Optimizing Parallel-Series Configurations for Reliability: Enhancing Integrated Redundancy

Sridhar Akiri, Bhavani Kapu, Peddi Phani Bushan Rao, Arun Kumar Saripalli, Christophe Chesneau

***Abstract*—In reliability theory, technical systems are often modeled using Series-Parallel configurations, which provide a structured framework to analyze the relationship between the lifetimes of individual components and the overall system reliability. These configurations build upon the foundational concept of Parallel-Series systems and are widely used in system design and optimization. Traditionally, system optimization focuses on constraints such as cost. However, additional factors like weight, volume, size, and space also play critical roles, particularly in applications such as AC motor control units. This paper investigates the impact of multiple constraints on optimizing system reliability. We explore an Integrated Redundant Reliability Parallel-Series configuration system, specifically designed to address these multidimensional constraints. The model is developed and solved using the Lagrangean multiplier method (LMM), providing real-valued solutions for critical parameters, including the number of components, component reliability, stage reliability, and overall system reliability. To ensure practical applicability, integer solutions are derived by employing the Newton-Raphson method during the analytical process. This comprehensive approach facilitates a deeper understanding of how multiple constraints influence system reliability and offers valuable insights for optimizing complex technical systems.**

***Index Terms*— Survival Theory, LMM Approach, IRR Model, Parallel-Series, Newton-Raphson Approach, System Efficiency.**

I. INTRODUCTION

IN classical reliability theory, the system and its constituent parts are constrained to exist in one of two states: functional or failed. However, within the framework of a multi-state system, a broader spectrum of possibilities emerges. In this context, both the overall system and its individual components can traverse a range of states beyond the binary distinction of operational or non-operational, introducing a more nuanced understanding of reliability. Redundancy optimization is defined as an integer programming problem with zero-one type variables, according to Mishra, K. B. [1]. An algorithm credited to Lawler and Bell are used to obtain the answer. Any arbitrary function can serve as the objective function and the constraints. We take into consideration three distinct iterations of the optimization issue. Using a digital computer, the answer is convenient and the formulation is simple. The number of restrictions does not limit the magnitude of the problem that can be addressed. A mathematical model was developed by Mishra, K. B. [2] to optimize a system's dependability under specified linear constraints. The system consists of multiple stages that are connected in series, with parallel redundancy at each level to increase reliability. Part I presents a novel application of Lagrange multipliers to convert the model of constrained optimization into a saddle point issue. The reliability function is maximized under certain conditions, and the resulting multidimensional nonlinear algebraic equations are solved by Newton's method. enormous systems can use this strategy since it avoids inverting the enormous Jacobian matrices through additional modifications. In Part II, the model of restricted optimization is transformed into a multistage decision process, and the optimal decision is reached by applying the Maximum principle. It is simple to create, implement, and program this strategy. The solution not only offers a significant reduction in computing time but may also be attained without the worry of non-convergence, which was frequently seen with previous solutions. Alternative designs are simple to think about.

To increase a structure's reliability, Agarwal, K. K. and Gupta, J. S. [3], proposed an integrated redundant reliability model. This model calls for additional resources and incorporates strategic considerations like integrating elements with higher reliability or incorporating redundant units. Agarwal, K. K. et al. [4], introduced numerous methods for assessing generic systems' reliability. Every method's benefit and drawback are examined. To compare the amount of computational work required and the size of the final derived dependability expression, an example is solved using each method. Using a variation technique, Fan, L. T. and Wang, T. [5], were able to determine the parallel system's optimal redundancy. To optimize the system profit is the goal function. Using this method, the optimum design of the multistage parallel systems can be found with a straightforward computational procedure. A pair of thorough numerical examples are provided. An overview of the approaches developed for solving different reliability optimization problems, as well as applications of these approaches to different kinds of design problems, heuristics, exact methods, reliability-redundancy allocation, multi-objective optimization, and assignment of interchangeable components in reliability systems, were established by Kuo, W., and Prasad, V. R. [6]. Like other applications, accurate solutions for dependability optimization problems are not always desirable due to their rarity and limited usefulness. accurate solutions can be challenging to find. Creating

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heuristic and metaheuristic algorithms to solve optimal redundancy-allocation problems constitutes most of the work in this field.

A general approach was proposed by Mettas, A. [7], to estimate the lowest reliability requirement for numerous components in a system that will result in the system's desired dependability value. There are two components to the model. The allocation problem is formulated using nonlinear programming in the first section. A cost function formulation for the nonlinear programming algorithm is presented in the second section. For this matter, it is assumed that the cost will behave generally as a function of the reliability of a component. Next, the cost of the system is lowered by finding an ideal component reliability that meets the dependability objective need of the system. Upon estimating the reliability required for every component, the choice of achieving this reliability through fault tolerance or fault avoidance can be made. The model can be applied to any kind of system, simple or complicated, and for a variety of distributions. It has produced highly positive results. This paradigm has the advantage of being very adaptable and requiring very little processing time. Sankaraiah, G., et al. [8], tried to investigate how different limitations affect system reliability. To conduct analysis, an integrated redundant reliability system is taken into consideration, modeled, and solved using a Lagrangian multiplier that yields a real-valued solution for the system's number of components as well as for each component's dependability at each stage. An integer solution is presented after the problem is further examined using a heuristic algorithm and an integer programming technique, which are validated by sensitivity analysis. Chunping Li and Huibing Hao [9], proposed an innovative reliability evaluation model that incorporates dependencies between two performance parameters through copula theory, enhancing accuracy compared to conventional methods that presume independence. The analysis of train wheel wear data indicates that disregarding PC dependencies might result in erroneous reliability findings, highlighting the necessity of dependency-aware methodologies in reliability engineering.

