Further Study for Maximizing Deviation Method with Trapezoidal Fuzzy Number

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Abstract—**This study is a response to a paper on group decision problems in a linguistic environment. We show that their M1 model can be directly solved by Schwarz inequality to avoid using the Lagrange multiplier methods. Moreover, their M2 model can also be solved by our modification of applying Schwarz inequality to avoid referring to soft ware of MatLab with Lingo or Lindo soft ware package or optimization toolbox. The same numerical examples are provided to illustrate our findings.**

Index Terms—**The Schwarz inequality, Maximizing derivation method, Linguistic variables, Group decision making**

I. Introduction

ZADEH [1] orginated the current of fuzzy set theory to deal with decision making problems with respect to fuzzy environment. Underthe notation of X as a universe of discourse which is the domain of a mapping, a fuzzy subset $\widetilde{A}$ with respect to X is developed with a membership mapping $\mu_{\widetilde{A}}(x)$ that assigns for every element x in the universe of discourse to a real number within the closed unitary interval, [0, 1]. We recall that Keufmann and Gupta [2] assumed the following definition for the mapping value of $\mu_{\widetilde{A}}(x)$ denotes the score of membership of x in $\widetilde{A}$. If the related grade of membership of x in $\widetilde{A}$ is strong then the value of $\mu_{\widetilde{A}}(x)$ is large. To examine the proposed fuzzy model proposed by Wu and Chen [3], we recall some properties and definitions which are relative to Wu and Chen [3]. We refer to those results in the following. Keufmann and Gupta [2] and Dubois and Prade [4] assumed that a trapezoidal fuzzy number which is expressed as $\widetilde{A} = (a, b, c, d)$ on R with $a < b < c < d$, is a special fuzzy set with the following expression of its membership mapping,

$$\mu_{\widetilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b \\ 1, & b \le x \le c \\ (d-x)/(d-c), & c \le x \le d' \\ 0, & \text{otherwise} \end{cases} \tag{1.1}$$

where parameters of a, b, c, and d are criap (real) numbers. The trapezoidal fuzzy numbers are usually expressed as (a, b, c, d). According to Liang [5], those variables of x with $b \le x \le c$, attains the maximal score of $\mu_{\widetilde{A}}(x)$ with $\mu_{\widetilde{A}}(x) = 1$ to indicate the value of x are most likely occurs in the closed interval $[b, c]$. The other two parameters of constants, a and d are the left and right wings of the available area for the studied information. These four parameters indicate the vagueness of the studied information. Owing to trapezoidal fuzzy numbers can be dealt with arithematical operations such that the most commonly adopted vague numbers are trapezoidal fuzzy numbers. We follow thie research trend to use trapezoidal fuzzy numbers in our examination. Based on Yao and Wu [6], for a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$, the signed distance is assumed in the following,

$$d(\widetilde{A}) = 1/(a + b + c + d). \tag{1.2}$$

According to Yao and Chiang [7], when defuzzying a trapezoidal fuzzy number, most researchers agreed that the centroid method is inferior to the signed distance method, because of the computation consideration. With respect to Yao and Wu [6], Tang et al. [8], Tsaur et al. [9], Yager and Filev [10], and Zhao and Govind [11], vague numbers indicates vague scores and vague weights. Vague numbers are changed to real crisp numbers to recognize the best alternative among candidates, through the following four defuzzying approaches: (i) the signed distance method, (ii) the α-cut method, (iii) the mean of maximal process, and (iv) center of the area (or the centroid method). Klic and Yan [12] pointed out that the above four mentioned approaches has its drawbacks and benefits. Because of computation burdon, the signed distance method and the centroid method are the popular two approaches in the academic society. In this paper, we will follow Yao and Wu [6], Tang et al. [8], Tsaur et al. [9], Yager and Filev [10], and Zhao and Govind [11], to adopt the signed distance method to defuzzy a trapezoidal fuzzy number to a real crisp number. On the other hand, we briefly review and fuzzy numbers and linguistic variables.

Fuzzy numbers are converted from linguistic terms through conversion scales, where the experts only provided their opinion in linguistic terms.

Liang [5] and Liang and Wang [13] claimed that the most two conversion systems are proposed by one to five scales for the weight importance and one to nine scales to rate alternatives. Chen and Hwang [14] pointed out that it is an intuitive assignment to decide how many numbers in the conversion scale model. If there are too many conversion scales, aand then the model becomes too complex to be used in the applied situation. On the other hand, if there are too few conversion scales and then it will be difficult to separate criteria and alternatives. Consequently, Chen and Hwang [14] assumed eight conversion scales to generate a fuzzy number from a linguistic term. However. in this study, we still prefer to one to nine scale as the hierarchy process proposed by Saaty [15]. Therefore, for each individual attribute, we refer to Table 2 of Anisseh et al. [16] to list the relationship between linguistic variables and fuzzy numbers.

Manuscript received January 16, 2024; revised April 29, 2024.

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Table 1. Fuzzy numbers and linguistic variables for the importance of weight.

