Research on the Coupling of Dual Layer Collaborative Network between Public Transport and Subway in Urban High Passenger Flow Areas

Han Pu, *Member, IAENG,* and Jiangpeng Wang

***Abstract*—The diversification of the social economy has highlighted the structural advantages of integrated, multi-modal urban transportation systems. Efficient coordination of bus and subway networks, the primary modes of urban transportation, is crucial for improving the overall efficiency of urban transport. This paper employs system dynamics principles to comprehensively investigate the coupling relationships and mutual influences between urban bus and subway networks. Considering the characteristics of multi-modal heterogeneous networks, this study identifies key indicators influencing bus-subway network coupling and establishes a corresponding evaluation system. The study employs a game theory-based combination weighting method to determine the weights of key indicators and subsequently applies a coupling coordination degree model to analyze the bus-subway network coupling. This paper investigates the coupling between bus and subway networks, providing an in-depth analysis of the synergy and coupling efficiency of these two primary urban transportation modes. Furthermore, it explores existing issues within the urban transportation system, focusing on addressing shortcomings, improving connectivity, optimizing network design, and enhancing overall efficiency. Taking Yanta District in Xi'an as a case study, this paper analyzes the coupling state of the bus-subway network within the district. Utilizing an obstacle diagnosis model, it quantitatively assesses the primary obstructive factors in the bus and subway dual-layer network system of Yanta District and proposes corresponding optimization strategies. This study provides a theoretical foundation for promoting integrated urban transportation and accelerating the development of green, low-carbon transport systems, while also offering practical guidance for future implementation.**

***Index Terms*—Urban transportation, Coupling, Obstruction degree, Bus-subway dual-layer network.**

I. INTRODUCTION

In recent years, w ith the continued development of public transportation, urban transportation systems operating under multimodal traffic structures have become increasingly complex. As key components of urban transportation, the coupling degree and coordination between bus and subway systems directly impact the efficiency and convenience of the overall urban transportation network. A thorough examination of the bus-subway coupling can reveal effective strategies for optimizing the operation of this dual-layer network, ultimately enhancing the overall efficiency of the urban transportation system.

Currently, scholars worldwide are actively researching network coupling. Regarding the definition of coupling, sociologist Mark Granovetter [1] pioneered the concept of "weak ties" in 1973 to describe coupling relationships within social networks. In 1975, Freeman W. J. [2] introduced the concept of "phase coupling" to describe the interaction and synchronization of neurons. Phan Van Long Em [3] applied coupling principles to different Hindmarsh-Rose neurons, demonstrating a mechanism for information transmission and interaction that transforms them from isolated individuals into interconnected entities capable of mutually influencing each other's states. Hou Zunzun [4] et al. proposed a method for optimizing passenger and freight cooperative transport in urban rail transit under virtual coupling conditions, applying the principles of information exchange and behavioral coordination to optimize collaborative transport. Furthermore, Albert-László Barabási and colleagues, through their research on the topological structure of complex networks, network dynamics, and the behavior of complex systems, have progressively developed a comprehensive understanding and definition of coupling.

In the analysis and evaluation of network coupling, Wu Yueming et al. [5] proposed a quantitative calculation model for coupling degree and employed grey system theory to predict it. Cheng Longxiang et al. [6] employed the Super-SBM model to measure agricultural production efficiency in China, assessing its coupling coordination with green technology innovation. They utilized a spatial autocorrelation model to describe and analyze the spatiotemporal evolution of the coupling coordination between the two. Bi Wentai et al. [7] selected data from agricultural companies listed on the A-share market in China and conducted a systematic analysis using coupling coordination models and fixed-effects models. They investigated the coordination trend between digital transformation and green development in Chinese agricultural enterprises. Bao Xueying et al. [8] established an evaluation index system for the coordinated development between mountain railway projects and material resources, based on their interaction, and analyzed and evaluated their coordination. Li Bo et al. [9] used the coupling coordination degree model and the resource-environment resilience impact model to evaluate and analyze the relationships

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between the 'production-resource environment,' 'consumption-resource environment,' and 'industrial chain structure-resource environment' triple systems. Fan Dongjun et al. [10] developed an improved coupling coordination model. They proposed classifying coupling coordination based on equal probabilities and enhanced the classification using kernel density estimation and Monte Carlo methods. Yan Zhigang et al. [11] used the Obstruction Degree analysis model to calculate the comprehensive evaluation value of sustainable development capacity and balanced development in China's railway transportation industry. They proposed improving indicators with high Obstruction Degrees to enhance overall operational capacity. Gu C et al. [12] introduced the concept of cooperation strength and established a model to describe emergent cooperation between dual-layer networks, clarifying the relationships within coupled networks.

In summary, current academic research has explored network coupling to some extent. While most studies focus on areas like biology and biological networks, brain science and neural networks, complex networks, and social networks, research on the coupling of multilayer networks in urban transportation systems remains relatively limited. The coupling of bus and subway dual-layer networks can be seen as a strategic measure to efficiently integrate and optimize urban transportation resources, enhance traffic flow, reduce congestion, and improve passengers' travel experiences. To address current urban traffic congestion and support the national call for low-carbon, green transportation, this paper studies the synergy and adaptability of a bus-subway coupled dual-layer network. This research holds significant academic value and practical importance. This paper focuses on the bus-subway dual-layer network as the research object and aims to explore the coupling mechanisms of dual-layer complex networks. It seeks to study the coupling, synergy, and adaptability of urban dual-layer transportation networks and propose optimization strategies to ensure more effective synergy between bus and subway networks within permissible limits. Eventually, it can promote sustainable urban transportation development and the construction of green, low-carbon transportation systems.

II. ANALYSIS OF INTERACTIONS BETWEEN URBAN BUS-SUBWAY

Based on the interaction between the integration and coupling cooperation of bus and subway networks, this paper summarizes 11 key indicators that affect the coupling synergy of the bus-subway dual-layer network, including route coverage and vehicle departure intervals. It also identifies 5 coupling relationship bridges: passenger transfer convenience, travel time, travel comfort, travel cost, and green travel. These key indicators and coupling relationship bridges will serve as the foundation for studying the interactive effects of the bus-subway dual-layer network coupling. Through a comprehensive analysis and evaluation of these factors, the interaction mechanisms between the bus and subway networks can be revealed, providing scientific evidence for optimizing network design and improving operational efficiency. Details are provided in Table 1.