Sridhar Akiri et.al. [10], conducted a comprehensive study, design, analysis, and optimization of an integrated coherent redundant reliability design that has not been reported in the literature. The system under investigation is initially designed and assessed using the Lagrangean multiplier, which provides a solution that is authentically accepted for the number of units, unit, and phase reliabilities, and thereby for the design's reliability.

An integer solution is derived to ensure the system's practical applicability. The system is analyzed while the design reliability is optimized using the integer and dynamic programming techniques. The swiftly expanding application areas of systems and software modeling, such as intelligent synthetic characters, human-machine interface, menu generators, user acceptance analysis, picture archiving, and software systems, were presented by Sridhar Akiri et.al. [11].

The book will be advantageous to students, research scholars, academicians, scientists, and industry practitioners, as it offers enhanced perspectives on contemporary global trends, issues, and practices. Offers optimization, simulation, and modeling of software reliability Provides practical applications, tools, and methodologies for resource allocation and reliability modeling. Demonstrates the optimization and cost modeling processes that are associated with intricate systems.

Chunping Li, Haiqing Zhao, and Huibing Hao [12], formulated novel partial dependency dependability models for intricate systems utilizing copula functions, offering explicit formulations for series and parallel configurations under diverse dependence scenarios. They proposed techniques to enhance system reliability through comparison analysis, including altering component dependencies, adjusting the number of dependent components, or building new dependent structures, supported by numerical examples illustrating these improvements.

Srinivasa Rao Velampudi et. al. [13], conducted a review of the literature on system reliability optimization with redundancy and integrated reliability models with redundancy, and they recommend additional enhancements. This investigation explores the optimization of structural reliability while considering resource constraints, including the price, weight, and volume of components. Although reliability is typically evaluated in terms of component price, real-world scenarios demonstrate the significant impact of other constraints, such as component weight and volume, resulting in a unique enhancement in structural reliability. The investigation examines a refined over-reliability model, navigating through numerous constraints to optimize the recommended configuration. The objective of Srinivasa Rao Velampudi et. al. [14], was to investigate and evaluate the influence of supplementary concealed restraints on the enhancement of structure reliability. The analysis is conducted with the integration of extraneous reliability from the structured structure. The Lagrangian multiplier method provides a solution for the reliability of elements, phases, and structures. Furthermore, The Dynamic programming method was implemented by employing a heuristic algorithm that generates an integer solution that is nearly optimal but not closed-bounded. The results obtained are illustrated by a numerical example. To optimize the efficiency of the system, Srinivasa Rao Velampudi et. al. [15], proposed an additional system that considers the number of factors in each phase, the efficiencies of the factors, and the various constraints. The authors utilized a variety of Lagrangean methods to ascertain the reliabilities of the phase and the numbers and efficiency of the factors under various parameters, including cost, size, and burden, to improve the efficiency of the system. The dynamic programming approach and simulation method have been modified to produce an integer result and to visualize the values as real.

The numerous proposed methods of system reliability evaluation of the consecutive-k-out-of-n: G systems were summarized by L Zhou et. al. [16], Nevertheless, these approaches are predicated on the premise that all components are identical and independent. Subsequently, we evaluate the system reliability of the linear consecutive-k-out-of-n: G system when the tenure of components is not required to be identical, and we introduce a domination method. A novel approach to evolutionary multi-task optimization in the reliability redundancy allocation problem was reported by R Nath et. Al. [17]. This approach leverages the concepts of the widely used multi-factorial evolutionary algorithm (MFEA). To optimize the overall performance of the system, Srinivasa Rao Velampudi et. al.

[18], developed a case study on the Muffle Box Furnace machine. The study employed Lagrangean methods to calculate the price-component, weight-component, and volume-component associated with various system configurations. This investigation concludes with the development of a United Reliability Model (URM) that addresses variables such as unknown elements ($t_{\alpha j}$), component-reliability ($r_{\alpha j}$), and stage-reliability (R_{SE}) at specific points. The integration of value constraints into IRR Models, which establishes a fixed relationship between the price of components and their reliability, is emphasized in the existing literature. The superfluous reliability system for the parallel-series structure composition is shaped and elevated by a novel approach that incorporates intended considerations of component's weight and component's volume as additional constraints alongside component's price.

