

Research on the Evolutionary Game of Safety Supervision in Coal Mining Enterprises under Different Reward and Punishment Mechanisms

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Abstract—Safety production is crucial for protecting lives and property, significantly promoting economic development and social stability. In response to the recurring safety incidents in China's coal mining industry, this current research investigates the strategic interactions between coal mine workers and enterprise managers concerning safety investments and regulatory compliance. Evolutionary game theory was employed to construct a dynamic model capturing the safety-related decision-making processes among the parties involved, exploring the effects of both static and dynamic reward and punishment mechanisms on strategic choices and system stability. These findings can be summarized as follows: (1) Under a static reward and punishment mechanism, the unsafe behavior strategies among workers and enterprise managers are influenced by government supervision and the intensity of rewards and punishments. (2) A dynamic reward and punishment mechanism overcomes the static approach's limitations by flexibly adjusting incentives and penalties to promote stable convergence. (3) Fluctuations in the intensity of governmental rewards and punishments, as well as changes in the regulatory environment, have a substantial impact on the safety behaviors of workers and the decision-making processes of managers. Future research should leverage artificial intelligence and big data technology to optimize reward and punishment strategies, enhance safety management efficiency, and provide more scientific decision-making support for safe production.

Index Terms—coal mine safety, safety management, dynamic mechanism, evolutionary game model, numerical simulation

I. INTRODUCTION

CHINA is the world's largest producer and consumer of coal, with its coal consumption reaching approximately 4.8 billion tons in 2024. As a pillar of the national economy, the coal industry has made significant contributions to economic and social development, while always prioritizing

safety production due to its nature as a traditional high-risk industry[1]. On May 1, 2024, the implementation of the "Coal Mine Safety Production Regulations" provided a more robust legal framework to foster safety in coal mining operations [2]. In this context, enterprises, as the primary agents of safety in coal mining, are obligated to fulfill their safety responsibilities at every stage of production. Enhancing safety management, technological innovation, and system development enables enterprises to significantly improve safety standards in coal mines, thereby fostering sustainable growth[3]. However, existing enterprises still face issues such as weak safety awareness, inadequate regulatory measures, and insufficient investment in technology, which not only limit their own development but also pose potential risks to society. In recent years, Chinese coal mining industry has experienced multiple safety accidents[4]. For instance, the coal and gas outburst at Pingdingshan Tian'an Coal Industry Co., Ltd. No. 12 Mine, Henan Province, on January 12, 2024, resulting in 16 fatalities and 5 injuries, directly revealed significant deficiencies in the company's safety monitoring and emergency response capabilities[5]. Therefore, it is particularly critical to establish a scientific and efficient coal mine safety supervision system.

Enterprise managers and coal mine workers each have own roles in safety production, but their different responsibilities and positions often lead to strategic choices that are divided[6]. The selective adherence to safety protocols by workers for personal interest and the cost-benefit analysis of increased safety investment by enterprise managers may result in conflicting management approaches that create safety oversight gaps and increase the risk of accidents [7], [8]. Based on incomplete statistics, there were 28 coal mine accidents nationwide from January to March 2024[9]. Reports indicate that most accidents were due to workers' lack of safety training and weak safety awareness, which leads to a failure of strictly following safety operating procedures. Meanwhile, enterprise managers, driven by cost considerations, may fail to fully invest in the upgrading and maintenance of safety facilities. The combination of these factors ultimately leads to tragedy[10]. To achieve continuous improvement and effective risk control in coal mine safety production, it is urgent to establish an effective safety supervision mechanism, clarify the rights and responsibilities of all parties involved, and promote cooperation between enterprise managers and workers in safety production[11].

In fact, evolutionary game theory integrates the concepts of game theory and dynamic evolution[12], [13]. It regards participants as individuals with bounded rationality who may make mistakes during continuous processes of learning

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and imitation, with their decision-making strategies subject to dynamic adjustment over time[14], [15], [16]. Many scholars have explored the evolutionary game research in the field of coal mine safety from different perspectives. To be specific, existing research by You et al. has discussed and analyzed the evolutionary patterns of group safety behaviors from the perspectives of management decision-making, environment, and management[17]. An evolutionary game model incorporating prospect theory has been constructed, which is characterized by having design firms, construction companies, and suppliers as key stakeholders[18]. Additionally, some studies have analyzed the interaction mechanism of safety behaviors among three parties and pointed out that factors such as participants' cognitive biases, risk preferences, safety investment costs, punishment intensity, and accident losses have an impact on safety behaviors. Research utilizing a three-party evolutionary game model indicates that government support and miners' emotions significantly influence the development of intelligent coal mines, recommending proactive government promotion and support[19]. Previous research has explored the coal mine safety evolution mechanism from different perspectives of the evolutionary game process, but there is relatively little research on the deep impact of the interaction between coal mine workers and enterprise managers on safety behavior. Hence, this study delves into the interaction between coal mine workers and enterprise managers, and investigates how this interaction influences coal mine safety behavior and accident prevention.