Key indicators serve as important considerations for building an efficient, convenient and sustainable bus-subway network. Besides, they significantly impact the coupling relationship bridges between the two networks. The following analysis examines the impact of each key indicator on the other network.

(1) Route and station planning: To ensure seamless connectivity between the bus and subway networks, the planning of both networks should consider the location of the other's stations, traffic demand, and citizens' main travel routes. By effectively integrating the route and station information of both networks, the urban transportation system can offer more convenient transfer options, reduce the number of transfers, and minimize passenger waiting times. Furthermore, it can enhance transfer convenience for citizens and reduce travel distances and time costs.

(2) Route Coverage: Generally, the route coverage of the bus network is larger than that of the subway network, especially in areas where the subway network is not fully covered. The extensive bus network can fill transportation gaps, offering passengers more travel options and enhancing transfer convenience by feeding passenger flow into the subway network. Expanding subway network coverage can alleviate bus network passenger flow, improve travel convenience and comfort, and reduce travel time and costs.

(3) Station Size: Station size considerations include platform space, facilities, and passenger capacity. For the bus network, considering factors like population, traffic volume, and travel demand when determining station size

TABLE I
Key Indicators and Coupling Relationship Bridges

Key indicators	Number of vehicles (trains) per unit route length	Route coverage area	Vehicle (train) departure interval	Passen-ger waiting time	Vehicle (train) speed distribution	Average vehicle (train) speed during peak hours	Passen-ger satisfac-tion	Station distrib-ution	Single ticket price	Passeng-er volume
Coupling Relationship Bridge	Passenger transfer convenience		Passenger travel time		Passenger travel comfort			Passenger travel cost		Green travel

can accommodate more buses, reduce traffic congestion, and alleviate pressure on the subway network during peak hours. This can also better serve transportation needs by improving the travel efficiency of the local population and reducing passenger travel time. Similarly, for the subway network, considering the same factors when determining station size allows stations to accommodate suitable passenger volumes, enhance convenience and comfort, and reduce travel time.

(4) Waiting Conditions and Transfer Facilities: Improving waiting and transfer facilities at stations can increase passenger comfort and satisfaction, and reduce unnecessary waiting time. The tendency of some passengers to linger in subway stations can lead to the inefficient use of public resources. The improvement of the waiting and transfer conditions at bus stops may yield better results than subway stations. These improvements also ensure more efficient use of resources, which enhances the overall comfort and satisfaction of passengers within the urban transport network.

(5) Vehicle Numbers and Operation Times: Bus and subway service frequency is typically adjusted during peak hours to meet demand. Mismatched service frequencies between networks can increase travel times, reduce comfort, and decrease convenience for passengers. Moreover, by reasonably planning vehicle operation times and coordinating bus and subway service hours, especially during peak and night times, passengers can benefit from convenient public transport services throughout the day.

(6) Increasing bus speeds could attract some passengers from the subway, thereby alleviating subway congestion during peak hours. However, this shift hinges on the magnitude of the speed difference and passenger travel preferences. Conversely, faster subway service could draw more riders, easing the strain on buses. However, a significant speed disparity between the two systems could create bottlenecks at transfer points and disrupt the overall network coherence.

(7) Bus network development should prioritize green travel by promoting electric buses, optimizing routes, and improving vehicle fuel efficiency. These measures can lower carbon emissions and air pollution, ultimately fostering sustainable urban development. Furthermore, these measures can attract citizens committed to green, low-carbon travel, encouraging active participation in the bus-subway network.

The foregoing analysis demonstrates that key performance indicators of the bus and subway networks are interconnected and exert mutual influence through established coupling mechanisms. To facilitate convenient transfers, key influencing factors encompass route and station planning strategies, route network coverage, and station capacity. With respect to travel time, key influencing factors include route optimization, station planning, and route network coverage. Regarding travel comfort, key influencing factors encompass station capacity, passenger waiting environment, and the availability of seamless transfer facilities. Concerning travel costs, key influencing factors are route and station planning, and route network coverage. Regarding environmentally sustainable travel, a key influencing factor is the adoption rate of new energy vehicles.

In summary, the mutual influence diagram of the city's "bus-subway network" is shown in Figure 1,2,3,4.

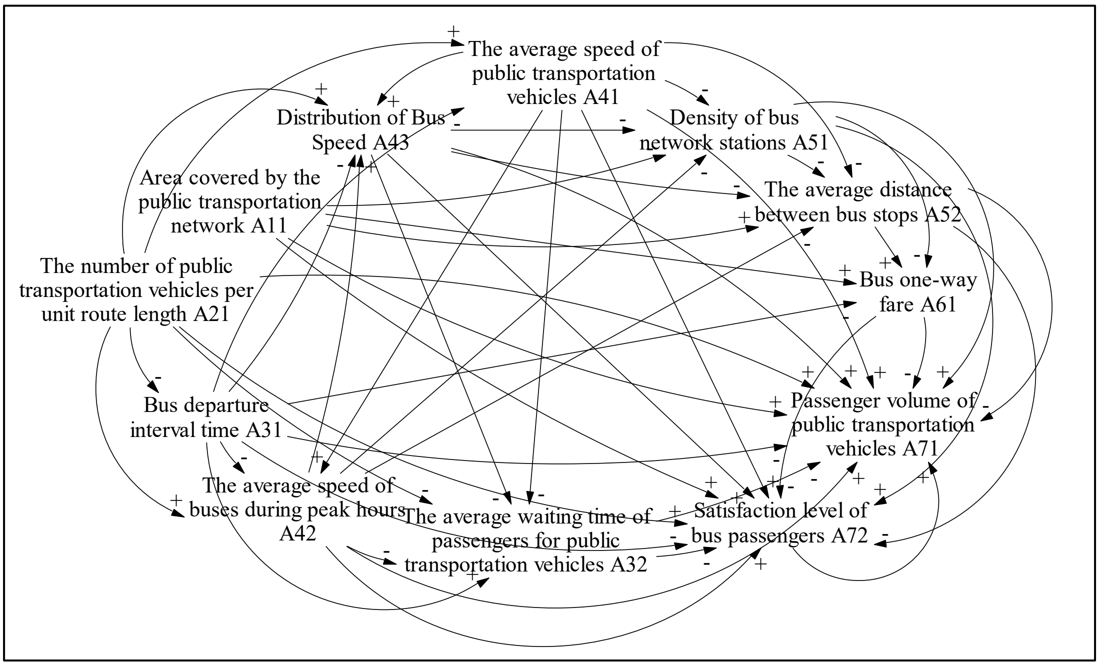

Fig.1. The mutual influence relationship between indicators within the public transportation network

