Improving of the Clarke and Wright's Heuristic for Solving Multiobjective Vehicles Routing Problem

Issouf Nikiéma, Joseph Poda, and Kounhinir Somé

Abstract—**In this paper, we propose a new multiobjective model for the Capacitated Vehicle Routing Problem with a single depot. On one hand, the model aims to optimize multiple conflicting objectives, such as minimizing the total travel distance and reducing the number of vehicles used. On the other hand, we develop a heuristic approach to effectively solve the problem. The Vehicle Routing Problem is one of the most widely studied combinatorial optimization problems, particularly in logistics and supply chain management. It encompasses numerous variants and is classified as NP-hard, making exact methods impractical for large-scale instances. Our proposed heuristic combines two well-known principles: the Hungarian method, typically used for solving assignment problems, and the Clarke-Wright savings algorithm, commonly applied in VRP contexts. This hybridization operates in two main stages: First, the Hungarian method is employed to identify the shortest paths within the cost matrix, thereby facilitating efficient pairing of nodes. Second, the Clarke-Wright savings principle is applied to construct low-cost vehicle routes. This combined approach enables us to achieve two key objectives: minimizing the size of the vehicle fleet and reducing the total distance traveled. A comparative study based on several benchmark instances demonstrates that the proposed method produces high-quality solutions, validating its effectiveness and potential for practical application.**

Index Terms—**Combinatorial-Optimization, Transportation-problems, Vehicle-routing, Efficient-solutions, Multiobjective-optimization.**

I. Introduction

COMBINATORIAL optimization problems form a specific category of optimization problems. Their particularity stems from the fact that the decision space is composed of vectors with integer components. In general, these problems admit a large number of feasible solutions, and there is no universal method to solve them with efficiently. They are classified as NP-hard problems [7]. The Vehicle Routing Problem (VRP) has attracted significant attention from researchers in recent years due to its wide range of applications in various fields. It has several variants including the Capacitated Vehicle Routing Problem (CVRP), the Vehicle Routing Problem with Time-Windows (VRPTW), and the Multi-Compartment Vehicle Routing Problem (MCVRP).

Among combinatorial optimization problems, VRP is one of the most extensively studied in the literature. It addresses optimal logistics and distribution management issues that companies must resolve, particularly regarding the transportation of raw materials and finished products. This class of problems arises in a wide variety of real-world applications, such as school bus routing, urban public transportation planning, mail collection and distribution, supply chain logistics, waste management, and many other domains. In the scientific literature, exact methods often struggle to provide efficient solutions for large-scale instances due to the inherent computational complexity of these problems, which are typically classified as NP-hard. As a result, most of the existing approaches rely on heuristic and metaheuristic algorithms, which are capable of producing high-quality approximate solutions within reasonable computation times. These methods offer a practical trade-off between solution quality and computational efficiency, making them particularly well-suited for addressing complex and large-scale problem instances commonly encountered in real-world applications. Building upon this foundation, this paper draws inspiration from a wide range of recent studies on the resolution of the VRP and its many variants, aiming to propose novel approaches or improvements that enhance both effectiveness and scalability in practical settings. Several recent studies have explored effective heuristic and metaheuristic approaches for solving various complex variants of the VRP. For instance, Matijević et al. [4] proposed a general Variable Neighbourhood Search (VNS) approach for asymmetric VRP with time windows and capacity constraints. Rosa et al. [17] addressed the VRP with asymmetric costs and heterogeneous fleets, highlighting the importance of fleet diversity in real-world applications.

In another study, Samira et al. [21] introduced a novel method for tackling large-scale instances of the asymmetric distance-constrained VRP, while Ha-Bang et al. [8] proposed a hybrid metaheuristic tailored to the same problem variant, achieving promising results in of solution quality and computational applied to logistics optimization.

Yane Hou et al. [9] developed a hybrid local search metaheuristic to solve the specifically designed for the school bus routing problem, demonstrating its effectiveness through extensive experimentation. Tan et al. [20] adapted the Ant System algorithm to solve capacitated VRPs, showing how bio-inspired heuristics can be effectively applied to logistics optimization.

