Reliability Estimation in Stress Strength Model for Ailamujia Distribution

Chunping Li, Huibing Hao, Xue Wang, Mingyu Shen

Abstract—**This paper deals with the estimation of stress strength reliability model *R* when the stress and strength follow Ailamujia distributions. Some point estimates of *R* are obtained via the maximum likelihood and Bayesian methods, and the delta and MCMC methods are utilized to obtain the interval estimators. Some Monte Carlo simulations are used to compare the different estimated methods, and one real data analysis is utilized to illustrate purposes.**

Index Terms—**Stress strength model, Ailamujia distribution, Bayesian estimation, Confidence interval, Maximum likelihood estimation**

I. Introduction

The Ailamujia distribution is a newly proposed lifetime model that has many engineering applications, such as queuing theory, reliability analysis and telecommunications networks [1-2]. For example, it can be used to simulate the time interval between the arrivals of telephone calls in telecommunications. In reliability analysis, it can be used to describe the life of a product. In the field of engineering maintenance, it can be used to model the repair time and characterize the distribution of delay time. From Refs. [1-2], the probability density function (PDF) of Ailamujia distribution is given by

$$f(x)=\frac{4x}{t_0^2}\exp\left(-\frac{2x}{t_0}\right),\ x>0,t_0>0 \tag{1}$$

For convenience, we reparametrized this distribution by defining $\alpha=2/t_0$. Then, we can get

$$f\left(x\right)=\alpha^2 x\exp\left(-\alpha x\right),\ x>0,\alpha>0 \tag{2}$$

where α is the scale parameter, and the corresponding cumulative distribution function (CDF) of the Ailamujia distribution is defined by

$$F(x)=1-\left(1+\alpha x\right)\exp\left(-\alpha x\right),\ x>0,\alpha>0 \tag{3}$$

Some studies have been given for the Ailamujia distributions, such as the maximum likelihood estimator, the Bayesian estimation, and the mini-max estimators, and so on (see in Refs. [3-5]).

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In the field of reliability engineering, the stress-strength model $R=P(Y<X)$ plays an important role in reliability analysis. In recent years, the stress-strength model has been extensively studied under various distributional assumptions, such as the generalized inverted exponential distribution [6], the exponentiated pareto distribution [7], the generalized lindley distribution [8], the inverse chen distribution [9], the lindley distribution [10], the inverse weibull distribution [11], the generalized exponentiated distribution [12], the power modified lindley distribution [13], the unit gompertz distribution [14], the generalized logistic distribution [15], the exponentiated generalized marshall olkin distribution [16], the weighted exponential distribution [17], the exponentiated logistic distribution [18], and the Burr Type distribution [19]. Although many stress-strength models have been studied, the Ailamujia distribution has not been explored in this context.

In this paper, we focus on the estimation of $R=P(Y<X)$ for the Ailamujia distribution. Some points estimator and interval estimator methods are proposed in the following.

II. System reliability model

Let X and Y be independent random variables following the Ailamujia distribution with parameters θ and β, respectively. That is, the PDFs of X and Y are

$$f_X\left(x\right)=\theta^2 x\exp\left(-\theta x\right),\ x>0,\theta>0 \tag{4}$$

and

$$f_Y\left(y\right)=\beta^2 y\exp\left(-\beta y\right),\ y>0,\beta>0 \tag{5}$$

Then, the reliability function can be obtained as

$$\begin{aligned}R=P(Y<X)&=\int_0^{+\infty}P(Y<X\mid X=x)f_X(x;\theta)dx\\&=\int_0^{+\infty}\theta^2 x\exp\left(-\theta x\right)\left[1-\left(1+\beta x\right)\exp\left(-\beta x\right)\right]dx\\&=1-\frac{\theta^2}{(\theta+\beta)^2}-\frac{2\theta^2\beta}{(\theta+\beta)^3}=\frac{\beta^2\left(\beta+3\theta\right)}{(\theta+\beta)^3}\end{aligned} \tag{6}$$

where the parameters θ and β are unknown. From the expression of R, if we get the estimator of the θ and β, then we can obtain the estimator of R.

III. The point estimates of R

A. The maximum likelihood estimator of R

Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ be two independent random samples of sizes n and m from the Ailamujia distribution with parameters θ and β, respectively. Then, the likelihood function is given by

$$L(\theta,\beta)=\prod_{i=1}^{n}\theta^2 x_i \exp\left(-\theta x_i\right)\prod_{j=1}^{m}\beta^2 y_j \exp\left(-\beta y_j\right) \tag{7}$$

Taking the logarithm for the above likelihood function in equation (7), we can get

$$InL(\theta,\beta)=\sum_{i=1}^{n}\ln x_i + 2n\ln\theta - \theta\sum_{i=1}^{n}x_i + \sum_{j=1}^{m}\ln y_i - 2m\ln\beta - \beta\sum_{j=1}^{m}y_j \tag{8}$$

Equating the partial derivative to zero to solve this equation, we can get

$$\frac{\partial InL(\theta,\beta)}{\partial\theta}=\frac{2n}{\theta}-\sum_{i=1}^{n}x_i=0 \tag{9}$$

and

$$\frac{\partial InL(\theta,\beta)}{\partial\beta}=\frac{2m}{\beta}-\sum_{j=1}^{m}y_j=0 \tag{10}$$

Then, the MLEs of θ and β, say $\hat{\theta}$ and $\hat{\beta}$, respectively, can be obtained as

$$\hat{\theta}=\frac{2n}{\sum_{i=1}^{n}x_i},\quad \hat{\beta}=\frac{2m}{\sum_{j=1}^{m}y_j} \tag{11}$$