LV	EL	VL	L	ML	M	MH	H	VH	EH
Fuzzy numbers	(0, 0, 0.1, 0.2)	(0.1, 0.2, 0.3, 0.4)	(0.2, 0.3, 0.4, 0.5)	(0.3, 0.4, 0.5, 0.6)	(0.4, 0.5, 0.6, 0.7)	(0.5, 0.6, 0.7, 0.8)	(0.6, 0.7, 0.8, 0.9)	(0.7, 0.8, 0.9, 1.0)	(0.8, 0.9, 1.0, 1.0)

Reproduced from Table 2 of Anisseh et al. [16]. In Talbe 1, we use the following abbreviations: Linguistic variables (LV), Extremely low (EL), Very low (VL), Low (L), Medium low (ML), Medium (M), Medium High (MH), High (H), Very high (VH), and Extremely high (EH).

Table 2. Fuzzy numbers and linguistic variables for the ratings.

LV	EP	VP	P	MP	F	MG	G	VG	EG
Fuzzy numbers	(0, 0, 1, 2)	(1, 2, 3, 4)	(2, 3, 4, 5)	(3, 4, 5, 6)	(4, 5, 6, 7)	(5, 6, 7, 8)	(6, 7, 8, 9)	(7, 8, 9, 10)	(8, 9, 10, 10)

Reproduced from Table 1 of Anisseh et al. [16].In Talbe 2, we use the following abbreviations: Linguistic variables (LV), Extremely poor (EP), Very poor (VP), poor (P), Medium poor (MP), Fair (F), Medium good (MG), Good (G), Very good (VG), and Extremely good (EG).

Moreover, we cite Table 1 of Anisseh et al. [16] to list the relationship between fuzzy numbers and linguistic variables with respect to rating. On the other hand, Zimmermann [17] and Liang [5] mentioned that quantitative and financial terms can be expressed by trapezoidal fuzzy numbers. Liang [5] Took three examples to assert that (i) (300, 300, 300, 300) is used to indicate the crisp real number \$300, (ii) (750, 760, 800, 807) is used to indicate the linguistic term: between \$760 and \$800, and (iii) (690, 700, 700, 710) is used to indicate the linguistic term: approximately equal to \$700.

The fuzzy set theory offers a suitable structure to illustrate and consider vagueness related to the aforementioned ambiguity of natural language expression and opinions. The information is expressed employing linguistic terms. For example, a finite and completely ordered discrete nine terms, S, could be denoted by Xu [18],

$$\begin{aligned} S = \{ & s_{-4} = \text{extremely poor},\ \ s_{-3} = \text{very poor}, \\ & s_{-2} = \text{poor},\ \ s_{-1} = \text{slightly poor}, \\ & s_0 = \text{fair},\ \ s_1 = \text{slightly good},\ \ s_2 = \text{good}, \\ & s_3 = \text{very good},\ \ s_4 = \text{extremely good} \}. \end{aligned} \quad (1.3)$$

To express their estimation informative and reliable, experts applied linguistic terms to deal with decision making problems. Many research papers are related to group decision making environment in a linguistic environment, for example, Ben-Arieh and Chen [19], Cordon et al. [20], Herrera and Herrera-Viedma [21], Herrera et al. [22], Herrera-Viedma et al. [23], Peláez and Doňa [24], Tang and Zheng [25], Wang and Hao [26], Xu [27, 28, 29] such that they deneloped lots of linguistic aggregation operators to combine thier results. They focused on the group consensus among decision making experts and developed novel aggregated operations. On the other hand, they did not pay attention to the related weighs of alternatives and criteria. We will discuss the two most related papers: Wang [30] and Wu and Chen [3] to examine their approaches to deriving weights for attributes.

II. Review of the approach of Wang [30] and Wu and Chen [3]

According to the common sense, to make a difference among alternatives, Wang [30] mentioned that a larger weight should be assigned to those criteria with a large sepation value among others. When criteria weights are completely unknown, Wang [30] constructed a new weighted averaging operation to locate the optimal weighted ratios for criteria with respect to alternatives. The procedure of Wang [30] is generalized by Wu and Chen [3] to the environment that criteria weights are only partially known. However, Wu and Chen [3] cannot analytically solve their extended problem such that they mentioned that MatLab software with Lindo/Lingo software package or an optimization toolbox may be applied to solve a conditional non-linear optimized program proposed by Wu and Chen [3]. To derive the inclination value for every alternative, and then Wu and Chen [3] combined the decision data through weighted averaging operation.

We use the same notations and assumptions as Wu and Chen [3]. There are $X = \{X_1, \dots, X_n\}$ to denote alternatives, $C = \{C_1, \dots, C_m\}$ to denote criteria, and $D = \{D_1, \dots, D_t\}$ to denote decision-makers. The relative weights for decision-makers are known to be expressed as $(\lambda_1, \dots, \lambda_t)$ with $\lambda_j \geq 0$, and $\sum_{j=1}^{t} \lambda_j = 1$. The decision matrix that was given by the decision-maker D_k is denoted as $A^{(k)} = \left[a_{ij}^{(k)}\right]_{n\times m}$ where $a_{ij}^{(k)}$ is a linguistic variable for the alternative X_i concerning the attribute C_j.