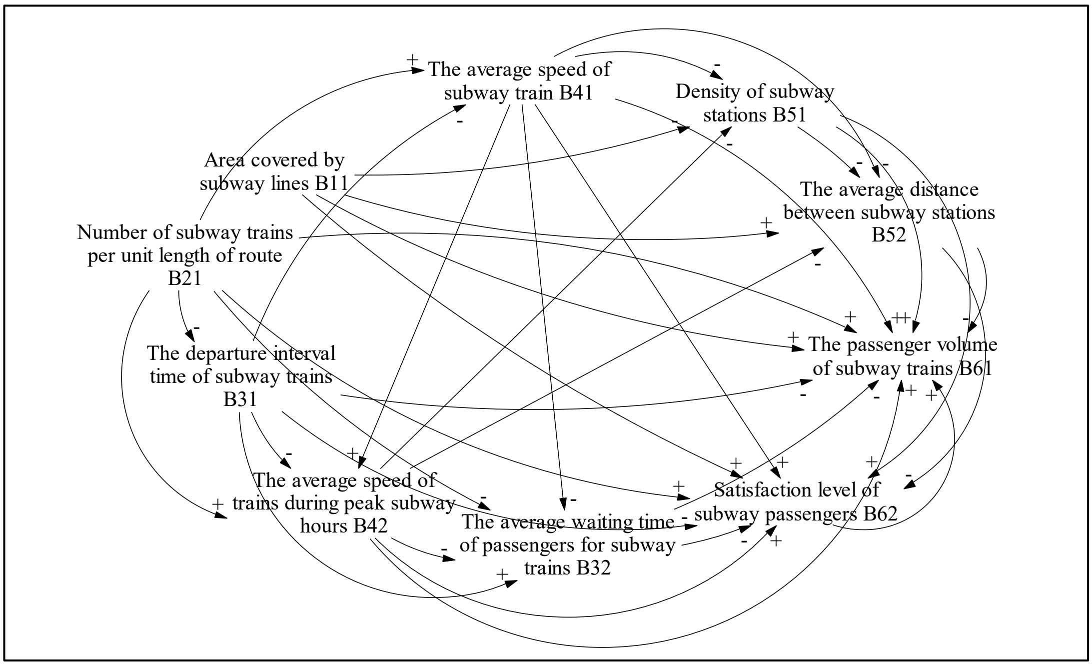

Fig.2. The influence relationship between indicators within the subway network

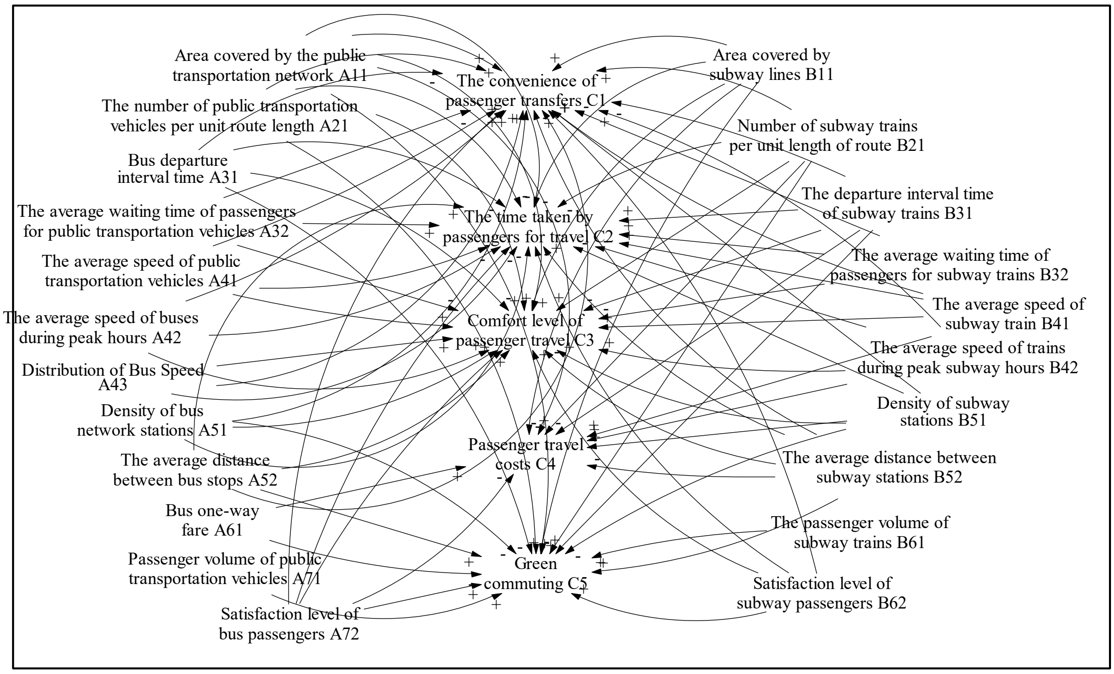

Fig.3.The mutual influence relationship between the "bus-subway" network

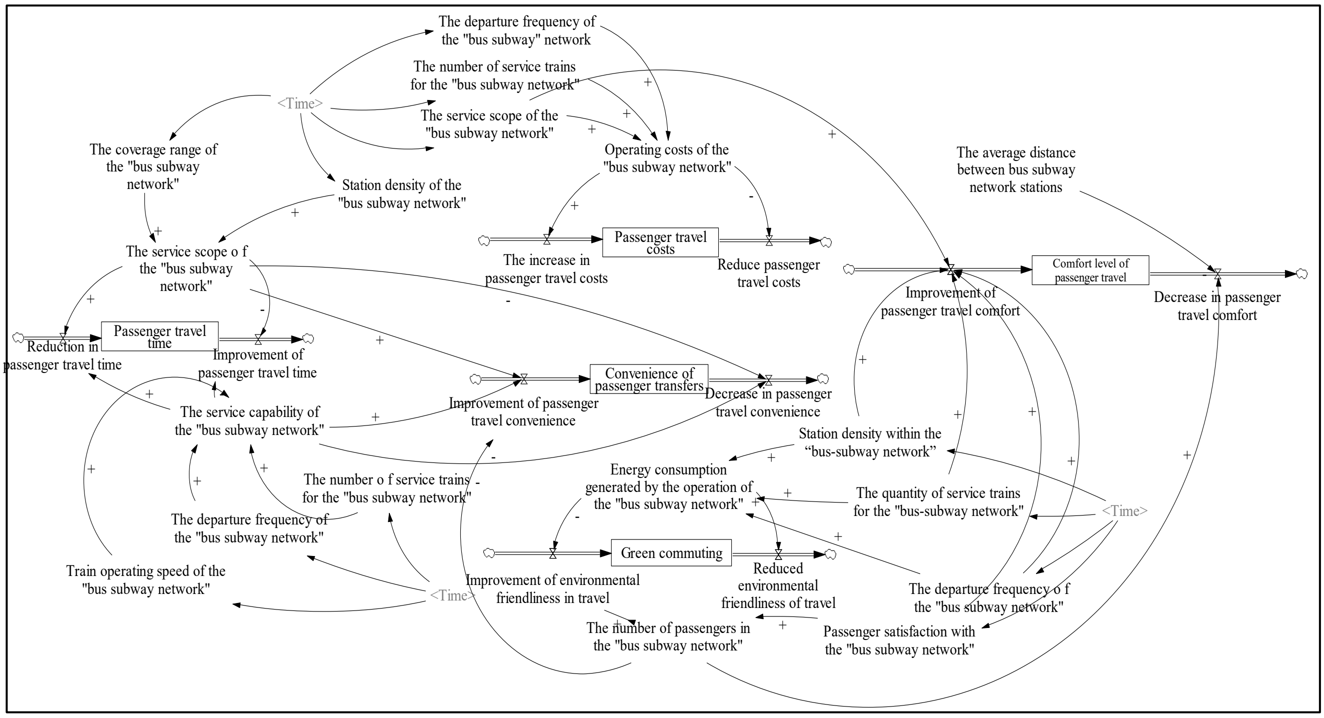

Fig.4. The mutual influence relationship between urban "bus-subway" networks

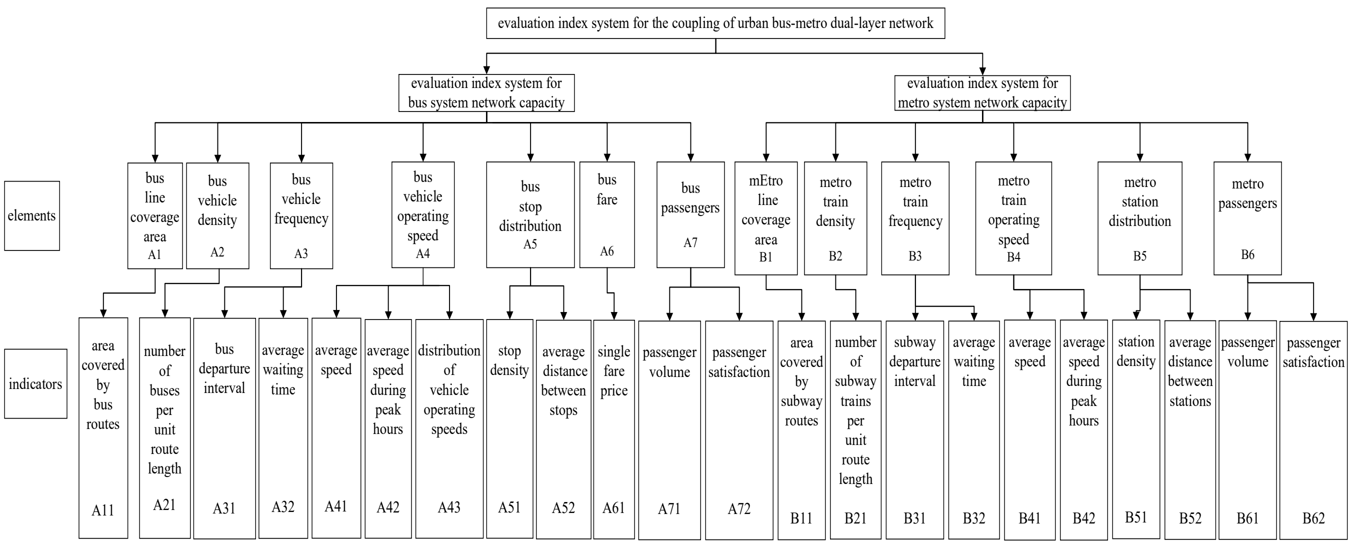

Fig. 5. Evaluation index system for the coupling of urban "bus subway network"

III. THE EVALUATION INDEX SYSTEM FOR THE COUPLING OF URBAN "BUS-SUBWAY NETWORK"

Based on the preceding analysis of the mutual influence between the bus network, subway network, and their coupling relationships, an evaluation index system for the coupling of the urban bus-subway network has been constructed, as depicted in Figure 5.

IV. COUPLING CALCULATION MODEL FOR THE BUS-SUBWAY DUAL-LAYER NETWORK

A. Subjective-Objective Weighting Method Based on Game Theory

(1) Determining Subjective Weights - G1 Method

For the evaluation index system of urban bus-subway network coupling, this paper employs the G1 method to determine the subjective weights. The G1 method is a novel subjective evaluation technique that determines indicator weights by ranking their relative importance based on expert judgment. The G1 method is an optimized version of the Analytic Hierarchy Process (AHP). It effectively avoids the consistency issues caused by a large number of indicators in AHP [13], simplifies calculations, and yields reliable results. The specific steps are as follows:

Step 1: Determine the order of indicators. The evaluated objects are the key indicators of the bus and subway networks. The corresponding evaluation indicators are: $A_{11}, A_{21}, \ldots, A_{72}, B_{11}, B_{21}, \ldots, B_{62}$, Experts are invited to evaluate the coupling capability of the assessment indicators within the city's bus-subway network. If Indicator A_{ij} is judged to be more important than Indicator A_{om} under a particular evaluation criterion or evaluation object, it is recorded as $A_{ij} \succ A_{om}$. After comparing the importance of all indicators pairwise, the most important indicator is marked as C_1, and the next most important as C_2 so $C_1 \succ C_2$, and so on.

