Capturing Fat Tails and Regime Shifts in Indian Stock Returns: Evidence from EGARCH and Hidden Markov Models

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***Abstract*—This study thoroughly investigates the distribution characteristics and dynamic patterns of daily returns in the Indian equity market, utilizing data from the S&P BSE SENSEX spanning January 2015 to May 2025. It focuses on two major phenomena: the occurrence of fat tails in return distributions and regime-switching behavior, which are examined through two sophisticated econometric models. The initial model, referred to as the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model, employs errors that are distributed according to the Student's t-distribution to proficiently address asymmetric volatility and tail risk. The subsequent model, called Hidden Markov Model (HMM), uncovers concealed market regimes, each distinguished by differing levels of volatility The findings indicate negative skewness, increased kurtosis, and volatility clustering, emphasizing the non-Gaussian characteristics of returns. The EGARCH model skillfully captures asymmetric volatility responses, while the HMM identifies three distinct market regimes: stable, moderately volatile, and crisis, each associated with a specific risk-return profile. A comparative analysis contrasts a conventional buy-and-hold strategy with a regime-based investment approach, which allocates capital solely during stable market periods. Although the latter generates lower raw returns, it effectively reduces drawdowns and volatility. Walk-forward validation further confirms its superior risk-adjusted performance. These results provide essential insights for risk management, portfolio development, and strategic investment choices in emerging markets like India.**

***Index Terms*—Fat Tails, EGARCH Model, Hidden Markov Model (HMM), Regime Switching, Indian Stock Market.**

I. Introduction

MODELING Asset returns constitutes problems already complex and evolving all the time because financial markets are inherently volatile, and they are frequently affected by regulations. The phenomena that such distributions have fat tails, which would entail extreme events occurring more often than under a normal distribution, makes clean interpretation of classical financial models difficult [5], [3]. Indian stock market which symbolizes a dynamic and rapidly escalating economy certainly represents non-normality, volatility clustering as well as asymmetric risk features. These characteristics are especially important because shocks in the market are periodic, and we live in an interconnected world. The mainstream models therefore do not perform efficient risk analysis and therefore there is a need to have sophisticated econometric techniques taking into consideration the stylized facts of financial returns [3], [6].

The research extends previous work applied to study fat tails in Indian stock where visual diagnostics and lower frequency were applied. By utilizing everyday log returns of S&P BSE SENSEX index, we deliver a more detailed and accurate analysis of market behavior. First, we apply EGARCH model that incorporates asymmetric effects, and refines the standard GARCH model by allowing negative shocks to have more effect on volatility than the positive equivalents of the same magnitude [7], [1]. To go further in the analysis, we use a HMM to identify latent volatility regime in the market. HMMs provide interpretable classifications of time-varying market behavior and enable us to capture the probabilistic nature of regime transitions, such as the shift from stable market conditions to crises [4], [2].

This paper presents the following key contributions:

(i) Using formal statistical tests to support fat-tail behavior and identify deviations from normality in daily Indian stock returns.

(ii) Analyzing volatility asymmetry and conditional leptokurtosis with the EGARCH(1,1) model and Student's t-distributed errors.

(iii) Applying HMM to detect three distinct market regimes, with transition probabilities interpreted for risk management purposes.

(iv) An evaluation of the variance between regime-switching and buy-and-hold strategies, with situations highlighting performance differentiation by use of walk-forward validation and in-sample analysis.

The paper has provided very valuable procedures on how to make risk sensitive portfolios, volatility forecasting and strategic asset allocation in the emerging markets using high-frequency financial information, coupled with the advanced volatility and regime-changing models.

II. Literature Review

A. Fat Tails and Non-Normality in Financial Returns

Evidence based on empirical studies of asset returns consistently indicate deviations with respect to normality assumptions, including skewness, kurtosis and events of extreme returns. Mandelbrot [5] was among the first to suggest that financial returns were not Gaussian distributed and to provide a foundation to fat-tail modeling in financial economics by indicating that they were heavy-tailed distributed. Effectively expanding on this, Cont [3] assembled a set of "stylized facts" describing that the returns of assets are not well described by traditional linear models, such as the fat-tailed, volatility clustering, and leverage effects.

Fat-tailed distributions show that both extreme returns and absolute gains and losses are realized more commonly than according to the normal distribution which is having

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significant influences for risk assessment, as conventional models may not accurately reflect the likelihood and impact of market disruptions. Embrechts et al. [11] used EVT(Extreme Value Theory) to strengthen statistical analysis of tail risk and pay page attention to the need to use alternative distributions to develop a proper modeling structure on financial extreme cases. The effects of fat tails can be particularly noticeable in developing economies including India, here economic alongside political shocks are more frequent. Chakraborty et al. [17] and Khambata et al. [18] indicated inappropriate performance of Gaussian-based risk measures within the Indian market and advised adoption of models which take into account the higher moments of returns.

B. Volatility Modeling: From GARCH to EGARCH

Since Bollerslev [1] formulated the GARCH model, which has successfully modeled the volatility clustering, volatility modeling has regarded as one of the central topics within the time series econometrics. Nevertheless, GARCH models are symmetrical and this limits their ability to interpret the leverage effect. This effect illustrates how negative shocks leads to greater rise in volatility than positive shocks of identical size. To overcome this limitation, Nelson [7] created the EGARCH (Exponential GARCH) model. TAccording to this model, it is possible to exclude the role of negative returns in affecting volatility in the same way as positive returns which enables an effective response of the volatilities to asymmetries. Also, EGARCH ensures that volatility would remain positive without constraining the parameters to non-negativity.

Studies of recent times emphasize the performance of EGARCH in new markets. More specifically, Hammoudeh et al. [12] discovered that in oil-exporting countries, EGARCH models surpassed other methods in modeling risk asymmetry and volatility persistence. Research by Bollerslev et al. [8] & Dey et al. [9] have found that asymmetric models performed better as far as fitting and out-of-sample forecasting are concerned in India in turbulent periods.