Current coal mine safety research mainly focuses on the causes and prevention of safety accidents. Previous studies have revealed the close relationship between the causes of coal mine accidents and safety management countermeasures through the analysis of accident cause theory and the systematic examination of accident reports, and has clearly identified that insufficient supervision is one of the key factors leading to coal mine accidents[20], [21]. Research on the prevention of coal mine safety accidents is relatively limited, while supervision in other fields is an important way of preventing safety accidents. Granting safety production rewards and honors, tax exemptions, and financial support to individuals or enterprises that perform outstandingly in coal mine safety production are common means of safety management supervision.

Yuan et al. surveyed the dynamic impact of government policies on the recycling strategies of construction contractors and recycling companies, and found that there is an optimal policy effect threshold for measures in the construction waste recycling system and government innovation subsidies[22]. Under the condition of strict safety quality supervision, government rewards can promote enterprises' compliance with standard practices, while excessively increasing the magnitude of rewards may weaken the self-motivation for fulfilling supervisory responsibilities[23]. Under bounded rationality conditions, this study analyzes the dynamic equilibrium scheme of Emergency Materials Allocation (EMA) through an evolutionary game model, discovering the impact of risk attitude, perceived probability, and preference on the allocation and transportation time of materials, thereby

providing a theoretical basis for improving the efficiency and rationality of decision-making in disaster situations[24]. Therefore, it is crucial to construct a rational reward mechanism, which can better meet the needs of various safety production entities and enhance the efficiency and effectiveness of safety management.

Based on the above analysis, this study initially constructs a dynamic model of unsafe behavior between coal mine workers and enterprise managers under the dual supervision of the government and enterprises, using evolutionary game theory. Subsequently, the stability of the strategies chosen by both parties under static and dynamic reward and punishment mechanisms is analyzed. Numerical simulation technology is then employed to simulate the impact of different reward and punishment mechanisms on the behavior of workers and managers, comparing the effects of static and dynamic reward and punishment mechanisms. Finally, targeted supervision strategies are proposed based on the simulation results.

II. MODEL CONSTRUCTION AND ANALYSIS

In this study, coal mine workers and enterprise managers are regarded as the main players in the game, and it is difficult for them to achieve complete rationality due to asymmetric information and other factors. Moreover, the behaviors of workers and enterprise managers in the game are not determined instantaneously but are continuously adjusted through ongoing interactions, eventually reaching an equilibrium state.

A. Model Assumptions and Variable Settings

Whether workers comply with safety behaviors may depend on their own emphasis on safety, or they may neglect safety due to the desire to save time and energy or be influenced by herd mentality. Therefore, the strategic choice space for workers is to follow safety behavior or to engage in unsafe behavior, denoted as P_1 (adhere to safety behavior, engage in unsafe behavior). The strategic choice of enterprise managers is influenced by factors such as safety education and training, as well as government supervision. On the one hand, they may strengthen safety supervision and increase safety investment; on the other hand, they may neglect safety investment due to cost considerations. Their strategic choice space is P_2 (increase safety investment, neglect safety investment).

In the game, the probability of workers choosing to adhere to safety behavior is x , and the probability of choosing to engage in unsafe behavior is $1 - x$. The probability of enterprise managers choosing to increase safety investment is y , and the probability of choosing to neglect safety investment is $1 - y$, where x and y both belong to the interval $[0, 1]$. If workers choose to adhere to safety behavior, they can obtain net benefits V_1 regardless of whether enterprise managers increase safety investment. However, when workers choose unsafe behavior to save time and energy or are influenced by herd mentality, they will obtain net benefits V_2 . It should be noted that engaging in unsafe behavior may lead to safety accidents with a probability β , and workers will bear the corresponding risk loss L_1 in such cases.

For enterprise managers, when choosing to increase safety investment, the enterprise can obtain net benefits W_1 , but it needs to pay safety investment costs C . At the same time, they will reward workers who adhere to safety behavior with R_y and penalize those who do not with F_1 . Additionally, strengthening safety management can effectively reduce the probability of safety accidents to α . When choosing to neglect safety investment, the enterprise can obtain net benefits W_2 . However, if workers do not adhere to safety behavior and cause safety accidents, the enterprise will face losses L_2 . Moreover, due to neglecting safety investment, the enterprise may face government penalties with a probability λ , with the penalty amount being F_2 .

B. Evolutionary Game Analysis under Static Reward Punishment Mechanism

Based on the above assumptions, the static reward and punishment mixed strategy evolutionary game matrix between coal mine workers and enterprise managers is shown in Table I.