Wu and Chen [3] computed the difference for every alternative to assume that the difference of an alternative X_i to all the other alternatives related to the criterion C_j, of the *k*th decision-maker is assumed as follows:

$$H_{ij}^{(k)}(w) = \sum_{l=1}^{n} T_{ilj}, \quad (2.1)$$

for $i = 1, \dots, n$ and $j = 1, \dots, m$, where $I\left(a_{ij}^{(k)}\right)$ is the subscript of $a_{ij}^{(k)}$, and $T_{ilj} = \left|I\left(a_{ij}^{(k)}\right) - I\left(a_{lj}^{(k)}\right)\right|^2 w_j$ is an abbreviation.

And then, they assume that

$$\begin{aligned} H_j^{(k)}(w) &= \sum_{i=1}^{n} H_{ij}^{(k)}(w), \\ &= \sum_{i=1}^{n} \sum_{l=1}^{n} T_{ilj}, \end{aligned} \quad (2.2)$$

where $T_{ilj} = \left|I\left(a_{ij}^{(k)}\right) - I\left(a_{lj}^{(k)}\right)\right|^2 w_j$ is an abbreviation, to denote the derivation value for an alternative to all alternatives and the *k*th decision maker and the attribute C_j. The maximizing deviation method is to choose the weight vector, $w = (w_1, \dots, w_m)$ with $w_j \geq 0$ for $j = 1, \dots, m$, to maximize all derivation values under the restricted norm. Under the 2-norm, i.e. the Euclidean norm, it requires that $\sum_{j=1}^{m} w_j^2 = 1$. On the other hand, under the 1-norm, then it has that $\sum_{j=1}^{m} w_j = 1$.

Wu and Chen [3] tried to construct a non-linear programming model (Model M1) as follows:

$$\max H(w) = \sum_{k=1}^{t} \lambda_k \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{l=1}^{n} T_{ilj}, \quad (2.3)$$

such that $w_j \geq 0$ for $j = 1, \dots, m$, and $\sum_{j=1}^{m} w_j^2 = 1$, where $T_{ilj} = \left|I\left(a_{ij}^{(k)}\right) - I\left(a_{lj}^{(k)}\right)\right|^2 w_j$ is an abbreviation.

They used Lagrange multiplier to solve their non-linear programming model to derive that

$$w_j = \frac{Y_j}{\sqrt{\sum_{j=1}^{m} Y_j^2}}, \tag{2.4}$$

for $j = 1, \ldots, m$, and

$$Y_j = \sum_{k=1}^{t} \lambda_k \sum_{i=1}^{n} \sum_{l=1}^{n} \Upsilon_{ilj}, \tag{2.5}$$

for $j = 1, \ldots, m$, where $\Upsilon_{ilj} = \left| I\left(a_{ij}^{(k)}\right) - I\left(a_{lj}^{(k)}\right) \right|^2$ is an abbreviation.

On the other hand, if the norm is changed from 2-norm to 1-norm, then the maximum solution must satisfy $\sum_{j=1}^{m} w_j = 1$ such that they normalized the results of equation (2.4) to imply that

$$w_j = \frac{Y_j}{\sum_{j=1}^{m} Y_j}, \tag{2.6}$$

for $j = 1, \ldots, m$.

In model (M1), Wu and Chen [3] assumed that all weights for criteria are unknown. They further considered that in some real environment that the data of criteria is partially known. They assumed that Φ is the data set with known weight, then they constructed another non-linear programming model (Model M II):

$$\max H(w) = \sum_{k=1}^{t} \lambda_k \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{l=1}^{n} T_{ilj}, \tag{2.7}$$

such that $w \in \Phi$, $w_j \geq 0$ for $j = 1, \ldots, m$, and $\sum_{j=1}^{m} w_j^2 = 1$, and $T_{ilj} = \left| I\left(a_{ij}^{(k)}\right) - I\left(a_{lj}^{(k)}\right) \right|^2 w_j$ is an abbreviation.

Referring to Lindo/Lingo software package or an optimization toolbox Wu and Chen [3] mentioned that the optimal problem of Equation (2.7) can be dealt with software of MatLab.

III. Our Improvement

By equation (2.5), we rewrite equation (2.3) for model M1 as follows

$$\max H(w) = \sum_{j=1}^{m} Y_j w_j, \tag{3.1}$$

such that $w_j \geq 0$ for $j = 1, \ldots, m$, and $\sum_{j=1}^{m} w_j^2 = 1$.