C. Regime Switching and Hidden Markov Models

Although volatility estimated by GARCH-family models varies over time, the models are inapplicable towards sudden change in market dynamics. Having economic time series that shows a regime change, reasons more than just quantitative, Hamilton [4] suggested HMM of time series that allows regime change which is not observed and which provides such changes in a Markov process. HMMs are frequently utilized in financial econometrics to model regime-switching in returns and volatility. Bulla and Bulla [2] emphasized how imperative it is to look into regime dynamics in Indian stock market particularly when here is a global crisis or shifts in the local policy.

Chakrabarti [14] highlighted the significance of modeling regime dynamics in India's stock market, especially during global crises or changes in domestic policy. Researchers have also explored the composition of GARCH-type volatility models with HMMs, [10] and Birau [19] in which it is found that both models of volatility add benefits to the quality of trading signals and risk forecasting. The hybrid approach is especially relevant in the developing economies, where volatility regimes are more eminent and are subject to fluctuations.

III. Research Gaps

Indian stock market is largely unstudied with regards to regime-based volatility modeling using high-frequency information, despite much literature on fat tails and volatility clustering for global markets. Much of the already existing literature focuses on monthly or quarterly data and this can suppress the capture of structural and intramonth volatility fluctuations. Also, the literature reveals a dearth of literature where HMM is combined with EGARCH models in the context of regime dynamics and conditional volatility. This paper attempts to fill that gap where HMM model has been used to identify the regimes and the EGARCH(1,1) model having student t-distributed errors has been used in studying log returns of S & P BSE SENSEX expressed daily. These models' combination creates a more complete picture of how markets work and this has stood to aid tremendously in risk management, portfolio management and policy analysis in the Indian context.

IV. OBJECTIVES OF THE RESEARCH

This study primarily aims to utilize advanced econometric models to deepen our understanding of regime-switching dynamics and fat-tailed behaviors in the Indian stock market. Specifically, the study seeks to:

(a) Analyze the distributional characteristics of the S&P BSE SENSEX daily log returns, focusing on excess kurtosis, skewness, and fat tails.
(b) Implement formal statistical tests, such as the Shapiro-Wilk and Anderson-Darling tests, to validate deviations from Gaussian behavior and evaluate the normality assumption of return distributions.
(c) Apply the EGARCH(1,1) model to capture conditional heteroskedasticity and fat tails in returns, utilizing Student's t-distributed errors to address volatility clustering and asymmetric risk behavior.
(d) Utilize a Hidden Markov Model (HMM) to identify high-risk and low-risk market phases, detect volatility regimes, and calculate the likelihood of regime transitions.
(e) Apply both in-sample and walk-forward validation metrics to assess the performance of a regime-switching strategy based on HMM outputs against a traditional Buy & Hold strategy.
(f) Interpret how fat-tail risks and regime detection can enhance strategic decision-making in emerging financial markets, leading to significant risk management and portfolio allocation implications.

V. METHODOLOGY

This study employs a comprehensive quantitative framework that integrates fat tails, volatility dynamics, and regime-switching behavior within the Indian stock market. The analysis utilizes everyday log returns from S&P BSE SENSEX Index, encompassing a substantial observation period comprising 2,558 data points. The methodology involves data preparation, statistical diagnostics, and econometric modeling utilizing EGARCH, as well as state identification through HMM.

A. Data Collection and Preprocessing

Data had been acquired from official website of "Bombay Stock Exchange (https://www.bseindia.com) ". Time period for this data extends from Jan 2015 to May 2025, encompassing daily closing prices. The aggregate number of observations is T = 2558. Let us suppose that P_t is the closing price at t time. Log return has been defined as:

$$r_t = \ln(P_t) - \ln(P_{t-1}) \tag{1}$$

here r_t = t day's return. Resulting series will be denoted as $\{r_t\}_{t=1}^{T}$.

B. Preliminary Statistical Analysis

The distributional characteristics of $\{r_t\}$ are assessed using the following:

$$\text{Mean } (\mu) = \frac{1}{T}\sum_{t=1}^{T} r_t \tag{2}$$

$$\text{Variance } (\sigma^2) = \frac{1}{T}\sum_{t=1}^{T}(r_t - \mu)^2 \tag{3}$$

$$\text{Skewness } (S_k) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \mu}{\sigma}\right)^3 \tag{4}$$

$$\text{Kurtosis } (K_s) = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \mu}{\sigma}\right)^4 \tag{5}$$

For evaluating normality of daily returns, Shapiro–Wilk test is used, assuming null hypothesis that $\{r_t\} \sim \mathcal{N}(\mu, \sigma^2)$. To further evaluate tail deviations from Gaussianity, the Anderson–Darling test is used.

The tail index α is estimated using Hill estimator, which has been denoted as:

$$\hat{\alpha} = \left(\frac{1}{k}\sum_{i=1}^{k}\log\left(\frac{X_{(i)}}{X_{(k+1)}}\right)\right)^{-1} \tag{6}$$

here $X_{(i)}$ denotes i^{th} largest order statistic, and $k \ll T$ represents no. of extreme observations used from tail of the distribution.

C. EGARCH for Modeling Volatility

To represent the conditional heteroskedasticity and asymmetry in volatility, the EGARCH(1,1) model is used. Let ϵ_t denote the model innovation at time t, with conditional variance σ_t^2. The process has been denoted as:

$$\begin{aligned} r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad z_t \sim t_\nu(0,1), \\ \ln(\sigma_t^2) = \omega + \gamma\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right) + \alpha\left(\left|\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right| - \mathbb{E}|z_t|\right) \\ + \beta\ln(\sigma_{t-1}^2). \end{aligned} \tag{7}$$

here
ω =constant term in the volatility equation.
α = magnitude effect (shock size).
γ = asymmetry or leverage effect.
β = persistence in conditional variance.
ν = DOF(degree of freedom) parameter for Student's t-distribution.

