouses, the cost of deterioration per unit of time remains constant, but the cost of holding increases linearly with time.

Assumption 4

The replenishment rate is infinite, and this assumption leads to a zero lead time, meaning there is no delay in the delivery of an order.

Assumption 5

The shortage is allowed at a time $t \in [t_2, T]$ which is partially backlogged with backordered rate $0 < \delta < 1$.

Assumption 6

Cost factors exhibit a deterministic behaviour, but their values are expected to rise when subjected to a constant rate of inflation.

IV. Mathematical Model Formulation

Consider a system that receives an initial delivery of H units. Out of these units, G is stored in OW, while the remaining (H-G) units are stored in RW. It is important to note that the goods in OW cannot be utilised until all the goods in RW have been consumed. We consider the period [0 t_1], during which the inventory level in RW tends towards zero as a result of the combined effects of demand and deterioration, whereas OW depletes solely due to deterioration.

For the duration [t_1 t_2], OW depletes until it becomes zero due to the incidence of demand and deterioration. As a result of this situation in both warehouses, shortages will worsen until the end of period T. At the time [t_2 T], shortages start as goods are being partially backlogged with constant demand. Therefore, we define the inventory level at time t as the negative inventory level, which indicates the degree of shortages. The inventory situation is summarised in Fig. 1, which shows the listing of the inventory levels over time.

The following governing differential equations are as follows:

$$\frac{dI_{rw}(t)}{dt} + \Omega I_r(t) = -e^{(\upsilon+\sigma t)} \qquad , 0 < t \le t_1 \tag{1}$$

$$\frac{dI_{ow}(t)}{dt} + \lambda I_o(t) = 0 \qquad , 0 < t \le t_1 \tag{2}$$

$$\frac{dI_{ow}(t)}{dt} + \lambda I_o(t) = -e^{(\upsilon+\sigma t)} \qquad , t_1 \le t \le t_2 \tag{3}$$

$$\frac{dI_{sw}(t)}{dt} = -\upsilon\chi \qquad , t_2 \le t \le T \tag{4}$$

On solving the above equations using boundary condition $I_{rw}(t_1) = 0$, $I_{ow}(t_1) = G$, $I_{ow}(t_2) = 0$ and $I_{sw}(t_2) = 0$ respectively, we get

$$I_{rw}(t) = \frac{e^{((\sigma+\Omega)t_1 - \Omega t + \upsilon)} - e^{(\upsilon+\sigma t)}}{\sigma + \Omega} \qquad , 0 < t \le t_1 \tag{5}$$

$$I_{ow}(t) = Ge^{-\lambda t} \qquad , 0 < t \le t_1 \tag{6}$$

$$I_{ow}(t) = \frac{e^{((\sigma+\lambda)t_2 - \lambda t + \upsilon)} - e^{(\upsilon+\sigma t)}}{\sigma + \lambda} \qquad , t_1 \le t \le t_2 \tag{7}$$

$$I_{sw}(t) = -\upsilon\eta(t - t_2) \qquad , t_2 \le t \le T \tag{8}$$

Now, considering the continuity in OW at t=t_2 it follows by (6) and (7) and we get,

$$G = \left(\frac{e^{(\sigma+\lambda)t_1+\upsilon} - e^{(\sigma+\lambda)t_2+\upsilon}}{\sigma + \lambda}\right) \tag{9}$$

$$\text{Also, } t_1 = \frac{\log\left(-e^{-\upsilon}\left(G(\sigma + \lambda) - e^{\upsilon+(\sigma+\lambda)t_2}\right)\right)}{\sigma + \lambda} \tag{10}$$

When t=0 is taken into account in (5), it allows us to express the maximum inventory level in RW as follows:

$$I_{rw}(0) = H - G = G - \frac{e^{\upsilon} + e^{(\Omega+\sigma)t_1+\upsilon}}{\sigma + \Omega} \tag{11}$$

Substituting the value of G into equation (11), we obtain:

$$H = \left(\frac{e^{(\sigma+\lambda)t_1+\upsilon} - e^{(\sigma+\lambda)t_2+\upsilon}}{\sigma + \lambda}\right) - \frac{e^{\upsilon} + e^{(\Omega+\sigma)t_1+\upsilon}}{\sigma + \Omega} \tag{12}$$

Upon substituting t=T into (4), the results corresponds to the maximum backordered quantity.

$$i.e.\ S = -\upsilon\eta(T - t_2) \tag{13}$$

The total quantity to be replenished in the next cycle can be expressed as:

$$TQR = I_{rw}(0) + I_{ow}(0) - I_{sw}(T)$$

$$TQR = G - \frac{e^{\upsilon} + e^{(\Omega+\sigma)t_1+\upsilon}}{\sigma + \Omega} - S$$

$$TQR = \left\{\left(\frac{e^{(\sigma+\lambda)t_1+\upsilon} - e^{(\sigma+\lambda)t_2+\upsilon}}{\sigma + \lambda}\right) + \upsilon\eta(T - t_2) - \left(\frac{e^{\upsilon} + e^{(\Omega+\sigma)t_1+\upsilon}}{\sigma + \Omega}\right)\right\} \tag{14}$$

The elements contributing to the total cost can be specified as follows:

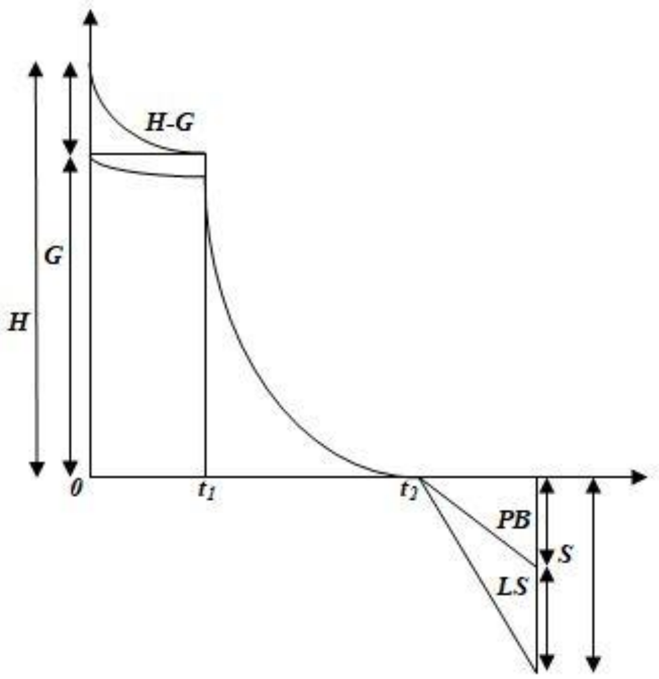

Fig. 1. Two-Warehouse Inventory Model for the Proposed Model

A. Ordering Cost

$$OC = A$$

B. Cost of holding inventory in RW

$$HC_{rw}=\int_0^{t_1} I_{rw}(t)(\upsilon_1+\omega_1 t)e^{-\varphi t}dt$$

$$HC_{rw}=\left\{\begin{array}{l}\dfrac{\omega_1 e^{(\sigma+\Omega)t_1+\upsilon}}{(\sigma+\Omega)}\left[\dfrac{1-e^{-(\Omega+\varphi)t_1}\left(t_1(\Omega+\varphi)+1\right)}{(\Omega+\varphi)^2}\right]\\ -\dfrac{\upsilon_1 e^{(\sigma+\Omega)t_1+\upsilon}\left(e^{-(\Omega+\varphi)t_1}-1\right)}{(\sigma+\Omega)(\Omega+\varphi)}-\dfrac{\upsilon_1 e^{\upsilon}\left(e^{(\sigma-\varphi)t_1}-1\right)}{(\sigma+\Omega)(\sigma-\varphi)}\\ -\dfrac{\omega_1 e^{\upsilon}}{(\sigma+\Omega)}\left[\dfrac{1+e^{(\sigma-\varphi)t_1}\left(t_1(\sigma-\varphi)-1\right)}{(\sigma-\varphi)^2}\right]\end{array}\right\}$$