Similarly, Yan-e Hou et al. [2] proposed a hybrid Max-Min Ant System algorithm (MMAS) for solving the Electric Vehicle Routing Problem (EVRP), addressing specific challenges related to battery range and charging infrastructure. In another work, Yan-e Hou et al. [1] investigated the multi-compartment VRP, proposing innovative strategies to handle multiple product types within a unified routing framework. Furthermore, Chunxiao Wang et al. [3] presented a hybrid genetic algorithm for solving

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the multi-compartment VRP with time windows, combining global exploration with local refinement technique to achieve high-quality solutions. These contributions highlight the growing interest in hybridization strategies that integrate classical heuristics with advanced search mechanisms to tackle increasingly complex VRP variants. Paolo et al. [22] introduced mathematical models, relaxations, and exact techniques aimed at addressing this challenging combinatorial optimization problem. Shubhechyya et al. [5] presented a unifying framework for the CVRP under conditions of risk and ambiguity, highlighting the need for robust approaches in incertain environments. Takwa et al. [19] developed a hybrid metaheuristic tailored to the distance-constrained CVRP, combining efficiency with solution quality. In related work, Okitonymbe et al. (2015) [14] extended the classical Clarke and Wright savings heuristic into a multiobjective setting, using the concept of referential dominance to handle multiple conflicting objectives simultaneously. Originally introduced by Clarke and Wright [25], this heuristic was designed to solve the single-objective version of the VRP and has since become a cornerstone in routing algorithms.
Another innovative approach based on the Cobweb algorithm was proposed by Okitonymbe et al. [15], offering an alternative clustering-based strategy for constructing efficient vehicle routes. Additionally, Mohamed Haouari et al. [24] reviewed state-of-the-art methods for solving VRPs, with time constraints providing valuable insights into current best practices and limitations. Despite the significant progress made in solving various VRP variants, the number of approaches specifically addressing multiobjective formulations remains limited in the literature. This highlights the relevance and timeliness of developing new methodologies that can effectively tackle real-world routing problems involving multiple, often competing, optimization criteria.

In this paper, we propose a new multiobjective model for the CVRP, with the dual objectives of minimizing both the total travel distance and the number of vehicles used. The problem under consideration involves the distribution of goods from a single depot to a set of geographically dispersed customers, a typical scenario in logistics and supply chain management. A key feature of our model is the dynamic integration of new customers into the routing process, which increases the complexity of path construction as the number of potential routes grows significantly. To address this variant efficiently, we propose a novel hybrid heuristic approach that combines two well-known methods: the Hungarian method, an exact algorithm widely used for solving assignment problems, and the Clarke-Wright savings method, a classical heuristic for constructing cost-effective vehicle routes. This hybridization leverages the strengths of both techniques: the Hungarian method is first applied to identify optimal pairings of customers based on cost efficiency, while the Clarke-Wright method is then used to merge these pairings into feasible and economical routes. The proposed approach aims to achieve high-quality solutions in terms of both fleet size minimization and total travel cost reduction. The Hungarian method, originally developed by Harold W. Kuhn [28], is employed in this study to identify the shortest pairwise distances that are prioritized during route construction. In combination with this, the Clarke-Wright savings algorithm is used to generate optimal vehicle tours by merging routes based on cost-saving principles. After presenting the theoretical foundation and step-by-step description of the proposed hybrid method, we illustrate its application through a didactic example. This is followed by its implementation on a set of benchmark test problems to evaluate and validate its performance. To enhance readability and facilitate understanding, the rest of this paper is structured as follows: Section II introduces the preliminaries and necessary background concepts. Section III presents the main contributions and results of this work. Finally, Section IV concludes the paper with a summary.

II. Preliminaries

A. Vehicle routing problem

The classical CVRP addressed in this study involves determining a set of routes with minimal total travel distance for delivering goods from a central depot to a group of geographically dispersed customers. In the standard formulation of the problem, the vehicle fleet is assumed to be homogeneous and based at a single depot, which holds sufficient inventory to satisfy all customer demands. Each vehicle has a maximum capacity of Qunits, and each customer i has a known demand q_i that must be fulfilled exactly once. The distance c_{ij} between each pair of customers (i, j) is considered to be symmetric and deterministic [16]. When all input data such as customer demands, the number of available vehicle, and travel distances are known and constant, the problem is classified as a deterministic the vehicle routing problem.

The goal is to determine the minimum number of vehicles required to serve all customers while simultaneously minimizing the total distance traveled by the fleet. In general terms, the CVRP can be described as the problem of routing a set of capacitated vehicles, stationed at a central depot, to supply a set of customers, each with a specific demand. Each customer must be visited exactly once, and each vehicle must complete its route without exceeding its capacity. A variety of models for CVRP have been proposed in the literature, with the most widely adopted being an integer linear programming (ILP) formulation that employs binary variables to represent route assignments [6], [10], [15], [23]. Before presenting the mathematical model, we first define the key parameters and notations used throughout this work.