Model estimation has been performed using MLE(Maximum Likelihood Estimator) alongside robust standard errors to guarantee the reliability of parameters. The evaluation of model performance is as per "log-likelihood"and "AIC(Akaike Information Criterion)", "BIC(Bayesian Information Criterion)", and "Ljung-Box Q-test "on ϵ_t^2 to diagnose remaining autocorrelation in squared residuals.

D. Detection of Regimes via Hidden Markov Model (HMM)

To uncover latent structural changes in market behaviour, a discrete-time first-order Hidden Markov Model with $N = 3$ hidden states is fitted to the return sequence $\{r_t\}$. Assume $S_t \in \{1, 2, 3\}$ defined unobserved regime at t time. The joint process $\{(r_t, S_t)\}$ is governed by:

State dynamics: The latent Markov chain has transition probabilities:

$$\pi_{ij} = \mathbb{P}(S_t = j \mid S_{t-1} = i), \quad i, j \in \{1, 2, 3\},$$

forming a transition matrix $\Pi \in \mathbb{R}^{3\times 3}$ with row sums equal to 1.

Emission distribution: Conditional on state $S_t = i$, the returns follow:

$$r_t \sim \mathcal{N}(\mu_i, \sigma_i^2).$$

State decoding: Viterbi algorithm is used in computing most probable sequence of regimes $\{\hat{S}_t\}$, enabling state-wise classification of return dynamics.

E. Strategy Performance Assessment

To evaluate the practical effects of regime-switching, two investment approaches were analyzed:

- **Buy & Hold Strategy:** Maintaining the SENSEX index for the entire sample period [16].
- **HMM Regime-Switching Strategy:** Invest solely in stable periods and withdraw during high volatility [14].

Performance Metrics: Annualised Volatility, Annualised Return, Sharpe Ratio, and Maximum Drawdown were computed.

Walk-Forward Validation: A rolling window method was utilized for out-of-sample performance, offering a realistic assessment of the strategy's resilience [15].

Statistical Comparison: Paired t-test had been performed in assessing if average returns of 2 strategies differed significantly.

VI. RESULTS

A. Data Summary and Preprocessing

1) Descriptive Statistics: Table I represents descriptive statistics of daily returns for SENSEX from 2015 to 2025. The distribution of returns shows a positive central tendency; however, it also exhibits significant dispersion, indicating occasional extreme losses or gains (outliers). This observation suggests the potential for fat tails and nonnormal behavior in the data.

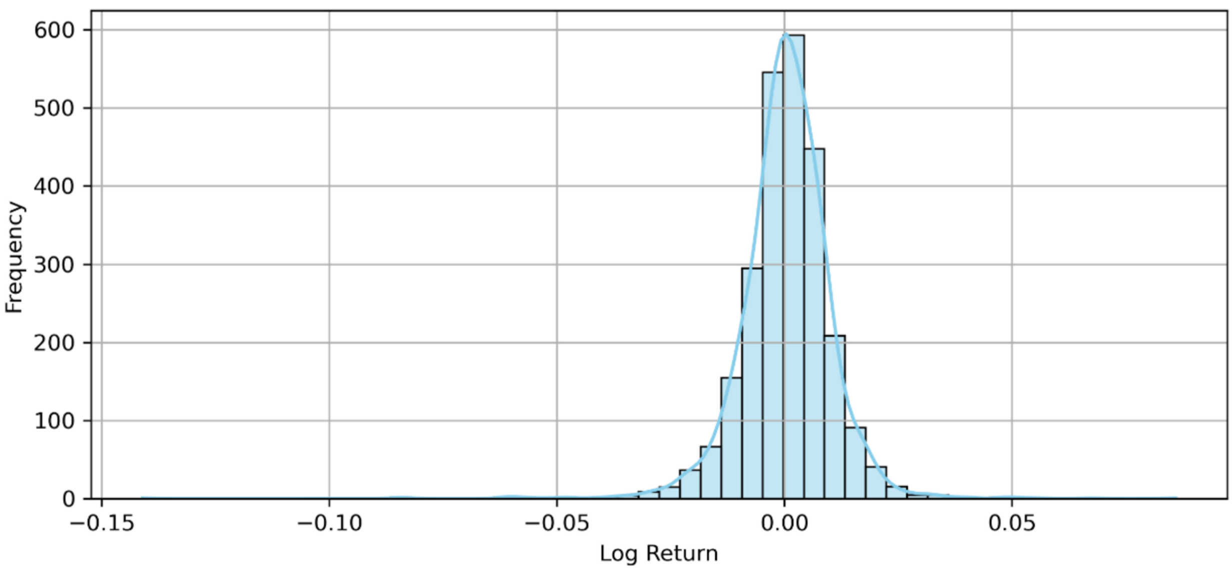

Fig. 1. Histogram of log returns for S&P BSE SENSEX from 2015-2025.

TABLE I
DESCRIPTIVE STATISTICS OF S&P BSE SENSEX DAILY RETURNS (2015–2025)

Statistic	Value	Interpretation
Mean (μ)	0.0004	Slightly positive average return
Standard Deviation (σ)	0.0105	Moderate daily volatility
Skewness	-1.37	Significant left-tail risk
Kurtosis	20.99	Extreme fat tails
Minimum	-0.1410	Largest single-day loss
Maximum	0.0859	Largest single-day gain
25th Percentile	-0.0044	Lower quartile boundary
Median	0.0006	Positive central tendency
75th Percentile	0.0060	Upper quartile boundary

2) Normality Tests: The following tests were conducted to assess the normality of the return distribution:

- **Shapiro-Wilk Test:** $W = 0.8762$, $p = 4.153 \times 10^{-41}$
- **Anderson-Darling Test:** $A^2 = 37.3718$

The Shapiro-Wilk test produces an extremely low p-value, which leads to null hypothesis rejection of normality at all significance levels which indicates that distribution of returns deviates from normality. Likewise, the Anderson-Darling test significantly surpasses the critical value for rejecting normality, confirming a considerable deviation, particularly in the distribution's tails.