C. Cost of holding inventory in OW

$$HC_{ow}=\int_0^{t_2} I_{ow}(t)(\upsilon_1+\omega_1 t)e^{-\Psi t}dt$$

$$HC_{ow}=\int_0^{t_1} I_{ow}(t)(\upsilon_1+\omega_1 t)e^{-\varphi t}dt+\int_{t_1}^{t_2} I_{ow}(t)(\upsilon_1+\omega_1 t)e^{-\varphi t}dt$$

$$HC_{ow}=\left\{\begin{array}{l}\dfrac{\omega_2 e^{\upsilon+(\sigma+\bar{\lambda})t_2}}{(\sigma+\bar{\lambda})}\left[\dfrac{e^{-t_1(\bar{\lambda}+\varphi)}\left((\bar{\lambda}+\varphi)t_1+1\right)-e^{-t_2(\bar{\lambda}+\varphi)}\left((\bar{\lambda}+\varphi)t_2+1\right)}{(\bar{\lambda}+\varphi)^2}\right]\\ -\dfrac{\omega_2 e^{\upsilon}}{\sigma+\bar{\lambda}}\left[\dfrac{e^{t_1(\sigma-\varphi)}\left((\varphi-\sigma)t_1+1\right)-e^{t_2(\sigma-\varphi)}\left((\varphi-\sigma)t_2+1\right)}{(\sigma-\varphi)^2}\right]\\ +\dfrac{\upsilon_2 e^{\upsilon+(\sigma+\bar{\lambda})t_2}\left(e^{-t_1(\bar{\lambda}+\varphi)}-e^{-t_2(\bar{\lambda}+\varphi)}\right)}{(\sigma+\bar{\lambda})(\bar{\lambda}+\varphi)}+\dfrac{\upsilon_2 e^{\upsilon}(e^{t_1(\sigma-\varphi)}-e^{t_2(\sigma-\varphi)})}{(\sigma+\bar{\lambda})(\sigma-\varphi)}\\ -\dfrac{G\omega_2 e^{-t_1(\bar{\lambda}+\varphi)}\left(\bar{\lambda}t_1-e^{t_1(\bar{\lambda}+\varphi)}+\varphi t_1+1\right)}{(\bar{\lambda}+\varphi)^2}-\dfrac{G\upsilon_2\left(e^{-t_1(\bar{\lambda}+\varphi)}-1\right)}{\bar{\lambda}+\varphi}\end{array}\right\}$$

D. Worth deterioration cost per cycle

$$DC=d\left[\Omega\int_0^{t_1} I_{rw}(t)e^{-\varphi t}dt+\bar{\lambda}\int_0^{t_2} I_{ow}(t)e^{-\varphi t}dt\right]$$

$$DC=-d\left\{\begin{array}{l}\dfrac{\Omega}{(\sigma+\Omega)}\left[\dfrac{e^{\sigma}\left(e^{t_1(\sigma-\varphi)}-1\right)}{(\sigma-\varphi)}+\dfrac{e^{\upsilon+(\sigma+\Omega)t_1}\left(e^{-t_1(\Omega+\varphi)}-1\right)}{(\Omega+\varphi)}\right]\\ -\dfrac{\bar{\lambda}}{(\sigma+\bar{\lambda})}\left[\begin{array}{l}\dfrac{e^{(\upsilon+\sigma t_2+\beta t_2)}\left(e^{-t_1(\bar{\lambda}+\varphi)}-e^{-t_2(\bar{\lambda}+\varphi)}\right)}{(\bar{\lambda}+r)}\\ +\dfrac{e^{\upsilon}\left(e^{t_1(\sigma-\varphi)}-e^{t_2(\sigma-\varphi)}\right)}{(\sigma-\varphi)}\end{array}\right]\\ +\dfrac{G\bar{\lambda}\left(e^{-t_1(\bar{\lambda}+\varphi)}-1\right)}{(\bar{\lambda}+\varphi)}\end{array}\right\}$$

E. Worth shortage cost per cycle

$$SC=-s_1\int_{t_2}^{T} I_s(t)e^{-\varphi t}dt$$

$$SC=-s_1\upsilon\chi\left[\frac{\left(e^{-\varphi T}-e^{-\varphi t_2}\right)+\varphi e^{-\varphi T}(T-t_2)}{\varphi^2}\right]$$

F. Worth lost sale cost

$$LS=s_2\int_{t_2}^{T}\upsilon(1-\eta)e^{-\varphi t}dt$$

$$LS=-s_2\upsilon\left(e^{-\varphi T}-e^{-\varphi t_2}\right)\left(\frac{1-\eta}{\varphi}\right)$$

Hence, the present value of the total inventory cost per unit of time in the proposed model is:

$$TCU(t_1,t_2,T)=\frac{OC+HC_{rw}+HC_{ow}+DC+SC+LS}{T} \quad (15)$$

The aforementioned equation representing the total average cost is thoroughly elucidated in equation (15).

By substituting the value of t_1 into equation (15) with reference to equation (10), we can derive:

$$TCU(t_1,t_2,T)=TCU(t_2,T)$$

To minimise the total cost of inventory per unit of time in present value, the necessary condition is to minimise:

$$\frac{\partial TCU(t_2,T)}{\partial t_2}=0 \quad \& \quad \frac{\partial TCU(t_2,T)}{\partial T}=0 \quad (16)$$

which also satisfy the conditions,

$$\frac{\partial^2 TCU(t_2,T)}{\partial t_2^2}>0 \quad \& \quad \frac{\partial^2 TCU(t_2,T)}{\partial T^2}>0 \quad (17)$$

Also,

$$\left(\frac{\partial^2 TCU(t_2,T)}{\partial t_2^2}\right)\left(\frac{\partial^2 TCU(t_2,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TCU(t_2,T)}{\partial t_2\partial T}\right)^2>0 \quad (18)$$

V. Solution Procedure

The optimisation process unfolds in several steps:

Step 1

Input all specific parameters into equation (15).