Step 2: Judging the relative importance ratio: Experts are invited to quantify the relative importance of adjacent indicators based on the importance ranking table of key indicators for the bus and metro networks. We assume that the expert judges the importance ratio of C_{k-1}/C_k to w_{k-1}/w_k (weight ratio) as r_k . In this formula, r_k is referred to as the "sentiment operator".

Step 3: Calculating the terminal weight: Suppose the expert provides the weight r_i, then the terminal indicator's weight is

$$w_m = \left(1 + \sum_{k=2}^{m} \prod_{i=k}^{m} r_i\right)^{-1} \tag{1}$$

Thus, the weight of w_{m-1} is determined as $w_{m-1} = r_k \cdot w_m$. Following this process, the weights of all key indicators for the urban " bus-subway network" can be obtained.

(2) Objective Weight Determination - Entropy Weight Method

The entropy weight method is an objective weighting method used to determine the weights of evaluation indicators. This method calculates the entropy of indicator values to measure the disorder of the indicators, thereby determining their importance and weights. The specific steps are as follows:

Step 1: Construct the indicator system. Identify the indicator matrix $X = [X_1, X_2, \ldots X_n]$ and differentiate whether the indicators are positive or negative.

Step 2: Data normalization. Standardize the data in each column of the matrix. The standardization formulas are as follows:

When indicator x_{ij} is a positive indicator:

$$Z_{ij} = \frac{x_{ij} - \min X_i}{\max X_i - \min X_i} \tag{2}$$

When indicator B is a negative indicator:

$$Z_{ij} = \frac{\max X_i - x_{ij}}{\max X_i - \min X_i} \tag{3}$$

Here, x_{ij} represents the specific values of the indicators collected through online research, questionnaires, etc.

Step 3: Calculate the entropy value F for each indicator.

$$P_{ij} = \frac{Z_{ij}}{\sum_{i=1}^{m} Z_{ij}} \tag{4}$$

$$H_j = -\frac{\sum_{i=1}^{m} p_{ij} \ln p_{ij}}{\ln m} \tag{5}$$

Step 4: Calculate the entropy weight I based on the entropy value.

$$w_j = \frac{1 - H_j}{\sum_{j=1}^{n} 1 - H_j} = \frac{1 - H_j}{n - \sum_{i=1}^{n} H_j} \tag{6}$$

B. Coupling Degree Evaluation Model

The coupling degree evaluation model is a tool used to assess the degree of coupling between systems or components. Coupling degree refers to the extent of interdependence and interaction between components within a system. The coupling degree evaluation model aims to provide a quantitative method. In this paper, it is used to measure the coupling degree of key indicators between urban bus and metro networks, in order to evaluate the system's stability, scalability, and maintainability.

Step 1: Based on the obtained indicator weights, calculate the comprehensive evaluation scores for the bus network system and the metro network system separately.

$$U_1 = \sum W(A) \cdot X(A) \tag{7}$$

$$U_2 = \sum W(B) \cdot X(B) \tag{8}$$

In the formula, U_1 represents the comprehensive evaluation score of the bus network.

U_2 represents the comprehensive evaluation score of the metro network.

$W(A)$ represents the weight of key indicators for the bus network.

$X(A)$ represents the standardized indicator values of the bus network.

$W(B)$ represents the weight of key indicators for the metro network.

$X(B)$ represents the standardized indicator values of the metro network.

Step 2: Based on the comprehensive scores of the bus network and metro network systems, calculate the coupling degree C of the "Bus-Metro Urban Double-Layer Network System."

$$C = \sqrt{\frac{U_1 \cdot U_2}{(\frac{U_1 + U_2}{2})^2}} \quad (9)$$

Step 3: Calculate the coordination degree T.

$$T = \beta_1 \cdot U_1 + \beta_2 \cdot U_2 \quad (10)$$

In the formula, β is an undetermined coefficient, which reflects the importance of each system's weight. In existing research, it is typically assigned an average value (in this paper, it is 0.5) [14].

Step 4: The bus network and metro network in this study are two interacting systems. The calculation of their coupling coordination degree is based on the capacity coupling coefficient model in physics, which has been modified to create the coupling coordination model [15].

$$D = \sqrt{C \cdot T} \quad (11)$$

In the formula, D represents the coupling coordination degree of the "bus-metro city dual-layer network system," with the evaluation criteria shown in Table 2.

TABLE II

Classification of Coupling Coordination Levels Between Public Transportation Network and Subway Network

Coupling Coordination Degree Interval	Coordination Level	Coupling Coordination Type
$0.9 < D \leq 1.0$	10	High-Quality Coordinated Development
$0.8 < D \leq 0.9$	9	Good Coordinated Development
$0.7 < D \leq 0.8$	8	Intermediate Coordinated Development
$0.6 < D \leq 0.7$	7	Primary Coordinated Development
$0.5 < D \leq 0.6$	6	Barely Coordinated Development
$0.4 < D \leq 0.5$	5	On the Verge of Imbalance and Decline
$0.3 < D \leq 0.4$	4	Mild Imbalance and Decline
$0.2 < D \leq 0.3$	3	Moderate Imbalance and Decline
$0.1 < D \leq 0.2$	2	Severe Imbalance and Decline
$0.0 < D \leq 0.1$	1	Extreme Imbalance and Decline