- $U = \{1, 2, \ldots, n\}$ denotes the set of customers to be visited;
- K is the number of objectives of the problem;
- x_{ijk} is the binary variable defined that $x_{ijk} = 1$, if vehicle k which customer i for customer j and $x_{ijk} = 0$ if not.
- "d_i", is the request expressed by the customer i another customer "i".
- Q is the capacity of each vehicle;
- c_{ij} is the distance between customer i and customer j.
- 0 is the depot, the starting point of any tour. One of the most common formulations of this problem in the literature is the following:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m} c_{ij}.^{t}x_{ijk} \qquad t=1,2,...,K.$$

$$sc: \begin{cases} \sum_{j=0}^{n}\sum_{k=1}^{m} x_{ijk}=1, & i=1,\dots,n. \quad (a)\\ \sum_{l=0}^{n} x_{ijk}=\sum_{j=0}^{n} x_{jlk}, & l=1,\dots,n,\\ & k=1,\dots,m. \quad (b)\\ \sum_{j=1}^{n} x_{0jk}=1, & k=1,\dots,m. \quad (c)\\ \sum_{i=0}^{n} x_{i0k}=1, & k=1,\dots,m. \quad (d)\\ \sum_{i=1}^{n}\sum_{j=0}^{n} d_i.x_{ijk}\le Q, & k=1,\dots,m. \quad (e)\\ \sum_{i\in U}\sum_{j\in U} x_{ijk}\ge \sum_{j=1}^{n} x_{ijk}, & U\subset\{1,2,\dots,n\},\\ & l\in U; k=1,\dots,m. \quad (f)\\ x_{ijk}\in\{0,1\},\ 0\le i,j\le n, & i\ne j, 0\le k\le m. \end{cases}$$

where:

i) Constraint (a) ensures that each customer i is visited exactly once across all vehicle routes.
ii) Constraint (b) guarantees that if a vehicle k arrives at a customer location, it must also depart from that location, preserving route continuity.
iii) Constraints (c) and (d) enforce that each vehicle which departs from the central depot must eventually return to it, ensuring closed routes.
iv) Constraint (e) imposes the capacity restriction on each vehicle k, requiring that the total demand of all customers assigned to a route does not exceed the vehicle's maximum capacity Q.
v) Constraint (f) eliminates sub-tours, thereby ensuring that the solution forms a single connected tour for each vehicle and avoids disconnected cycles that do not include the depot.

B. Clarke-Wright heuristics

1) Principle: The Clarke-Wright heuristic is one of the earliest and most influential heuristics introduced by G. Clarke et al. [11], [25] for solving vehicle routing problems. The core idea of this method is to construct cost-effective routes by merging elementary tours, aiming to minimize the total travel distance — hence its designation as an "economic heuristic".Consider two customers, i and j, both requiring service from the depot located at node 0. If each customer is served individually using a separate vehicle, the resulting routes are $0-i-0$ and $0-j-0$. Under the assumption of symmetric distances, the total distance traveled in this scenario is given by [13]:

$$D_1 = 2.d(0,i) + 2.d(0,j) \tag{1}$$

However, if both customers i and j are served sequentially by a single vehicle, the resulting route becomes $0-i-j-0$. Under the same assumption of symmetric distances, the total distance traveled in this case is given by [13]:

$$D_2 = d(0,i) + d(i,j) + d(j,0). \tag{2}$$

The distance saved by including both customers i and j in the same route is expressed by the following formula [13], [14], [16]:

$$\begin{aligned}\delta_{ij} &= 2.d(0,i)+2.d(0,j)-d(0,i)-d(0,j)-d(i,j)\\ &= d(0,i)+d(0,j)-d(i,j).\end{aligned} \tag{3}$$

This quantity, known as the savings value, represents the reduction in total travel distance achieved by merging the two individual routes into a single one. The condition $\delta_{ij} > 0$ holds due to the triangle inequality, which ensures that the direction path between any two points is always shorter than or equal to any detour [14], [16]. Graphically, this merging process can be visualized as illustrated in Figure 1. First, Clarke and Wright construct a distance matrix to represent the travel distances between all pairs of customer locations and the depot. For two customers i and j with respective geographic coordinates (x_i, y_i) and (x_j, y_j), the Euclidean distance between them is computed using the following formula [11], [18]:

$$d(i,j) = \sqrt{(x_i-x_j)^2 + (y_i-y_j)^2}. \tag{4}$$

Based on this distance matrix, Clarke and Wright proposed two strategies for constructing distribution routes: the parallel version and the sequential version. In the parallel approach, multiple routes are built simultaneously by merging several customer pairs at each iteration. In contrast, the sequential approach constructs one route at a time until all customers are included in a vehicle route.