Step 2

Calculate the initial partial derivative of equation (15) as described in (16). Subsequently, solve these equation systems to determine critical points.

Step 3

Validate equations (17) and (18) by substituting the critical points.

Step 4

If the solution fails to satisfy either equation (17) or (18), it indicates that the proposed model is invalid, and it will not be possible to minimise the total cost.

Step 5

If the solution meets the criteria outlined in the equations, these critical points represent the optimal decision variables.

Step 6

Calculate the average total inventory cost per unit time by substituting the values of the decision variables into (15).

The total cost function in our analysis exhibits non-linear characteristics, necessitating the use of specialised computational tools for optimization. To determine a unique optimal solution, we rely on the cost function's convexity, a fundamental property in mathematical optimization. The specific determination of this optimal solution is accomplished using MATLAB R2017b software, known for its robust optimisation capabilities.

In our optimisation procedure, we utilise the "fmincon" function within MATLAB, a powerful tool for handling non-linear and constrained optimisation problems. This choice ensures effective navigation of the intricate landscape of our non-linear cost function, reliably identifying the optimal solution.

By leveraging these computational resources and methodologies, we derive the most favourable set of decision variables that minimise our total inventory cost. This optimisation process enhances decision-making in inventory management, providing valuable insights into the intricacies of the cost function.

VI. Numerical Example

The proposed model has undergone numerical analysis utilising the data presented in Table III. These data have been suitably expressed in units relevant to the study.

TABLE III
Values and Units of Parameters of the Proposed Model

Parameters	Values	Units
A	400	\$/order
σ	0.1	-----
υ	5.121	-----
ω_1	0.5	\$/unit
υ_1	2.5	\$/unit
ω_2	0.8	\$/unit
υ_2	5	\$/unit
Ω	0.09	%
λ	0.13	%
G	800	unit
η	0.9	%
d	5	\$/unit
φ	0.8	%
s_1	20	\$/unit
s_2	22	\$/unit

The optimal total inventory cost per unit time and ordering quantity are determined as $148.0671 \approx \$148$ and $148.0671 \approx \$148$ units respectively. The optimal cycle interval is determined to be t_1=2.0872 days, $t_2=4.3420$ *days* and T=43.7040 days.

The fulfillment of equations (17) and (18) serves as an indicator that the average of total costs within the model display convex characteristics.

VII. Sensitivity Analysis

A sensitivity analysis has been conducted to show the robustness of the proposed inventory model and the impact of the parameters on the optimal solution. Percentage changes are considered in analyzing sensitivity, and the criteria are modified by -10% to 10%. The consequences are determined by changing one parameter at a time while keeping the original values of the other parameters.

After analyzing the parameters, several important features are observed. Firstly, it is found that increasing the ordering cost led to an increase in the total cost without affecting other factors. Secondly, an increase in the demand's parameter (σ) resulted in a decrease in the total cost and quantity while increasing the cycle length, as evidenced by a decrease in t_1^* and t_2^* while increase in the demand's parameter (υ) resulted the same but the quantity is increased.

Furthermore, increasing the holding cost's parameters ($\omega_1 \& \upsilon_1$) of *RW* led to a decrease in the total cost, quantity, t_1^* and t_2^*, as well as an increase in the total cycle length. The total cost, quantity, t_1^*, t_2^* and T^* increased as the holding cost parameters ($\omega_2 \& \upsilon_2$) of *OW* were increased.

Moreover, with an escalation in the rate of deterioration (Ω) in RW, the total cost, quantity, and time ($t_1^* \& t_2^*$) decreased, while the overall cycle time (T^*) increased. Conversely, as the rate of OW deterioration increased, the total cost, quantity, t_1^* and T^* experienced an increase, leading to a decrease in time t_2^*.

Increasing the quantity in OW led to an increase in all factors while, increasing the shortage rate resulted in a decrease in the all the factors. An increase in the deterioration cost per unit time resulted in an increase in the total cost and cycle length, while decreasing quantity, t_1^* and t_2^*.

Increasing the quantity in OW led to an upsurge in all factors, while augmenting the shortage rate resulted in a decrease in all factors. A rise in the deterioration cost per unit time led to an increase in the total cost and cycle length, concurrently decreasing quantity and time ($t_1^* \& t_2^*$).

Furthermore, an increase in the rate of inflation led to a decrease in the total cost and cycle length, while resulting in an increase in time ($t_1^* \& t_2^*$).and quantity. Finally, an increase in the shortage and lost sale cost per unit time resulted in a decrease in all factors, although the decrease is relatively small. The percentage variation in these parameters is depicted in Fig. 2 through 15.

References

[1] Hartley RV. Operations research: A managerial emphasis. Goodyear; 1976.

[2] Ghare, P. M., & Schrader, G. F. (1963). A model for exponentially decaying inventory. Journal of Industrial Engineering, 14(5), 238-243.

[3] Jaggi CK, Verma P. An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. International Journal of Services Operations and Informatics. 2010 Jan 1;5(3):209-30. https://doi.org/10.1504/IJSOI.2010.034036

[4] Jaggi CK, Khanna A, Verma P. Two-warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationary conditions. International Journal of Systems Science. 2011 Jul 1;42(7):1185-96. https://doi.org/10.1080/00207720903353674