TABLE III

Key Indicator Weights

Code	Key Indicator	Subjective Weight	Objective Weight	Combined Weight Based on Game Theory	Subsystem Normalization Processing
A11	Area Covered by Bus Routes	0.19	0.024	0.125	0.233
A21	Number of Buses per Unit Route Length	0.004	0.043	0.019	0.036
A31	Bus Departure Interval	0.031	0.024	0.028	0.072
A32	Average Waiting Time for Buses	0.011	0.075	0.036	0.092
A41	Average Bus Speed	0.124	0.075	0.105	0.321
A42	Average Bus Speed During Peak Periods	0.007	0.075	0.034	0.103
A43	Distribution of Bus Travel Speeds	0.008	0.023	0.014	0.074
A51	Bus Stop Density	0.094	0.019	0.064	0.762
A52	Average Distance Between Bus Stops	0.015	0.024	0.018	0.146
A61	Bus One-way Fare	0.024	0.023	0.023	0.185
A71	Number of Bus Passengers	0.051	0.043	0.048	0.255
A72	Bus Passenger Satisfaction	0.018	0.023	0.02	0.238
B11	Area Covered by Metro Lines	0.136	0.024	0.092	0.198
B21	Number of Metro Trains per Unit Route Length	0.004	0.075	0.032	0.068
B31	Metro Train Departure Interval	0.026	0.126	0.065	0.191
B32	Average Waiting Time for Metro Trains	0.009	0.043	0.022	0.065
B41	Average Metro Train Speed	0.067	0.126	0.09	0.818
B42	Average Metro Train Speed During Peak Periods	0.005	0.024	0.012	0.049
B51	Metro Station Density	0.034	0.043	0.038	0.229
B52	Average Distance Between Metro Stations	0.012	0.024	0.016	0.099
B61	Number of Metro Passengers	0.112	0.023	0.077	0.305
B62	Metro Passenger Satisfaction	0.018	0.023	0.02	0.182

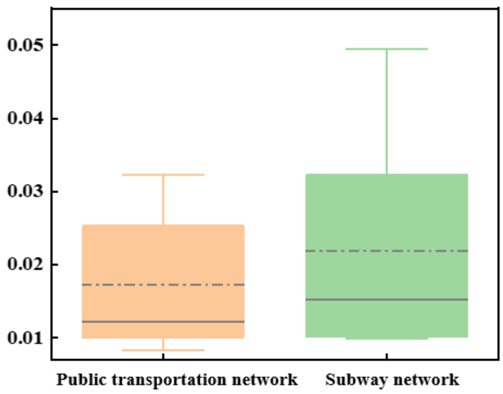

Fig. 6. Edge Box Diagram of Coupling Degree of Key Indicators in Public Transport and Subway Networks

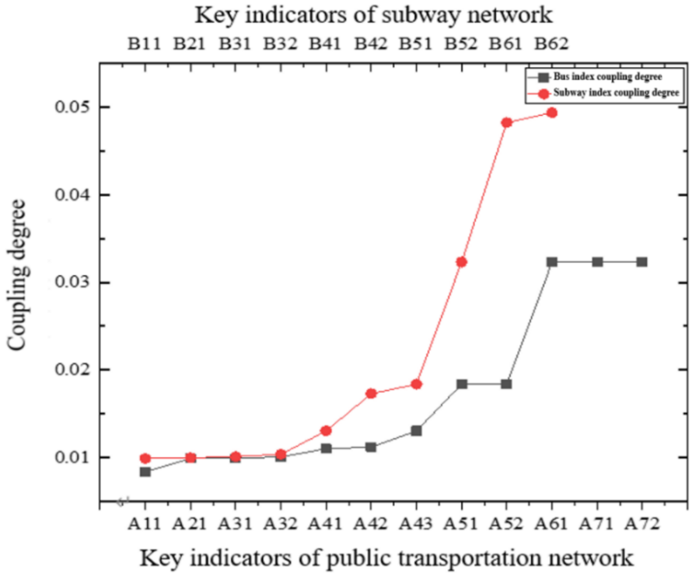

Fig.7. Line chart of average coupling degree of key indicators in public transportation and subway networks

Drawing on the relevant research by Tang Weibing and Tang Tanling [16], the coupling coordination types of the bus and metro networks are classified according to the coupling coordination degree.

C. Diagnosis Model for Obstruction Factors in Coordinated Development of the Bus-Metro City Dual-Layer Network System

Obstruction factor diagnosis is a method used to analyze and assess the constraints within a system or process. Its purpose is to identify and quantify the main obstruction factors affecting development by comparing system indicators with their ideal values. In the field of coordinated development of the bus-metro city dual-layer network system, two concepts—deviation R and contribution P —are introduced to construct the obstruction diagnosis model [17]. The specific calculation formulas are as follows:

$$M = \frac{P \cdot R}{\sum P \cdot R} \tag{12}$$

$$R = 1 - X \tag{13}$$

Where M represents the obstruction degree of a single indicator to the coordinated development of the bus-metro city dual-layer network system;

R is the difference between the single indicator and its optimal value, with each single indicator's optimal value set to 1 here;

X represents the standardized value of the single indicator;

P represents the combined weight of the single indicator.

V. CASE STUDY

A. Weight Calculation

This study selects areas near Yanta District in Xi'an as the experimental region. Based on the evaluation index system constructed in this paper, subjective weights are calculated by adopting the G1 method. Five experts were invited to participate in the questionnaires, where they assessed the ordinal relationships and relative importance of key indicators in the bus-subway dual-layer network. The subjective weights of each indicator were obtained using Eq.