Based on the calculation of evolutionary game theory and the payoff matrix, the expected utility for workers to choose safe behavior is $E_x = y(V_1 + R_y) + (1 - y)V_1$. The expected utility for choosing unsafe behavior is $E_{1-x} = y[V_2 - (\beta - \alpha)L_1 - F_1] + (1 - y)(V_2 - \beta L_1)$. Further, the average expected utility is $\bar{E}_x = xE_x + (1 - x)E_{1-x}$. The expected utilities for enterprise managers to increase safety investment and neglect safety investment are $E_y = x(W_1 - C - R_y) + (1 - x)[W_1 - C - (\beta - \alpha)L_2 + F_1]$, $E_{1-y} = x(W_2 - \lambda F_2) + (1 - x)(W_2 - \beta L_2 - \lambda F_2)$ respectively. Further, the average expected utility is $\bar{E}_y = yE_y + (1 - y)E_{1-y}$. The replicator dynamic equations for the strategy choices of workers and enterprise managers are obtained as follows:

$$\begin{cases} F_1(x) = dx/dt = x(E_x - \bar{E}_x) \\ \quad = x(1 - x)(V_1 - V_2 + \beta L_1 + y(R_y + F_1 - \alpha L_1)) \\ F_1(y) = dy/dt = y(E_y - \bar{E}_y) \\ \quad = y(1 - y)[x(-R_y - \alpha L_2 - F_1) + W_1 - W_2 - C + \alpha L_2 + F_1 + \lambda F_2] \end{cases} \quad (1)$$

Setting $F_1(x) = 0$, $F_1(y) = 0$, according to the theory of differential equations, the system has at least four equilibrium points $E_1(0, 0)$, $E_2(0, 1)$, $E_3(1, 0)$, $E_4(1, 1)$. Additionally, when $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $W_2 - W_1 + C + R_y < \lambda F_2 < W_2 - W_1 + C + R_y$, it is obtained that $0 < x^* < 1$, $0 < y^* < 1$, where $x^* = (W_1 - W_2 - C + \alpha L_2 + F_1 + \lambda F_2) / (R_y + \alpha L_2 + F_1)$, $y^* = (V_2 - V_1 - \beta L_2) / (R_y - \alpha L_1 + F_1)$. Moreover, (x^*, y^*) satisfy the equation.

Not all equilibrium points are evolutionary stable states (ESS). Only when the determinant of the system's Jacobian matrix is greater than 0 and the trace is less than 0, the equilibrium point is an ESS. The Jacobian matrix of the system is:

$$J = \begin{bmatrix} \frac{\partial F_1(x)}{\partial x} & \frac{\partial F_1(x)}{\partial y} \\ \frac{\partial F_1(y)}{\partial x} & \frac{\partial F_1(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \quad (2)$$

The determinant and trace expressions for each equilibrium point can be derived as shown in Table II.

From the determinants and traces of each equilibrium point, we can see that their signs are related to $V_1 + R_y$, $V_2 - (\beta - \alpha)L_1 - F_1$, λF_2 , $W_2 - W_1 + C + R_y$, $W_2 - W_1 + C - \alpha L_2 - F_1$. To facilitate a more comprehensive comparison, the supervision situations faced by workers can be categorized into six distinct scenarios based on the relative magnitudes of these expressions:

(1) Weak-Strong Supervision (Weak enterprise managers supervise—Strong government supervision), where the relationship is $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 > W_2 - W_1 + C + R_y$. That is, the rewards and punishments for workers by enterprise managers are relatively small, but the government's punishment for enterprise managers is relatively large.

(2) Weak-Weak Supervision (Weak enterprise managers supervise—Weak government supervision), where the relationship is $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 < W_2 - W_1 + C - \alpha L_2 - F_1$. That is, the rewards and punishments for workers by enterprise managers are relatively small, and the government's punishment for enterprise managers is also relatively small.

(3) Only enterprise managers supervision is relatively weak, and there is an upper and lower limit for the government's punishment of enterprise managers, where the relationship is: $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $W_2 - W_1 + C - \alpha L_2 - F_1 < \lambda F_2 < W_2 - W_1 + C + R_y$.

(4) Strong-Strong Supervision (Strong enterprise managers supervise—Strong government supervision), where the relationship is $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 > W_2 - W_1 + C + R_y$. That is, the rewards and punishments for workers by enterprise managers are relatively large, and the government's punishment for enterprise managers is also relatively large.

(5) Strong-Weak Supervision (Strong enterprise managers supervise—Weak government supervision), where the relationship is $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 < W_2 - W_1 + C - \alpha L_2 - F_1$. That is, the rewards and punishments for workers by enterprise managers are relatively large, but the government's punishment for enterprise managers is relatively small.

(6) Only enterprise managers supervision is relatively strong, and there is an upper and lower limit for the government's punishment of enterprise managers, where the relationship is: $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $W_2 - W_1 + C - \alpha L_2 - F_1 < \lambda F_2 < W_2 - W_1 + C + R_y$.

