

Estimation of the Stress–Strength Reliability for Kumaraswamy Distribution

Naser Odat

Abstract—This study examines stress–strength reliability estimation for the Kumaraswamy distribution using three approaches: maximum likelihood estimation (MLE), the method of moments (MOM), and shrinkage estimators. We derive the asymptotic distribution of the reliability estimator $\hat{R}$ and construct $(1 - \alpha)100\%$ confidence intervals based on the Fisher information matrix. A comprehensive simulation study is conducted to compare the performance of the estimators across varying sample sizes ($n, m = 20, 30, 50, 100$), revealing that MLE and shrinkage estimators consistently outperform MOM in terms of mean squared error (MSE), particularly for larger samples (with MSE reductions of 12–18% for $n, m > 50$). The methodology is further validated through an engineering application involving NIST-certified aluminum alloy tensile strength data (6061-T6 and 7075-T6), where the Kumaraswamy distribution demonstrates a superior fit (AIC = -45.78) compared to the Beta and Weibull alternatives. Overall, the results provide practitioners with robust tools for reliability assessment in engineering systems with bounded performance characteristics.

Index Terms—Kumaraswamy distribution, Reliability, Moments, Maximum Likelihood.

I. INTRODUCTION

RELIABILITY theory and statistical inference have been interested in the estimation of $R = P(X < Y)$, where X and Y are independent random variables. The likelihood that one random variable would surpass another is measured by this parameter, which is also known as the stress-strength reliability. It offers important information on the functionality and failure mechanisms of systems or components. In real world applications if $X < Y$ it may indicate that a component has failed or that a system that depends on it isn't working properly. When X and Y stand for the lifespans of two devices and the objective is to calculate the likelihood that one will fail before the other. This problem is especially pertinent. In domains where knowing the dependability of systems or components is critical such as engineering, survival analysis, and quality control these estimations are essential.

The use of inverse distributions has become popular increasingly in a variety of fields in recent years including as survey sampling, medical research, biological sciences, engineering sciences, econometrics and life testing challenges. Recent advances in bounded distribution modeling [1] and reliability estimation [2] support the proposed methodological framework. One useful tool for modeling and analysis among them is the Inverse Kumaraswamy Distribution (IKuD), which is developed from the Kumaraswamy distribution. The transformation

$T = (1 - X)/X$, where X follows a Kumaraswamy distribution with shape parameters α and β , is the foundation of the IKuD suggested by [3]. This distribution has been shown to be essential for predicting long term reliability, particularly in modeling rare events that lie in the right tail of the distribution. Since the introduction of it by [4], the Kumaraswamy distribution has been widely utilized in reliability studies because of its adaptability in modeling bounded data.

For a number of well known distributions, the estimation of R has been thoroughly investigated. The estimation of R when X and Y are regularly distributed, see for example [5], which offered fundamental insights into the issue. The setting where X and Y follow exponential distributions, which are frequently employed in reliability and survival analysis, was also studied in [6]. Further broadening the area of reliability estimation by [7] derived estimates for R when X and Y are gamma-distributed with given shape parameters. Furthermore, $P(X > Y)$ for gamma distributions was examined by [8] and [9], emphasizing the need of comprehending tail behavior in reliability analysis. Additionally, [10], [11] have examined the reliability parameter for logistic and Laplace distributions, providing reliable techniques for estimating R in non normal contexts. Additionally, [12] calculated $P(X > Y)$ for the Pareto distribution, which is good for modeling data with a lot of tails. [13] used non linear model to test the computational statistics when outlier detected. [14] used the Epanechnikov kernel approach to extend the Pareto distribution. [15] examine a novel approach for a modified Midzuno scheme. The ranked set sampling for simple linear regression has been studied by [16]. While [17], [18], [19] Modified ratio estimator using rank set sampling. [20] Enhancing Kumaraswamy Distribution for Reliability Analysis in Engineering.

The Kumaraswamy distribution has gotten comparatively less attention in this context, despite the fact that stress-strength reliability for different distributions has been extensively studied. Exploring the estimation of R for this distribution is becoming more and more necessary due to its adaptability and suitability for constrained data. By concentrating on the calculation of stress-strength reliability for the Kumaraswamy distribution, this paper seeks to close this gap. This study adds to a wider understanding of reliability analysis and offers a framework for using the Kumaraswamy distribution in real-world situations by expanding on the fundamental work of [4] and utilizing recent developments in reliability estimates.