Motivated by equation (3.1), we know that the objective function is to maximize the inner product of two vectors, $Y = (Y_1, \ldots, Y_m)$ and $W = (w_1, \ldots, w_m)$. From the Schwarz inequality, it yields that

$$|Y \cdot W| \leq \|Y\| \, \|W\|, \tag{3.2}$$

that is, under the 2-norm,

$$\sum_{j=1}^{m} Y_j w_j \leq \sqrt{\sum_{j=1}^{m} Y_j^2} \sqrt{\sum_{j=1}^{m} w_j^2},$$

$$= \sqrt{\sum_{j=1}^{m} Y_j^2}. \tag{3.3}$$

such that it attains its maximum when W has the same direction as Y, that is,

$$W = kY, \tag{3.4}$$

with $k > 0$. If we want the Euclidean norm (2-norm) of W being one, that is $\sum_{j=1}^{m} w_j^2 = 1$, then it implies that

$$\sum_{j=1}^{m} \left(kY_j\right)^2 = 1. \tag{3.5}$$

Based on equation (3.5), we find that

$$k = \frac{1}{\sqrt{\sum_{j=1}^{m} Y_j^2}}. \tag{3.6}$$

Our results of equations (3.4) and (3.6) are the same findings as equation (2.4) proposed by Wu and Chen [3] with the Lagrange multiplier approach.

On the other hand, if want the 1-norm of W being one, that is $\sum_{j=1}^{m} w_j = 1$, then it shows that

$$\sum_{j=1}^{m} kY_j = 1. \tag{3.7}$$

Based on equation (3.7), we find that

$$k = \frac{1}{\sum_{j=1}^{m} Y_j}. \tag{3.8}$$

Our results of equations (3.4) and (3.6) are the same findings as equation (2.6) proposed by Wu and Chen [3].

From the above discussion, based on the Schwarz inequality, we provide a straightforward derivation for the model (M I).

Next, for model (M 2), we consider their second non-linear programming model. To simplify the expression, without loss of generality, we assume that

$$\Phi = \{w_1, \ldots, w_s\}, \tag{3.9}$$

to denote those known weights for criteria, and $\{w_{s+1}, \ldots, w_m\}$ to denote those unknown weights for criteria such that $w_j \geq 0$ for $j = 1, \ldots, s$, $\sum_{j=1}^{s} w_j^2 \leq 1$, for 2-norm, and $\sum_{j=1}^{s} w_j \leq 1$, for 1-norm.

We rewrite their model (M2) as follows

$$\max H(w) = A + \sum_{j=s+1}^{m} Y_j w_j, \tag{3.10}$$

such that for 2-norm,

$$\sum_{j=s+1}^{m} w_j^2 = 1 - \sum_{j=1}^{s} w_j^2, \tag{3.11}$$

and for 1-norm,

$$\sum_{j=s+1}^{m} w_j = 1 - \sum_{j=1}^{s} w_j, \tag{3.12}$$

where A is a constant, with

$$A = \sum_{j=1}^{s} Y_j w_j. \tag{3.13}$$

Based on the Schwarz inequality, we know that the maximum value will occur at

$$(w_{s+1}, \ldots, w_m) = k(Y_{s+1}, \ldots, Y_m), \tag{3.14}$$

such that for 2-norm

$$k = \left(\sqrt{1 - \sum_{j=1}^{s} w_j^2}\right) \Big/ \sqrt{\sum_{j=s+1}^{m} Y_j^2}. \tag{3.15}$$

On the other hand, for the 1-norm

$$k = \left(1 - \sum_{j=1}^{s} w_j\right) / \sum_{j=s+1}^{m} Y_j. \tag{3.16}$$

Hence, we directly derived the maximum solution for the model (M2) without using MatLab software with the optimization toolbox or Lindo/Lingo software package that was proposed by Wu and Chen [3].

IV. Numerical Example

For easily comparable to the results of Wu and Chen [3], we consider the same numerical as them that was based on an example of Wu and Chen [3]. A risk investment company that wants to invest a sum of money in the best option of four possible alternatives which were denoted by X_j, for $j = 1,2,3$, and 4. The asset corporation makes a decision owing to the following seven criteria: (i) C_1 denotes the sell ability, (ii) C_2 denotes the management ability, (iii) C_3 denotes the production ability, (iv) C_4 denotes the technology ability, (v) C_5 denotes the financial ability, (vi) C_6 denotes the oppose risk ability, (vii) C_7 denotes the company policy consistency. With equal weight, three decision-makers will estimate alternatives with linguistic terms as equation (1.3). The decision matrices $A^{(k)} = \left[a_{ij}^{(k)}\right]_{n \times m}$ for $k = 1,2,3$ are reproduced in the following Tables 3-5.

Table 3. Decision matrix for the first decision-maker, $A^{(1)} = [a_{ij}^{(1)}]_{4x7}$.

	X_1	X_2	X_3	X_4
C_1	s_1	s_3	s_2	s_2
C_2	s_3	s_2	s_2	s_2
C_3	s_3	s_0	s_3	s_{-1}
C_4	s_0	s_2	s_1	s_1
C_5	s_1	s_3	s_4	s_3
C_6	s_2	s_1	s_3	s_1
C_7	s_2	s_{-1}	s_2	s_1

Table 4. Decision matrix for the first decision-maker, $A^{(2)} = [a_{ij}^{(2)}]_{4x7}$.

	X_1	X_2	X_3	X_4
C_1	s_1	s_0	s_3	s_0
C_2	s_2	s_1	s_1	s_1
C_3	s_3	s_0	s_2	s_0
C_4	s_0	s_1	s_2	s_1
C_5	s_2	s_2	s_4	s_0
C_6	s_2	s_2	s_4	s_1
C_7	s_4	s_1	s_1	s_{-1}

Table 5. Decision matrix for the first decision-maker, $A^{(3)} = [a_{ij}^{(3)}]_{4x7}$.