2) Algorithm: According to Ulungu et al., the sequential version of the Clarke-Wright algorithm typically produces a solution relatively quickly, but it often lacks overall efficiency due to its greedy nature and limited exploration of route combinations. In contrast, the parallel version requires more computational effort and multiple iterations, but it generally yields higher-quality solutions with greater total distance savings. Therefore, it can be concluded that the parallel version prioritizes efficiency, while the sequential version emphasizes effectiveness in terms of speed. The primary objective in designing vehicle routes using this method is to maximize the reduction in total travel distance through optimal route merging. The Clarke-Wright (C-W) algorithm can be summarized as follows [12]:

Algorithm 1 Algorithm C-W

1. Compute the savings matrix $\delta_{ij} = d(0,i)+d(0,j)-d(i,j)$ for all customer pairs.
2. Sort the savings values in descending order.
3. Starting from the top of the list, attempt to merge routes associated with the highest savings, provided that:
   - The customers involved are not already part of another route,
   - The total demand of the merged route does not exceed the vehicle capacity Q,
   - No sub-tours are formed.
4. Repeat step 3 until no more feasible merges can be performed, or all customers are routed.

C. The Hungarian method

1) The Hungarian method principle: The Hungarian method is one of the most effective techniques for solving

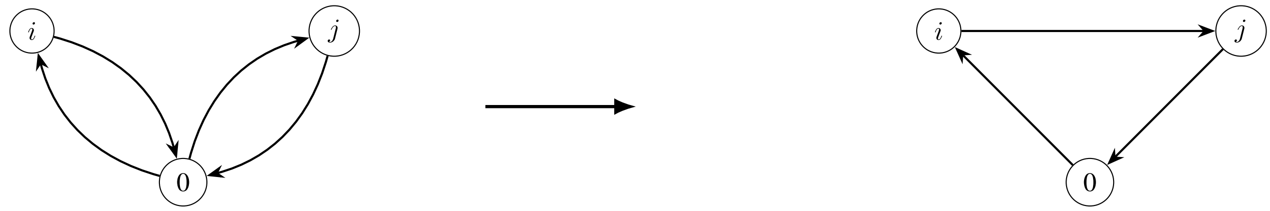

Fig. 1. C-W heuristic principle

assignment problems. It was first developed in 1955 by the American mathematician Harold. W. Kuhn [28], who drew inspiration from the earlier works of Dénes Köing and Jenö Egerváry. This method is specifically designed for cases where the number of tasks to be assigned is equal to the number of available resources. Due to its effectiveness, the method has been widely extended by researchers. Ford and Fulkerson [26] just like James Munkres [27] adapting it to solve a transportation problem.

2) The Hungarian method algorithm: The Hungarian algorithm can be summarized in the following five steps:

Algorithm 2 Hungarian method algorithm

1. **Subtract row minimums**
   For each row, subtract the smallest element in that row from all elements in the same row
2. **Subtract column minimums**
   For each column, subtract the smallest element in that column from all elements in the same column.
3. **Cover all zeros with a minimum number of Lines**
   Try to cover all zeros in the matrix using the minimum number of horizontal and vertical lines.
   If the number of lines equals n, an optimal assignment is possible. If not, proceed to Step 4.
4. **Adjust the matrix**
   Find the smallest uncovered element, then: subtract it from all uncovered elements add it to all elements covered twice leave elements covered once unchanged, Return to Step 3.
5. **Make the optimal assignment**
   Select a zero per row and per column.

Steps 1 and 2 identify the lowest costs and create zeros in the matrix. If the assignment is acceptable at the end of this stage, it will be the minimum-cost assignment. If no admissible assignment is found, zeros will be added in subsequent stages (Echelon 2, 3, etc.) until an admissible assignment is obtained. We use and adapt this principle in our approach to facilitate selecting customers to integrate into an elementary tour.

Remark 1: The Hungarian method is one of the most effective methods for solving assignment problems, and it has been adapted to solve transportation problems. However, it is not suitable for solving vehicle tour problems because tours must be constructed based on vehicle capacities before vehicles can be assigned to them. Furthermore, since a vehicle is assigned to a route that includes a customer, the method does not respect the principle of classical assignment, which establishes a bijection between two sets of the same cardinality.