The authors provided a comprehensive examination of the evolution and design of parallel-series systems as a component of the IRR Models research contribution, with a particular emphasis on redundant reliability configurations. Furthermore, the authors explore the design components of a redundant reliability system that is integrated. Additionally, their research encompasses a thorough case study, as demonstrated. Collectively, these contributions enhance our comprehension of parallel-series configuration by providing insights into their growth patterns, design considerations, and integrated reliability models. This corpus of work is a valuable contribution to the field of engineering applications and reliability theory.

The authors of this paper focus on the application of the regular Lagrangean multipliers method to derive real-valued solutions, both with and without rounding-off. The "Newton-Raphson" method is a novel scientific approach that is employed to derive integer values. This method is implemented to compare the solutions obtained through the Lagrangean method and to obtain scientifically sound solutions, thereby guaranteeing the retention of the necessary number of components ($t_{\alpha j}$) in each stage. Concurrently, this methodology contributes to the improvement of the overall system reliability (R_{SE}).

II. Methods

A. Consideration Symbols

Uniformity is assumed among elements within each stage, signifying that all elements share an equivalent level of reliability. Statistical independence is attributed to all elements, implying that the failure of one element exerts no influence on the functionality of other elements within the structure.

R_{SE} = System-Reliability

$R_{\alpha j}$ = Reliability of Phase 'j', $0 < R_{\alpha j} < 1$

$r_{\alpha j}$ = Reliability of each component in phase 'j';

Where $0 < r_{\alpha j} < 1$

$t_{\alpha j}$ = Number of components in phase 'αj'

CP_{α} = Component's-Price factor for each element in the phase 'αj'

WP_{α} = Component's-Weight factor for each element in the phase 'αj'

VP_{α} = Component's-Volume factor for each element in the phase 'αj'

$C_{\alpha 0}$ = Maximum permissible system–Component's-Price

$W_{\alpha 0}$ = Maximum permissible system-Component's-Weight

$V_{\alpha 0}$ = Maximum permissible system-Component's-Volume

LMM Lagrangean Multiplier Method

NRMA Newton-Raphson Method Approach

IRRM Integrated Redundant Reliability Model

b_{α}, f_{α}, i_{α}, d_{α}, g_{α}, q_{α} are Constants.

B. Mathematical Examination

The system's dependability concerning the given value function

$$\text{Maximize } R_{SE} = 1 - \prod_{\alpha=1}^{k}\left[1 - \prod_{j=1}^{n} R_{\alpha j}\right] \quad (1)$$

The subsequent correlation between value and efficiency is employed to determine the value coefficient of each unit in the phase 'αj'.

$$r_{\alpha} = \tanh^{-1}\left(\frac{C_{\alpha}}{b_{\alpha}}\right)^{\frac{1}{d_{\alpha}}} \quad (2)$$

$$\text{Therefore} \quad C_{\alpha} = b_{\alpha} \tanh\,[r_{\alpha}]^{\,d_{\alpha}} \quad (3)$$

$$\text{Similarly,} \quad L_{\alpha} = f_{\alpha} \tanh\,[r_{\alpha}]^{\,g_{\alpha}} \quad (4)$$

$$S_{\alpha} = p_{\alpha} \tanh\,[r_{\alpha}]^{\,q_{\alpha}} \quad (5)$$

Since component's-price is linear in 'αj',

$$\sum_{\alpha=1}^{n} C_{\alpha}.t_{\alpha} \le C_{\alpha 0} \quad (6)$$

Similarly, component's-weight and component's-volume are also linear in 'αj'.

$$\sum_{\alpha=1}^{n} L_{\alpha}.t_{\alpha} \le W_{\alpha 0} \quad (7)$$

$$\sum_{\alpha=1}^{n} S_{\alpha}.t_{\alpha} \le V_{\alpha 0} \quad (8)$$

Substituting (2) in (3)

$$\sum_{\alpha=1}^{n} b_{\alpha}[\tanh(r_{\alpha})]^{d_{\alpha}}.t_{\alpha} \le C_{\alpha 0} \quad (9)$$

$$\sum_{\alpha=1}^{n} f_{\alpha}[\tanh(r_{\alpha})]^{g_{\alpha}}.t_{\alpha} \le W_{\alpha 0} \quad (10)$$

$$\sum_{\alpha=1}^{n} p_{\alpha}[\tanh(r_{\alpha})]^{q_{\alpha}}.t_{\alpha} \le V_{\alpha 0} \quad (11)$$

The transformed equation through the relation

$$t_{\alpha} = \frac{\log R_{\alpha}}{\log r_{\alpha}} \quad (12)$$

$$\text{Where } R_{\alpha j} = \prod_{\alpha=1}^{k}\left[1 - (1 - r_{\alpha})^{t_{\alpha}}\right] \quad (13)$$

Subject to the constraints

$$\sum_{\alpha=1}^{n} b_{\alpha}[\tanh(r_{\alpha})]^{d_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - C_{\alpha 0} \le 0 \quad (14)$$

$$\sum_{\alpha=1}^{n} f_{\alpha}[\tanh(r_{\alpha})]^{g_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - W_{\alpha 0} \le 0 \quad (15)$$

$$\sum_{\alpha=1}^{n} p_{\alpha}[\tanh(r_{\alpha})]^{q_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - V_{\alpha 0} \le 0 \quad (16)$$