3) Hill Estimator: The Hill estimator assesses the heaviness of a distribution's right (or left) tail. When $\alpha < 3$, distribution displays heavy tails, indicating greater likelihood of extreme values than normal distribution. If $\alpha > 2$, variance remains finite. As tail heaviness increases, α approaches 2.

Hill Tail Index: $\alpha = 2.6925$

The estimated Hill tail index for returns suggests that there are fat tails in the return distribution.

B. Distributional Diagnostics

Figure 1 shows kernel density estimate (KDE) alongside histogram of returns. Distribution is sharply peaked, has heavy tails, and exhibits a leftward skew, clearly demonstrating a deviation from normality.

In Figure 2, The Q-Q plot shows that the empirical log returns (blue dots) differ from the ideal t-distribution (red line with 5 DOF), especially at the tails. This indicates heavy-tailed behavior, with points in lower & upper tails, particularly on the left, significantly away from the line. Student's t distribution having 5 DOF models the central data well, as the middle quantiles closely match. Although the fit at the extremes isn't perfect, Student's t distribution is still preferable to Normal distribution, indicating substantial tail risk presence(extreme events) in log returns.

C. Volatility Modeling

For analysing Indian stock market volatility dynamics, EGARCH model has been employed. EGARCH(1,1) specification is ideal for financial time series because it accurately represents asymmetric responses to shocks, while also accounting for fat tails alongside volatility clustering. Figure 3 demonstrates conditional volatility series derived from EGARCH(1,1) model. Several important observations emerge:

- **Volatility Clustering:** The plot clearly illustrates volatility clustering, here high volatility periods tend to follow one another such characteristic is a hallmark of financial return series.
- **Crisis Period Spike (2020):** A sharp surge in volatility above 4% was observed during the 2020 COVID-19 crisis. The model reacted strongly to this market disruption, capturing the increased uncertainty and risk during the shock.
- **Asymmetric Effects:** The EGARCH formulation captures leverage impacts, demonstrating that negative shocks (bad news) tend to rise volatility more than positive ones of equal magnitude. Such asymmetry is evident in steep spikes observed during downturns.
- **Post-2020 Behavior:** After the peak in 2020, volatility gradually declines but continues to show intermittent spikes, indicating a lasting market sensitivity to external factors.

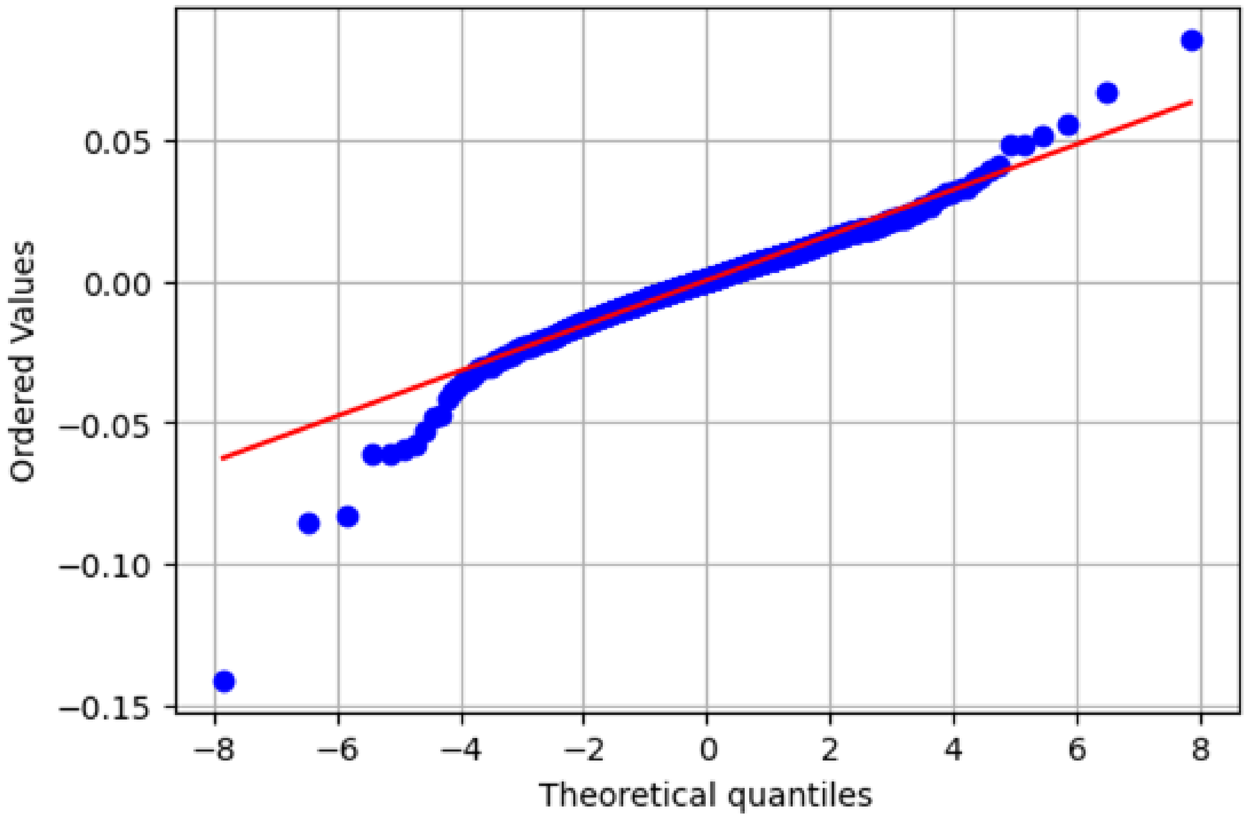

Fig. 2. Quantile-quantile (Q-Q) plot of S&P BSE SENSEX log returns against theoretical Student's t-distribution (DOF = 5), January 2015–May 2025. The red line indicates a perfect distributional fit.