III. Main results

A. Multiobjective CVRP model

The previous formulation of the multiobjective vehicle routing problem did not account for minimizing the number of vehicles. The constraint (b) states that a vehicle k arriving at a customer's location must leave again. However, in this formulation, the depot visited at the end of the tour is treated as a customer. In reality, a vehicle that arrives at the depot does not leave again because it is supposed to complete only one tour. This discrepancy led us to propose an alternative formulation in the form of a linear program to address the issue. This model includes two objective functions. The first objective function is defined as follows:

$$Z_1 = \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} c_{ij}.x_{ijk} \qquad k = 1,\ldots,m \tag{5}$$

This function translates the total distance traveled by the deployed fleet. The second equation defined by

$$Z_2 = \sum_{j=1}^{n}\sum_{k=1}^{m} x_{0jk} \qquad k \in 1,\ldots,m \tag{6}$$

This function counts the number of vehicles deployed. This program is as follows:

$$\min(Z_1, Z_2)$$
$$sc : \begin{cases} \sum_{i=0}^{n}\sum_{k=1}^{m} x_{ijk} = 1, & j = 1,\ldots,n \quad (i) \\ \sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x_{j\ell k}, & \ell = 1,\ldots,n, \\ & k = 1,\ldots,m \quad (ii) \\ \sum_{j=1}^{n} x_{0jk} = \sum_{i=1}^{n} x_{i0k} = 1, & k = 1,\ldots,m \quad (iii) \\ \sum_{i=0}^{n}\sum_{j=0}^{n} d_i.x_{ijk} \leq Q, & k = 1,\ldots,m \quad (iv) \\ \sum_{i\in U}\sum_{j\in U} x_{ijk} \geq \sum_{j=1}^{n} x_{\ell jk}, & U \subset \{1,2,\ldots,n\}, \\ & \ell \in U, k = 1,\ldots,m \quad (v) \\ x_{ijk} \in \{0,1\},\ 0 \leq i,j \leq n, & i \neq j, 1 \leq k \leq m \end{cases}$$

Where the relationship :

- (i) stipulates that a customer "j" is visited once and only once,
- (ii) stipulates that a "k" vehicle arriving at a customer's premises will leave again,
- (iii) stipulates that each vehicle leaving the depot returns to it,
- (iv) stipulates that the sum of requests from customers visited by a vehicle "k" must be less than or equal to the vehicle's capacity,
- (v) eliminates sub-tours to guarantee tour connectivity.

B. Resolution approach

1) Description: This approach is based on identifying the shortest distances in the cost matrix and constructing routes based on these minimum costs. These minimum costs are marked with ”0”. Since the distances are symmetrical, the cost matrix is also symmetrical. For calculation purposes, we focus on the upper triangular part of the matrix. We use the principle of the Hungarian method to identify the ”0”s. For each row and column of the cost matrix (i.e., distances), we determine the smallest cost and subtract it from all other elements in the same row or column. This process allows us to locate the minimum costs (i.e., the ”0”s in echelon 1) and build routes using the Clarke-Wright savings heuristic whenever possible. If a set of admissible routes that includes all customers cannot be constructed, additional ”0”s are created by subtracting the smallest non-zero element from each non-zero element of the row (resulting in 0s of echelon 2). Then, we attempt to build the routes using the heuristic again, giving preference to the smallest echelon ”0”s. If this is still not possible, we create ”0”s of echelon 3 and so on until route construction is feasible. In our approach, when building routes using the Clarke-Wright heuristic, we ensure that capacity constraints are satisfied and prioritize integrating customers that offer the greatest distance savings, i.e., those closest to the current route.

2) Algorithm of our approach: The algorithm of our approach, which we call the hybrid algorithm, is as follows:

Algorithm 3 Hybrid algorithm

Input: n, C Upper triangular matrix of order n with positive values

Output: (Eligible tours)

1. For j from 1 to n do
   $\alpha_j \longleftarrow \min(C(1:j,j))$
   $C(1:j,j) \longleftarrow C(1:j,j) - \alpha_j$
   End of for
2. For i from 1 to n do
   $\beta_i \longleftarrow \min(C(i,i:n))$
   $C(i,i:n) \longleftarrow C(i,i:n) - \beta_i$
   End of for
3. If possible build tours
4. Otherwise do
   For i from 1 to n do
   $\delta_i \longleftarrow \min(C(i,i:n)) > 0$
   $C(i,i:n) \longleftarrow C(i,i:n) - \delta_i$
   End of for
5. Return to step 3.