Based on the equations (6) and (11), the estimator of R can be obtained as

$$\hat{R}=\frac{\hat{\beta}^2\left(\hat{\beta}+3\hat{\theta}\right)}{(\hat{\theta}+\hat{\beta})^3} \tag{12}$$

B. The bayesian estimator of R

In this section, we derive the bayesian estimator of R under the mean squared error loss function. Suppose that $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ are two independent random samples of sizes n and m, drawn from the Ailamujia distribution with parameters θ and β. Then, the likelihood functions of each sample can be obtained as

$$L_1(\theta)=\prod_{i=1}^{n}\theta^2 x_i \exp\left(-\theta x_i\right)\propto\theta^{2n}\exp\left(-\theta\sum_{i=1}^{n}x_i\right) \tag{13}$$

and

$$L_2(\beta)=\prod_{j=1}^{m}\beta^2 y_j \exp\left(-\beta y_j\right)\propto\beta^{2m}\exp\left(-\beta\sum_{j=1}^{m}y_j\right) \tag{14}$$

It is assumed that θ and β are independent random variables having gamma prior distributions with the following PDFs as

$$\pi(\theta)=\frac{b_1^{a_1}}{\Gamma(a_1)}\theta^{a_1-1}\exp(-b_1\theta),\ \theta>0,\ a_1>0, b_1>0 \tag{15}$$

and

$$\pi(\beta)=\frac{b_2^{a_2}}{\Gamma(a_2)}\beta^{a_2-1}\exp(-b_2\beta),\ \beta>0,\ a_2>0, b_2>0 \tag{16}$$

Therefore, the joint density of the $\theta, \beta, x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_m$ can be obtained as

$$L\left(\theta,\beta\middle|x_1,\cdots,x_n;y_1,\cdots,y_m\right)=L_1(\theta)\times L_2(\beta)\times\pi(\theta)\times\pi(\beta) \tag{17}$$

Based on the bayesian theorem, given the $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_m$, the joint posterior density of θ and β can be obtained as

$$\begin{aligned}\pi(\theta,\beta\mid X;Y)&=\pi\left(\theta,\beta\mid x_1,\cdots,x_n;y_1,\cdots,y_m\right)\\&=\frac{L\left(\theta,\beta\middle|x_1,\cdots,x_n;y_1,\cdots,y_m\right)}{\int_0^{+\infty}\int_0^{+\infty}L\left(\theta,\beta\middle|x_1,\cdots,x_n;y_1,\cdots,y_m\right)d\theta d\beta}\\&=\frac{d_1^{c_1}d_2^{c_2}}{\Gamma(c_1)\Gamma(c_2)}\theta^{c_1-1}\beta^{c_2-1}\exp\left(-d_1\theta-d_2\beta\right)\end{aligned} \tag{18}$$

Then, we can get the posterior PDFs of θ and β are as

$$\theta\mid x_1,\cdots,x_n\sim Gamma(c_1,d_1) \tag{19}$$

and

$$\beta\mid y_1,\cdots,y_m\sim Gamma(c_2,d_2) \tag{20}$$

where

$c_1=2n+a_1$, $d_1=\sum_{i=1}^{n}x_i+b_1$, $c_2=2m+a_2$, $d_2=\sum_{j=1}^{m}y_j+b_2$.

To obtain the Bayesian estimator of R, we use the idea of Smith and Roberrs [20] to generate a sample from the conditional posterior densities via the MCMC method. We adopt the following scheme:

Step 1): Generate θ_1 from $Gamma(c_1,d_1)$,

Step 2): Generate β_1 from $Gamma(c_2,d_2)$,

Step 3): Repeat Steps 1 and 2, M times.

Step 4): Let $R_k=\frac{\beta_k^2\left(\beta_k+3\theta_k\right)}{(\theta_k+\beta_k)^3}$, $k = 1, 2, \cdots, M$.

Under the squared error loss function, the approximate posterior mean, and posterior variance of R become

$$\widehat{E}(R)=\frac{1}{M}\sum_{k=1}^{M}R_k,\ \widehat{V}(R)=\frac{1}{M}\sum_{k=1}^{M}\left(R_k-\widehat{E}(R)\right)^2 \tag{21}$$

Similar to Chen and Shao [25], we can get the credible intervals of R as

$$\left(\hat{R}\left(\frac{\gamma}{2}\right),\hat{R}\left(1-\frac{\gamma}{2}\right)\right) \tag{22}$$

where $\hat{R}\left(\frac{\gamma}{2}\right)$ and $\hat{R}\left(1-\frac{\gamma}{2}\right)$ are the $\frac{\gamma}{2}$ and $1-\frac{\gamma}{2}$ quantiles of R in equation (12).