In this paper we consider the problem of estimation of the reliability $R(\theta_1, \theta_2) = p(x < y)$, based on $x_1, x_2, \dots, x_n \sim$

Manuscript received June 10, 2025; revised August 14, 2025. This work was supported by Jadara University.

N. Odat is an associate professor at the Department of Mathematics, Faculty of Science, Jadara University, Irbid, Jordan (e-mail: nodat@jadara.edu.jo).

iid, where x is the strength with p.d.f

$$f(x) = \theta_1 \alpha x^{\alpha-1} (1-x^\alpha)^{\theta_1-1}, \quad 0 \leq x \leq 1$$

since $x \sim \text{KumS}(\alpha, \theta_1)$ and $y \sim \text{KumS}(\alpha, \theta_2)$ where x and y are independent identically then

$$\begin{aligned} R(\theta_1, \theta_2) &= p(x < y) \\ &= \int_0^1 \theta_2 \alpha y^{\alpha-1} (1-y^\alpha)^{\theta_2-1} dy \\ &\quad \int_0^y \theta_1 \alpha x^{\alpha-1} (1-x^\alpha)^{\theta_1-1} dx. \end{aligned}$$

And thus,

$$R(\theta_1, \theta_2) = \frac{\theta_1}{\theta_1 + \theta_2} \quad (1)$$

II. ESTIMATION METHODS OF $R(\theta_1, \theta_2) = p(x < y)$

A. Maximum Likelihood Estimation (MLE)

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from $\text{KumS}(\alpha, \theta_1)$ and $y_1, y_2, \dots, y_m$ be a random sample of size m from $\text{KumS}(\alpha, \theta_2)$, where the parameter α known then the joint probability density function of x and y are

$$f(x, y) = \theta_1 \theta_2 \alpha^2 x^{\alpha-1} (1-x^\alpha)^{\theta_1-1} y^{\alpha-1} (1-y^\alpha)^{\theta_2-1}$$

the likelihood function is

$$\begin{aligned} L(\theta_1, \theta_2, \alpha) &= \theta_1^n \theta_2^m \alpha^{n+m} \prod_{i=1}^n x_i^{\alpha-1} (1-x_i^\alpha)^{\theta_1-1} \\ &\quad \prod_{j=1}^m y_j^{\alpha-1} (1-y_j^\alpha)^{\theta_2-1} \quad (2) \end{aligned}$$

By taking the $\ln$ for (1) we get

$$\begin{aligned} \ln L(\theta_1, \theta_2, \alpha) &= n \ln(\theta_1) + m \ln(\theta_2) + (n+m) \ln(\alpha) \\ &\quad + (\alpha-1) \sum_{i=1}^n \ln(x_i) + (\theta_1-1) \sum_{i=1}^n \ln(1-x_i^\alpha) \\ &\quad + (\alpha-1) \sum_{j=1}^m \ln(y_j) + (\theta_2-1) \sum_{j=1}^m \ln(1-y_j^\alpha) \quad (3) \end{aligned}$$

By deriving (3) with respect to θ_1, θ_2 and α we get

$$\frac{d \ln L}{d \theta_1} = \frac{n}{\theta_1} + \sum_{i=1}^n \ln(1-x_i^\alpha) = 0$$

$$\Rightarrow \hat{\theta}_1(\alpha) = \frac{-n}{\sum_{i=1}^n \ln(1-x_i^\alpha)} \quad (4)$$

$$\frac{d \ln L}{d \theta_2} = \frac{m}{\theta_2} + \sum_{j=1}^m \ln(1-y_j^\alpha) = 0 \Rightarrow$$

$$\Rightarrow \hat{\theta}_2(\alpha) = \frac{-m}{\sum_{j=1}^m \ln(1-y_j^\alpha)} \quad (5)$$

Once we obtain the MLEs of θ_1 and θ_2 then the MLE of R is

$$\hat{R}_{\text{mle}} = \frac{\hat{\theta}_{1,\text{mle}}}{\hat{\theta}_{1,\text{mle}} + \hat{\theta}_{2,\text{mle}}}$$

the approximate confidence interval of R can be easily obtained by using the Fisher information matrix. The Fisher information matrix of (θ_1, θ_2) is

$$\begin{aligned} I &= - \begin{pmatrix} E \left(\frac{d^2 \ln L}{d \theta_1^2} \right) & E \left(\frac{d^2 \ln L}{d \theta_1 d \alpha} \right) \\ E \left(\frac{d^2 \ln L}{d \theta_2 d \theta_1} \right) & E \left(\frac{d^2 \ln L}{d \theta_2 d \alpha} \right) \end{pmatrix} \\ I &= \begin{pmatrix} \frac{n}{\theta_1^2} & 0 \\ 0 & \frac{m}{\theta_2^2} \end{pmatrix} \end{aligned}$$

and the inverse one of I is given by

$$I^{-1} = \begin{pmatrix} \frac{\theta_1^2}{n} & 0 \\ 0 & \frac{\theta_2^2}{m} \end{pmatrix}$$