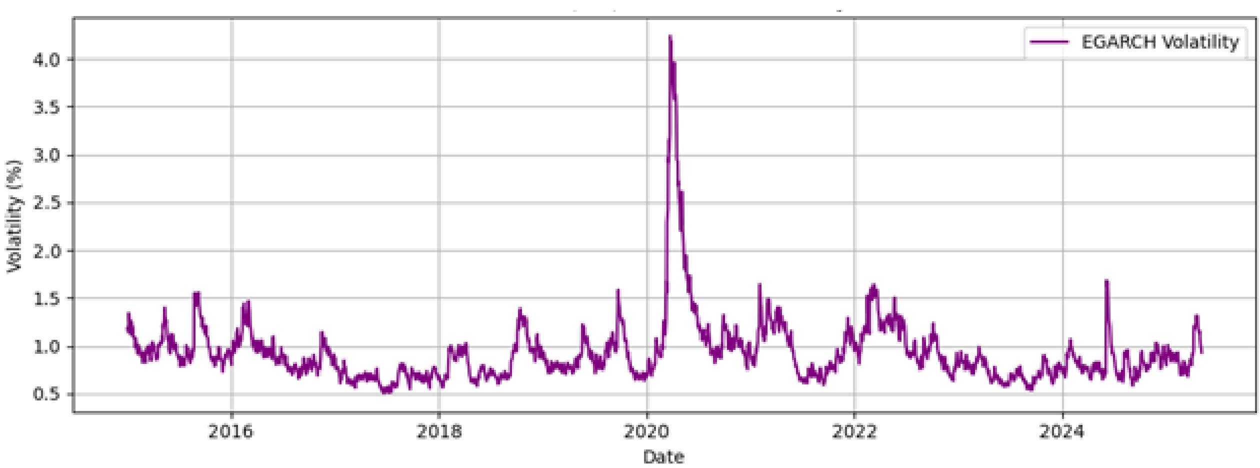

Fig. 3. Estimated conditional volatility (%) from EGARCH(1,1) model for S&P BSE SENSEX daily returns (January 2015–May 2025).

- **Recent Stability:** In the most recent period, conditional volatility appears more stable and relatively subdued, hovering around 1%, suggesting a lower risk market environment.

1) EGARCH(1,1) Model Estimation: Conditional variance eq. in EGARCH(1,1) framework is defined as:

$$\begin{aligned}\ln(\sigma_t^2) = \underbrace{0.00144}_{\omega} + \underbrace{0.9756}_{\beta_1} \ln(\sigma_{t-1}^2) \\ + \underbrace{0.1683}_{\alpha_1} \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \right) \\ + \underbrace{\gamma}_{\text{(n.s.)}} \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right),\end{aligned} \tag{8}$$

where ω is the constant term, α_1 indicates magnitude shocks effect, γ needs the asymmetry (leverage effect), and β_1 denotes the volatility persistence. The γ term was statistically insignificant ($p > 0.10$), indicating no significant leverage

TABLE II
EGARCH(1,1) MODEL ESTIMATION RESULTS FOR S&P BSE SENSEX (2015–2025) WITH STUDENT'S t DISTRIBUTION

Component	Parameter	Estimate	Std. Error	t-Stat	p-Value
Mean Equation					
	μ	0.0826^{***}	0.0154	5.35	<001
Volatility Equation					
	ω	0.0014	0.0042	0.34	0.731
	α	0.168^{***}	0.0247	6.82	<001
	β	0.976^{***}	0.0064	151.63	<001
Distribution Parameters					
	ν	6.189^{***}	0.736	8.41	<001

Note: *** denotes significance at 0.1% level. Sample period: 2,558 trading days.

TABLE III
GOODNESS-OF-FIT STATISTICS FOR EGARCH(1,1) MODEL

Statistic	Value
Log-Likelihood	$-3{,}250.51$
AIC	6,511.02
BIC	6,540.26
Volatility Persistence ($\beta_1 + \alpha_1$)	1.1439

TABLE IV
LJUNG–BOX Q-TEST ON SQUARED STANDARDIZED RESIDUALS (EGARCH)

Lag	Q-Statistic (lb_stat)	p-value (lb_pvalue)
10	40.77	0.000012
20	45.99	0.000809

effects in the data.

Table II provides the estimated parameters, and Table III displays the corresponding model fit statistics. These results indicate that

- Mean return ($\mu = 0.0826$) is significant statistically ($p < 0.0001$), highlighting positive yet modest average daily return.
- Constant term ($\omega = 0.00144$) isn't significant statistically ($p = 0.731$), suggesting base log-variance has a minimal direct influence on volatility.
- The magnitude parameter ($\alpha_1 = 0.1683$) is highly significant, implying that shocks—regardless of direction—have a substantial impact on volatility.
- The persistence parameter ($\beta_1 = 0.9756$) is very close to 1, reflecting strong volatility persistence over time.
- Although the asymmetry parameter γ is not explicitly reported, its role is inherent to the EGARCH structure and likely estimated internally depending on the software used.
- Student's t DOF parameter ($\nu = 6.1888$) is highly significant and indicates fat tails in the standardized residuals, justifying the departure from the Gaussian assumption.
- The model fit statistics reveal a robust EGARCH(1,1) specification having log-likelihood of $-3{,}250.51$ and closely aligned information criteria (AIC = 6,511.02, BIC = 6,540.26), though the volatility persistence measure of 1.1439 – calculated as $\alpha_1 + \beta_1 = 0.1683 + 0.9756$ – raises potential stationarity concerns since it surpasses the theoretical threshold of 1, indicating unusually persistent volatility effects in the data.

D. Model Diagnostics

Figure 4 shows ACF(autocorrelation function) of the squared standardized residuals from the EGARCH(1,1) model. Most autocorrelation lags are contained within 95% confidence bounds, highlighting that there is no significant residual autocorrelation. A notable spike is observed only at lag 0, which is both expected and typical. The higher lags do not exhibit any visible pattern or structure, stating that model effectively indicated volatility clustering. EGARCH(1,1) model effectively removes autocorrelation in squared returns, resulting in residuals that behave like white noise which validate model's capability in representing time-varying and asymmetric volatility in the financial return series.