IV. THE INTERVAL ESTIMATES OF R

A. The asymptotic distribution approach

In this section, the asymptotic distribution of $\hat{R}$ is obtained. Based on the asymptotic distribution of $\hat{R}$, the asymptotic confidence interval of R is derived. By using the central limit theoremand and the delta method in equation (13), we can get

$$\hat{R}\sim N(R,\sigma_{\hat{R}}^2) \tag{23}$$

where $\sigma_{\hat{R}}^2=\left[G'I^{-1}G\right]_{\hat{\theta},\hat{\beta}}$, $G'=\left(\partial R/\partial\theta,\partial R/\partial\beta\right)$, and I^{-1} is the inverse of the Fisher's information matrix I about the parameters (θ, β), and

$$I=\begin{pmatrix}-E\dfrac{\partial^2 InL(\theta,\beta)}{\partial\theta^2} & -E\dfrac{\partial^2 InL(\theta,\beta)}{\partial\theta\partial\beta}\\ -E\dfrac{\partial^2 InL(\theta,\beta)}{\partial\beta\partial\theta} & -E\dfrac{\partial^2 InL(\theta,\beta)}{\partial\beta^2}\end{pmatrix} \tag{24}$$

where the partial derivatives are

$$E\frac{\partial^2 InL(\theta,\beta)}{\partial\theta^2}=-\frac{2n}{\theta^2},\ E\frac{\partial^2 InL(\theta,\beta)}{\partial\beta^2}=-\frac{2m}{\beta^2}$$

$$E\frac{\partial^2 InL(\theta,\beta)}{\partial\theta\partial\beta}=E\frac{\partial^2 InL(\theta,\beta)}{\partial\beta\partial\theta}=0,$$

$$G'=\left(\frac{\partial R}{\partial\theta},\frac{\partial R}{\partial\beta}\right)=\left(\frac{-6\theta\beta^2}{(\theta+\beta)^4},\quad\frac{6\beta\theta^2}{(\theta+\beta)^4}\right)$$

$$\sigma_{\hat{R}}^2=\left[G'I^{-1}G\right]_{\hat{\theta},\hat{\beta}}=\frac{\theta^2}{2n}\left(\frac{\partial R}{\partial\theta}\right)^2+\frac{\beta^2}{2m}\left(\frac{\partial R}{\partial\beta}\right)^2$$

$$=\frac{\theta^2}{2n}\left(\frac{-6\theta\beta^2}{(\theta+\beta)^4}\right)^2+\frac{\beta^2}{2m}\left(\frac{6\beta\theta^2}{(\theta+\beta)^4}\right)^2$$

$$=\frac{12\theta^4\beta^4}{(\theta+\beta)^8}\left(\frac{n+m}{nm}\right)$$

Then, we get the following asymptotically distribution as

$$\frac{\hat{R}-R}{\sigma_{\hat{R}}}\sim N(0,1) \tag{25}$$

We can use this asymptotic normality to construct an approximate $100(1-\gamma)\%$ confidence interval for R as

$$\left(\hat{R}-Z_{1-\gamma/2}\sigma_{\hat{R}},\hat{R}+Z_{1-\gamma/2}\sigma_{\hat{R}}\right) \tag{26}$$

where $Z_{1-\gamma/2}$ is the $(1-\gamma/2)$ quantile of the standard normal distribution.

B. *The bootstrap approach*

It is known that asymptotic confidence intervals based on the asymptotic results do not perform very well for small sample sizes. In this subsection, two bootstrap confidence intervals are proposed in this paper: (1) bootstrap-p method of Efron [21], and (2) bootstrap-t method of Hall [22].

Boot-p algorithm:

Step1: Generate random samples $x_1, x_2, \cdots, x_n$ from the Ailamujia distribution with parameters θ, and $y_1, y_2,\cdots,y_m$ from the same distribution with parameters β. Then, compute maximum likelihood estimators as $\hat{\theta}$ and $\hat{\beta}$.

Step2: Using parameter $\hat{\theta}$ generate a bootstrap sample $x_1^*,x_2^*,\cdots,x_n^*$, and similarly using parameter $\hat{\beta}$ generate a bootstrap sample $y_1^*,y_2^*,\cdots,y_m^*$. Based on these bootstrap samples compute bootstrap estimate of R using equation (23), say $\hat{R}^*$.

Step3: Repeat step 2, N boot times.

Step4: Let $H(x)=P(\hat{R}^*\le x)$ be the cumulative distribution function of $\hat{R}^*$. Define $\hat{R}_{Boot\_p}=H^{-1}(x)$ for a given x. The approximate $100(1-\gamma/2)\%$ bootstrap confidence interval of R is

$$\left(\hat{R}_{Boot\_p}\left(\frac{\gamma}{2}\right),\hat{R}_{Boot\_p}\left(1-\frac{\gamma}{2}\right)\right) \tag{27}$$

Boot-t algorithm:

Step1: Generate random samples $x_1, x_2, \cdots, x_n$ from the Ailamujia distribution with parameters θ, and $y_1, y_2, \cdots, y_m$ from the same distribution with parameters β. Then, compute maximum likelihood estimators as $\hat{\theta}$ and $\hat{\beta}$.

Step2: Using parameter $\hat{\theta}$ generate a bootstrap sample $x_1^*,x_2^*,\cdots,x_n^*$, and similarly using parameter $\hat{\beta}$ generate a bootstrap sample $y_1^*,y_2^*,\cdots,y_m^*$. Based on these bootstrap samples, compute the bootstrap estimate of R using equation (23), denoted as $\hat{R}^*$.

Step 3: Repeat Step 2, N times, and calculate the following statistic:

$$T^*=\frac{\sqrt{N}\left(\hat{R}^*-\hat{R}\right)}{\sqrt{V\left(\hat{R}^*\right)}}$$

where $\hat{R}$ and Var($\hat{R}^*$) are the mean and variance of $\{\hat{R}_1^*,\hat{R}_2^*,\cdots,\hat{R}_N^*\}$, respectively.

Step4: Let $H(x)=P(T^*\le x)$ be the cumulative distribution function of T^*.