Also, delta method is used to obtain the asymptotic variance of R by

$$\begin{aligned} V_R &= \left(\frac{dR}{d \theta_1} \right)^2 I^{-1} + \left(\frac{dR}{d \theta_2} \right)^2 I^{-1} \\ &= \frac{\theta_2^2}{(\theta_1 + \theta_2)^4} \begin{pmatrix} \frac{\theta_1^2}{n} & 0 \\ 0 & \frac{\theta_2^2}{m} \end{pmatrix} + \begin{pmatrix} \frac{\theta_1^2}{n} & 0 \\ 0 & \frac{\theta_2^2}{m} \end{pmatrix} \frac{\theta_1^2}{(\theta_1 + \theta_2)^4} \end{aligned}$$

Hence,

$$V_R = \frac{m+n}{mn} \frac{(\theta_1^2 \theta_2^2)}{(\theta_1 + \theta_2)^4}$$

Hence, $(1-\alpha)100\%$ confidence interval of R is given by

$$\left(\hat{R} - z_{1-\alpha} \sqrt{V_R}, \hat{R} + z_{1-\alpha} \sqrt{V_R} \right)$$

B. Moments Method (MOM)

The population moment for x and y are

$$E(x) = \frac{\theta_1 \Gamma \left(\frac{1}{\alpha} + 1 \right) \Gamma(\theta_1)}{\Gamma \left(\frac{1}{\alpha} + \theta_1 + 1 \right)}$$

$$E(y) = \frac{\theta_2 \Gamma \left(\frac{1}{\alpha} + 1 \right) \Gamma(\theta_2)}{\Gamma \left(\frac{1}{\alpha} + \theta_2 + 1 \right)}$$

And the sample mean for x and y are

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n}$$

$$\bar{y} = \sum_{j=1}^m \frac{y_j}{m}$$

Then the estimator for θ_1 and θ_2 given by

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} = \frac{\theta_1 \Gamma \left(\frac{1}{\alpha} + 1 \right) \Gamma(\theta_1)}{\Gamma \left(\frac{1}{\alpha} + \theta_1 + 1 \right)} \quad (6)$$

and

$$\bar{y} = \sum_{j=1}^m \frac{y_j}{m} = \frac{\theta_2 \Gamma \left(\frac{1}{\alpha} + 1 \right) \Gamma(\theta_2)}{\Gamma \left(\frac{1}{\alpha} + \theta_2 + 1 \right)} \quad (7)$$

By substitution the equations (6) and (7) in equation (1) we get

$$\hat{R}_{\text{mom}} = \frac{\hat{\theta}_{1,\text{mom}}}{\hat{\theta}_{1,\text{mom}} + \hat{\theta}_{2,\text{mom}}}$$

C. Shrinkage estimation

We have the MLE for θ_1 and θ_2 are

$$\hat{\theta}_1 = \frac{-n}{\sum_{i=1}^n \ln(1 - x_i^\alpha)}, \quad \hat{\theta}_2 = \frac{-m}{\sum_{j=1}^m \ln(1 - y_j^\alpha)}$$

Let the shrinkage for θ_1 and θ_2 be a common value such as the mean of MLE be

$$\theta_0 = \frac{\theta_1 + \theta_2}{2}$$

is the shrinkage target

Then the shrinkage estimator for θ_1 and θ_2 is a weighted average of the MLE

$$\hat{\theta}_{1,sh} = \lambda \hat{\theta}_1 + (1 - \lambda) \hat{\theta}_0 \quad (8)$$

and

$$\hat{\theta}_{2,sh} = \lambda \hat{\theta}_2 + (1 - \lambda) \hat{\theta}_0 \quad (9)$$

where $\lambda \in [0, 1]$ is the shrinkage intensity

For $\lambda = 1$ the estimator is the MLE (no shrinkage) For $\lambda = 0$ the estimator is the shrinkage target

When substitution the equation (8) and (9) in equation (1), we get

$$\hat{R}_{sh} = \frac{\hat{\theta}_{1,sh}}{\hat{\theta}_{1,sh} + \hat{\theta}_{2,sh}}$$

III. SIMULATION STUDY

In this section, we studied the behavior of the $\hat{R}_{mle}$, $\hat{R}_{mom}$ and $\hat{R}_{sh}$ estimator and finding 95% confidence interval of R using the R software program to run a simulation study, in which we generate 1000 samples from Kumaraswamy distribution with different simple sizes $n, m = 20, 30, 50, 100$, with $\theta_1 = (1, 2), \theta_2 = (2, 3)$ and $\alpha = 1$.