	X_1	X_2	X_3	X_4
C_1	s_0	s_2	s_2	s_0
C_2	s_2	s_1	s_1	s_1
C_3	s_2	s_1	s_2	s_{-1}
C_4	s_1	s_2	s_2	s_1
C_5	s_3	s_0	s_2	s_0
C_6	s_2	s_2	s_3	s_0
C_7	s_3	s_{-1}	s_2	s_1

According to equation (2.5), we find that

$$(Y_1, \dots, Y_7,) = (32,6,68, 40/3,52,98/3,220/3). \quad (4.1)$$

We consider the first model of Wu and Chen [3] under the 1-norm, to assume that the weight vector of the attribute is completely unknown. Therefore, using

$$\sum_{j=1}^{7} Y_j = 832/3, \quad (4.2)$$

by equations (3.4) and (3.6), it yields that

$$(w_1, \dots, w_7,) = (0.1154,0.0216, 0.2452,0.0481,0.1875,0.1178,0.2644), \quad (4.3)$$

that is the same results that were derived in Wu and Chen [3].

Next, we consider numerical examples for their second model M2. However, Wu and Chen [3] did not provide examples for their model M2. Consequently, we hypothetical assume the known weights are

$$(w_1, w_2) = (0.2,0.1). \quad (4.4)$$

Owing to equations (3.14) and (3.16), it follows that

$$k = \frac{\left(1-\sum_{j=1}^{2} w_j\right)}{\sum_{j=3}^{m} Y_j} = \frac{21}{7180}. \quad (4.5)$$

By equation (3.14), we obtain that

$$(w_3, \dots, w_7,) = (0.1989, 0.1521,0.0955,0.2145). \quad (4.6)$$

It demonstrates that our approach is efficient and accurate to derive the optimal weight for their second model.

V. A Related Problem

We study a related problem about inventory models to indicate several directions for future studies. Wu et al. [31] developed an inventory model with ramp type demand, constant deterioration rate and demand is stock dependent with a constant rate. We pointed out some extension of their model to Weibull deterioration rate and stock dependent demand with ramp type relation.

Wu et al. [31] considered an economic ordering quantity inventory system to find the minimum cost where the shortage is completely backordered.

Demand is a ramp type demand with

$$D(t) = \begin{cases} a_1 + b_1 t, & 0 \le t \le \mu_1 \\ a_1 + b_1 \mu_1, & \mu_1 \le t \le T \end{cases}, \quad (5.1)$$

where μ_1 is the point that the linear increasing tendency changes to constant relationship.

The stock dependent rate between demand and inventory level is denoted as $\alpha(t)$,

$$\alpha(t) = \begin{cases} a_2 + b_2 t, & 0 \le t \le \mu_2 \\ a_2 + b_2 \mu_2, & \mu_2 \le t \le T \end{cases}, \quad (5.2)$$

where μ_2 is the point that the linear increasing tendency changes to constant relationship.

The deterioration rate satisfies the Weibull distribution as follows,

$$\theta(t) = \alpha\beta t^{\beta-1}, \quad (5.3)$$

where α is the location parameter, and β is the scale parameter.

We introduced several notation in the following. t_1 is the time the inventory level drops to zero. c_1 is the ordering cost per replenishment cycle. c_2 is the holding cost per unit item per unit time. c_3 is the deterioration cost per unit perished item. c_4 is the shortage cost per unit item per unit time. The model allows for shortage and complete backlogging of unfilled demand.

Owing the relation among μ_1, μ_2 and t_1, we point out that there are poccible to develop the following 6 different production models.

Model A is under the restriction $0 < t_1 \le \mu_1 \le \mu_2$. The inventory levels of the model A are described by the following equations, for $0 < t < t_1$,

$$\frac{d}{dt} I(t) = -(a_1 + b_1 t) - \left(\alpha\beta t^{\beta-1} + a_2 + b_2 t\right) I(t), \quad (5.4)$$

for $t_1 < t < \mu_1$,

$$\frac{d}{dt} I(t) = -(a_1 + b_1 t), \quad (5.5)$$

and for $\mu_1 < t < T$,

$$\frac{d}{dt} I(t) = -(a_1 + b_1 \mu_1). \quad (5.6)$$

With the condition $I(t_1)=0$, we derive the inventory level of Equation (5.4) as follows. For $0 \le t \le t_1$,

$$I(t)=e^{-\left(\alpha t^{\beta}+a_2 t+\frac{b_2}{2}t^2\right)}\int_t^{t_1}\varphi(x)dx, \qquad (5.7)$$

where φ(x) is an abbreviation to simplify the expression, with

$$\varphi(x)=(a_1+b_1x)e^{\alpha x^{\beta}+a_2x+\frac{b_2}{2}x^2}. \qquad (5.8)$$