Define $\hat{R}_{Boot\_t}=\hat{R}+N^{-\frac{1}{2}}\sqrt{V\left(\hat{R}\right)}H^{-1}(x)$ for a given x. The approximate $100(1-\gamma/2)\%$ bootstrap confidence interval of R is

$$\left(\hat{R}_{Boot\_t}\left(\frac{\gamma}{2}\right),\hat{R}_{Boot\_t}\left(1-\frac{\gamma}{2}\right)\right) \tag{28}$$

V. MONTE-CARLO SIMULATION

In this section, we use Monte Carlo simulations to compare the performance of different methods, mainly focusing on small sample sizes and different parameter values, such as (n, m)=(10, 10), (10, 20), (10, 30), (20, 10), (20, 20), (20, 30), (30, 10), (30, 20), (30, 30), and (θ, β)=(2, 0.5), (1.5, 0.5), (1.0, 0.5), (0.5, 0.5), (0.5, 0.1), (0.5, 1.0), (0.5, 1.5), and (0.5, 2.0). Under the different cases, we compare the performances of different estimators of R by using the average biases, mean squares errors, and the average confidence lengths.

For the MLE and Bayes estimator of R, we report the average biases and mean squared errors (MSEs) over 1000 replications. Based on the MLE of R, we also compute the 95% confidence intervals using the asymptotic distribution and report the average confidence lengths across 1000 replications.

Given the poor performance of asymptotic confidence intervals for small sample sizes, we additionally report 95% confidence intervals via Boot-p and Boot-t methods. The bootstrap intervals are obtained using 1000 bootstrap replications for both methods, and their average confidence lengths are reported.

Under the squared error loss function and Gamma prior distribution, the Bayes estimator of R and the associated credible interval are derived using 10000 replications. In this case, we compute the average biases, mean squared errors, and average lengths of simulated intervals for R. All results are reported in Tables I–VIII.

TABLE I
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=2.0, B=0.5, R=0.1040

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias) (MSE)	AVR(Bias) (MSE)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)
10	10	0.1159(0.0437) (0.0030)	0.1262(0.0576) (0.0052)	(0.0207,0.2111) (0.1904)	(0.0146,0.3028) (0.2882)	(0.0817,0.1262) (0.0445)	(0.0391,0.2841) (0.2450)
	20	0.1150(0.0334) (0.0019)	0.1269(0.0516) (0.0040)	(0.0325,0.1974) (0.1649)	(0.0369,0.2407) (0.2038)	(0.0925,0.1155) (0.0230)	(0.0454,0.2228) (0.1774)
	30	0.1066(0.0319) (0.0017)	0.1162(0.0471) (0.0043)	(0.0289,0.1843) (0.1554)	(0.0314,0.2914) (0.2600)	(0.0952,0.1128) (0.0176)	(0.0493,0.2164) (0.1671)
20	10	0.1197(0.0363) (0.0024)	0.1253(0.0511) (0.1253)	(0.0372,0.2021) (0.1649)	(0.0358,0.2839) (0.2481)	(0.0764,0.1316) (0.0552)	(0.0471,0.2413) (0.1942)
	20	0.1156(0.0306) (0.0014)	0.1160(0.0419) (0.0031)	(0.0482,0.1829) (0.1347)	(0.0345,0.2426) (0.2081)	(0.0924,0.1156) (0.0232)	(0.0516,0.2032) (0.1516)
	30	0.1088(0.0298) (0.0013)	0.1155(0.0365) (0.0020)	(0.0473,0.1702) (0.1229)	(0.0443,0.2201) (0.1758)	(0.0949,0.1130) (0.0181)	(0.0598,0.2078) (0.1480)
30	10	0.1165(0.0346) (0.0020)	0.1279(0.0470) (0.0040)	(0.0388,0.1943) (0.1555)	(0.0343,0.2643) (0.2300)	(0.0817,0.1263) (0.0446)	(0.0512,0.2484) (0.1972)
	20	0.1175(0.0292) (0.0013)	0.1213(0.0386) (0.0026)	(0.0560,0.1789) (0.1229)	(0.0518,0.2380) (0.1862)	(0.0927,0.1153) (0.0226)	(0.0534,0.2078) (0.1544)
	30	0.1151(0.0268) (0.0011)	0.1159(0.0341) (0.0020)	(0.0602,0.1701) (0.1099)	(0.0521,0.2269) (0.1748)	(0.0930,0.1150) (0.0220)	(0.0632,0.1850) (0.1218)

TABLE II
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=1.5, B=0.5, R=0.15625

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias) (MSE)	AVR(Bias) (MSE)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)
10	10	0.1789(0.0576) (0.0064)	0.1901(0.0897) (0.0143)	(0.048,0.3096) (0.2616)	(0.0544,0.4741) (0.4197)	(0.0914,0.2211) (0.1297)	(0.0665,0.3601) (0.2936)
	20	0.1732(0.0458) (0.0034)	0.1846(0.0715) (0.0085)	(0.0600,0.2865) (0.2265)	(0.0405,0.3876) (0.3471)	(0.1267,0.1858) (0.0591)	(0.0691,0.3209) (0.2518)
	30	0.1718(0.0432) (0.0029)	0.1768(0.0578) (0.0059)	(0.0651,0.2786) (0.2135)	(0.0574,0.3581) (0.3007)	(0.1324,0.1801) (0.0477)	(0.0738,0.2929) (0.2191)
20	10	0.17735(0.0598) 0(.00549)	0.17858(0.0801) (0.0113)	(0.0641,0.2906) (0.2265)	(0.0422,0.4512) (0.4090)	(0.1003,0.2122) (0.1119)	(0.0775,0.3473) (0.2698)
	20	0.1691(0.0440) (0.0029)	0.1608(0.0546) (0.0045)	(0.0767,0.2616) (0.1849)	(0.0657,0.2990) (0.2333)	(0.1331,0.1794) (0.0463)	(0.0884,0.2837) (0.1953)
	30	0.1690(0.0356) (0.0018)	0.1704(0.0482) (0.0039)	(0.0846,0.2534) (0.1688)	(0.0693,0.3269) (0.2576)	(0.1357,0.1768) (0.0411)	(0.0827,0.2828) (0.2001)
30	10	0.1897(0.0593) (0.0045)	0.1839(0.0703) (0.0077)	(0.0830,0.2965) (0.2135)	(0.0622,0.3849) (0.3227)	(0.1236,0.1889) (0.0653)	(0.0780,0.3445) (0.2665)
	20	0.1740(0.0415) (0.0030)	0.1769(0.0578) (0.0049)	(0.0897,0.2584) (0.1687)	(0.0773,0.3019) (0.2246)	(0.1368,0.1757) (0.0389)	(0.0921,0.2662) (0.1741)
	30	0.1698(0.0354) (0.0018)	0.1735(0.0493) (0.0037)	(0.0943,2453) (0.1510)	(0.0768,0.3043) (0.2275)	(0.1446,0.1679) (0.0233)	(0.1036,0.2695) (0.1659)