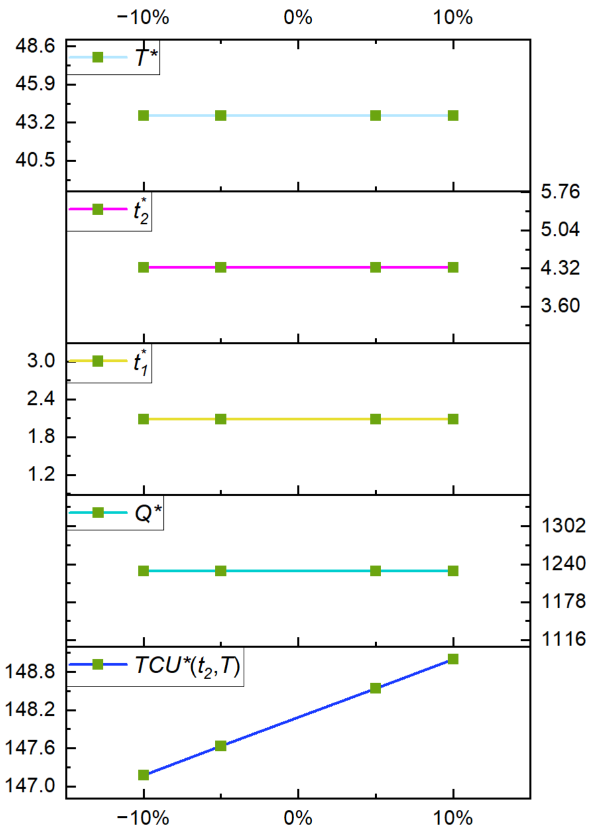

Fig. 2. Variation of parameter A

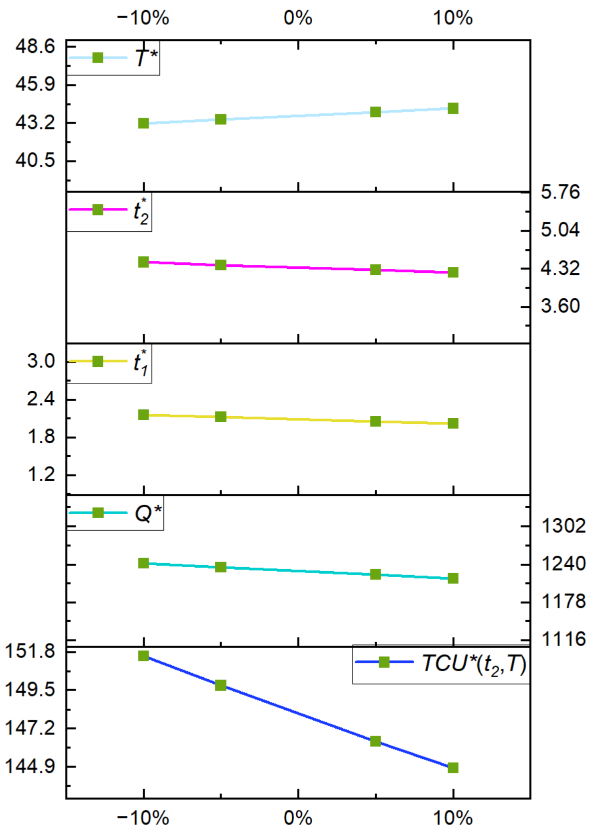

Fig. 3. Variation of parameter σ

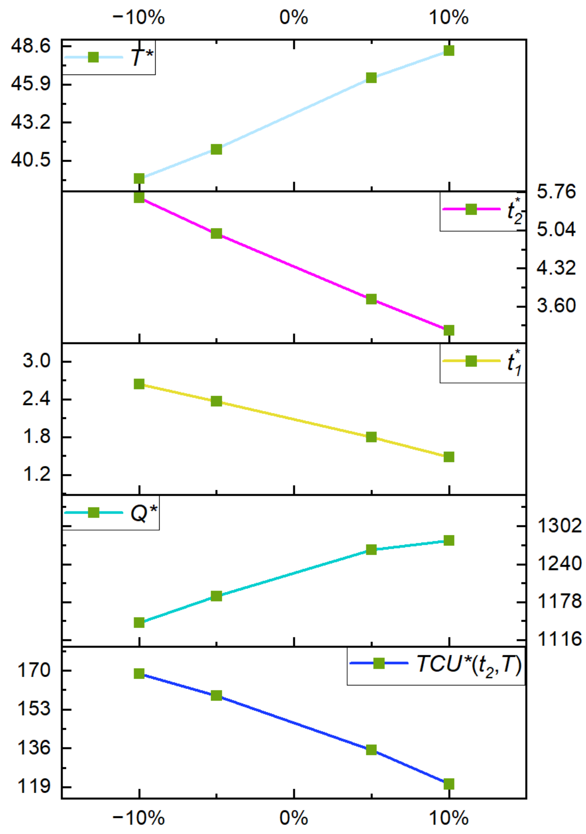

Fig. 4. Variation of parameter υ

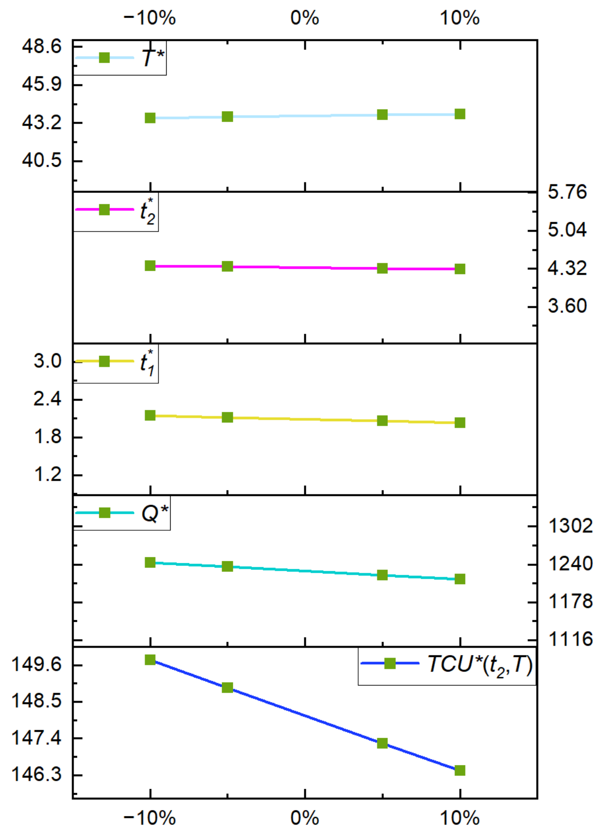

Fig. 5. Variation of parameter ω_1

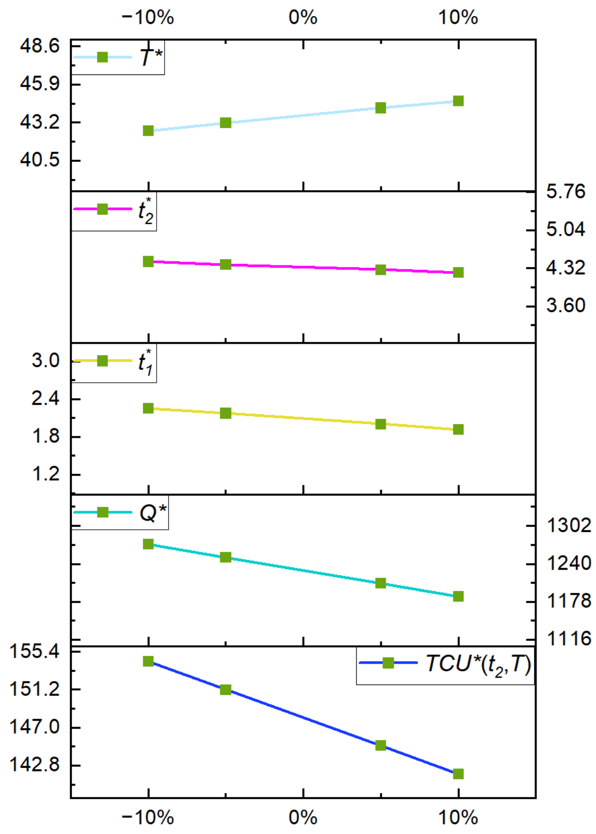

Fig. 6. Variation of parameter υ_1

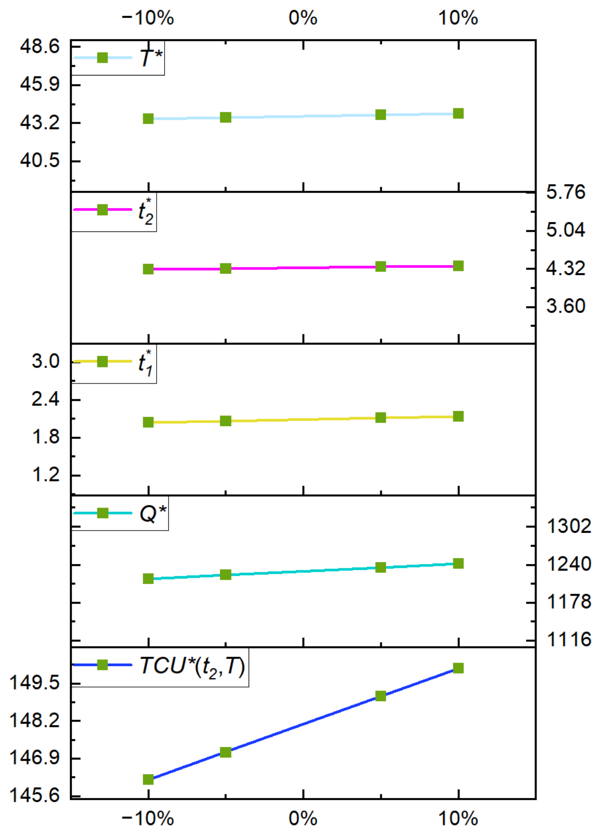

Fig. 7. Variation of parameter ω_2

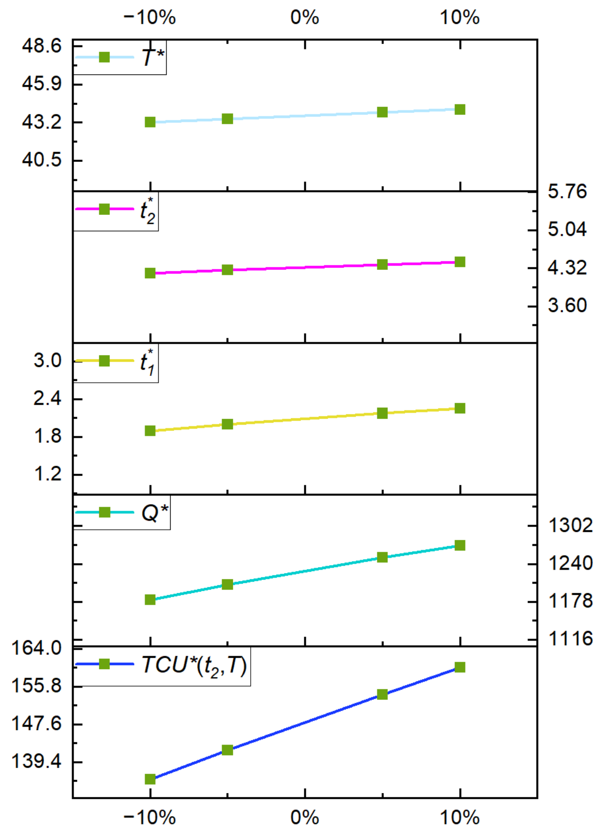

Fig. 8. Variation of parameter υ_2

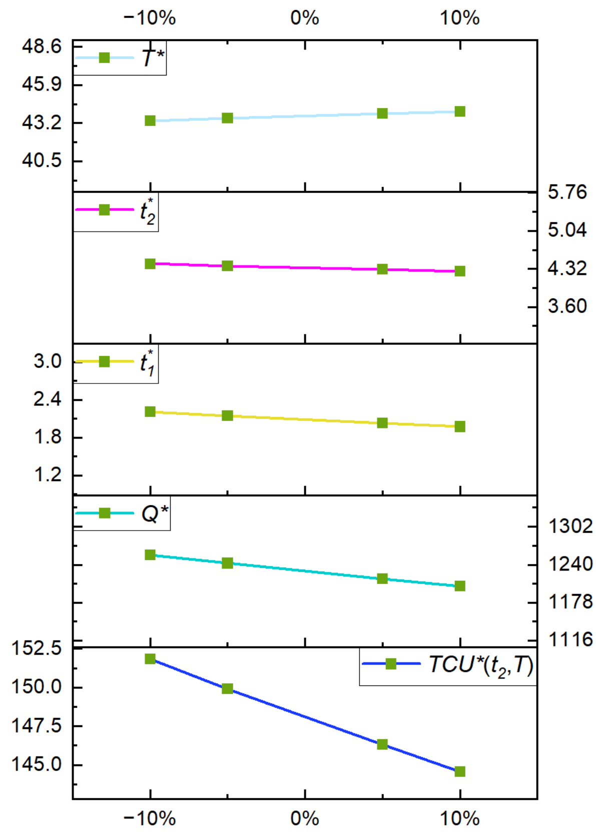

Fig. 9. Variation of parameter Ω

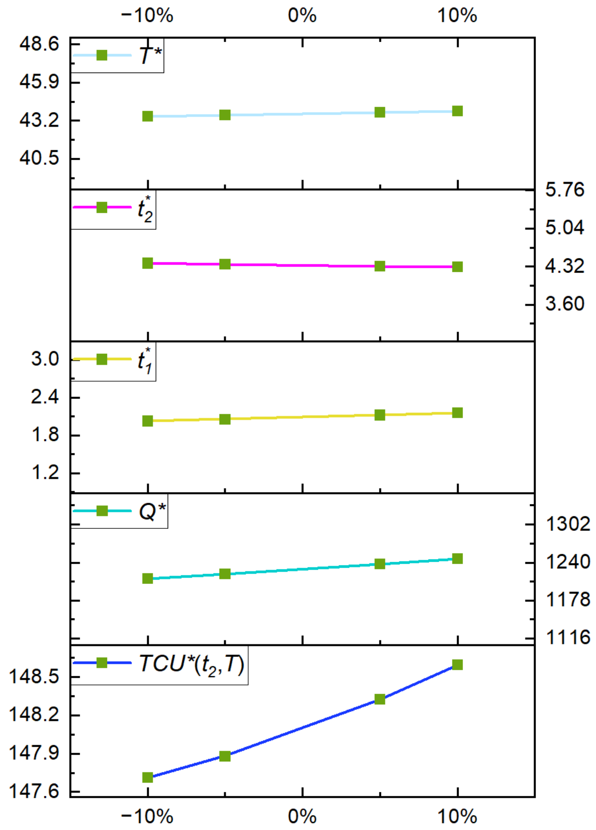

Fig. 10. Variation of parameter λ

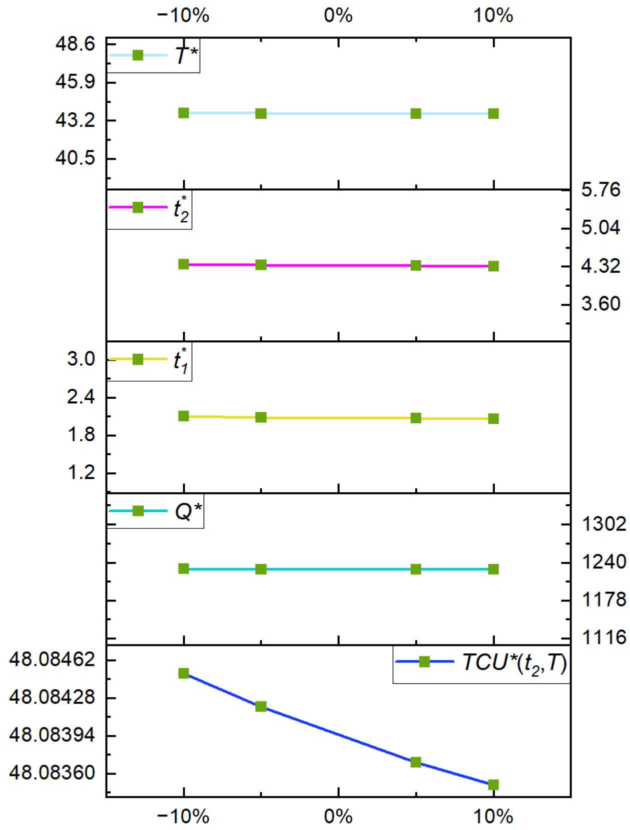

Fig. 12. Variation of parameter η

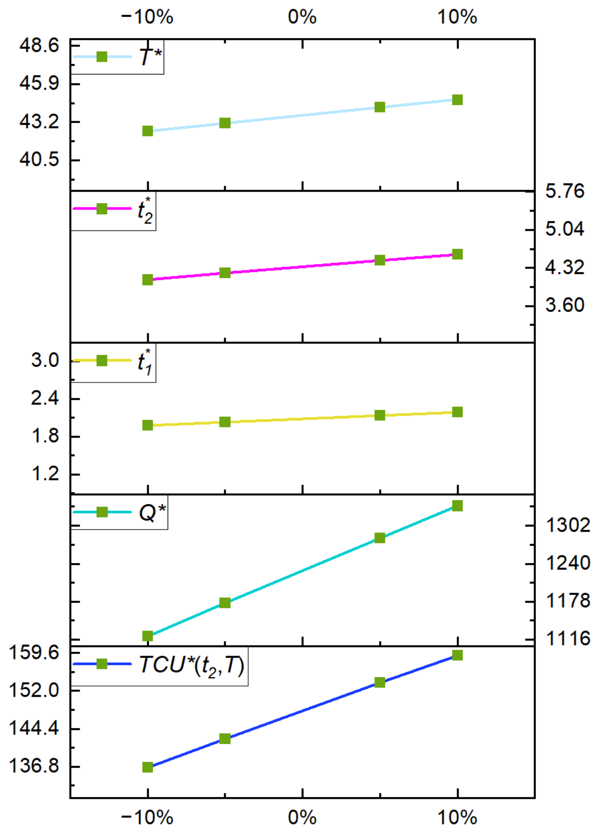

Fig. 11. Variation of parameter G

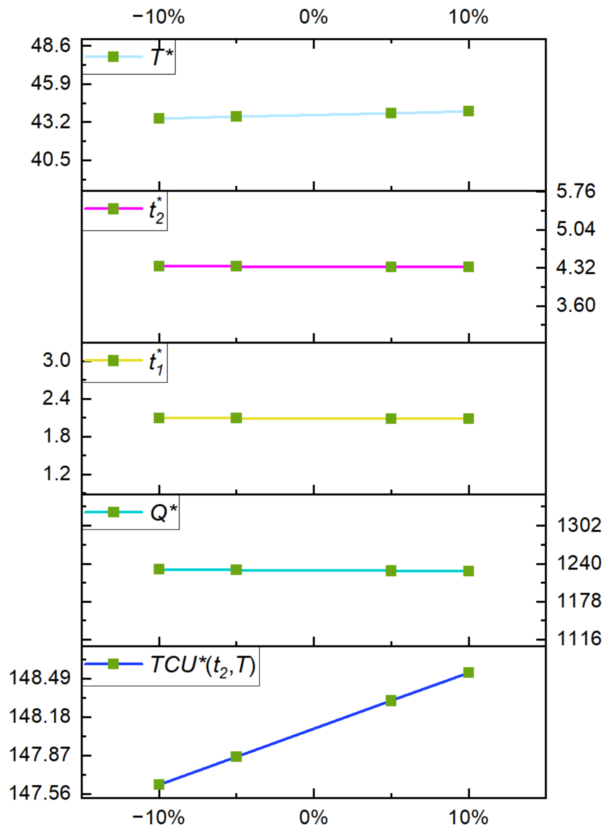

Fig. 13. Variation of parameter d

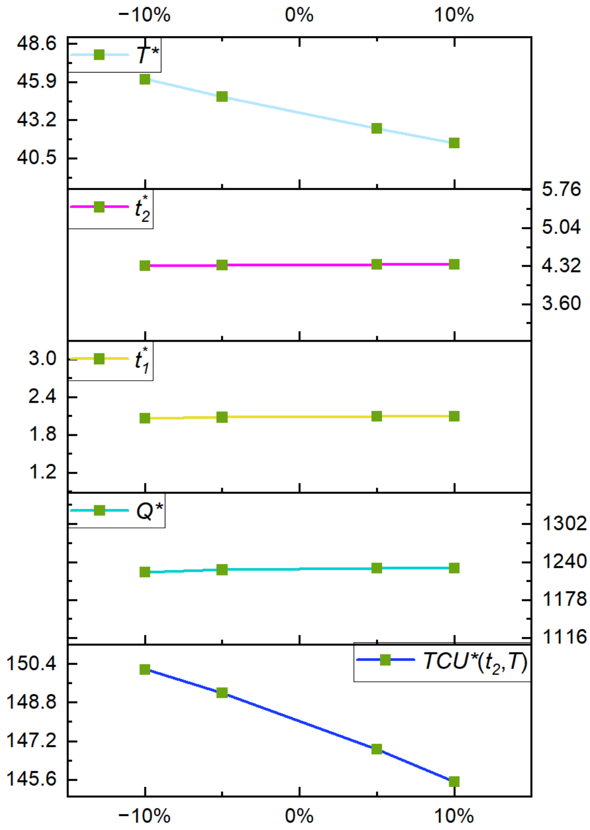

Fig. 14. Variation of parameter φ

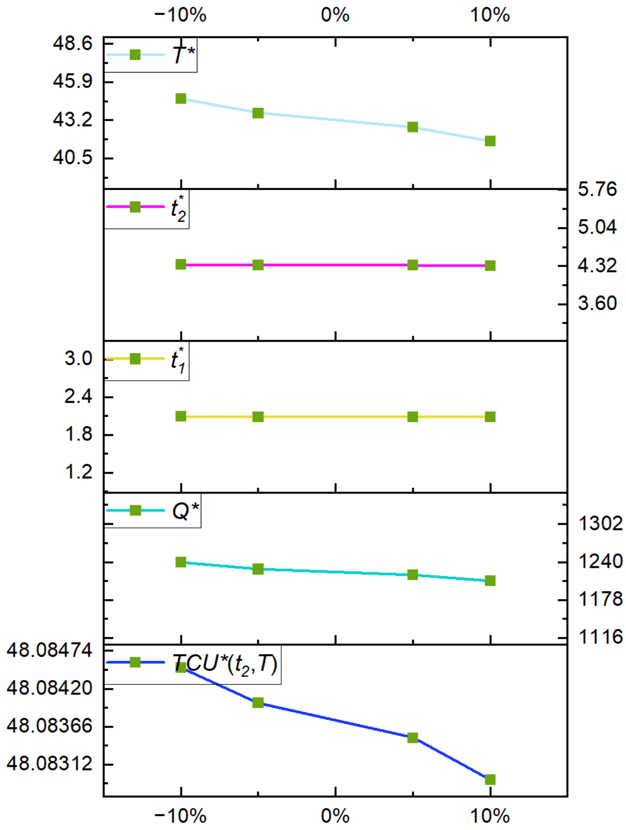

Fig. 16. Variation of parameter s_2

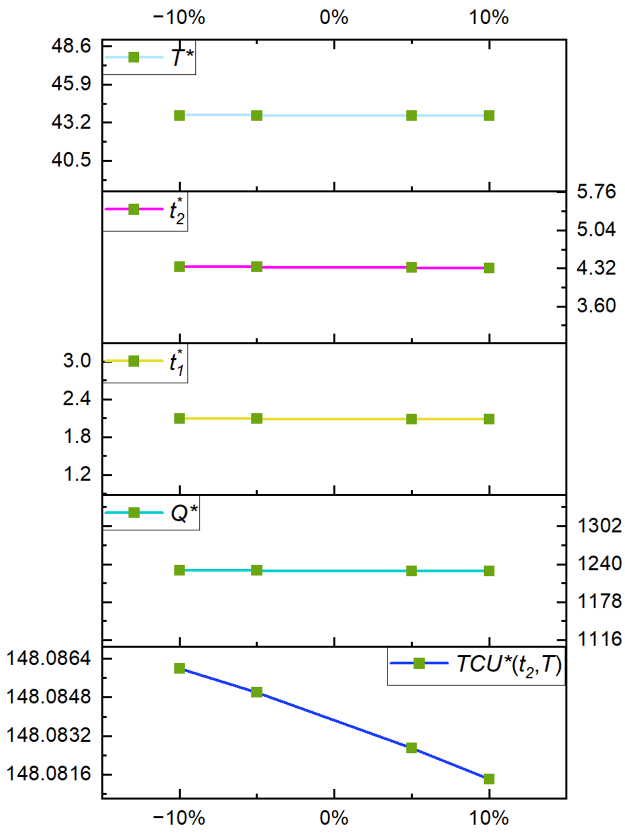

Fig. 15. Variation of parameter s_1