The stability of each equilibrium point is shown in Table III. When workers and enterprise managers face weak-strong supervision, and only management supervision is relatively weak, the system has a unique evolutionary stable point $(0, 1)$, corresponding to Scenarios 1 and 3. Meanwhile, when the penalties imposed by a third-party government on enterprise managers exceed a certain level, enterprises will inevitably choose to increase safety investment to ensure their profits. However, due to the cost constraints, the extent of the increase in safety investment by enterprise managers is relatively weak. This leads to the benefits of workers adhering to safe behaviors being less than those of engaging in unsafe behaviors. Consequently, workers will inevitably choose to engage in unsafe behaviors. Therefore, (implement unsafe behavior, increase safety investment) are the only stable strategies. When workers and enterprise managers are

TABLE I
STATIC EVOLUTIONARY GAME PAYOFF MATRIX BETWEEN WORKERS AND ENTERPRISE MANAGERS

Workers / Enterprise Manager	Increase Safety Investment (y)	Neglect Safety Investment($1 - y$)
Adhere to Safety Behavior (x)	$V_1 + R_y, W_1 - C - R_y$	$V_1, W_2 - \lambda F_2$
Implement Unsafe Behavior ($1 - x$)	$V_2 - (\beta - \alpha)L_1 - F_1, W_1 - C + F_1 + (\beta - \alpha)L_2$	$V_2 - \beta L_1, W_2 - \lambda F_2 - \beta L_2$

TABLE II
EXPRESSIONS FOR THE DETERMINANTS AND TRACES AT EACH EQUILIBRIUM POINT

Equilibrium Point	det J	tr J
$E_1(0, 0)$	$s_1(F_1 + \alpha L_2 + s_2)$	$s_1 + (F_1 + \alpha L_2 + s_2)$
$E_2(0, 1)$	$-(R_y + F_1 - \alpha L_1 + s_1)(F_1 + \alpha L_2 + s_2)$	$(R_y + F_1 - \alpha L_1 + s_1) - (F_1 + \alpha L_2 + s_2)$
$E_3(1, 0)$	$-s_1(s_2 - R_y)$	$-s_1 + (s_2 - R_y)$
$E_4(1, 1)$	$(s_1 - \alpha L_2 + R_y + F_1)(s_2 - R_y)$	$-(s_1 - \alpha L_2 + R_y + F_1) - (s_2 - R_y)$
$E_5(x^*, y^*)$	$a_1 d_1 - b_1 c_1$	0

Note: $s_1 = V_1 - V_2 + \beta L_1$, $s_2 = W_1 - W_2 - C + \lambda F_2$; $a_1 d_1 - b_1 c_1 = (1 - 2x)(1 - 2y)[V_1 - V_2 + \beta L_1 + y(R_y + F_1 - \alpha L_1)] \times [x(-R_y - F_1 - \alpha L_2) + W_1 - W_2 - C + \alpha L_2 + F_1 + \lambda F_2] + xy(1 - x)(1 - y)(R_y + F_1 - \alpha L_1)(R_y + F_1 + \alpha L_2)$.

TABLE III
EVOLUTIONARY STABILITY OF EQUILIBRIUM POINTS IN THE SYSTEM

Scenario 1: $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 > W_2 - W_1 + C + R_y$				Scenario 4: $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 > W_2 - W_1 + C + R_y$			
Equilibrium Point	det J	tr J	Stability	Equilibrium Point	det J	tr J	Stability
(0, 0)	< 0	Uncertain	Saddle Point	(0, 0)	< 0	Uncertain	Saddle Point
(0, 1)	> 0	< 0	ESS	(0, 1)	< 0	Uncertain	Saddle Point
(1, 0)	> 0	> 0	Unstable Point	(1, 0)	> 0	> 0	Unstable Point
(1, 1)	< 0	Uncertain	Saddle Point	(1, 1)	> 0	< 0	ESS
Scenario 2: $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 < W_2 - W_1 + C - \alpha L_2 - F_1$				Scenario 5: $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $\lambda F_2 < W_2 - W_1 + C - \alpha L_2 - F_1$			
(0, 0)	> 0	< 0	ESS	(0, 0)	> 0	< 0	ESS
(0, 1)	< 0	Uncertain	Saddle Point	(0, 1)	> 0	> 0	Unstable Point
(1, 0)	< 0	Uncertain	Saddle Point	(1, 0)	< 0	Uncertain	Saddle Point
(1, 1)	> 0	> 0	Unstable Point	(1, 1)	< 0	Uncertain	Saddle Point
Scenario 3: $V_1 + R_y < V_2 - (\beta - \alpha)L_1 - F_1$, $W_2 - W_1 + C - \alpha L_2 - F_1 < \lambda F_2 < W_2 - W_1 + C + R_y$				Scenario 6: $V_1 + R_y > V_2 - (\beta - \alpha)L_1 - F_1$, $W_2 - W_1 + C - \alpha L_2 - F_1 < \lambda F_2 < W_2 - W_1 + C + R_y$			
(0, 0)	< 0	Uncertain	Saddle Point	(0, 0)	< 0	Uncertain	Saddle Point
(0, 1)	> 0	< 0	ESS	(0, 1)	< 0	Uncertain	Saddle Point
(1, 0)	< 0	Uncertain	Saddle Point	(1, 0)	< 0	Uncertain	Saddle Point
(1, 1)	> 0	> 0	Unstable Point	(1, 1)	< 0	Uncertain	Saddle Point
				(x^*, y^*)	> 0	0	Center Point