TABLE III
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=1.0, B=0.5, R=0.2593

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias) (MSE)	AVR(Bias) (MSE)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)
10	10	0.2867(0.0779) (0.0088)	0.3051(0.0116) (0.0168)	(0.1034,0.4703) (0.3669)	(0.0865,0.5718) (0.4853)	(0.1871,0.3314) (0.1443)	(0.1202,0.5160) (0.3958)
	20	0.2762(0.0603) (0.0062)	0.2981(0.0768) (0.0094)	(0.1022,0.4377) (0.3355)	(0.1119,0.4772) (0.3653)	(0.2144,0.3041) (0.0897)	(0.1297,0.4486) (0.3189)
	30	0.2612(0.0432) (0.0078)	0.2685(0.0578) (0.0059)	(0.1178,0.4203) (0.3025)	(0.1126,0.5544) (0.4418)	(0.2350,0.2835) (0.0205)	(0.1306,0.4283) (0.2977)
20	10	0.2780(0.0653) (0.0061)	0.2793(0.0872) (0.0116)	(0.1189,0.4370) (0.3181)	(0.0786,0.4753) (0.3967)	(0.2020,0.3165) (0.1145)	(0.1278,0.5131) (0.3853)
	20	0.2734(0.0610) (0.0061)	0.2723(0.0844) (0.0116)	(0.1436,0.4033) (0.2597)	(0.1252,0.5485) (0.4233)	(0.2278,0.2907) (0.0629)	(0.1645,0.4575) (0.2930)
	30	0.2808(0.0515) (0.0039)	0.2965(0.0758) (0.0074)	(0.1622,0.3993 (0.2371)	(0.1530,0.4619) (0.3089)	(0.2324,0.2861) (0.0537)	(0.1582,0.4136) (0.2554)
30	10	0.2856(0.0721) (0.0077)	0.2875(0.0933) (0.0141)	(0.1357,0.4356) (0.2999)	(0.0973,0.5943) (0.4970)	(0.2012,0.3173) (0.1161)	(0.1499,0.4872) (0.3373)
	20	0.2779(0.0511) (0.0039)	0.2807(0.0704) (0.0077)	(0.1594,0.3964) (0.2370)	(0.1351,0.4772) (0.3421)	(0.2291,0.2894) (0.0603)	(0.1666,0.4157) (0.2491)
	30	0.2723(0.0482) (0.0035)	0.2851(0.0677) (0.0067)	(0.1733,0.3854) (0.2121)	(0.1493,0.4700) (0.3207)	(0.2444,0.2741) (0.0297)	(0.1623,0.3916) (0.2293)

TABLE IV
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=0.5, Β=0.5, R=0.5

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias)	AVR(Bias)	(EL,CP)	(EL,CP)	(EL,CP)	(EL,CP)
		(MSE)	(MSE)	(Length)	(Length)	(Length)	(Length)
10	10	0.5008(0.1025)	0.5091(0.1399)	0.2684,0.7333)	(0.1993,0.8102)	(0.4267,0.5733)	0.2716,0.7623)
		(0.0161)	(0.0279)	(0.4649)	(0.6109)	(0.1466)	(0.4907)
	20	0.4922(0.0853)	0.4959(0.1145)	0.2909,0.6935)	(0.2409,0.7602)	(0.4701,0.5299)	0.2681,0.6958)
		(0.0111)	(0.0195)	(0.4026)	(0.5193)	(0.0598)	(0.4277)
	30	0.4953(0.0860)	0.4976(0.1046)	0.3055,0.6851)	(0.2419,0.7584)	(0.4699,0.5301)	0.2905,0.6883)
		(0.0115)	(0.0170)	(0.3796)	(0.5165)	(0.0602)	(0.3978)
20	10	0.4856(0.0721)	0.4875(0.0933)	(0.3357,0.5356)	(0.0973,0.5943)	(0.4374,0.5626)	0.3091,0.7203)
		(0.0077)	(0.0141)	(0.2999)	(0.4970)	(0.1252)	(0.4112)
	20	0.4779(0.0511)	0.4807(0.0704)	(0.3594,0.5964)	(0.1351,0.4772)	(0.4637,0.5363)	0.3097,0.6880)
		(0.0039)	(0.0077)	(0.2370)	(0.3421)	(0.0726)	(0.3783)
	30	0.4723(0.0482)	0.4851(0.0677)	(0.3733,0.5854)	(0.1493,0.4700)	(0.4726,0.5274)	0.3520,0.6562)
		(0.0035)	(0.0067)	(0.2121)	(0.3207)	(0.0548)	(0.3042)
30	10	0.5064(0.0881)	0.4977(0.1176)	(0.3166,0.6962)	(0.2391,0.7593)	(0.4438,0.5562)	0.3080,0.7037)
		(0.0119)	(0.0205)	(0.3796)	(0.5202)	(0.1124)	(0.3957)
	20	0.4998(0.0609)	0.4958(0.0849)	(0.3498,0.6499)	(0.2969,0.6952)	(0.4681,0.5319)	0.3369,0.6729)
		(0.0058)	(0.0111)	(0.3001)	(0.3983)	(0.0638)	(0.3360)
	30	0.5047(0.0595)	0.4988(0.0768)	(0.3705,0.6389)	(0.3102,0.6800)	(0.4755,0.5245)	(0.3568,0.6381)
		(0.0055)	(0.0090)	(0.2684)	(0.3698)	(0.0490)	(0.2813)