Positivity restrictions $\alpha j \ge 0$

A Lagrangean function is defined as

$$L = R_{\alpha} + \varepsilon_1\left[\sum_{\alpha=1}^{n} b_{\alpha}[\tanh(r_{\alpha})]^{d_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - C_{\alpha 0}\right] + \varepsilon_2\left[\sum_{\alpha=1}^{n} f_{\alpha}[\tanh(r_{\alpha})]^{g_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - W_{\alpha 0}\right] + \varepsilon_3\left[\sum_{\alpha=1}^{n} p_{\alpha}[\tanh(r_{\alpha})]^{q_{\alpha}}.\frac{\log R_{\alpha}}{\log r_{\alpha}} - V_{\alpha 0}\right] \quad (17)$$

Utilizing the Lagrangean function enables the identification of the optimal point and its separation by $R_{\alpha j}$, $r_{\alpha j}$, $\mathscr{E}_1$, $\mathscr{E}_2$ and $\mathscr{E}_3$.

$$\frac{\partial L}{\partial R_{\alpha}} = 1 + \varepsilon_1\left[\sum_{\alpha=1}^{n} b_{\alpha}[\tanh(r_{\alpha})]^{d_{\alpha}}.\frac{1}{R_{\alpha}\log r_{\alpha}}\right] + \varepsilon_2\left[\sum_{\alpha=1}^{n} f_{\alpha}[\tanh(r_{\alpha})]^{g_{\alpha}}.\frac{1}{R_{\alpha}\log r_{\alpha}}\right] + \varepsilon_3\left[\sum_{\alpha=1}^{n} p_{\alpha}[\tanh(r_{\alpha})]^{q_{\alpha}}.\frac{1}{R_{\alpha}\log r_{\alpha}}\right] \quad (18)$$

$$\frac{\partial L}{\partial r_\propto} = \varepsilon_1\left[\left(\sum_{\propto=1}^{n} b_\propto[\tanh(r_\propto)]^{d_\propto}.\frac{\log R_\propto}{\log r_\propto}\right)\left[\frac{\operatorname{sech}^2(r_\propto)}{\tanh(r_\propto)} - \frac{1}{r_\propto \log r_\propto}\right]\right] + \varepsilon_2\left[\left(\sum_{\propto=1}^{n} f_\propto[\tanh(r_\propto)]^{g_\propto}.\frac{\log R_\propto}{\log r_\propto}\right)\left[\frac{\operatorname{sech}^2(r_\propto)}{\tanh(r_\propto)} - \frac{1}{r_\propto \log r_\propto}\right]\right] + \varepsilon_3\left[\left(\sum_{\propto=1}^{n} p_\propto[\tanh(r_\propto)]^{q_\propto}.\frac{\log R_\propto}{\log r_\propto}\right)\left[\frac{\operatorname{sech}^2(r_\propto)}{\tanh(r_\propto)} - \frac{1}{r_\propto \log r_\propto}\right]\right] \quad (19)$$

$$\frac{\partial L}{\partial \varepsilon_1} = \sum_{\propto=1}^{n} b_\propto[\tanh(r_\propto)]^{d_\propto}.\frac{\log R_\propto}{\log r_\propto} - C_{\alpha 0} \quad (20)$$

$$\frac{\partial L}{\partial \varepsilon_2} = \sum_{\propto=1}^{n} f_\propto[\tanh(r_\propto)]^{g_\propto}.\frac{\log R_\propto}{\log r_\propto} - W_{\alpha 0} \quad (21)$$

$$\frac{\partial L}{\partial \varepsilon_3} = \sum_{\propto=1}^{n} p_\propto[\tanh(r_\propto)]^{q_\propto}.\frac{\log R_\propto}{\log r_\propto} - V\alpha_0 \quad (22)$$

Where $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ are Lagrangean multipliers.

C. Case Problem

In the pursuit of utilizing optimization techniques [9] to derive various parameters for a specific mechanical system, this research incorporates assumptions where factors such as component's- price, component's-weight, and component's-volume are presumed to be directly proportional to system reliability. It is important to note that this assumption may not hold true for electronic systems. Consequently, the assessment of maximum component's-reliability ($r_{\alpha j}$), stage-reliability ($R_{\alpha j}$), quantity of elements per stage ($t_{\alpha j}$), and structure accuracy (R_{SE}) is applicable to any given mechanical system [10]. This research specifically centers on assessing the structural accuracy [13] of a dedicated apparatus engineered for assembling single-phase industrial AC motor control circuits.

In the context of this analysis, the AC motor is used to assemble a power generator with 3 to 5 foundational elements. The machine, valued at approximately $750, represents the structure's component's-price, while the component's-weight, it bears is 650 pounds, constituting the structural component's-weight. Additionally, the component's-volume occupied by the machine volume to 750 cm^3, signifying the dimension of the component or overall structure size. To engage authors originating in diverse backgrounds, hypothetical numbers are employed, offering flexibility for adjustment based on varying environmental considerations. The schematic diagram of the AC motor control circuits in Figure 1.