confronted with weak-weak and strong-weak supervision, the relatively low intensity of government oversight results in lower penalties for enterprises that do not increase safety investment. This makes the benefits of not increasing safety investment higher than those of increasing it. In such circumstances, enterprise managers will inevitably choose to neglect safety investment. Consequently, workers, due to the lack of safety investment by enterprises, will choose behaviors with higher benefits, i.e., non-compliant behavior, corresponding to Scenarios 2 and 5, where (implement unsafe behavior, neglect safety investment) is the only stable strategy. When workers and enterprise managers face strong-strong supervision, the high penalties imposed by

the government on enterprise managers lead to a definite choice of increasing safety investment. Meanwhile, the strong system of rewards and punishments implemented by enterprise managers encourages workers to adhere to safety behaviors, corresponding to Scenario 4. When workers and enterprise managers are confronted with a situation where only the managers are under strong government supervision, and the degree of government supervision over the managers is dynamically changing, but at this time the managers have stronger supervision over the workers, it leads to changes in the enterprise's regulatory benefits. Consequently, the managers' choices are also constantly changing, and the workers' strategic choices will change accordingly. In this

case, the system has no evolutionary stable point, and the strategies of both parties are interdependent and in a state of dynamic change, corresponding to Scenario 6.

In summary, under the static reward punishment mechanism, when government supervision is relatively weak, enterprise management will choose to neglect safety investment, and workers will not choose to adhere to safety behaviors (corresponding to Scenarios 2 and 5). When government supervision is relatively strong, enterprise managers will choose to increase safety investment, and workers will choose to adhere to safety behaviors (corresponding to Scenario 4). When government supervision is within a dynamic range, neither party has a stable strategy choice. In real life, due to the influence of various factors such as fiscal resources, personnel, and public opinion on the intensity of government supervision, Scenario 6 is more common. Based on the above, it is difficult to stabilize the behavior strategies of both parties under static conditions. The following section will construct a dynamic reward punishment mechanism based on the conditions of Scenario 6.

C. Evolutionary Game Analysis under Dynamic Reward Punishment Mechanism

The dynamic reward punishment mechanism refers to the strategy where enterprise managers, under the “increase safety investment” policy, dynamically adjust the upper limits of rewards and punishments based on the strategic choices of workers. Specifically, enterprise managers reward workers who adhere to safety behavior with $(1-x)R_y$, and punish workers who engage unsafe behavior with $(1-x)F_1$. Under the dynamic reward punishment mechanism, the evolutionary game payoff matrix between workers and enterprise managers is shown in Table IV.

Under the condition that all other parameters remain unchanged, the replicator dynamic equations for the strategy choices of workers and enterprise managers can be obtained as follows:

$$\begin{cases} F_2(x) = x(1-x)[(1-x)(R_y + F_1)y - \alpha L_1 y \\ \quad + V_1 - V_2 + \beta L_1] \\ F_2(y) = y(1-y)[(1-x)(F_1 - xF_1 - xR_y \\ \quad + \alpha L_2) + W_1 - W_2 - C + \lambda F_2] \end{cases} \quad (3)$$

According to the stability theory of differential equations, the probability of workers choosing to adhere to safety behavior and enterprise managers increasing safety investment needs to satisfy $F_2(x) = 0$, $F_2(y) = 0$. From this, five equilibrium points $E'_1(0,0)$, $E'_2(0,1)$, $E'_3(1,0)$, $E'_4(1,1)$, $E'_5(x^*, y^*)$ are obtained. Based on Friedman's Lyapunov first method, the stability of the equilibrium points is judged. The Jacobian matrix J of the evolutionary system is:

$$J = \begin{bmatrix} \frac{\partial F_2(x)}{\partial x} & \frac{\partial F_2(x)}{\partial y} \\ \frac{\partial F_2(y)}{\partial x} & \frac{\partial F_2(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \quad (4)$$

Substituting the five equilibrium points of the worker and enterprise manager evolutionary system $E'_1(0,0)$, $E'_2(0,1)$, $E'_3(1,0)$, $E'_4(1,1)$, $E'_5(x^*, y^*)$ into the Jacobian matrix J , the determinant and trace of each equilibrium point are obtained, as shown in Table V.

The stability of each equilibrium point, as determined by the determinants and traces, is shown in Table VI. It can be seen that, unlike the static reward punishment mechanism, which lacks evolutionary stable points, the dynamic reward punishment mechanism results in a unique evolutionary stable point $E'_5(x^*, y^*)$. At this point, after long-term dynamic adjustments, the probabilities of workers adhering to safety behavior and enterprise managers increasing safety investment approach a stable state. Additionally, the government can adjust the punishment intensity for managers, thereby increasing the probability of enterprise managers increasing safety investment and workers adhering to safety behavior. Therefore, the dynamic reward punishment mechanism is more effective than the static reward punishment mechanism in enhancing mine safety supervision.