Clarke and Wright [14], [16] proposed two versions of their cost-based algorithm for constructing distribution routes: the sequential version and the parallel version.

a. Sequential version: routes are constructed one at a time.
b. Parallel version: routes are constructed simultaneously.

In our approach, whether sequential or parallel, we ensure that the next customer to be included in a route is the closest (i.e., the one offering the greatest savings) so that the vehicle’s capacity constraint is respected.

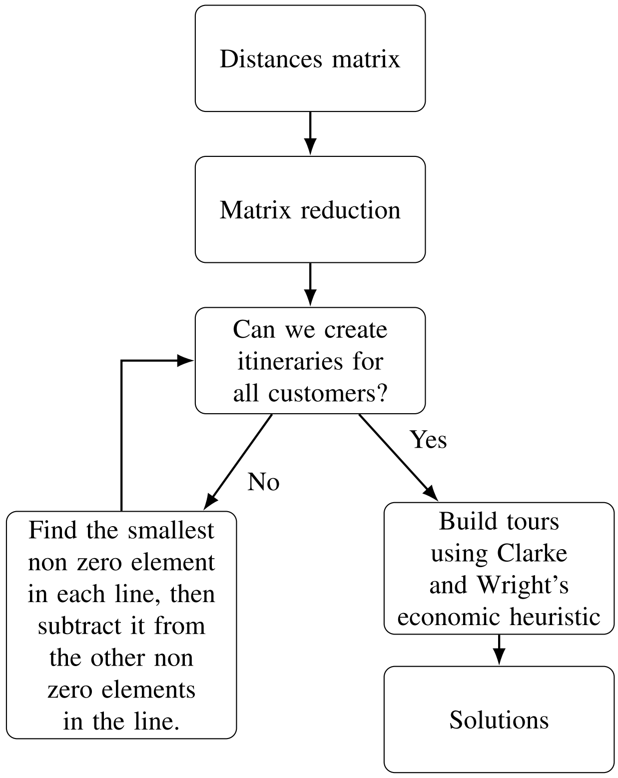

Fig. 2. Approach flowchart

This approach belongs to the class of heuristics and has been successfully applied to a test problem.

3) Approach flowchart: The flowchart for this approach is shown in Figure 2.

4) Didactic example: All that has been said is illustrated in a didactic example. This example is one of the frequently encountered test problems in the literature [14], [16].

A. Statement of problem

A pharmaceutical company needs to distribute its products to 15 customers. The demand d_i of each customer, along with the in table I. The company operates from a single warehouse and has access to a fleet of vehicles, each with a capacity of 8 tonnes. The distances from the warehouse to each customer are listed in the first row of Table I. Customers are ranked in descending order of priority. The company aims to organize its distribution with two main objectives:

1. Minimize the total distance traveled,
2. Minimize the number of vehicles used, while ensuring that all customer priorities are respected and vehicle capacities are not exceeded.

The cost associated with transportation is 25 UM per kilometer, and the fixed cost of using a vehicle is 2500 UM.

B. Reduction of the distance matrix

Matrix reduction consists of generating echelon zeros within it. After the reduction, we obtain Table II, in which 0^i corresponds to a zero of echelon i. The echelon indicates the order in which the zero appears.

C. Tours construction

Routes are constructed using the parallel version, which

TABLE I
MATRIX OF DISTANCES AND DEMANDS

N°	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	-	15	28	30	22	27	21	22	20	36	63	120	22	63	12	27
1		-	21	32	32	41	35	32	22	48	25	37	18	25	22	22
2			-	18	30	46	47	50	42	45	54	40	45	54	18	20
3				-	18	36	43	52	20	24	42	56	49	42	40	36
4					-	18	27	40	40	21	12	43	38	12	30	45
5						-	16	33	42	15	51	72	45	51	38	37
6							-	18	30	32	23	65	40	23	58	40
7								-	15	35	53	37	39	53	30	46
8									-	28	52	38	40	52	32	43
9										-	43	25	42	43	39	61
10											-	40	53	35	64	65
11												-	62	26	42	37
12													-	33	33	38
13														-	62	25
14															-	36
d_j	-	3	3	4	2	4	2	3	4	5	3	4	2	5	4	3
Priority	-	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