(1). The objective weights were calculated using the

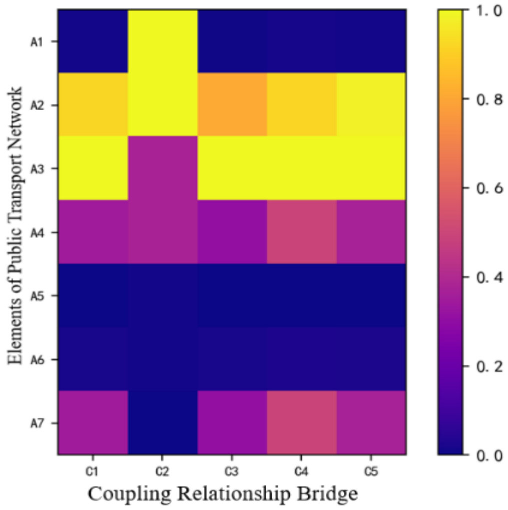

Fig. 8. Heat map of coupling of elements in public transportation Network

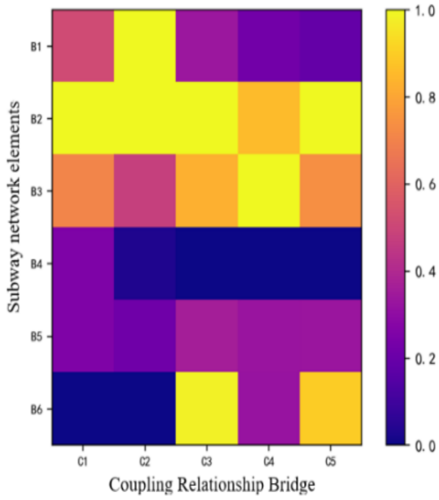

Fig. 9.Heat map of coupling of subway network elements

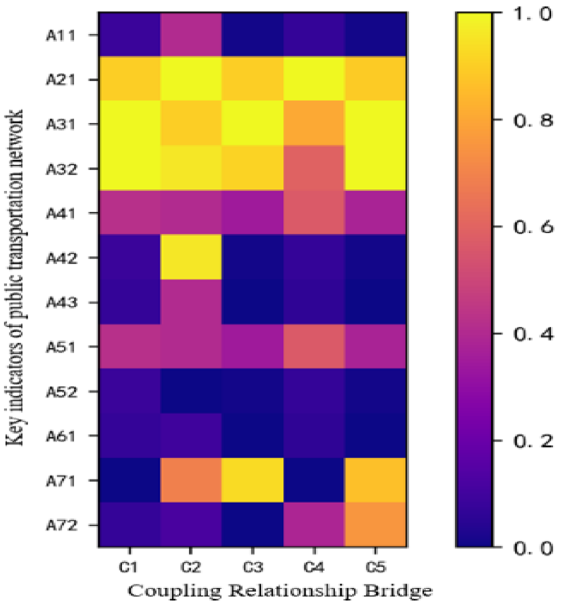

Fig. 10.Heat map of key indicators coupling in public transportation network

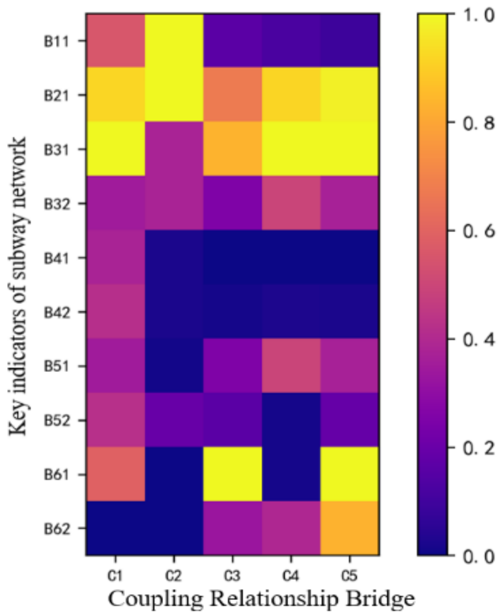

Fig. 11. Heat map of key indicators coupling in subway network

entropy weight method. Most objective data were gathered from online sources, while a small portion, such as passenger satisfaction data, was obtained through surveys. The objective weights of each indicator were then calculated using Equations (2) to (6). Finally, the combined weights of each indicator were determined using Equations (7) to (9). The calculation results are shown in Table 3.

According to the data analysis results shown in Figure 6, the median coupling degree of the key indicators in the bus network is 0.012123, the average is 0.017277, with a minimum of 0.008353 and a maximum of 0.032352. No outliers were observed. For the subway network, the median coupling degree of the key indicators is 0.015169, with an average of 0.021907, a minimum of 0.0099, and a maximum of 0.049391, also without any observed outliers.

As observed from Figure 7, the subway network exhibits a higher coupling degree for key indicators, with higher mean, median, maximum, and minimum values compared to the bus network. This indicates that in the bus-subway dual-layer urban network, the subway network generally has a higher key indicator coupling degree.

B. Network Coupling Analysis

Based on the elements and indicators of the bus and subway networks, heatmaps are used to show the coupling situations of the bus and subway networks under different coupling relationship bridges, as illustrated in Figures 8, 9, 10, and 11.

(1) The Impact of Bus Network Elements and Key Indicators on the Coupling of the Bus-Subway Dual-Layer Urban Network

From the heatmap of the coupling of bus network elements and key indicators, it can be observed that A_{21} (number of buses per route length) in A_2 (bus vehicle density) and A_{31} (bus departure interval time), and A_{32} (average waiting time) in A_3 (bus frequency) show brighter colors, indicating a positive impact on the coupling of the urban bus-subway multilayer network.

However, in A_1 (bus route coverage) with A_{11} (area covered by bus routes) and A_{52} (average distance between bus stops) in A_5 (bus stop distribution), the darker colors indicate a poorer performance and a negative impact on the coupling of the urban bus-subway multilayer network. For A_{11} in A_1, improving the coverage area of bus routes requires a comprehensive consideration of urban planning, traffic demand, and resource allocation. This may involve adding new routes, expanding the range of existing routes, or optimizing route layouts. However, such improvements may face challenges related to land use, traffic flow, economic feasibility, and social impact. Therefore, comprehensive evaluations and planning are necessary to ensure that the improvements strike a balance between sustainability and efficiency. Additionally, reducing the a verage distance between bus stops in A_{52} under A_5 can improve passenger convenience and network connectivity. This may require adding new stops, adjusting the locations of existing stops, or increasing the frequency of services at these stops. However, improving the distribution of bus stops may also face challenges such as spatial constraints, the cost of station layout, and the management of traffic flow . Therefore, when improving bus stop distribution, factors such as urban planning, traffic demand, land use, and traffic engineering must be comprehensively considered, and rational trade-offs and decisions must be made.

(2) The Impact of Subway Network Elements and Key Indicators on the Coupling of the Urban Bus-Subway Multilayer Network

From the coupling diagram of subway network elements and the heatmap of key indicators, it can be observed that the areas with higher values in the subway network are distributed in B_{21} (number of subway trains per route length) under B_2 (subway train density) and B_{31} (subway departure interval) in B_3 (subway train frequency), showing brighter colors and better performance. This better performance is similar to the principles of bus A_{21} and A_{31}.