III. NUMERICAL SIMULATION ANALYSIS

To verify the theoretical results, a numerical simulation of the system is conducted. The specific parameters are as follows: $\alpha = 0.2$, $\beta = 0.1$, $R_y = 2$, $L_1 = 4$, $V_1 = 2$, $V_2 = 5$, $W_1 = 4$, $W_2 = 5$, $C = 1.5$, $F_1 = 1$, $\lambda = 0.2$, $F_2 = 33$, $L_2 = 6$.

A. Evolutionary Path Analysis under Static Reward Punishment Mechanism

Under the static reward punishment mechanism, the system eventually converges to $(0,1)$ as shown in Fig. 1, where the parameters satisfy condition 1. When F_2 is set to 25, the parameters meet condition 3, and the simulation results are presented in Fig. 2.

As illustrated in Fig. 1 and 2, both systems ultimately converge to the point $(0,1)$. In Scenario 1, the convergence speeds of x and y are relatively fast, whereas in Scenario 3, although the convergence speed of x remains relatively fast, the convergence speed of y is slower. The primary reason for this difference is as follows: In Scenario 1, implementing unsafe behavior is the dominant strategy for workers, while increasing safety investment is the dominated strategy for enterprise managers. Consequently, the equilibrium point $(0,1)$ is easily achieved. In Scenario 3, the initial higher likelihood of workers adhering to safety behavior leads to a potential reduction in the intensity of safety investment by enterprise managers. As the intensity of safety investment decreases, workers, noticing that the payoff for adhering to safety behavior is lower than that for engaging in unsafe behavior, gradually shift towards unsafe behavior. However, as the behavior of the workers changes, enterprise managers also adjust their strategies accordingly, resulting in a relatively slower convergence speed for the managers' behavior.

By further modifying the parameters to $V_1 = 3$, $W_1 = 2$, $C = 0.5$, $F_1 = 2$, $F_2 = 20$, the parameters satisfy condition 6. Under these conditions, the system's evolutionary path is illustrated in Fig. 3. The system's evolutionary trajectory exhibits periodic closed-loop motion, making it difficult for the strategies of both parties to reach a stable state. At this point, implementing a dynamic reward punishment mechanism results in a spiral convergence trend in the system's evolutionary path. From

TABLE IV
EVOLUTIONARY GAME PAYOFF MATRIX BETWEEN WORKERS AND MANAGEMENT PERSONNEL UNDER DYNAMIC CONDITIONS

Workers / Enterprise Manager	Increase Safety Investment (y)	Neglect Safety Investment ($1 - y$)
Adhere to Safety Behavior (x)	$V_1 + (1 - x)R_y, W_1 - C - (1 - x)R_y$	$V_1, W_2 - \lambda F_2$
Implement Unsafe Behavior ($1 - x$)	$V_2 - (\beta - \alpha)L_1 - (1 - x)F_1, W_1 - C + (1 - x)F_1 - (\beta - \alpha)L_2$	$V_2 - \beta L_1, W_2 - \lambda F_2 - \beta L_2$

TABLE V
EXPRESSIONS FOR THE DETERMINANTS AND TRACES AT EACH EQUILIBRIUM POINT AND STABILITY ANALYSIS

Equilibrium Point	det J	tr J
$E_1'(0, 0)$	$s_1(F_1 + \alpha L_2 + s_2)$	$s_1 + (F_1 + \alpha L_2 + s_2)$
$E_2'(0, 1)$	$-(R_y + F_1 - \alpha L_1 + s_1)(F_1 + \alpha L_2 + s_2)$	$(R_y + F_1 - \alpha L_1 + s_1) - (F_1 + \alpha L_2 + s_2)$
$E_3'(1, 0)$	$-s_1 s_2$	$-s_1 + s_2$
$E_4'(1, 1)$	$(s_1 - \alpha L_2)s_2$	$-(s_1 - \alpha L_2) - s_2$
$E_5'(x^*, y^*)$	$a_2 d_2 - b_2 c_2$	0

Note: $s_1 = V_1 - V_2 + \beta L_1$, $s_2 = W_1 - W_2 - C + \lambda F_2$; $a_2 d_2 - b_2 c_2 = [(1 - 4x + 3x^2)(R_y + F_1)y - (1 - x^2)\alpha L_1 y + (1 - x)(V_1 - V_2 + \beta L_1)] \times [(1 - y)(1 - x)(F_1 - xF_1 - xR_y + \alpha L_2) + (1 - y)(W_1 - W_2 - C + \lambda F_2) + y(1 - y)(1 - 2y)] + x(1 - x)y(1 - y)[(1 - x)(R_y + F_1) - \alpha L_1] \times [F_1 - xF_1 - xR_y + \alpha L_2]$

TABLE VI
STABILITY ANALYSIS OF EACH EQUILIBRIUM POINT

Equilibrium Point	det J	tr J	Stability
(0, 0)	< 0	Uncertain	Saddle Point
(0, 1)	< 0	Uncertain	Saddle Point
(1, 0)	> 0	> 0	Unstable Point
(1, 1)	< 0	Uncertain	Saddle Point
(x^*, y^*)	> 0	0	ESS