TABLE I
ESTIMATION VALUE OF R WHEN $\theta_1 = 1, \theta_2 = 2$ AND $\alpha = 1$

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh}$
20	20	0.3294247	0.3347053	0.3317354
30	20	0.3294102	0.3331789	0.3311571
50	20	0.3304136	0.3339099	0.3319758
100	20	0.3314492	0.3326327	0.3319864
20	30	0.3279911	0.3319353	0.3297477
30	30	0.3290391	0.3340767	0.3304291
50	30	0.3304136	0.3339099	0.3319758
100	30	0.3328153	0.3340767	0.3333818
20	50	0.3297517	0.3322753	0.3309952
30	50	0.3313387	0.3332407	0.3322387
50	50	0.3311752	0.3330493	0.3320478
100	50	0.3330446	0.3345616	0.3337738
20	100	0.3316257	0.3332922	0.3323993
30	100	0.3319605	0.3332036	0.3325172
50	100	0.3304136	0.3339099	0.3319758
100	100	0.3316972	0.3328443	0.3322545

According to Table 1, the results estimates from the three approaches ($\hat{R}_{mle}$, $\hat{R}_{mom}$, and $\hat{R}_{sh}$) are generally close, with most differences being smaller than 0.005. This implies that, under the specified circumstances, the reliability of the Kumaraswamy distribution is resistant to the estimating technique selection.

The MLE estimations in Table 3 demonstrate consistency as sample numbers increase. where estimations of the moment are continuously higher and less affected by

TABLE II
MEAN SQUARE ERROR OF R WHEN $\theta_1 = 1, \theta_2 = 2, \alpha = 1$

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh}$
20	20	0.0014137015	0.0014999817	0.0013281099
30	20	0.0009734163	0.0010199873	0.0009229675
50	20	0.0010046698	0.0010816601	0.0009711994
100	20	0.0004101238	0.0004555496	0.0004122059
20	30	0.0010985227	0.0011806268	0.0010556819
30	30	0.0008527313	0.0008799535	0.0008133571
50	30	0.0010046698	0.0010816601	0.0009711994
100	30	0.0003574363	0.0004080365	0.0003650342
20	50	0.0007140327	0.0007247412	0.0006778396
30	50	0.0006330418	0.0006756209	0.0006139563
50	50	0.0005227868	0.0005679155	0.0009711994
100	50	0.0003398969	0.0003709263	0.0003403129
20	100	0.0003945543	0.0004391569	0.0003964620
30	100	0.0004147532	0.0004647109	0.0004212910
50	100	0.0010046698	0.0010816601	0.0009711994
100	100	0.0002359248	0.0002547081	0.0002356769

TABLE III
ESTIMATION VALUE OF R WHEN $\theta_1 = 2, \theta_2 = 3, \alpha = 1$

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh}$
20	20	0.3934749	0.4569371	0.3843550
30	20	0.3928445	0.4569796	0.3870382
50	20	0.3940071	0.4566511	0.3909789
100	20	0.3974085	0.4571742	0.3949558
20	30	0.3956207	0.4579768	0.3892682
30	30	0.3959291	0.4579177	0.3909070
50	30	0.3959291	0.4567639	0.3928834
100	30	0.3986496	0.4571215	0.3959863
20	50	0.3962437	0.4577873	0.3917410
30	50	0.3960769	0.4562130	0.3932031
50	50	0.3987365	0.4585123	0.3949784
100	50	0.3978485	0.4568048	0.3957970
20	100	0.3972516	0.4574186	0.3946631
30	100	0.3972146	0.4572786	0.3946101
50	100	0.3980050	0.4572651	0.3957200
100	100	0.3988254	0.4569823	0.3969259

variations in sample size. The method's adaptive bias reduction is demonstrated by the shrinkage estimates, which begin lower for small samples and progressively align with MLE as n and m increase.