TABLE V
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=0.5, Β=0.1, R=0.0741

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias)	AVR(Bias)	(EL,CP)	(EL,CP)	(EL,CP)	(EL,CP)
		(MSE)	(MSE)	(Length)	(Length)	(Length)	(Length)
10	10	0.0747(0.0289)	0.0839(0.0430)	(0.0030,0.1464)	(0.0148,0.2506)	(0.0430,0.1052)	(0.0205,0.1976)
		(0.0015)	(0.0036)	(0.1434)	(0.2358)	(0.0622)	(0.1771)
	20	0.0743(0.0288)	0.0824(0.0397)	(0.0122,0.1365)	(0.0160,0.2306)	(0.0614,0.0868)	(0.0256,0.1479)
		(0.0015)	(0.0030)	(0.1243)	(0.2146)	(0.0254)	(0.1223)
	30	0.0753(0.0253)	0.0784(0.0342)	(0.0167,0.1338)	(0.0172,0.1729)	(0.0614,0.0868)	(0.0256,0.1479)
		(0.0011)	(0.0019)	(0.1171)	(0.1557)	(0.0254)	(0.1223)
20	10	0.0788(0.0267)	0.0850(0.0376)	(0.0167,0.1409)	(0.0212,0.2270)	(0.0419,0.1062)	(0.0267,0.1564)
		(0.0012)	(0.0027)	(0.1242)	(0.2058)	(0.0643)	(0.1297)
	20	0.0735(0.0205)	0.0777(0.0292)	(0.0227,0.1242)	(0.0212,0.1745)	(0.0659,0.0823)	(0.0318,0.1349)
		(0.0007)	(0.0015)	(0.1015)	(0.1533)	(0.0164)	(0.1031)
	30	0.0712(0.0185)	0.0765(0.0267)	(0.0247,0.1175)	(0.0265,0.1637)	(0.0658,0.0823)	(0.0325,0.1206)
		(0.0005)	(0.0013)	(0.0928)	(0.1372)	(0.0165)	(0.0881)
30	10	0.0758(0.0244)	0.0771(0.0344)	(0.0172,0.1343)	(0.0184,0.1887)	(0.0550,0.0932)	(0.0311,0.1530)
		(0.0011)	(0.0019)	(0.1171)	(0.1703)	(0.0382)	(0.1219)
	20	0.0714(0.0191)	0.0741(0.0269)	(0.0251,0.1177)	(0.0266,0.1552)	(0.0661,0.0820)	(0.0356,0.1284)
		(0.0006)	(0.0012)	(0.0926)	(0.1286)	(0.0159)	(0.0928)
	30	0.0708(0.0175)	0.0743(0.0256)	(0.0283,0.1122)	(0.0272,0.1529)	(0.0653,0.0829)	(0.0347,0.1289)
		(0.0005)	(0.0011)	(0.0839)	(0.1257)	(0.0176)	(0.0942)

TABLE VI
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=0.5, Β=1.0, R=0.7407

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias)	AVR(Bias)	(EL,CP)	(EL,CP)	(EL,CP)	(EL,CP)
		(MSE)	(MSE)	(Length)	(Length)	(Length)	(Length)
10	10	0.7175(0.0814)	0.7108(0.1116)	(0.5338,0.9011)	(0.3662,0.9194)	(0.6841,0.7974)	(0.4740,0.8682)
		(0.0106)	(0.0202)	(0.3673)	(0.5532)	(0.1133)	(0.3942)
	20	0.7075(0.0762)	0.70340.0962)	(0.5485,0.8665)	(0.4332,0.8880)	(0.7007,0.7808)	(0.5028,0.8726)
		(0.0092)	0.0147)	(0.3180)	(0.4548)	(0.0801)	(0.3698)
	30	0.7221(0.0619)	0.71950.0865)	(0.5722,0.8721)	(0.4824,0.9003)	(0.7151,0.7664)	(0.5390,0.8654)
		(0.0061)	0.0117)	(0.2999)	(0.4179)	(0.0513)	(0.3264)
20	10	0.7291(0.0670)	0.7234(0.0849)	(0.5700,0.8881)	(0.4655,0.9072)	(0.6895,0.7920)	(0.5189,0.8663)
		(0.0070)	(0.0117)	(0.3181)	(0.4417)	(0.1025)	(0.3474)
	20	0.7145(0.0605)	0.7075(0.0843)	(0.6034,0.8445)	(0.4752,0.8800)	(0.7105,0.7710)	(0.5797,0.8398)
		(0.0053)	(0.0104)	(0.2411)	(0.4048)	(0.0605)	(0.2601)
	30	0.7219(0.0540)	0.7199(0.0704)	(0.5134,0.8405)	(0.5134,0.8616)	(0.7177,0.7638)	(0.5873,0.8394)
		(0.0043)	(0.0078)	(0.3271)	(0.3482)	(0.0461)	(0.2521)
30	10	0.7273(0.0639)	0.7148(0.0907)	(0.5774,0.8773)	(0.4649,0.8882)	(0.6843,0.7971)	(0.5493,0.8435)
		(0.0066)	(0.0127)	(0.2999)	(0.4233)	(0.1123)	(0.2942)
	20	0.7225(0.0516)	0.7147(0.0670)	(0.6039,0.8410)	(0.5397,0.8609)	(0.7084,0.7731)	(0.5639,0.8435)
		(0.0040)	(0.0071)	(0.2371)	(0.3212)	(0.0647)	(0.2796)
	30	0.7245(0.0491)	0.7232(0.0656)	(0.6189,0.8309)	(0.5196,0.8532)	(0.7190,0.7625)	(0.5975,0.8223)
		(0.0037)	(0.0067)	(0.2120)	(0.3336)	(0.0435)	(0.2248)