TABLE II
REDUCED MATRIX

N°	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	-	0^1	0^5	0^7	0^4	0^3	0^2	0^4	0^2	0^8	0^9	7	0^4	0^9	0^4	0^6
1		-	0^1	0^4	0^4	0^6	0^5	0^4	0^2	13	0^3	2	0^1	0^3	0^2	0^2
2			-	0^1	0^3	0^4	1	5	0^5	0^6	9	0^4	0^6	9	0^1	0^2
3				-	0^3	0^4	0^6	10	0^2	0^3	0^6	14	17	0^6	0^5	0^5
4					-	0^1	0^3	11	11	0^2	0^1	13	8	0^1	0^4	15
5						-	0^1	0^2	0^5	0^1	0^6	0^8	0^7	0^6	0^4	0^3
6							-	0^1	0^3	0^4	0^2	28	0^5	0^2	18	0^5
7								-	0^1	0^3	14	0^4	0^5	10	0^2	15
8									-	0^1	0^5	0^3	0^4	10	0^2	3
9										-	0^1	0^5	0^3	0^4	0^2	16
10											-	0^2	0^3	0^4	0^2	1
11												-	0^4	0^1	0^3	0^2
12													-	0^1	0^1	0^2
13														-	0^2	0^1
14															-	0^1
d_j	-	3	3	4	2	4	2	3	4	5	3	4	2	5	4	3

converges faster than the sequential version. During the construction process, we prioritize the zeros with the lowest echelon, as they yield the greatest savings. After construction, the admissible tours obtained are presented in Table III. There are several admissible solutions given by the set of these tours. The admissible solutions we obtain are dominated by the following: (0-4-9-0), (0-6-11-12-0), (0-13-15-0), (0-1-10-0), (0-8-14-0), (0-5-7-0), (0-2-3-0). The total distance traveled is 689 km, priority is 120, and the fleet size is 7 vehicles.

D. Comment

Our approach yielded the solution (689, 120, 7). In 2015, Okkitonumbe et al. addressed the same issue using a hybrid approach based on Clarke and Wright's heuristic, referred to as the Dominance Preference Benchmark Method (DPBM). They obtained the following outcomes: (750, 120, 7) for the sequential version and (725, 120, 8) for the parallel version. The same team suggested a hybrid method based on the spider-web algorithm and Clarke and Wright's savings heuristic [15]. When this method was applied to the same test problem, the result was (723, 120, 7). These results are the best we have seen in the literature thus far. A comparison of our results with those of the aforementioned authors is presented in Table IV.

TABLE IV
COMPARATIVE TABLE OF SOLUTIONS

Method	Solutions	Version	Cost
MRPD	(750,120,7)	Sequential	36250UM
	(725,120,8)	Parallel	38125UM
Cobweb Heuristic	(723,120,7)	Sequential	35575UM
Our hybrid method	(689,120,7)	Parallel	34725UM

TABLE III
ELIGIBLE TOURS

N°	Tour	Distance (km)	Priority	(T)
1	(0-1-6-10-0)	136	31	8
2	(0-1-10-6-0)	84	30	8
3	(0-1-10-0)	103	21	6
4	(0-1-10-0)	103	21	6
5	(0-1-2-4-0)	88	41	8
6	(0-1-8-0)	57	23	7
7	(0-1-14-0)	49	17	7
8	(0-1-12-0)	55	19	7
9	(0-1-8-0)	57	23	7
10	(0-1-14-0)	37	17	7
11	(0-1-4-10-0)	122	33	8
12	(0-2-3-0)	76	27	7
13	(0-2-14-0)	46	16	7
14	(0-4-5-6-0)	77	35	8
15	(0-4-6-0)	70	22	4
16	(0-4-6-7-0)	89	31	7
17	(0-4-6-10-0)	135	28	7
18	(0-4-9-0)	79	19	7
19	(0-5-14-0)	77	13	8
20	(0-5-7-0)	82	20	7
21	(0-5-4-6-0)	93	34	8
21	(0-5-11-12-0)	183	20	8
22	(0-5-6-0)	64	21	6
23	(0-6-11-12-0)	170	19	8
24	(0-7-9-0)	93	16	8
25	(0-7-1-12-0)	94	27	8
26	(0-7-8-0)	57	17	7
27	(0-7-14-0)	52	11	7
28	(0-8-14-0)	64	10	8
29	(0-8-12-0)	80	12	6
30	(0-8-11-0)	178	13	8
31	(0-12-14-0)	67	6	6
32	(0-12-13-0)	118	7	7
33	(0-13-15-0)	115	4	8
34	(0-14-15-0)	52	3	7

This is the most economical method in the table. It offers a 3.4% reduction compared to the Cobweb method and a 9.4% reduction compared to the MRPD sequential version. Although it results in costs of 689 instead of 723 or 750, the fleet size remains at 7. The cost of our solution shows that the solution we have found is better than the others.