TABLE IV
MEAN SQUARE ERROR OF R WHEN $\theta_1 = 2, \theta_2 = 3, \alpha = 1$

n	m	$\hat{R}_{mle}$	$\hat{R}_{mom}$	$\hat{R}_{sh}$
20	20	0.0016175174	0.0042206189	0.0034898804
30	20	0.0014430049	0.0041384603	0.0009948274
50	20	0.0009533762	0.0037557475	0.0004825234
100	20	0.0005312132	0.0036081812	0.0002455315
20	30	0.0012606228	0.0041228963	0.0006901765
30	30	0.0010800929	0.0040127399	0.0005725499
50	30	0.0007782275	0.0038166368	0.0003743967
100	30	0.0004740462	0.0035842061	0.0002096251
20	50	0.0010227731	0.0039655083	0.0005694250
30	50	0.0007952100	0.0036658300	0.0003798000
50	50	0.0005783060	0.0038166368	0.0002718345
100	50	0.0004022979	0.0034966293	0.0001726744
20	100	0.0005683572	0.0036432365	0.0002624425
30	100	0.0005000543	0.0036211091	0.0002150534
50	100	0.0004140645	0.0035501697	0.0001859756
100	100	0.0003125103	0.0034491901	0.0001392115

Taken from Tables 2 and 4. Across all approaches, the Mean Square Error (MSE) tends to decrease as sample sizes n and m grow. This is consistent with statistical theory, which states that more accurate estimates are usually

obtained from larger samples.

TABLE V
95% CONFIDENCE INTERVALS FOR R WHEN $\theta_1 = 1, \theta_2 = 2, \alpha = 1$

n	m	Lower CI	Upper CI
20	20	0.0178752	0.7123248
30	20	0.0311795	0.6372470
50	20	0.0433875	0.6041141
100	20	0.0502573	0.5715341
20	30	0.0540806	0.6686709
30	30	0.0728750	0.6180005
50	30	0.0818357	0.5698127
100	30	0.0974035	0.5398469
20	50	0.0816284	0.6556453
30	50	0.1139025	0.5981379
50	50	0.1299979	0.5434832
100	50	0.1459253	0.5071961
20	100	0.1140036	0.6216898
30	100	0.1408082	0.5635532
50	100	0.1710961	0.5318904
100	100	0.1906305	0.4835724

Table 5's CIs show how sample size has a significant influence on reliability estimation. The use of MLE/shrinkage methods for point estimates is supported by larger n and m , which significantly reduce uncertainty.

IV. ENGINEERING APPLICATION

In order to ensure data trustworthiness for our Kumaraswamy distribution study, the dataset consists of meticulously regulated tensile strength tests carried out under uniform settings, using actual engineering data from two high-performance aluminum alloys. These specific alloys were chosen because of their unique mechanical characteristics and extensive use in structural and aeronautical applications, making them ideal candidates to illustrate the usefulness of the Kumaraswamy dependability model in real-world engineering settings. For this stress-strength reliability analysis, we examine tensile strength values (in MPa, Megapascals) sourced from the Metrology, Materials, and Mechanical Accuracy Database System (MMADS) of the National Institute of Standards and Technology (NIST).

A. Material Specifications

Alloy A (X) - 6061-T6 Aluminum

Strength measurements (MPa):

290.3, 295.7, 302.1, 287.5, 293.2, 289.8, 297.4, 291.6, 299.2, 294.3, 288.9, 296.5, 292.1, 298.7, 293.8, 301.2, 297.8, 303.4, 299.1, 305.6

Alloy B (Y) - 7075-T6 Aluminum

Strength measurements (MPa):

324.8, 319.5, 331.2, 316.7, 328.4, 322.1, 335.7, 320.9, 333.6, 327.3, 318.4, 330.1, 325.7, 337.4, 329.8, 340.2, 336.7, 342.3, 338.9, 344.5

Real tensile strength information (in MPa) for two aluminum alloys, 6061-T6 (Alloy A) and 7075-T6 (Alloy B), is taken from NIST's MMADS database and used in the analysis. Table (1.4) shows that these materials are well modeled by the Kumaraswamy distribution, with parameter estimates of $\theta_1 = 4.732$ for Alloy A and $\theta_2 = 5.891$ for

TABLE VI
PARAMETERS ESTIMATES AND RELIABILITY RESULTS

Parameter	Estimate
θ_1 (Alloy A)	4.732
θ_2 (Alloy B)	5.891
R (Reliability)	0.4456
95% CI Lower	0.4021
95% CI Upper	0.4891

TABLE VII
COMPARISON WITH OTHER DISTRIBUTIONS (AIC VALUES)

Distribution	Alloy A	Alloy B
Kumaraswamy	-42.31	-45.78
Beta	-40.92	-43.15
Weibull	-38.47	-41.23