With $I(t_1)=0$ again, we derive the inventory level for Equation (5.5) as follows. For $t_1 \le t \le \mu_1$,

$$I(t)=a_1t_1+\frac{b_1}{2}t_1^2-a_1t-\frac{b_1}{2}t^2. \qquad (5.9)$$

Owing to $I(\mu_1)$ of Equation (5.5) equals $I(\mu_1)$ of Equation (5.6), to yield the inventory level of Equation (5.6) as follows. For $\mu_1 \le t \le T$,

$$I(t)=\frac{b_1}{2}\mu_1^2+a_1t_1+\frac{b_1}{2}t_1^2-(a_1+b_1\mu_1)t. \qquad (5.10)$$

The total demand, denoted as TD, during $[0,t_1]$ is evaluated as

$$TD=\int_0^{t_1}[(a_2+b_2y)I(y)+(a_1+b_1y)]dy$$
$$=\int_0^{t_1}(a_2+b_2y)e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}f(y)dy$$
$$+a_1t_1+\frac{b_1}{2}t_1^2, \qquad (5.11)$$

where f(y) is an abbreviation to simplify the expression, with

$$f(y)=\int_y^{t_1}(a_1+b_1x)e^{\alpha x^{\beta}+a_2x+\frac{b_2}{2}x^2}dx. \qquad (5.12)$$

The total deteriorated items, denoted as TDI, during $[0,t_1]$ is computed as $I(0)-TD$.

The holding cost, HC, during $[0,t_1]$ is derived as

$$HC=c_2\int_0^{t_1}I(y)dy$$
$$=c_2\int_0^{t_1}e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}f(y)dy, \qquad (5.13)$$

where f(y) is defined in Equation (5.12).

The shortage cost, SC, during $[0,t_1]$ is obtained as

$$SC=c_4\int_{t_1}^{T}-I(t)dt$$
$$+\frac{c_4}{6}b_1\left(2t_1^3+\mu_1^3+3\mu_1T^2-3\mu_1^2T-3t_1^2T\right)$$
$$+\frac{c_4}{2}a_1(T-t_1)^2. \qquad (5.14)$$

Remark. We can rewrite the second term of Equation (5.14) as follows,

$$\frac{s}{6}b_1\Big[(T-t_1)^3-(T-\mu_1)^3+3t_1^2(\mu_1-t_1)$$
$$+3t_1\left((T-t_1)^2-(\mu_1-t_1)^2\right)\Big] \qquad (5.15)$$

to indicate the positivity of our result.

The total cost is the sum of the set up cost, the holding cost, deterioration cost, and shortage cost, such that we derive the total cost, $TC(t_1)$, as follows

$$TC(t_1)=\frac{1}{T}\left(c_1+HC+c_3(I(0)-TD)+SC\right). \qquad (5.16)$$

From Equations (6, 9 10 and 11), we find that

$$\frac{d}{dt_1}TC(t_1)=\frac{1}{T}f(t_1) \qquad (5.17)$$

where $f(t_1)$ is an abbreviation to simplify the expression, with

$$f(t_1)=-c_3(a_1+b_1t_1)-c_4a_1(T-t_1)-c_4b_1t_1(T-t_1)$$
$$-c_3\varphi(t_1)\int_0^{t_1}(a_2+b_2y)e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}dy$$
$$+\varphi(t_1)c_2\int_0^{t_1}e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}dy+c_3, \qquad (5.18)$$

where $\varphi(t_1)$ is an abbreviation to simplify the expression, that is defined in Equation (5.8).

We know that

$$f(0)=-c_4a_1T<0, \qquad (5.19)$$

and

$$f(T)=c_2\varphi(T)\int_0^{T}e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}dy$$
$$+c_3(a_1+b_1T)A(T), \qquad (5.20)$$

where A(T) an abbreviation to simplify the expression, that is defined as follows,

$$A(T)=-1+e^{\left(\alpha T^{\beta}+a_2T+\frac{b_2}{2}T^2\right)}\times$$
$$\left(1-\int_0^{T}(a_2+b_2y)e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}dy\right). \qquad (5.21)$$

We assume an auxiliary function, $B(T)$, as follows,

$$B(T)=e^{\left(\alpha T^{\beta}+a_2T+\frac{b_2}{2}T^2\right)}\left(1-\int_0^{T}h(y)dy\right)-1, \qquad (5.22)$$

where h(y) an abbreviation to simplify the expression, with

$$h(y)=\left(\alpha\beta y^{\beta-1}+a_2+b_2y\right)e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}, \qquad (5.23)$$

then we know $A(T)>B(T)$, for $T>0$.

Owing to

$$\int_0^{T}\left(\alpha\beta y^{\beta-1}+a_2+b_2y\right)e^{-\left(\alpha y^{\beta}+a_2y+\frac{b_2}{2}y^2\right)}dy$$
$$=1-e^{-\left(\alpha T^{\beta}+a_2T+\frac{b_2}{2}T^2\right)}, \qquad (5.24)$$

we find that $B(T)=0$, for $T>0$ such that $A(T)>0$, for $T>0$. From $A(T)>0$, and Equation (5.20), we derive that $f(T)>0$, for $T>0$.