D. Parameters

Tables below showcase the efficacy of individual factors, phases, the number of factors in each stage, and the overall structural effectiveness [2, 3]. The information needed for the parameters relevant to the case problem is presented in the table I.

E. Utilizing the Lagrangean multiplier method (LMM) without rounding off to address component-price constraints with precision.

The efficiency design related to values is outlined in the table II.

F. Utilizing the Lagrangean multiplier method without rounding off to address component-weight constraints with precision

The equivalent results for the load are shown in the Table III.

G. Utilizing the Lagrangean multiplier method without rounding off to address component-volume constraints with precision

The equivalent results for size are described in the table IV.

Structure Efficiency (R_{SE}), devoid of rounding-off, remains directly proportional to the individual components of component's-price, component's-weight, and component's volume with a constant of proportionality set at 0.8934.

TABLE I
PRE-FIXED COMPONENTS VALUES

Phase	Price and Its Reliability		Weight and Its Reliability		Volume and Its Reliability	
	$b\alpha j$	$f\alpha j$	$i\alpha j$	$d\alpha j$	$g\alpha j$	$q\alpha j$
1	80	0.85	100	0.92	90	0.94
2	55	0.88	60	0.88	75	0.89
3	65	0.91	75	0.91	105	0.86

TABLE II
ANALYZING PRICE CONSTRAINTS FOR COMPONENTS THROUGH THE LAGRANGE MULTIPLIER METHOD

Phase	$b\alpha j$	$f\alpha j$	$r\alpha j$	$Logr\alpha j$	$R\alpha j$	$LogR\alpha j$	$t\alpha j$	$CP\alpha$	$CP\alpha.t\alpha j$
01	80	0.85	0.9452	-0.0245	0.8424	-0.0745	3.0408	68	206.9240
02	55	0.88	0.9574	-0.0189	0.8243	-0.0839	4.4392	48.4	214.8186
03	65	0.91	0.9654	-0.0153	0.8353	-0.0782	5.1111	59.15	302.3036
Final Component's-price									724.0462

TABLE III
ANALYZING WEIGHT CONSTRAINTS FOR COMPONENTS THROUGH THE LAGRANGE MULTIPLIER METHOD

Phase	Pi	ki	$r\alpha j$	$Logr\alpha j$	$R\alpha j$	$LogR\alpha j$	$t\alpha j$	WPo	$WP\alpha.t\alpha j$
01	100	0.92	0.8741	-0.0584	0.6777	-0.1690	2.8938	92	265.2296
02	60	0.88	0.8445	-0.0734	0.6487	-0.1880	2.5613	52.8	135.2366
03	75	0.91	0.8456	-0.0728	0.5461	-0.2627	3.6085	68.25	246.1914
Final Component's-weight									647.6576

TABLE IV
ANALYZING VOLUME CONSTRAINTS FOR COMPONENTS THROUGH THE LAGRANGE MULTIPLIER METHOD

Phase	ϑj	qj	$r\alpha j$	$Logr\alpha j$	$R\alpha j$	$LogR\alpha j$	$t\alpha j$	$VP\alpha$	$VP\alpha.t\alpha j$
01	90	0.94	0.8741	-0.0584	0.6777	-0.1690	2.8938	84.6	244.8155
02	75	0.89	0.8445	-0.0734	0.6487	-0.1880	2.5613	66.75	170.9668
03	105	0.86	0.8456	-0.0728	0.5461	-0.2627	3.6085	90.3	325.8476
Final Component's-volume									741.6299

III. Optimization of Efficiency Through the Application the Lagrangean Multiplier Method

The design efficiency [11] compiles the values of 'α_j' as integers, rounding each 'α_j' value to the nearest whole number. Tables delineate the permissible results for, component's-price, component's-weight, and component's-volume. The task at hand involves computing variances attributable to component's-price, component's-weight, component's-volume, and examining construction capacity, both with 'α_j' rounded to the nearest integer and in its original form, to extract comprehensive insights.

A. Designing Efficiency Through LMM Considering Components-Price, Components-Weight, and Components-Volume with Rounding Precision

The equivalent results for price, weight, size, and system reliability are described in the Table V. and the following are the variations due to component's-price, components-weight, components-volume

Mutation in Component's-price = 04.56%
Mutation in Component's-weight = 09.16%
Mutation in Component's-volume = 10.03%
Mutation in Structure Efficiecy = 05.85%

IV. Newton-Raphson Approach

Employing the Lagrangean technique [5], which possesses several drawbacks, including the requirement to specify the quantity of components needed at each stage (α_j) in real numbers, can prove challenging to implement. The conventional practice of rounding down values may lead to alterations in component's-price, component's-weight, and component's-volume, influencing system reliability and significantly impacting the efficiency design of the model. Recognizing this limitation, the author proposes an alternative empirical approach that employs the Newton-Raphson method to derive an integer solution. This method utilizes the solutions generated by the Lagrangean approach as parameters for the proposed Newton-Raphson technique.