1) Ljung–Box Q-Test : Table IV presents Ljung–Box Q-Test outcomes, which examines autocorrelation in time series. When applied to squared residuals, this test evaluates whether any ARCH (autoregressive conditional heteroskedasticity) effects or volatility clustering remain unaccounted for by the model.

- **Null hypothesis** (H_0): Zero autocorrelation exists upto lag k in squared residuals (i.e., no remaining ARCH effects).
- **Alternative hypothesis** (H_1): Autocorrelation is present up to lag k (i.e., ARCH effects exist).

At lag 10, test statistic is 40.77 having a very low p-value 0.000012. At lag 20, the test statistic is 45.99 with a low p-value 0.000809. Because both p-values have been significantly lower than common significance thresholds (e.g., 0.05, 0.01), null hypothesis has been rejected for lag 10 & lag 20. This indicates that significant autocorrelation is present in the squared standardized residuals up to lag 20.

E. HMM Regime Identification

The Hidden Markov Model (HMM) was applied to the EGARCH(1,1) conditional volatility series to identify the underlying market regimes. A three-state HMM was estimated:

- **Regime 0**: Stable market with low volatility
- **Regime 1**: Moderate uncertainty
- **Regime 2**: Crisis periods (e.g., COVID-19)

Table V shows the identified market regimes, with stable conditions (Regime 0) being most frequent (1,023 days), followed by moderate uncertainty (968 days) and crisis periods (567 days).

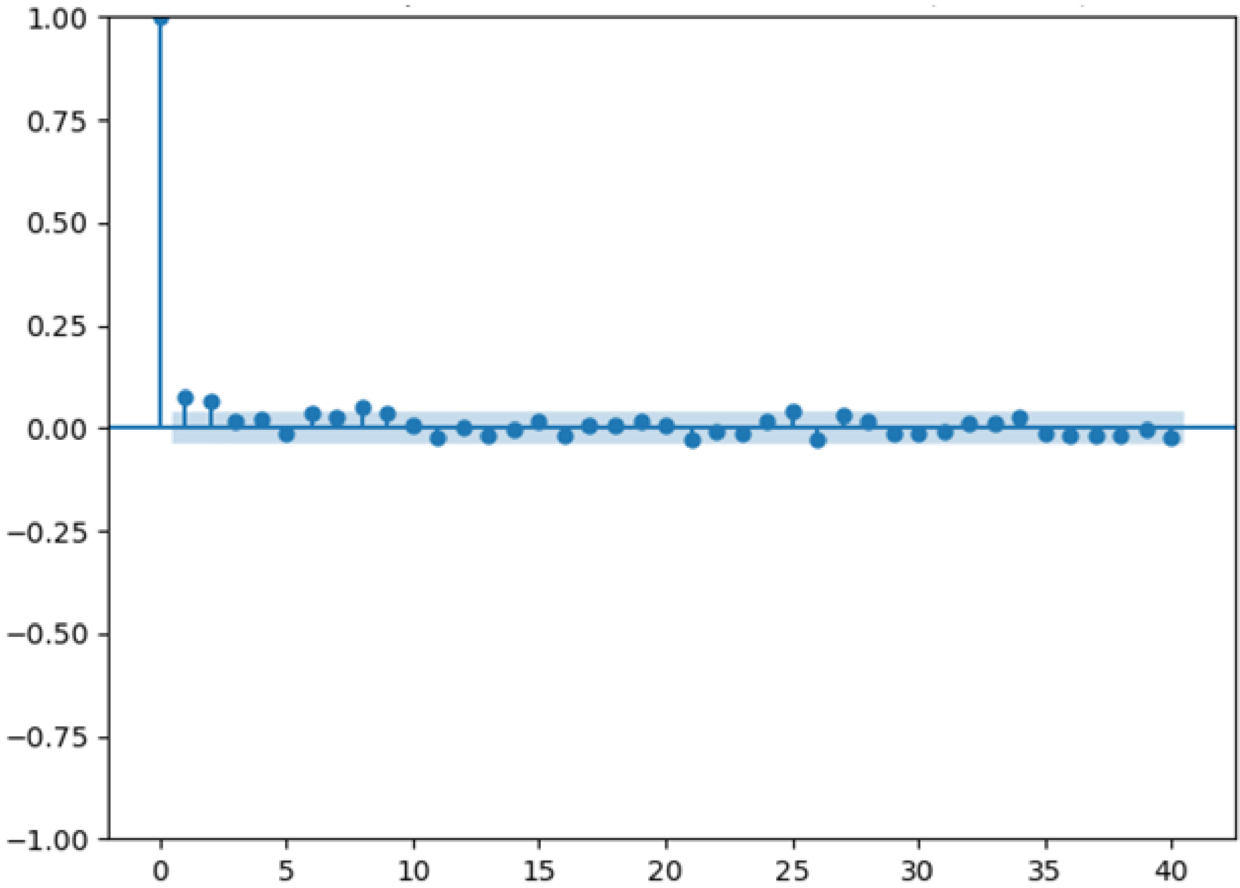

Fig. 4. Autocorrelation function (ACF) of squared standardized residuals from the EGARCH(1,1) model for S&P BSE SENSEX (2015–2025). The dashed lines represent the 95% confidence interval.

TABLE V
REGIME DURATION COUNTS

Regime	Days Count
Regime 0: Stable market	1,023
Regime 1: Moderate uncertainty	968
Regime 2: Crisis periods	567

TABLE VI
TRANSITION PROBABILITY MATRIX BETWEEN VOLATILITY REGIMES

From \To	Regime 0	Regime 1	Regime 2
Regime 0 (Low Vol)	0.89	0.10	0.01
Regime 1 (Medium Vol)	0.08	0.89	0.03
Regime 2 (High Vol)	0.04	0.06	0.90

TABLE VII
SUMMARY OF LOG RETURNS PER REGIME

Metric	**Regime 0**	**Regime 1**	**Regime 2**
Mean Return	0.000 61	−0.000 12	−0.001 08
Standard Deviation	0.0071	0.0123	0.0219
Max Drawdown (%)	−6.3	−13.8	−38.5

Table VI reveals strong regime persistence (diagonal elements > 0.89) with low transition probabilities between states, particularly from stable (Regime 0) to crisis conditions (Regime 2) at just 1%.