[5] Kumar N, Singh SR, Kumari R. Three echelon supply chain inventory model for deteriorating items with limited storage facility and lead-time under inflation. International Journal of Services and Operations Management. 2012 Jan 1;13(1):98-118. https://doi.org/10.1504/IJSOM.2012.048277

[6] Sett BK, Sarkar B, Goswami A. A two-warehouse inventory model with increasing demand and time varying deterioration. Scientia Iranica. 2012 Dec 1;19(6):1969-77. https://doi.org/10.1016/j.scient.2012.10.040

[7] Dash BP, Singh T, Pattnayak H. An inventory model for deteriorating items with exponential declining demand and time-varying holding cost. American Journal of Operations Research. 2014 Jan 6;2014. https://doi.org/10.4236/ajor.2014.41001

[8] Jaggi C, Tiwari S, Goel S. Replenishment policy for non-instantaneous deteriorating items in two storage facilities under inflationary conditions. International Journal of Industrial Engineering Computations. 2016;7(3):489-506. https://doi.org/10.5267/j.ijiec.2015.12.001

[9] Dutta D, Kumar P. A partial backlogging inventory model for deteriorating items with time-varying demand and holding cost. International Journal of Mathematics in Operational Research. 2015;7(3):281-96. https://doi.org/10.1504/IJMOR.2015.069144