A. Newton-Raphson Method (NRM)

The graph of $y = f(t)$ crosses the $x - axis$ at the point R corresponding to the equation $f(t) = 0$ (Fig 4.1). Suppose the current approximation to the root is t_r in the rth iteration. Let p be the corresponding point on the curve. The tangent to the curve at p cuts the $x - axis$ at T, where $t = t_{r+1,}$ say giving us the next approximate to the required root.

Let $PM = f(t_r)$ (23)

and $TM = t_r - t_{r+1}$ (24)

so that the slope of the tangent at p $(t = t_r)$ is

$$\tan P\widehat{TM} = f(t_r) = f(t_r)/(t_r - t_{r+1}) \quad (25)$$

Therefore,

$$t_{r+1} - t_r = -\frac{f(t_r)}{f'(t_r)} \quad (26)$$

i.e., $$t_{r+1} = t_r - \left[\frac{f(t_r)}{f'(t_r)}\right] \quad r = 1,2,3, \dots \dots \quad (27)$$

The process can be continued till the absolute value of the difference between two successive approximations, say t_n and t_{n+1} is less than the prescribed degree of accuracy Q.

i.e., $|t_{n+1} - t_n| < Q$ (28)

In this method, slide down a tangent line along the curve y=f(t), But when the slope of a tangent corresponding to an approximate root in one of the iterations is zero (parallel to the x-axis) or numerically very small, Newton's method fails to give the solution.

B. Convergence of Newton's Method

The sequence of approximate solutions T_1, T_2, T_3 obtained by the Newton -Raphson formula converges except in few situations, quite rapidly to the actual solution. Suppose $T = a$ is the actual root of the equation $f(t) = 0$ so that $f(a) = 0$. If t_n is an approximation obtained in the nth iteration, let us write, $\varepsilon_n = a - t_n$ (29)

Where ε_n is small when t_n is close to a. Now from the Newton-Raphson formula, we have

$$t_{n+1} = t_n - \left[\frac{f(t_n)}{f'(t_n)}\right] \quad (30)$$

Using the above two equations

$$t_{n+1} = (a - \varepsilon_n) - \left[\frac{f(a-\varepsilon_n)}{f'(a-\varepsilon_n)}\right] \quad (31)$$

From Taylor's theorem

$$t_{n+1} = (a-\varepsilon_n) - \left[\frac{f(a) - \varepsilon_n f'(a) + \frac{1}{2}\varepsilon_n^2 f''(a) - \cdots}{f'(a) - \varepsilon_n f''(a) + \frac{1}{2}\varepsilon_n^2 f''(a) - \cdots}\right] \tag{32}$$

$$= a - \varepsilon_n + \varepsilon_n \left[1 - \frac{1}{2}\varepsilon_n \frac{f''(a)}{f'(a)} + \cdots \ldots\right]\left[1 + \varepsilon_n \frac{f''(a)}{f'(a)} - \cdots \ldots \ldots\right] \tag{33}$$

(Since $f(a) = 0$ and using the binomial theorem)

$$= a - \varepsilon_n + \varepsilon_n \left[1 - \frac{1}{2}\varepsilon_n \frac{f''(a)}{f'(a)} + \varepsilon_n \frac{f''(a)}{f'(a)} + \cdots \ldots \ldots \ldots\right] \tag{34}$$

$$\approx a + \frac{1}{2}\varepsilon_n^2 \frac{f''(a)}{f'(a)} \tag{35}$$

Therefore,

$$t_{n+1} \approx a + \frac{1}{2}\varepsilon_n^2 \frac{f''(a)}{f'(a)} \tag{36}$$

$$a - t_{n+1} = -\frac{1}{2}\varepsilon_n^2 \frac{f''(a)}{f'(a)} \tag{37}$$

i.e., $\varepsilon_{n+1} = -\frac{1}{2}\varepsilon_n^2 \frac{f''(a)}{f'(a)}$ (38)

Thus, the Newton's iteration formula is a second order process which means that the solution is the one of quadratic convergence.

C. Algorithm of Newton's Method

Step 1: Choose a trail solution t_α, *find* $f(t_0)$ *and* $f'(t_0)$ (39)

Step 2: Next approximation x_i is obtained from

$t = t_0 - f(t_0) / f'(t_0)$ (40)

Step 3: Follow the above procedure to find successive approximation t_{r+1}, using the formula

$t_{r+1} = t_r - \frac{f(t_r)}{f'(t_r)}$ $r = 1, 2, 3, \ldots \ldots$ (41)

Step 4: stop when $|t_{r+1} - t_r| < Q,$ (42)

Where Q is the prescribed accuracy.

The author applied a Newton-Raphson approach to compute the new phase reliability (R_{sj}), resulting in values of 0.8424, 0.8243, and 0.8353 for stage reliability (R_{sj}). Employing the NRM, the study presents the outcomes for the mathematical function presented successively in tables 6, 7, and 8, facilitating the derivation of necessary conclusions.

V. RESULTS

The implementation of the LMM has yielded a solution in real numbers for the proposed Comprehensive. Redundant Reliability Systems in the mathematical models investigated, fulfilling the need for an integer solution.