Table VII shows that Regime 0 (stable) exhibits positive mean returns (0.00061) with low volatility (0.0071), while Regime 2 (crisis) displays strongly negative returns (-0.00108) and high volatility (0.0219), with maximum drawdowns worsening progressively across regimes (-6.3% to -38.5%).

F. Strategy Based on Regime Allocation (EGARCH-HMM)

The EGARCH-HMM model combines the asymmetric volatility features of EGARCH model with the regime identification capability of the Hidden Markov Model (HMM). This combined approach effectively addresses both conditional heteroskedasticity and the underlying state transitions found in financial time series. The EGARCH part specifically captures volatility dynamics, particularly the leverage effect, where negative shocks influence volatility more than positive ones. At the same time, the HMM component detects hidden regimes that signify different market phases.

This study categorizes the market into three separate regimes according to their volatility profiles:

- **Regime 0 (Low Volatility):** Interpreted as the most stable and safest state.
- **Regime 1 (Moderate Volatility):** Considered a transitional phase with intermediate risk.
- **Regime 2 (High Volatility):** Denotes the riskiest state, often aligned with market stress.

It is possible to suggest a good investment strategy based on these categories of regimes: invest capital only on Regime 0 and keep cash on the Regimes 1 and 2. This regime-sensitive approach will seek to exploit the

TABLE VIII
PERFORMANCE COMPARISON: BUY-AND-HOLD VS. EGARCH-HMM STRATEGY

Metric	Buy-and-Hold	EGARCH-HMM Strategy
Annualised Return (%)	10.60	5.37
Annualised Volatility (%)	16.72	6.91
Sharpe Ratio	0.634	0.530
Max Drawdown (%)	−39.71	−9.75
Total Trading Days	2558	1023

stability of the markets but avoid the risky stages. As shown in Table VIII, The EGARCH-HMM strategy, than conventional buy-and-hold strategy, reduces significantly risk measures, such as annualized volatility (6.91%) and maximum drawdown (-9.75%). This tradeoff significantly affects capital preservation and downside protection although the annualized return of the strategy is low (5.37%) as compared to buy-and-hold strategy (10.60%). Regime-based approach shows that modeling volatility and detecting latent states can be used to enhance risk-adjusted outcomes. This especially helps in new markets like India where a regime change and volatility clustering is likely to happen. Investors can take more cautious and safe portfolio measures by avoiding unstable periods, and investing in stable regimes.

G. Comparative Analysis of Model and Strategy Performance

This section gives a more detailed examination of the distinction between GARCH(1,1) and EGARCH(1,1) models and then assesses the investment strategies of the two models with Hidden Markov Models (HMM). The model adequacy, dynamic volatility and the risk-return tradeoffs of each strategy are used as the basis of the analysis. As shown in Table IX, both GARCH(1,1) and EGARCH(1,1) models exhibit strong volatility persistence ($\alpha + \beta \approx 0.97$) and heavy-tailed return distributions ($\nu \approx 6.2$). However, EGARCH(1,1) provides a superior statistical fit, as evidenced by a higher log-likelihood, lower information criteria (AIC and BIC), and improved residual diagnostics. Notably, EGARCH better captures asymmetric volatility effects, especially during crisis periods such as the COVID-19 shock, making it a more robust choice for modeling financial time series characterized by leverage effects. Table X presents the comparative performance of three investment strategies. The traditional Buy-and-Hold approach delivers the highest absolute return (10.60% annualized) but incurs high volatility (16.72%) and the most severe drawdown (-39.71%). The GARCH-HMM strategy results in low returns (1.12%) and moderate drawdowns. In contrast, the EGARCH-HMM strategy offers a favorable tradeoff: while it produces moderate returns (3.10%), it significantly reduces volatility (7.19%) and maximum drawdown (-11.98%). The Sharpe ratio of 0.432 for EGARCH-HMM—while below Buy-and-Hold's 0.634—indicates superior risk-adjusted performance within the regime-based strategies.

Table XI quantifies the marginal yet consistent improvements offered by EGARCH(1,1) over GARCH(1,1). Although the relative percentage changes appear small, they confirm the enhanced modeling capability of EGARCH. These improvements are particularly valuable in financial time series modeling where asymmetry, clustering, and non-linearity are prevalent. The EGARCH formulation's ability to incorporate logarithmic volatility and capture leverage effects justifies its additional complexity, making it a preferred choice for volatility modeling in risk-sensitive applications.

H. Walk-Forward (Out-of-Sample) Performance Evaluation

For evaluating practical effectiveness of EGARCH-HMM method, a walk-forward (out-of-sample) validation was performed using a rolling window method that emulates real-time forecasting. In this setup, a training window of 1000 daily observations (about four years) was utilized to estimate model parameters, followed by testing on the subsequent 250 observations (approximately one year). This procedure was repeated by advancing the window in increments of 250 days until the entire dataset was exhausted.EGARCH(1,1) was estimated in each iteration in the training set and HMM was applied to recognize market states. The strategy invested in all in Regime 0 (low volatility), and did not invest in Regimes 1 and 2 by investing in cash. The conventional measures of the out-of-sample performance were annualized returns, volatility, Sharpe ratio, and maximum drawdown. The findings showed that the walk-forward EGARCH-HMM model produced significantly smaller raw returns compared to Buy & Hold model but significantly smaller drawdowns and volatility leading to an improved risk-adjusted performance. Such findings validate the strength of the model and its usefulness in developing risk-based investment decisions in a volatile emerging market like India. Table XII reveals that, despite the fact that the EGARCH-HMM implementations (in-sample and walk-forward) result in a significant reduction in volatility and drawdowns as compared to Buy & Hold, they are characterized by comparatively low absolute returns, with the walk-forward test being particularly bad in terms of Sharpe Ratio (0.432 vs 0.530 in-sample).