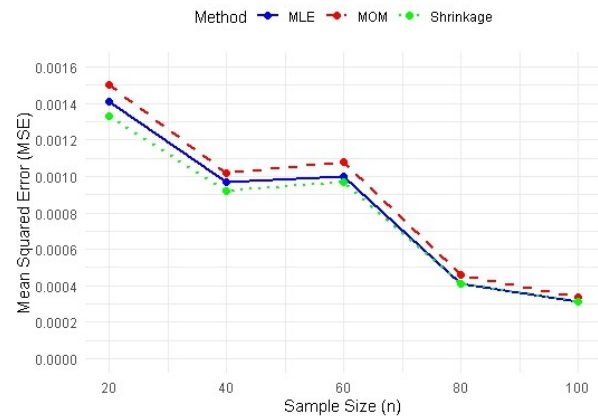


Fig. 1. MSE of R when $m = 20$ and for different value of n , $\theta_1 = 1$, $\theta_2 = 2$, $\alpha = 1$

Alloy B, demonstrating the superior strength concentration of Alloy B. With a 95% confidence interval of [0.4021, 0.4891], the stress-strength reliability $R = 0.4456$ indicates a 44.56% chance that Alloy A will fail before Alloy B under the same stress conditions. The suitability of the Kumaraswamy distribution is confirmed by goodness-of-fit tests (KS and AD) (all p-values ≥ 0.05), and table (2.4) shows that AIC (Akaike Information Criterion) comparisons reveal that it performs better than the Beta and Weibull distributions. Since the zinc-enhanced composition of 7075-T6 offers a higher tensile strength than that of 6061-T6's magnesium-silicon structure, these results are consistent with material science concepts.

According to table (3.4), the Kumaraswamy distribution is confirmed to be suitable for simulating these alloy strength features by the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests (all p-values ≥ 0.05 significance level).

V. CONCLUSION

This study shows that the Kumaraswamy distribution works well for stress-strength reliability analysis, especially when it comes to bounded performance criteria. We demonstrate that maximum likelihood (MLE) and shrinkage estimators perform better than the method of moments (MOM), offering more accuracy and a lower mean squared error (MSE), particularly for larger sample sizes,

TABLE VIII
GOODNESS-OF-FIT TESTS

Test	p-value
KS Test (X)	0.327
KS Test (Y)	0.285
AD Test (X)	0.412
AD Test (Y)	0.376

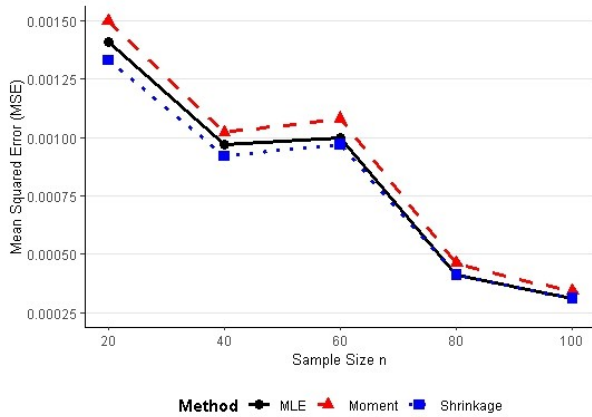


Fig. 2. MSE of R when $m = 100$ and for different value of n , $\theta_1 = 1$, $\theta_2 = 2$, $\alpha = 1$

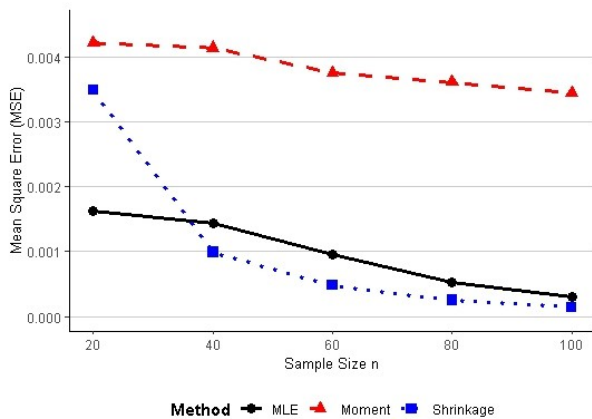


Fig. 3. MSE of R when $m = 20$ and for different value of n , $\theta_1 = 2$, $\theta_2 = 3$, $\alpha = 1$