E. Other results and discussion

To validate our method, we applied it to other test problems described in the Table V.

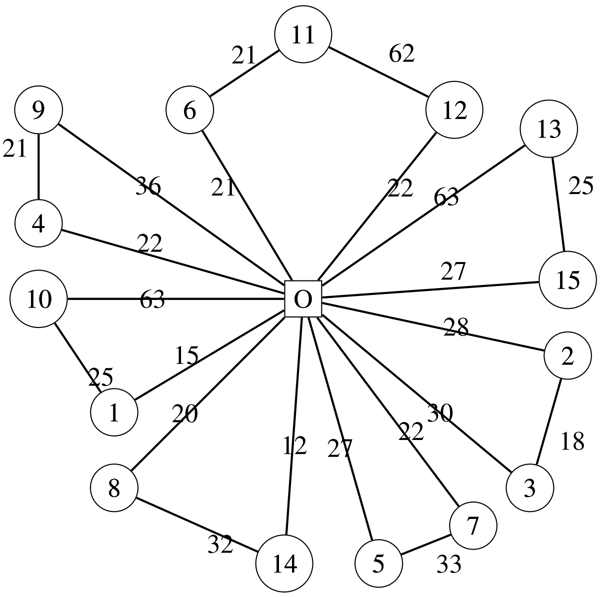

Fig. 3. Representation of the Solution

TABLE V
DESCRIPTION OF TEST PROBLEMS

N°	Benchmark	Instance	n	Q
1	Set A (Augerat, 1995)	A-n32-k5	32	100
2	Set A (Augerat, 1995)	A-n33-k5	33	100
3	Set A (Augerat, 1995)	A-n33-k6	33	100
4	Set A (Augerat, 1995)	A-n34-k5	34	100
5	Set A (Augerat, 1995)	A-n36-k5	36	100
6	Set E (Christofides and Eilon, 1969)	E-n13-k4	12	6000
7	Set E (Christofides and Eilon, 1969)	E-n22-k4	21	6000
8	Set P (Augerat, 1995)	P-n16-k8	15	35
9	Set P (Augerat, 1995)	P-n19-k2	18	160
10	Set P (Augerat, 1995)	P-n22-k8	21	3000

Where n is the number of customers and Q is the capacity of each vehicle.

TABLE VI
COMPARISON OF OUR SOLUTIONS WITH SOURCE SOLUTIONS

Instance	Our solution		Source solution	
	k	Distance	k	Best-distance
A-n32-k5	5	784	5	784
A-n33-k5	5	661	5	661
A-n33-k6	5	742	5	742
A-n34-k5	5	778	5	778
A-n36-k5	5	799	5	799
E-n13-k4	4	**296**	4	**247**
E-n22-k4	4	**381**	4	**375**
P-n16-k8	8	450	8	450
P-n19-k2	2	212	2	212
P-n22-k8	8	603	8	603

Where k denotes the number of routes. Out of the ten test problems to which we applied our method, we obtained eight exact solutions and two approximate solutions. Our method belongs to the category of heuristic methods and allows for quickly finding a near-optimal or optimal solution.

However, we observe that when vehicle capacity is high and customer demand is low, our method does not yield the exact solution, but rather an approximate one. This is due to the fact that the number of potential routes increases dramatically under these conditions, making it difficult for the method to converge efficiently.

We therefore conclude that our method is less effective in cases where vehicle capacity is high and individual customer demand is low.

IV. Conclusion

In this paper, we proposed a hybrid approach to solve the Capacitated Vehicle Routing Problem (CVRP). The method combines Clarke and Wright's savings heuristic with the Hungarian algorithm to improve the efficiency of route construction. The routes are built using the parallel version of Clarke and Wright's method. Indeed, after multiple tests, the parallel version proved to be more efficient and faster than the sequential version.

Experimental results on classical benchmark instances show that the proposed approach provides competitive solutions, outperforming certain existing heuristics in specific cases. However, convergence difficulties were observed when vehicle capacities were very high and customer demands were low. This limitation is mainly due to a combinatorial explosion in the number of feasible routes, which increases computational complexity.

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