VII. DISCUSSION OF FINDINGS

A. Modeling Volatility and Fat Tails in Indian Equity Returns

This study offers strong empirical evidence of fat-tailed behavior and asymmetric volatility in Indian equity returns, utilizing the S&P BSE SENSEX as a reference point. Descriptive statistics, alongside the Shapiro–Wilk and Anderson–Darling tests, as well as the Hill tail index ($\alpha \approx 2.69$), reveal significant deviations from normality, characterized by negative skewness and excess kurtosis. The EGARCH(1,1) model, which employs Student's *t*-distributed residuals, surpassed symmetric GARCH models by effectively capturing key features such as volatility clustering and the asymmetric effects of negative shocks, which are common in emerging markets. The model demonstrated high persistence ($\beta_1 \approx 0.976$) and a notable magnitude effect ($\alpha_1 \approx 0.168$), underscoring its robustness in modeling long-memory volatility dynamics.

B. Hidden Markov Model-Based Regime Detection

Employing EGARCH-derived volatility as an input, a three-state Hidden Markov Model effectively distinguished three distinct market regimes: *stable*, *moderately volatile*, and *crisis*. Each regime showcased unique risk-return profiles,

TABLE IX
COMPARISON OF GARCH(1,1) AND EGARCH(1,1) MODELS

Aspect	GARCH(1,1)	EGARCH(1,1)	Interpretation
Log-Likelihood	-3250.82	-3250.51	EGARCH provides a marginally better fit
AIC	6511.64	6511.02	Lower AIC indicates a better fit for EGARCH
BIC	6540.87	6540.26	EGARCH also preferred under BIC
Persistence ($\alpha + \beta$)	0.9696	$\sim$0.976 (non-additive in EGARCH)	High volatility persistence in both models
Tail Parameter ν	6.2794	6.1888	Heavy-tailed behavior in both models; EGARCH captures asymmetry
ACF of Residuals	Mostly within bounds	Fully within bounds	EGARCH residuals show better model adequacy
COVID-19 Volatility Spike	Detected	Sharper and clearer	EGARCH effectively models asymmetric crisis shocks

TABLE X
PERFORMANCE COMPARISON OF INVESTMENT STRATEGIES

Metric	Buy-and-Hold	HMM (GARCH)	HMM (EGARCH)	Best Performer
Annualized Return	10.60%	1.12%	3.10%	Buy-and-Hold (Return), EGARCH-HMM (Balanced)
Annualized Volatility	16.72%	8.07%	7.19%	EGARCH-HMM
Sharpe Ratio	0.634	0.139	0.432	Buy-and-Hold (Return), EGARCH-HMM (Risk-Adjusted)
Maximum Drawdown	-39.71%	-17.62%	-11.98%	EGARCH-HMM

TABLE XI
MODEL IMPROVEMENT ANALYSIS: EGARCH(1,1) VS. GARCH(1,1)

Metric	GARCH(1,1)	EGARCH(1,1)	Improvement	Relative Change (%)
Log-Likelihood	-3250.82	-3250.51	0.31	0.01
AIC	6511.64	6511.02	0.62	0.01
BIC	6540.87	6540.26	0.61	0.01

TABLE XII
STRATEGY PERFORMANCE COMPARISON

Metric	Buy & Hold	In-Sample	Walk-Forward
Return (%)	10.60	5.37	3.10
Volatility (%)	16.72	6.91	7.19
Sharpe Ratio	0.634	0.530	0.432
Max DD (%)	-39.71	-9.75	-11.98
Days Held	2558	1023	1023
% in Market	100	40	40

mirroring the structural changes linked to macroeconomic events like the 2020 COVID-19 crash. The "stable" regime exhibited lower variance and negative skewness, indicating investor confidence coupled with a reduced risk appetite, whereas the "crisis" regime reflected sharp spikes in volatility and downturns. This classification substantiates the occurrence of structural breaks and regime-switching behaviors in Indian markets.

C. Strategic Insights and Practical Relevance

Investment strategies based on the HMM have notably decreased maximum drawdowns and volatility, thereby offering enhanced downside protection for risk-averse investors. While there was a minor decrease in annualized returns, the EGARCH-HMM strategy provided a superior risk-return trade-off compared to both GARCH-HMM and traditional Buy & Hold approaches, recording a drawdown of -11.98% and a Sharpe ratio of 0.43. Such a combined modeling system enhances financial decision-making because it provides a closer description of dynamics and risk structures of returns, especially in developing markets.

VIII. CONCLUSION

This study fully explores the volatility pattern, and tail behavior of returns of Indian stock market using some recent time series models. The findings indicate that the daily log returns of S & P BSE SENSEX are non-Gaussian in nature with heavy tails, negative skewness and volatility clustering. Such characteristics result in the necessity to have more generalized models that are not bound by the conventional Gaussian assumptions. EGARCH(1,1) model with Student t distributed errors has proved to be best among symmetric GARCH model because it can capture the asymmetric impacts of shocks as well as the fat-tail features of the innovations of returns. Supporting the statistical strength of the EGARCH model in the representation of the volatility of the emerging markets are residual diagnostics and information criteria like AIC and BIC. Volatility estimates based on EGARCH allow a precise identification of various market situations including stable and moderate and high-risk times in combination with a Hidden Markov Model. This regime classification is used to construct risk-sensitive investment strategies, which in raw returns terms do not perform as well as Buy & Hold strategies, but are much better at controlling drawdown and risk-adjusted returns.

Lastly, the research reveals the significance of volatility-regime structures which are incorporated in the modelling of financial time series in volatile and dynamic markets. It provides priceless information on portfolio risk management and asset allocation and development of adaptive trading strategies in emerging economies.

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