[10] Saha S, Sen N. An inventory model for deteriorating items with time and price dependent demand and shortages under the effect of inflation. International Journal of Mathematics in Operational Research.2019;14(3):377-88. https://doi.org/10.1504/IJMOR.2019.099385

[11] Yadav AS, Swami A. An inventory model for non-instantaneous deteriorating items with variable holding cost under two-storage. International Journal of Procurement Management. 2019;12(6):690-710. https://doi.org/10.1504/IJPM.2019.102928

[12] Khan MA, Shaikh AA, Panda GC, Bhunia AK, Konstantaras I. Non-instantaneous deterioration effect in ordering decisions for a two-warehouse inventory system under advance payment and backlogging. Annals of Operations Research. 2020 Jun;289:243-75. https://doi.org/10.1007/s10479-020-03568-x

[13] Sahoo CK, Paul KC, Kumar S. Two warehouses EOQ inventory model of degrading matter having exponential decreasing order, limited suspension in price including salvage value. SN Computer Science. 2020 Nov;1(6):1-9. https://doi.org/10.1007/s42979-020-00346-1

[14] Das S, Manna AK, Mahmoud EE, Abualnaja KM, Abdel-Aty AH, Shaikh AA. Product replacement policy in a production inventory model with replacement period-, stock-, and price-dependent demand. Journal of Mathematics. 2020 Dec 29;2020. https://doi.org/10.1155/2020/6697279

[15] Chakraborty D, Jana DK, Roy TK. Multi-warehouse partial backlogging inventory system with inflation for non-instantaneous deteriorating multi-item under imprecise environment. Soft Computing. 2020 Oct;24(19):14471-90. https://doi.org/10.1007/s00500-020-04800-3

[16] Kumar V. To develop a deterministic inventory model for deteriorating items with biquadratic demand rate and constant deterioration rate. International Journal of Mathematics Trends and Technology-IJMTT. 2021;67. https://doi.org/10.14445/22315373/IJMTT-V67I9P511

[17] Pathak, K., Yadav, A.S., Agarwal, P. (2024). Optimizing two-warehouse inventory for shelf-life stock with time-varying bi-quadratic demand under shortages and inflation. Mathematical Modelling of Engineering Problems, Vol. 11, No. 2, pp. 446-456. https://doi.org/10.18280/mmep.110216

[18] Malik AK, Garg H. An improved fuzzy inventory model under two warehouses. Journal of Artificial Intelligence and Systems. 2021 Sep 8;3(1):115-29. https://doi.org/10.33969/AIS.2021.31008