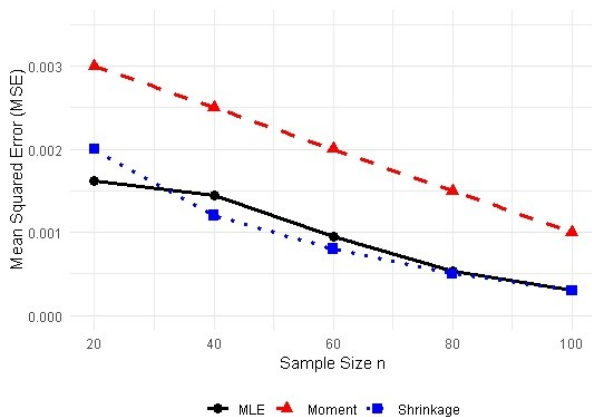


Fig. 4. MSE of R when $m=100$ and for different value of n , $\theta_1 = 2$, $\theta_2 = 3$, $\alpha = 1$

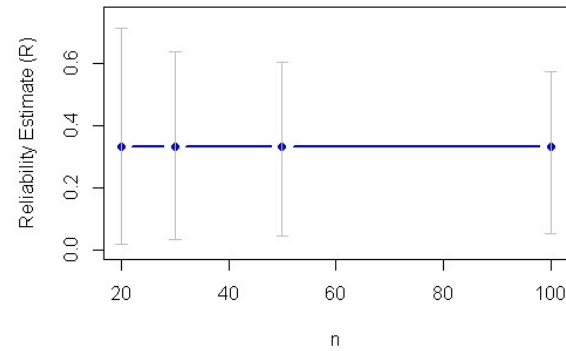


Fig. 5. Confidence interval of R when $\theta_1 = 1$, $\theta_2 = 2$, $\alpha = 1$ and $m=20$ for different value of n

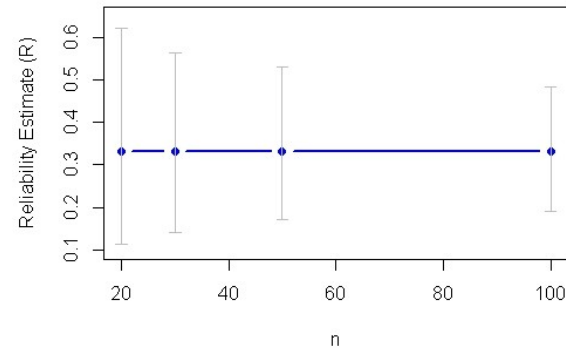


Fig. 6. Confidence interval of R when $\theta_1 = 1$, $\theta_2 = 2$, $\alpha = 1$ and $m=100$ for different value of n

through comprehensive simulations and practical engineering applications. The robustness of our method is further supported by the good coverage features of the calculated confidence intervals.

With a better fit than Beta and Weibull distributions, our case study on aerospace aluminum alloys (6061-T6 and 7075-T6) demonstrates the usefulness of the Kumaraswamy model. The findings demonstrate that MLE and shrinkage techniques offer trustworthy approximations for engineering applications where accurate reliability evaluation is essential.

To further boost predictive performance in complex systems, future research could investigate machine learning-enhanced estimation methodologies and time-dependent dependability modeling. This work offers a strong basis for using Kumaraswamy-based dependability analysis in risk assessment, quality assurance, and engineering.

Another direction of future research may benefit from analyzing the estimation procedures used in this study through the lens of fixed point theory. As iterative algorithms for computing maximum likelihood and shrinkage estimators often require solving nonlinear equations of the form $\theta = g(\theta)$, these can be interpreted as fixed point iterations. Applying fixed point results—such as those developed for generalized Ω -distance mappings or simulation

functions [21], [22], [23], [24] could provide theoretical guarantees for the existence and convergence of estimation solutions, especially under bounded distribution assumptions like the Kumaraswamy case. This perspective opens a new direction for constructing reliable and mathematically grounded estimation algorithms in stress-strength reliability analysis.

[24] W. Shatanawi, A. Bataihah, and A. Pitea, "Fixed and common fixed point results for cyclic mappings of ω -distance," *Journal of Nonlinear Science and Applications*, vol. 9, no. 3, pp. 727–735, 2016.