TABLE VII
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=0.5, B=1.5, R=0.84375

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias) (MSE)	AVR(Bias) (MSE)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)
10	10	0.8221(0.0609) (0.0061)	0.8129(0.0861) (0.0117)	(0.6914,0.9528) (0.2614)	(0.5339,0.9651) (0.4312)	(0.7909,0.8966) (0.1057)	(0.6488,0.9325) (0.2837)
	20	0.8199(0.0564) (0.0052)	0.8184(0.0744) (0.0096)	(0.7067,0.9331) (0.2264)	(0.5756,0.9516) (0.3760)	(0.8189,0.8686) (0.0497)	(0.6509,0.9195) (0.2686)
	30	0.8215(0.0521) (0.0045)	0.8203(0.0691) (0.0079)	(0.7147,0.9282) (0.2135)	(0.6183,0.9461) (0.3278)	(0.8138,0.8737) (0.0599)	(0.6673,0.9197) (0.2524)
20	10	0.8254(0.0520) (0.0044)	0.8153(0.0737) (0.0087)	(0.7122,0.9386) (0.2264)	(0.5988,0.9520) (0.3532)	(0.8125,0.8750) (0.0625)	(0.7129,0.9155) (0.2026)
	20	0.8261(0.0449) (0.0032)	0.8192(0.0624) (0.0064)	(0.7337,0.9186) (0.1849)	(0.6374,0.9371) (0.2997)	(0.8292,0.8583) (0.0291)	(0.7259,0.9032) (0.1733)
	30	0.8259(0.0401) (0.0025)	08260(0.0539) (0.0046)	(0.7415,0.9103) (0.1688)	(0.6613,0.9342) (0.2729)	(0.8284,0.8591) (0.0307)	(0.7238,0.9098) (0.1852)
30	10	0.8260(0.0498) (0.0038)	0.8192(0.0646) (0.0068)	(0.7192,0.9327) (0.2135)	(0.6264,0.9432) (0.3168)	(0.8172,0.8703) (0.0531)	(0.7017,0.9344) (0.2327)
	20	0.8280(0.0393) (0.0022)	0.8234(0.0538) (0.0044)	(0.7436,0.9124) (0.1688)	(0.6746,0.9283) (0.2537)	(0.8021,0.8854) (0.0833)	(0.6982,0.9291) (0.2309)
	30	0.8292(0.0347) (0.0018)	0.8248(0.0483) (0.0034)	(0.7537,0.9047) (0.1510)	(0.6883,0.9207) (0.2324)	(0.8258,0.8617) (0.0359)	(0.7478,0.8971) (0.1493)

TABLE VIII
AVR, BIAS, MSE, EL AND CP UNDER TRUE VALUES Θ=0.5, B=2.0, R=0.8960

		MLE	MCMC	MLE	MCMC	Boot-t	Boot-p
n	m	AVR(Bias) (MSE)	AVR(Bias) (MSE)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)	(EL,CP) (Length)
10	10	0.8806(0.0446) (0.0035)	0.8766(0.0613) (0.0069)	(0.7854,0.9758) (0.1904)	(0.6435,0.9756) (0.3321)	(0.8466,0.9454) (0.0988)	(0.7319,0.9632) (0.2313)
	20	0.8779(0.0405) (0.0029)	0.8738(0.0536) (0.0051)	(0.7954,0.9603) (0.1649)	(0.6936,0.9669) (0.2733)	(0.8759,0.9161) (0.0402)	(0.7412,0.9563) (0.2151)
	30	0.8789(0.0377) (0.0023)	0.8746(0.0509) (0.0047)	(0.8012,0.9567) (0.1555)	(0.6895,0.9708) (0.2813)	(0.8752,0.9168) (0.0416)	(0.7559,0.9546) (0.1987)
20	10	0.8838(0.0357) (0.0020)	0.8730(0.0567) (0.0057)	(0.8013,0.9662) (0.1649)	(0.6768,0.9694) (0.2926)	(0.8679,0.9241) (0.0562)	(0.7934,0.9536) (0.1602)
	20	0.8853(0.0310) (0.0016)	0.8784(0.0419) (0.0029)	(0.8179,0.9186) (0.1007)	(0.7468,0.9569) (0.2101)	(0.8649,0.9271) (0.0622)	(0.7931,0.9461) (0.1530)
	30	0.8837(0.0291) (0.0013)	08808(0.0408) (0.0025)	(0.8222,0.9451) (0.1229)	(0.7664,0.9575) (0.1911)	(0.8836,0.9084) (0.0248)	(0.8012,0.9412) (0.1400)
30	10	0.8814(0.0306) (0.0014)	0.8787(0.0427) (0.0028)	(0.8199,0.9428) (0.1229)	(0.7540,0.9555) (0.2015)	(0.8765,0.9155) (0.0390)	(0.7854,0.9523) (0.1669)
	20	0.8856(0.0294) (0.0013)	0.8795(0.0416) (0.0029)	(0.8242,0.9471) (0.1229)	(0.7584,0.9575) (0.1991)	(0.8820,0.9100) (0.0280)	(0.7914,0.9405) (0.1491)
	30	0.8844(0.0265) (0.0011)	0.8790(0.0366) (0.0021)	(0.8294,0.9393) (0.1099)	(0.7657,0.9493) (0.1836)	(0.8834,0.9086) (0.0252)	(0.8108,0.9375) (0.1267)