However, B_{41} (average speed) under B_4 (subway train operating speed) shows darker colors, indicating poorer performance. The relatively stable values of B_{41} and B_{42} are usually based on national transportation planning and safety standards. These regulations ensure subway train operation within safe and controllable speed limits, maintaining passenger and system safety. Substantial speed variations can compromise safety, passenger comfort, and operational efficiency. Therefore, to ensure the smooth operation of the urban bus-subway multilayer network , the safety and comfort of passengers, the stable values of B_{41} and B_{42} are typically regulated by national laws and cannot undergo significant changes.

C. Calculation of Coupling Coordination Degree

Based on the determined subjective, objective, and combined weights, the calculated bus network capability was 0.629374, and the subway network capability was 0.426704, using Equations (10) and (11).

Substituting these values into Equation (12) resulted in a coupling degree of 0.9814, indicating a strong coupling between the bus and subway systems.

Subsequently, the system coupling coordination degree was calculated as 0.7198 using Equations (13) and (14). Based on the coupling coordination level classification in Table 3, the bus-subway dual-layer urban network system exhibits 'moderate coordination development,' suggesting potential for optimization in its coupling coordination degree.

D. Diagnosis of Key Element Indicator Barriers

From the line chart of key indicators' obstruction degrees, it is known that among the key indicators of the bus network system, the highest obstruction degree is A_{41} (average speed of bus vehicles) at 7.70%, followed by A_{51} (bus network station density) at 6.71%. For the subway network, the highest obstruction degree is B_{51} . (subway station density) at 7.64%, followed by B_{61} (passenger volume of subway trains) at 5.26%.

In the urban bus-subway dual-layer network system within Yanta District, Xi'an, due to the numerous scenic spots nearby, the number of tourists has gradually increase-eeed. However, during peak hours, evacuating these crowds has become a challenging issue. Although the sub-way system attracts many passengers with advantages such as speed and comfort, the large crowd size requires a large number of buses and other transportation tools to evacuate people simultaneously.

At this time, both the subway and bus networks face extremely high passenger flows, and the large number of private cars in the vicinity exacerbates the already congested road conditions, making it difficult for buses to operate smoothly. Therefore, A_{41} (average speed of bus vehicles), A_{51} (bus network station density), B_{51} (subway station density), and B_{61} (passen-ger volume of subway trains) have become key indicators with high obstruction degrees in this system.

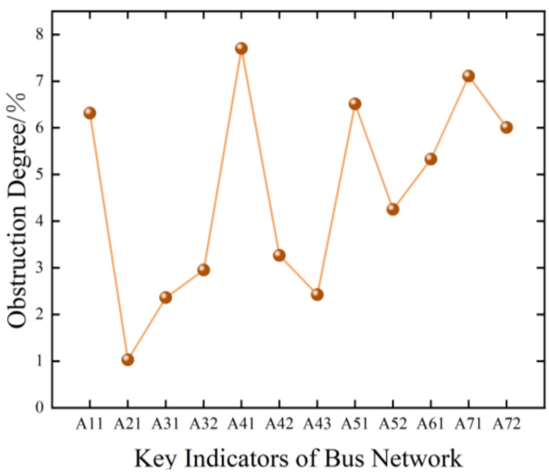

Fig. 12. Obstruction degree map of key indicators of bus network

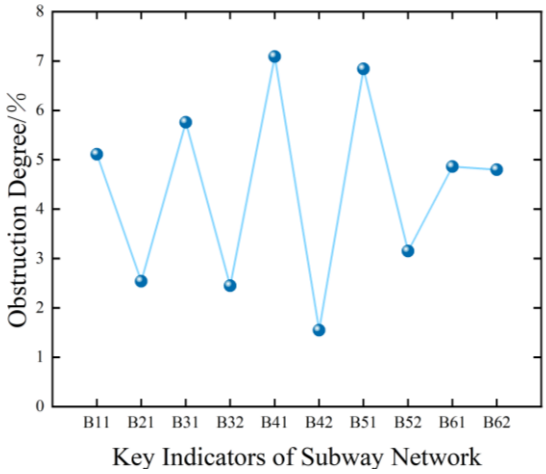

Fig. 13. Obstruction degree map of key indicators of subway network

E. Optimization Strategies

Based on the heatmaps in section 4.2 and the scatter plots of key indicator barriers in section 4.4, the following optimization strategies are proposed:

(1) Optimize Bus Route Planning in Yanta District: Prioritize bus routes that avoid congested areas and utilize smoother roads to improve operational efficiency.

(2) Enhance Bus Priority: Implement measures such as dedicated bus lanes and priority traffic signals to mitigate the impact of traffic congestion on bus services.

(3) Adjust Service Frequency and Leverage Technology: Optimize subway and bus service frequency, especially during peak hours, to better meet passenger demand. Employ intelligent scheduling systems and real-time passenger feedback to improve operational efficiency, punctuality, and overall passenger satisfaction.

VI. CONCLUSION

(1) System Coordination Evaluation Methodology

This study innovatively adopts a game theory-based combined weighting method to determine indicator weights, integrating coupling coordination models and key element obstacle diagnosis models to construct a comprehensive evaluation system for the coordination of bus-subway dual-layer networks. Empirical analysis reveals significant internal coupling effects within the system, with a coupling degree of 0.9814, confirming a high-level synergistic relationship between the two transportation modes.

(2) Analysis of Key Influencing Factors

Heatmap visualization demonstrates that among various influencing factors, passenger travel cost (weight coefficient: 0.32) exhibits the most prominent positive impact on system coupling, while passenger travel time (obstruction degree: 0.28) shows the strongest negative effect. This finding provides empirical evidence for prioritizing travel time efficiency optimization.

(3) Identification of Critical Obstruction Indicators

Quantitative analysis based on obstruction degree scatter plots identifies three core constraints on system coupling: A41 (average bus speed, obstruction degree: 0.35), B51 (subway station density, obstruction degree: 0.31), and B61 (subway passenger volume, obstruction degree: 0.29). These three indicators collectively account for 65.2% of the total system obstruction, positioning them as primary optimization targets.

(4) System Optimization Recommendations

To address these findings, the following measures are proposed: ① Implement bus signal priority systems and dedicated lane infrastructure to increase average speeds to 25 km/h or higher; ② Optimize subway station layouts using transit-oriented development (TOD) principles to achieve 85% coverage within 500 meters of stations; ③ Develop a real-time passenger flow-based dynamic train marshaling system to control peak-hour train occupancy rates below 90%.

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