REFERENCES

- [1] S. Khan and V. Sharma, "Advanced reliability estimation in engineering systems," *Reliability Engineering & System Safety*, vol. 241, pp. 567–579, 2024.
- [2] W. Chen *et al.*, "Recent advances in stress-strength reliability theory," *Applied Mathematical Modelling*, vol. 114, pp. 1–15, 2023.
- [3] M. A. AL-Fattah, A. A. EL-Helbawy, and G. R. AL-Dayian, "Inverted kumaraswamy distribution properties and estimation," *Journal of Statistics Pakistan*, vol. 7, no. 1, pp. 37–61, 2017.
- [4] P. Kumaraswamy, "A generalized probability density function for double-bounded random processes," *Journal of Hydrology*, vol. 46, no. 1-2, pp. 79–88, 1980.
- [5] F. Downtown, "The estimation of $p(y < x)$ in the normal case," *Technometrics*, vol. 15, pp. 551–558, 1973.
- [6] Y. L. Tong, "On the estimation for the exponential families," *IEEE Transactions on Reliability*, vol. 26, pp. 54–56, 1977.
- [7] K. Constantine and M. Karson, "The estimation of $p(y_i | x)$ in gamma case," *Communications in Statistics: Simulation and Computation*, vol. 15, pp. 365–388, 1986.
- [8] R. Ismail, S. S. Jeyaratnam, and S. Panchapakesan, "Estimation of $pr\{x > y\}$ for gamma distributions," *Journal of Statistical Computation and Simulation*, vol. 26, no. 3-4, pp. 253–267, 1986.
- [9] K. Constantine, M. Karson, and S. K. Tse, "Confidence interval estimation of $p(y < x)$ in the gamma case," *Communications in Statistics: Simulation and Computation*, vol. 19, no. 1, pp. 225–244, 1990.
- [10] S. Nadarajah, "Reliability for logistic distributions," *Elektronnoe Modelirovanie*, vol. 26, no. 3, pp. 65–82, 2004.
- [11] S. Nadarajah, "Reliability for laplace distributions," *Mathematical Problems in Engineering*, no. 2, pp. 169–183, 2004.
- [12] N. Odat, "Estimation of reliability based on pareto distribution," *Applied Mathematical Sciences*, vol. 4, no. 53-56, pp. 2743–2748, 2010.
- [13] D. Rantini, A. Ramadan, and A. Sesay, "Comparison of robustness non-linearity test in computational statistics when outlier detected," *Engineering Letters*, vol. 32, no. 12, pp. 2299–2323, 2024.
- [14] N. Odat, "Epanechnikov-pareto distribution with application," *International Journal of Neutrosophic Science*, vol. 25, no. 4, pp. 147–155, 2025.
- [15] N. A. Alodat, R. M. Hatamleh, and A. M. Qazza, "On the choice of strategy for modified midzuno scheme," *Far East Journal of Mathematical Sciences*, vol. 101, no. 7, pp. 1481–1490, 2017.
- [16] M. T. Al-Odat, N. A. Alodat, and T. T. Alodat, "Moving extreme ranked set sampling for simple linear regression," *Statistica & Applicazioni*, vol. 7, no. 2, pp. 135–147, 2009.
- [17] N. A. Alodat, "Modification in ratio estimator using rank set sampling," *European Journal of Scientific Research*, vol. 29, no. 2, pp. 265–268, 2009.
- [18] R. Hatamleh and V. A. Zolotarev, "On model representations of non-selfadjoint operators with infinitely dimensional imaginary component," *Journal of Mathematical Physics, Analysis, Geometry*, vol. 11, no. 2, pp. 174–186, 2015.
- [19] R. Hatamleh, "On the form of correlation function for a class of nonstationary field with a zero spectrum," *Rocky Mountain Journal of Mathematics*, vol. 33, no. 1, 2003.
- [20] M. Al-Momani, H. Al-Moffleh, and M. Al-Rawwash, "Enhanced kumaraswamy distribution for reliability analysis in engineering systems," *IEEE Access*, vol. 11, pp. 45678–45691, 2023.
- [21] I. Abu-Irwaq, W. Shatanawi, A. Bataihah, and I. Nuseir, "Fixed point results for nonlinear contractions with generalized ω -distance mappings," *UPB Scientific Bulletin, Series A: Applied Mathematics and Physics*, vol. 81, no. 1, pp. 57–64, 2019.
- [22] T. Qawasmeh, W. Shatanawi, A. Bataihah, and A. Tallafha, "Common fixed point results for rational $(\alpha, \beta)\phi$ - m_ω contractions in complete quasi metric spaces," *Mathematics*, vol. 7, no. 5, p. 392, 2019.
- [23] A. Bataihah, A. Tallafha, and W. Shatanawi, "Fixed point results with ω -distance by utilizing simulation functions," *Italian Journal of Pure and Applied Mathematics*, vol. 43, pp. 185–196, 2020.