TABLE IX
GOODNESS OF FIT FOR GIVEN DATA SET

sample	K-S	P value
X	0.1786	0.3165
Y	0.1225	0.7319

TABLE X
DIFFERENT POINT ESTIMATE VALUE FOR GIVEN DATA SET

	MLE	MCMC
Alpha x	0.0260	0.0309
Alpha y	0.0239	0.0282
R	0.4687	0.4657

TABLE XI
DIFFERENT INTERVAL ESTIMATE VALUE FOR GIVEN DATA SET

	MLE interval of R	MCMC interval of R	Boot-t interval of R	Boot-p interval of R
R	(0.4326,0.4846)	(0.5282,0.5947)	(0.4429,0.4945)	(0.5460,0.6054)
Length	0.0520	0.0665	0.0516	0.0594

From the above simulation results, we can find that: (1) For the point estimation of R, the MLE value is very close to the Bayesian estimate value. However, compared to the Bayesian estimator, the MLE exhibits smaller bias and MSE. Thus, we conclude that the MLE outperforms the Bayesian estimator in small sample scenarios. (2) For the interval estimation of R, all true reliability values are contained within the estimated intervals, demonstrating the effectiveness of the interval estimation methods.

By analyzing the length of different interval estimates, we find that the Boot-t method produces the shortest intervals, followed by the MLE method, then the Boot-p method; the Bayesian method has the longest interval length. Thus, we recommend the Boot-t method for interval estimation. (3) When fixing n and increasing m, or fixing m and increasing n, both point estimators (MLE and Bayesian estimator) approach the true value more closely, but no clear trend in R is observed. For interval estimation, however, the lengths of all interval estimates consistently decrease as n or m increases in most cases. (4) When fixing β and increasing θ for the same n and m, the bias and MSE of both point estimators (MLE and Bayesian estimator) increase, and the lengths of all four interval estimators also increase. (5) When fixing θ and increasing β for the same n and m, the bias and MSE of both point estimators decrease, and the lengths of all four interval estimators also decrease.

VI. Real data analysis

The following real-life data sets, from Ref [23], represent the monthly concentration of sulfur dioxide in Long Beach, California, from 1956 to 1974. In Ref [24], the Weibull distribution is used to fit the data sets, where X and Y denote the sulfur dioxide concentrations in March and August as follow:

X: 97, 51, 11, 4, 141, 18, 142, 24, 191, 68, 77, 80, 1, 16, 106, 206, 163, 18, 82, 54, 31, 216, 46, 111, 39, 18, 63

Y: 90, 10, 60, 186, 61, 49, 14, 24, 208, 130, 56, 20, 79, 84, 44, 59, 29, 118, 101, 208, 25, 156, 310, 76, 26, 44, 23, 62, 70

Let X and Y denote the 1-hour average concentrations of sulphur dioxide in May and October, respectively. Then R=P(Y<X) represents the probability that the concentration in May is lower than that in October. In this paper, we use the Ailamujia distribution to fit the data sets.

We also tested goodness-of-fit for each data set separately using the Kolmogorov-Smirnov (K-S) test. We observe that for the data set of X, the K-S distance is 0.1786 with a p-value of 0.3165, and for the data set of Y, the K-S distance is 0.1225 with a p-value of 0.7319. Since both p-values exceed the common significance level of 0.05, the two data sets show a reasonable fit to the Ailamujia distribution based on the K-S test results.

Table IX gives the K-S test statistics with p-values for each data set. From the Table IX, we do not reject the null hypotheses that the data sets are drawn from the Ailamujia distributions at the significance level α=0.05.

Also, the Table X gives the point estimates of R, and Table XI gives the 95% confidence interval for R. From Tables X and XI, we find that the Boot-t method produces the shortest intervals among all methods. This finding from the real data analysis is consistent with the results obtained from the simulation study. Based on the above analysis, there is no sufficient evidence to conclude that the sulfur dioxide concentration in May is lower than that in October.

V. Conclusions

In this article, the reliability estimation of R=P(Y<X) for the Ailamujia distribution is discussed. The MLE and Bayesian estimator of R are derived, and several confidence intervals for R are also obtained. Some Monte Carlo simulations are conducted to compare different estimation methods. By analyzing the simulation results, we find that the MLE outperforms the Bayesian estimator in small sample scenarios. In addition, the Boot-t interval estimation method has the shortest intervals and it is recommended. Finally, a real data set is analyzed to illustrate the proposed model and methods.

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