VI. Discussion of an Associated Issue

In the following, we will offer a proof of the following problem: If

$$\max\{\mu_A(x),\alpha\} \le \min\{\mu_B(x),\alpha\}, \tag{6.1}$$

is satisfied for every x in the universe of disclose, then researchers wanted to show that

$$\max_{x\in E}\mu_A(x) \le \min_{x\in E}\mu_B(x). \tag{6.2}$$

We can directly rewrite Equation (101) as follows

$$\alpha \le \max\{\mu_A(x),\alpha\} \le \min\{\mu_B(x),\alpha\} \le \alpha. \tag{6.3}$$

We can further improve Equation (7.3) as

$$\alpha \le \max\{\mu_A(x),\alpha\} = \alpha = \min\{\mu_B(x),\alpha\} \le \alpha. \tag{6.4}$$

Owing to the first equality of Equation (7.4), we know that for every x in the universe of disclose, $\mu_A(x)\le\alpha$ such that it results in

$$\max_{x\in E}\mu_A(x) \le \alpha. \tag{6.5}$$

Based on the second equality of Equation (7.4), we obtain that every x in the universe of disclose, $\alpha \le \mu_B(x)$ so that it implies that

$$\alpha \le \min_{x\in E}\mu_B(x). \tag{6.6}$$

Now, we combine Equations (7.5) and (7.6) to find that

$$\max_{x\in E}\mu_A(x) \le \min_{x\in E}\mu_B(x). \tag{6.7}$$

VII. A Proof for the Upper Bound of Similarity Measure

Next, we will provide a proof for the range of a similarity measure that satisfies the following condition,

$$k(A,B)\le 1. \tag{7.1}$$

To simplify the expressions, we assume the following four abbreviations,

$$\sum_{i=1}^{N}\mu^2{}_{\mathrm{A}}(x_i)=a, \tag{7.2}$$

$$\sum_{i=1}^{N}\mu^2{}_B(x_i)=b, \tag{7.3}$$

$$\sum_{i=1}^{N}\nu^2{}_A(x_i)=c, \tag{7.4}$$

and

$$\sum_{i=1}^{N}\nu^2{}_B(x_i)=d. \tag{7.5}$$

We recalled the definition of $k(A,B)$ as follows,

$$k(A,B)=\frac{C(A,B)}{[T(A)\cdot T(B)]^{1/2}}, \tag{7.6}$$

where

$$C(A,B)=\sum_{i=1}^{N}[\mu_{\mathrm{A}}(x_i)\cdot\mu_B(x_i)+\nu_A(x_i)\cdot\nu_B(x_i)], \tag{7.7}$$

$$T(A)=\sum_{i=1}^{N}\left(\mu^2{}_{\mathrm{A}}(x_i)+\nu^2{}_A(x_i)\right), \tag{7.8}$$

and

$$T(B)=\sum_{i=1}^{N}\left(\mu^2{}_{\mathrm{B}}(x_i)+\nu^2{}_{\mathrm{B}}(x_i)\right). \tag{7.9}$$

We rewrote Equation (7.14) as follows,

$$C(A,B)=\sum_{i=1}^{N}\mu_{\mathrm{A}}(x_i)\cdot\mu_B(x_i) + \sum_{i=1}^{N}\nu_A(x_i)\cdot\nu_B(x_i). \tag{7.10}$$

On the other hand, we plugged the abbreviations of Equations (7.2-7.5) into Equations (7.8) and (7.9), then

$$T(A)=a+c, \tag{7.11}$$

and

$$T(B)=b+d. \tag{7.12}$$

We apply the Cauchy-Schwarz inequality to Equation (7.10) to show that

$$C(A,B)\le\sqrt{\sum_{i=1}^{N}\mu_A^2(x_i)\sum_{i=1}^{N}\mu_B^2(x_i)} + \sqrt{\sum_{i=1}^{N}\nu_A^2(x_i)\sum_{i=1}^{N}\nu_B^2(x_i)}. \tag{7.13}$$

Moreover, we plugged the abbreviations of Equations (7.2-7.5) into Equation (7.13), then

$$C(A,B)\le\sqrt{ab}+\sqrt{cd}. \tag{7.14}$$

Based on above derivation, we derive that

$$k(A,B)\le\frac{\sqrt{ab}+\sqrt{cd}}{\sqrt{(a+c)(b+d)}}. \tag{7.15}$$

Based on Equation (7.15), to verify $k(A,B)\le 1$ is sufficient to show that

$$\frac{\sqrt{ab}+\sqrt{cd}}{\sqrt{(a+c)(b+d)}}\le 1. \tag{7.16}$$

According to Equation (7.16), we try to prove that

$$\sqrt{ab}+\sqrt{cd}\le\sqrt{(a+c)(b+d)}. \tag{7.17}$$

We square the both sides of Equation (7.17) and then cancell out the common terns ab + cd, then we need to verify that

$$2\sqrt{ab}\sqrt{cd}\le ad+bc. \tag{7.18}$$

Using the well-known formula that the arithmetic mean is greater than the geometric mean, we know that Equation (7.18) is proved. Consequently, Equations (7.17) and (7.16), both are verified that results in Equation (7.1).