A. Elaboration on the Constraints of Component's-Price, Component's-Weight, and Component's-Volume Utilizing the Newton's Raphson Method (NRM).

The component's-price, component's-weight and component's-volume related efficiency design is described in the Table V and VI.

B. *Comparison of Optimization of Integrated Redundant Reliability Parallel-Series Systems* (IRRPS) *– LMM with rounding-off and NRM Approach for Component's-Price.*

The comparison between the LMM with rounding-off and NRM for components of price-related efficiency design is presented in Table VII.

C. *Analyzing the Optimal Enhancement of* IRRPS *– LMM with rounding-off and NRM approach for Component's-Weight.*

The comparison between the LMM with rounding-off and NRM for components of weight related efficiency design in Table VIII.

D. Analyzing the optimal enhancement of IRRPS using the LMM with rounding-off and NRM for component volume.

The comparison between the LMM with rounding-off and NRM for components of volume-related efficiency design is presented in Table IX.

VI. DISCUSSION

The authors have undertaken a comprehensive analysis of the components involved in an AC motor control unit, specifically focusing on the rectifier, inverter, and DC link. By employing the Integrated Redundant Reliability Model, the study successfully identifies the number of components, the reliability of each component, the reliability at various stages, and the overall system reliability through a parallel-series configuration. The ensuing discussion details the performance metrics of the AC motor control unit in relation to these critical components.

This research presents a novel reliability model designed specifically for a parallel-series configuration system with multiple efficiency criteria. When data is presented in real numbers, the LMM is employed to ascertain the values of components ($t_{\alpha j}$), component's-reliability ($r_{\alpha j}$), stage-reliability ($R_{\alpha j}$), and system-reliability (R_{SE}). The resulting component-efficiencies ($r_{\alpha j}$) are 0.9452, 0.9574, and 0.9654, stage-reliabilities ($R_{\alpha j}$), are 0.8424, 0.8243, and 0.8353, and structure-reliability (R_{SE}) is 0.9457.

For practical implementation, the Newton's Raphson method is employed to obtain integer-solutions, leading to component-reliabilities ($r_{\alpha j}$) of 0.9551, 0.9638, and 0.9952, stage reliabilities ($R_{\alpha j}$) of 0.8849, 0.8482, and 0.8461, and system reliability (R_{SE}) of 0.9672 These integer solutions are derived using inputs derived from the Lagrangean method, ensuring the applicability of the model in practical scenarios.

The analysis reveals that variations in component's-price, component's-weight, and component's-volume are evident, although minor. However, these fluctuations, when juxtaposed with stage reliability, exert a positive influence on overall system reliability. The developed Integrated Reliability Model (IRM) demonstrates significant utility, particularly in practical situations demanding the incorporation of redundancy by reliability engineers within a parallel-series configuration. This proves particularly advantageous when the system's intrinsic value is on the lower side. In forthcoming studies, the authors propose investigating an innovative methodology that imposes constraints on reliability values at both the lower and upper bounds of components while simultaneously maximizing system dependability. Leveraging existing heuristic processes, the aim is to formulate analogous IRMs.

TABLE V
EFFICIENCY DESIGN RELATING TO COMPONENT'S-PRICE, COMPONENT'S-WEIGHT, AND COMPONENT'S-VOLUME, THE TABLE BELOW ILLUSTRATES CONSTRAINT ANALYSIS USING THE LAGRANGE MULTIPLIER METHOD, INCORPORATING THE ROUNDING OFF PROCESS

Phase	$r\alpha j$	$R\alpha j$	$t\alpha j$	$CP\alpha$	$CP\alpha.t\alpha j$	$t\alpha j$	$WP\alpha$	$WP\alpha.t\alpha j$	$t\alpha j$	$VP\alpha Sej$	$VP\alpha.t\alpha j$
1	0.9452	0.8424	3	68	204	3	92	276	3	85	255
2	0.9574	0.8243	4	48	192	3	53	159	3	67	201
3	0.9654	0.8353	5	59	295	4	68	272	4	90	360
Total Component's-price, Component's-weight, Component's-volume					691			707			816
System Reliability (RSE)											0.9457

TABLE VI
EFFICIENCY DESIGN RELATING TO COMPONENT'S-PRICE, COMPONENT'S-WEIGHT, AND COMPONENT'S-VOLUME, CONSTRAINT ANALYSIS BY USING NEWTON'S RAPHSON METHOD IS SHOWN IN THE FOLLOWING

Phase	$r\alpha j$	$R\alpha j$	$t\alpha j$	$CP\alpha$	$CP\alpha.t\alpha j$	$t\alpha j$	$WP\alpha$	$WP\alpha.t\alpha j$	$t\alpha j$	$VP\alpha Sej$	$VP\alpha.t\alpha j$
01	0.9551	0.8849	3	66	198	3	91	273	3	83	249
02	0.9638	0.8482	4	47	188	3	50	150	3	67	201
03	0.9952	0.8461	4	55	220	4	64	256	4	88	352
Total unit's - price, unit's - weight, unit's - volume					606			679			802
System Reliability (RSE)											0.9672