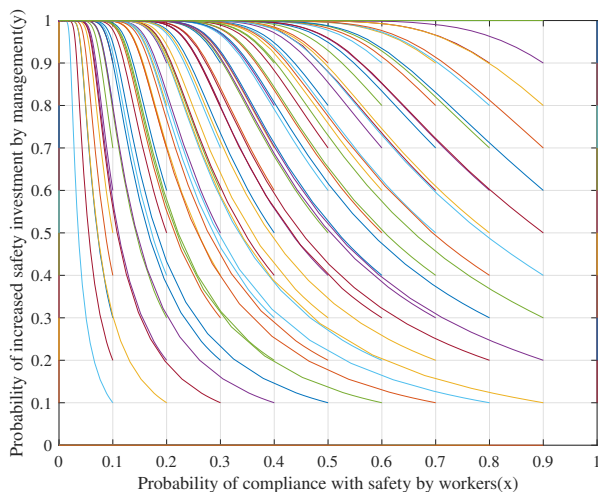


Fig. 1: Evolution Path of Scenario 1.

a policy-making perspective, as shown in Fig. 4, enterprise managers can enhance the probability of workers adhering to safety behavior by adjusting the levels of rewards and punishments. Additionally, government departments can encourage management personnel to increase safety investment by imposing specific penalties. Therefore, compared with the static reward punishment mechanism, the dynamic situation is more effective in reducing or controlling the occurrence of coal mine accidents.

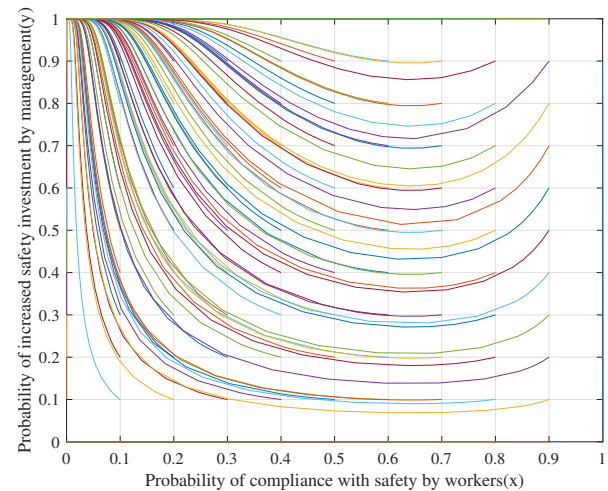


Fig. 2: Evolution Path of Scenario 3.

B. The Impact of Management and Government Supervision on System Evolutionary Paths

To explore the role of management supervision and government oversight in the strategic behavior of both parties, within the framework of the dynamic reward punishment mechanism, we further investigate the impact of F_2 and R_y on the system's evolutionary paths under different values.

From Fig. 5, it can be observed that when the value of F_2 increases from 20 to 21, the evolutionary stable point of the system shifts from (0.18, 0.47) to (0.24, 0.57). The probability x that workers choose to adopt safe behaviors increases, as does the probability y that enterprise managers choose to increase safety investment. The primary reason is that the increased government penalties reduce the payoffs for managers who neglect safety investment, making it more advantageous for them to increase safety investment. Meanwhile, the heightened safety investment by enterprise managers encourages workers to comply with safety behaviors to ensure their own benefits.

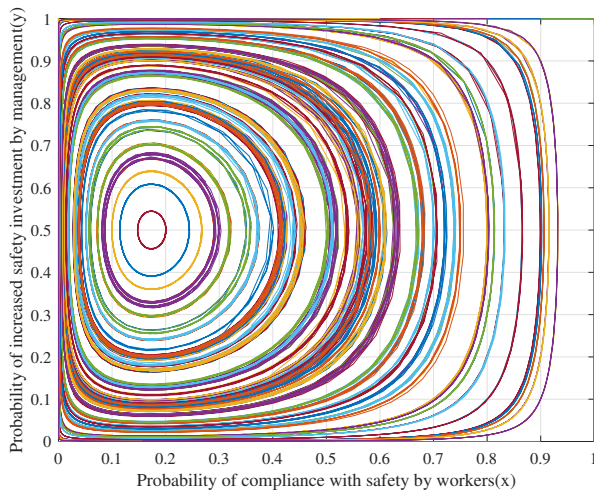


Fig. 3: Evolution Path of Scenario 6.