[19] Neeraj Kumar, Sweta Dahiya, Sanjey Kumar, Two warehouse inventory model for deteriorating items with fixed shelf-life stock-dependent demand and partial backlogging, J. Math. Comput. Sci., 12 (2022), Article ID 85. https://doi.org/10.28919/jmcs/7142

$$
TCU(t_1,t_2,T)=\frac{1}{T}\left\{\begin{array}{l}
A+\left\{\begin{array}{l}
\dfrac{\omega_1 e^{(\sigma+\Omega)t_1+\upsilon}}{(\sigma+\Omega)}\left[\dfrac{1-e^{-(\Omega+\varphi)t_1}\left(t_1(\Omega+\varphi)+1\right)}{(\Omega+\varphi)^2}\right]-\dfrac{\upsilon_1 e^{(\sigma+\Omega)t_1+\upsilon}\left(e^{-(\Omega+\varphi)t_1}-1\right)}{(\sigma+\Omega)(\Omega+\varphi)} \\
-\dfrac{\omega_1 e^{\upsilon}}{(\sigma+\Omega)}\left[\dfrac{1+e^{(\sigma-\varphi)t_1}\left(t_1(\sigma-\varphi)-1\right)}{(\sigma-\varphi)^2}\right]-\dfrac{\upsilon_1 e^{\upsilon}\left(e^{(\sigma-\varphi)t_1}-1\right)}{(\sigma+\Omega)(\sigma-\varphi)}
\end{array}\right\} \\
+\left\{\begin{array}{l}
\dfrac{\omega_2 e^{\upsilon+(\sigma+\bar{\lambda})t_2}}{(\sigma+\bar{\lambda})}\left[\dfrac{e^{-t_1(\bar{\lambda}+\varphi)}\left((\bar{\lambda}+\varphi)t_1+1\right)-e^{-t_2(\bar{\lambda}+\varphi)}\left((\bar{\lambda}+\varphi)t_2+1\right)}{(\bar{\lambda}+\varphi)^2}\right] \\
-\dfrac{\omega_2 e^{\upsilon}}{\sigma+\bar{\lambda}}\left[\dfrac{e^{t_1(\sigma-\varphi)}\left((\varphi-\sigma)t_1+1\right)-e^{t_2(\sigma-\varphi)}\left((\varphi-\sigma)t_2+1\right)}{(\sigma-\varphi)^2}\right] \\
+\dfrac{\upsilon_2 e^{\upsilon+(\sigma+\bar{\lambda})t_2}\left(e^{-t_1(\bar{\lambda}+\varphi)}-e^{-t_2(\bar{\lambda}+\varphi)}\right)}{(\sigma+\bar{\lambda})(\bar{\lambda}+\varphi)}+\dfrac{\upsilon_2 e^{\upsilon}(e^{t_1(\sigma-\varphi)}-e^{t_2(\sigma-\varphi)})}{(\sigma+\bar{\lambda})(\sigma-\varphi)} \\
-\dfrac{G\omega_2 e^{-t_1(\bar{\lambda}+\varphi)}\left(\bar{\lambda}t_1-e^{t_1(\bar{\lambda}+\varphi)}+\varphi t_1+1\right)}{(\bar{\lambda}+\varphi)^2}-\dfrac{G\upsilon_2\left(e^{-t_1(\bar{\lambda}+\varphi)}-1\right)}{\bar{\lambda}+\varphi}
\end{array}\right\} \\
-d\left\{\begin{array}{l}
\dfrac{\Omega}{(\sigma+\Omega)}\left[\dfrac{e^{\sigma}\left(e^{t_1(\sigma-\varphi)}-1\right)}{(\sigma-\varphi)}+\dfrac{e^{\upsilon+(\sigma+\Omega)t_1}\left(e^{-t_1(\Omega+\varphi)}-1\right)}{(\Omega+\varphi)}\right] \\
-\dfrac{\bar{\lambda}}{(\sigma+\bar{\lambda})}\left[\dfrac{e^{(\upsilon+\sigma t_2+\beta t_2)}\left(e^{-t_1(\bar{\lambda}+\varphi)}-e^{-t_2(\bar{\lambda}+\varphi)}\right)}{(\bar{\lambda}+r)}+\dfrac{e^{\upsilon}\left(e^{t_1(\sigma-\varphi)}-e^{t_2(\sigma-\varphi)}\right)}{(\sigma-\varphi)}\right] \\
+\dfrac{G\bar{\lambda}\left(e^{-t_1(\bar{\lambda}+\varphi)}-1\right)}{(\bar{\lambda}+\varphi)}
\end{array}\right\} \\
-s_1\upsilon\chi\left[\dfrac{\left(e^{-\varphi T}-e^{-\varphi t_2}\right)+\varphi e^{-\varphi T}(T-t_2)}{\varphi^2}\right]-s_2\upsilon\left(e^{-\varphi T}-e^{-\varphi t_2}\right)\left(\dfrac{1-\eta}{\varphi}\right)
\end{array}\right\} \quad (15)
$$

TABLE I
LITERATURE REVIEW FOR THE PROPOSED MODEL

Author	Warehouse System	Holding Cost	Inflation	Shortages	Demand	Deterioration
Jaggi CK & Verma P. (2010)	Two warehouses	Constant	Not allowed	Completely backlogged	Constant	Non-Instantaneous
Jaggi CK *et al*. (2011)	Two warehouses	Constant	Allowed	Partial backlog	Varying linearly with time	Non-Instantaneous
Kumar N *et al*. (2012)	Two warehouses	Constant	Allowed	Partial backlog	Stock-dependent demand	Non-Instantaneous
Sett BK *et al.* (2012)	Two warehouses	Constant	Not allowed	Not allowed	Quadratic time dependent	Instantaneous
Dash BP *et al.* (2014)	Single	Varying with time	Not allowed	Not allowed	Exponential decline in demand	Instantaneous
Chandra K. Jaggi et al. (2015)	Two warehouses	Constant	Allowed	Partial backlog	Constant	Non-Instantaneous
Dutta D & Kumar P. (2015)	Single	Linear time dependent	Not allowed	Partial backlog	Linear time dependent	Instantaneous
Saha S & Sen N. (2019)	Single	Constant	Allowed	Partial backlog	Time and price dependent demand	Instantaneous
Yadav AS & Swami A. (2019)	Two warehouses	Linear Time-dependent	Not allowed	Completely backlogged	Linear time dependent	Non-Instantaneous
Khan MA *et.* al.(2020)	Two warehouses	Constant	Not allowed	Partial backlog	Depending on the price	Non-Instantaneous
Sahoo CK *et al*. (2020)	Two warehouses	Constant	Not allowed	Not allowed	Exponential decreasing	Instantaneous
Das S. (2020)	Single	Constant	Not allowed	Not allowed	Replacement period, depending on stock and price	Not considered
Chakraborty D (2020)	Multi-warehouse	Constant	Allowed	Partial backlog	Stock-dependent	Non-Instantaneous
Suman VK. (2021)	Single	Constant	Not allowed	Completely backlogged	Bi-quadratic time dependent	Instantaneous
Pathak K. *et al.*(2024)	Two warehouses	Constant	Allowed	Partial backlog	Bi-quadratic time dependent	Non-Instantaneous
Malik AK & Garg H. (2021)	Two warehouses	Constant	Not allowed	Not allowed	Linear demand function	Instantaneous
Kumar N *et al.* (2022)	Two warehouses	Constant	Not allowed	Partial backlog	Stock-dependent demand	Non-Instantaneous
In this article	Two warehouses	Linearly time dependent	Allowed	Partial backlog	Exponential variation of time	Instantaneous