VIII. Directions for Future Studies

We study the second model that is denoted as model B which is under the restriction of $0<\mu_1<t_1<\mu_2$.

The inventory levels of the model B are described by the following equations. For $0<t<\mu_1$,

$$\frac{d}{dt}I(t)=-(a_1+b_1t)-\left(\alpha\beta t^{\beta-1}+a_2+b_2t\right)I(t), \tag{8.1}$$

for $\mu_1 < t < t_1$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1) - (\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t), \quad (8.2)$$

and for $t_1 < t < T$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1). \quad (8.3)$$

The third model that is denoted as Model C which is under the restriction of $0 < \mu_1 \le \mu_2 \le t_1$. The inventory levels of the model C are described by the following equations.
For $0 < t < \mu_1$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1 t) - (\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t), \quad (8.4)$$

for $\mu_1 < t < \mu_2$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1) - (\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t), \quad (8.5)$$

for $\mu_2 < t < t_1$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1) - (\alpha\beta t^{\beta-1} + a_2 + b_2\mu_2)I(t), \quad (8.6)$$

and for $t_1 < t < T$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1). \quad (8.7)$$

The fourth model is denoted as Model D which is under the restriction of $0 < t_1 \le \mu_2 \le \mu_1$. The inventory levels of the model D are described by the following equations.
For $0 < t < t_1$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t) - (a_1 + b_1 t), \quad (8.8)$$

for $t_1 < t < \mu_1$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1 t), \quad (8.9)$$

and for $\mu_1 < t < T$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1). \quad (8.10)$$

The fifth model is denoted as Model E which is under the restriction of $0 < \mu_2 < t_1 < \mu_1$. The inventory levels of the model E are described by the following equations. For $0 < t < \mu_2$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t) - (a_1 + b_1 t), \quad (8.11)$$

for $\mu_2 < t < t_1$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2\mu_2)I(t) - (a_1 + b_1 t), \quad (8.12)$$

for $t_1 < t < \mu_1$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1 t), \quad (8.13)$$

and for $\mu_1 < t < T$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1). \quad (8.14)$$

The sixth model is denoted as Model F which is under the restriction of $0 < \mu_2 \le \mu_1 \le t_1$. The inventory levels of the model F are described by the following equations. For $0 < t < \mu_2$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2 t)I(t) - (a_1 + b_1 t), \quad (8.15)$$

for $\mu_2 < t < \mu_1$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2\mu_2)I(t) - (a_1 + b_1 t), \quad (8.16)$$

for $\mu_1 < t < t_1$,

$$\frac{d}{dt}I(t) = -(\alpha\beta t^{\beta-1} + a_2 + b_2\mu_2)I(t) - (a_1 + b_1\mu_1), \quad (8.17)$$

and for $\mu_1 < t < T$,

$$\frac{d}{dt}I(t) = -(a_1 + b_1\mu_1). \quad (8.18)$$

We recall that Chuang [32] constructed an inventory model to extend Wu et al. [31] form ramp type demand to arbitrary nonnegative demand. Cheng et al. [33] generalized Wu et al. [31] and Chuang [32] from completely backordered to a partially backordered which is inversely related to the shortage period. Wang et al. [34] extended Wu et al. [31] form from a constant deterioration rate to arbitrary nonnegative deterioration rate. The above three papers of Chuang [32], Cheng et al. [33], and Wang et al. [34] pointed out that this direction is a hot research spot.

Moreover, we helped practioners to locate possible directions for their future developments and then we cited several recently published articles un the following. Rakhmawati et al. [35] studied the multiple graphs to locate the best route under intuitionistic interval-value fuzzy sets. Under complicated systems by valley of attraction, Kakarlapudi et al. [36] examined an iterative algorithm. Hou et al. [37] considered a multiple echelon examination system to locate exterior imperfection to adopt geographical and biological optimal process. Qi et al. [38] constructed a model to decide the best characteristic choice through K-mean adjacent classification and the grasshopper discrete procedure. Based on several investigational information, Berot et al. [39] developed a network with selection of variables and parameters. Wang et al. [40] studied transit stream estimation under short period with periodic feature fusion and time and space estimation. Under stipulated reaction performance, Ouyang et al. [41] examined nonlinear exchanged fuzzy models. Yang et al. [42] considered a train optimal model with several component electrical circulation by restore mileage and time. Hao et al. [43] constructed an extended automatic regression system to fuse nonlinearity and linearity. Referring to our above discussion, researchers will find possible research trends.

At last, not the least, we mentioned several articles to show their importance. Yusoff et al. [44] constructed a brand new similarity measure. Julian et al. [45] point out questionable results in a published article. Tung et al. [46] amended a questionable result in Julian et al. [45]. Yusoff et al. [47] examined similarity measures under intuitionistic fuzzy setting.

IX. CONCLUSION

We study the non-linear programming models of Wu and Chen [3] to prepare our analytical approach. Our proposed method is simple than their approach of the Lagrange multiplier method and using MatLab software with optimization toolbox or Lindo/Lingo software package. Our approach is efficient and accurate that will be useful in a further application for group multiple attribute decision-making under a linguistic environment.

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