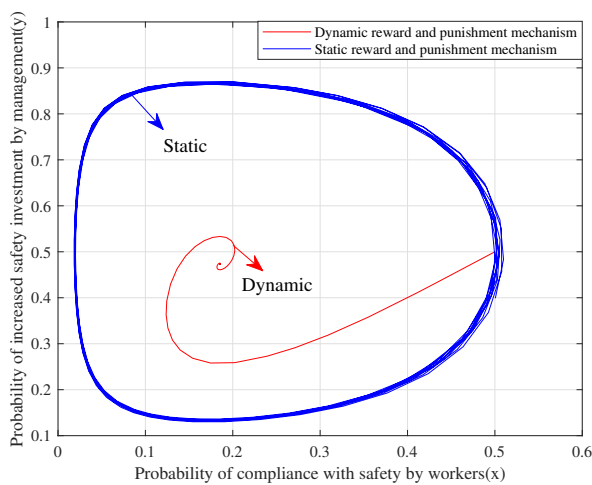


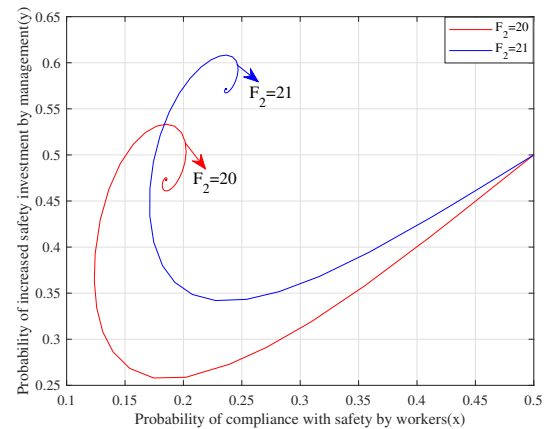
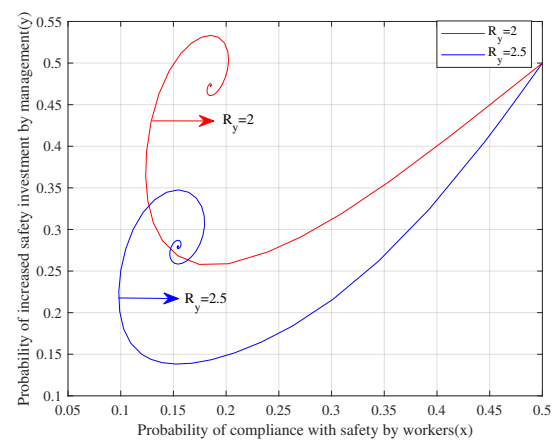
Fig. 4: Evolutionary paths of behaviors under different reward punishment mechanisms.

From Fig. 6, it can be seen that the evolutionary stable point of the system shifts from $(0.18, 0.47)$ to $(0.16, 0.28)$, when the value of R_y increases from 2 to 2.5. The probability x of workers choosing to adhere to safety behavior and the probability y of enterprise managers choosing to increase safety investment both decrease. The main reason is that the increase in the reward intensity for enterprise managers leads to a decrease in the payoffs when they increase safety investment, resulting in an increase in the financial pressure of enterprise safety supervision. Therefore, the probability of enterprise managers strengthening safety supervision decreases, and workers may engage in opportunistic behavior, that is, choosing behaviors with higher payoffs to pursue higher benefits, leading to a decrease in the probability of adhering to safety behavior.

IV. CONCLUSIONS AND IMPLICATIONS

A. Research Conclusions

This study employs an evolutionary game model and simulation analysis to investigate the behavioral strategies of coal mine workers and enterprise managers, comparing


 Fig. 5: The impact of F_2 on the evolution of strategic behaviors.

 Fig. 6: The impact of R_y on the evolution of strategic behaviors.

the evolutionary paths under static and dynamic reward punishment mechanisms. The findings were revealed through this research. First, under the static reward punishment mechanism, when government supervision is strong, enterprise managers can influence workers' behavior strategies by adjusting the intensity of rewards and punishments. Conversely, when government supervision is weak, enterprise managers are less likely to increase safety investment, leading to workers not adhering to safety behavior. Second, within a certain range of government supervision, the probabilities of workers adhering to safety behavior and enterprise managers increasing safety investment exhibit oscillating changes under the static reward punishment mechanism. However, under the dynamic mechanism, these strategies tend to converge. Therefore, the dynamic reward punishment mechanism is more effective in controlling the probability of coal mine safety accidents. Finally, under the dynamic reward punishment mechanism, enterprises should primarily use punishment as the main incentive, supplemented by rewards. Moreover, the punishments imposed by both enterprises and the government should be kept within an appropriate range to avoid unintended consequences.

B. Management Implications

Based on the research findings, the following management implications can be summarized:

(1) Strengthen supervision. Understanding the impact of supervision intensity on behavior, it is evident that under the static reward punishment mechanism, when workers face strong oversight from enterprises and enterprises are under significant scrutiny, workers are compelled to adhere to safety practices, and enterprise managers increase safety investments. This underscores the need for enhanced enterprise regulation and governmental oversight.

(2) Implement flexible rewards and punishments. Observations indicate that when governmental supervision falls within a specific range, a dynamic reward punishment system is more effective for managing worker behavior. Timely rewards for adherence to safety protocols are necessary, yet they must be kept moderate to avoid financial strain. Conversely, penalties for unsafe actions are essential, with the severity of these penalties subject to government oversight.

(3) Emphasize light rewards. It is important to recognize that overly generous rewards may not achieve the intended goals and can lead to enterprises overlooking safety investments due to financial constraints, potentially causing workers to act opportunistically. However, increasing the government's penalty for enterprises positively influences workers' adherence to safety practices and encourages enterprise managers to boost safety investments.

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