TABLE VII
FINDINGS FROM LAGRANGE MULTIPLIER METHOD WITH ROUNDED VALUES AND NEWTON'S RAPHSON METHOD APPROACH FOR COMPONENT'S-PRICE

		LMM With Rounding Off				Newton's Raphson Method			
Phase	$t\alpha j$	$r\alpha j$	$R\alpha j$	$CP\alpha$	$CP\alpha.t\alpha j$	$r\alpha j$	$R\alpha j$	$CP\alpha$	$CP\alpha.t\alpha j$
01	3	0.9452	0.8424	68	204	0.9551	0.8849	66	198
02	4	0.9574	0.8243	48	192	0.9638	0.8482	47	188
03	5	0.9654	0.8353	59	295	0.9952	0.8461	55	220
Final Unit's Price					691				606
System Reliability		Applying the LMM (RSE)			0.9457	Applying the NRM (RSE)			0.9672

TABLE VIII
FINDINGS FROM THE LAGRANGE MULTIPLIER METHOD WITH ROUNDING-OFF TECHNIQUE AND NEWTON'S RAPHSON METHOD APPROACH FOR COMPONENT'S-WEIGHT

		LMM With Rounding Off				Newton's Raphson Method			
Phase	$t\alpha j$	$r\alpha j$	$R\alpha j$	$WP\alpha$	$WP\alpha.t\alpha j$	$r\alpha j$	$R\alpha j$	$WP\alpha$	$WP\alpha.t\alpha j$
01	3	0.9452	0.8424	92	276	0.9551	0.8849	91	273
02	3	0.9574	0.8243	53	159	0.9638	0.8482	50	150
03	4	0.9654	0.8353	68	272	0.9952	0.8461	64	256
Final Unit's Weight					707				679
System Reliability		Applying the LMM (RSE)			0.9457	Applying the NRM (RSE)			0.9672

TABLE IX
FINDINGS FROM THE LAGRANGE MULTIPLIER METHOD WITH ROUNDING-OFF TECHNIQUE AND NEWTON'S RAPHSON METHOD FOR COMPONENT'S-VOLUME

		LMM With Rounding Off				Newton's Raphson Method			
Phase	$t\alpha j$	$r\alpha j$	$R\alpha j$	$VP\alpha$	$VP\alpha.t\alpha j$	$r\alpha j$	$R\alpha j$	$VP\alpha$	$VP\alpha.t\alpha j$
01	3	0.9452	0.8424	85	255	0.9551	0.8849	83	249
02	3	0.9574	0.8243	67	201	0.9638	0.8482	67	201
03	4	0.9654	0.8353	90	360	0.9952	0.8461	88	352
Final Unit's Volume					816				802
System Reliability		Applying the LMM (RSE)			0.9457	Applying the NRM (RSE)			0.9672

Mutation in Component's-price = 14.02%
Mutation in Component's-weight = 04.12%
Mutation in Component's-volume = 04.75%
Mutation in Structure Efficiecy = 02.22%

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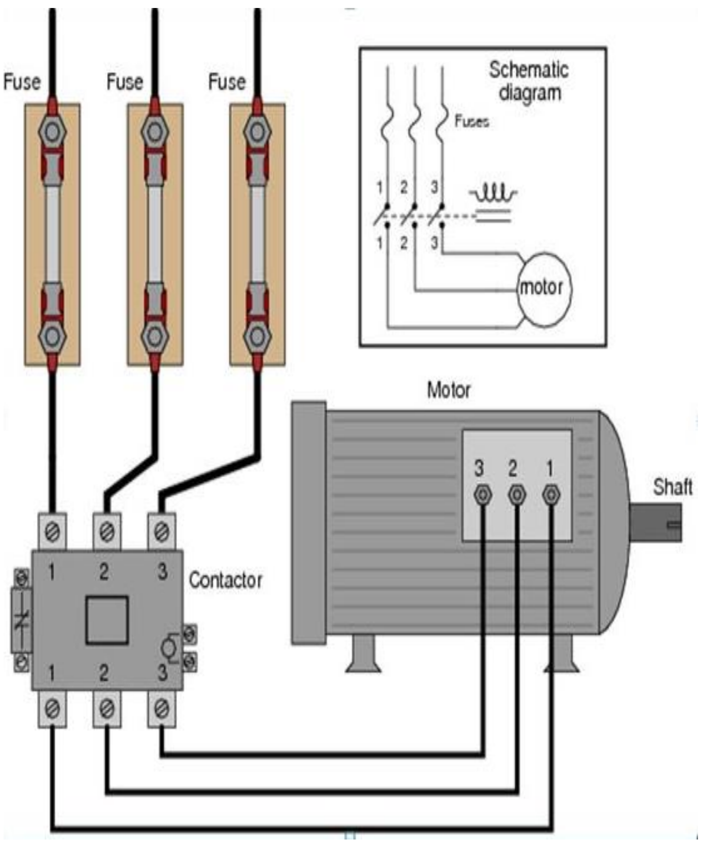

Fig. 1. AC Motor control units