

Strategic Outsourcing and Value Chain Climbing Based on Pricing Competition under Stochastic Demand

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Abstract-This paper examines strategic interactions between outsourcing and value chain climbing in a supply chain, analyzing decision-making processes and outcomes for a supplier and buyer under competitive pricing. Using a two-stage game-theoretic model, we investigate how stochastic demand and price competition influence sequential decisions. In Stage 1, the supplier sets outsourcing prices; in Stage 2, the buyer determines retail prices and production quantities. Equilibrium analysis identifies two dominant strategies: (1) market maintaining (full outsourcing) and (2) cost advantage seeking (full in-house production). Numerical studies (baseline and nonlinear experiments) confirm buyer's persistently outsource across all stochastic demand types and parameter combinations. This resilience stems from their ability to dynamically adjust prices, countering supplier threats. Meanwhile, shifts in the production cost ratio redistribute profits asymmetrically: the supplier's profit always gains with the increase in production cost ratio in both two experiments, while the buyer's profit shows more complex variation patterns. The increased in competition intensity increases profit volatility of the buyer in nonlinear experiment yet consistently benefits supplier. Our findings endogenize vertical competition, bridge operations-marketing integration, and offer strategic guidance for managing supplier encroachment in uncertain markets.

Keywords-outsourcing strategy, value chain climbing, price competition, stochastic demand

I. INTRODUCTION

IN the context of global value chain restructuring, outsourcing and in-sourcing decisions are pivotal in economics and business strategy, addressing key strategic issues. Firms often outsource less profitable activities (e.g., manufacturing) to suppliers with economies of scale in developing countries. These suppliers, known as original equipment manufacturers (OEMs) or contract manufacturers (CMs), enable technology giants like Apple, Dell, and Intel to focus on core, high-profit activities (e.g., R&D and

marketing), thereby enhancing their competitive advantages and financial flexibility. This strategy aligns with the well-known smile curve theory, which posits that the value chain's ends (R&D and marketing) are more profitable than the middle segment (manufacturing). For outsourcing buyers, crucial strategies include cost reduction and a focus on patents, branding, and services.

However, developing-country suppliers can acquire market entry capabilities through learning - by - doing, termed value chain climbing, a key industrial upgrading routewith significant implications for global competition [1,2]. This process may undermine industry leaders' profits. Samsung's foray into the smartphone market exemplifies this: after supplying Apple, it harnessed its tech edge to launch the Galaxy series, becoming a global leader and threatening Apple's share [3]. Despite legal disputes, Samsung maintained its competitive edge and continued to supply high-quality, cost-effective components to Apple. This might spur other developing - country suppliers to emulate such strategies. However, some suppliers avoid value chain climbing due to fears of buyer retaliation or transaction dependence, which can impede technological advancement and market competitiveness [4].

While existing literature has extensively explored outsourcing motivations and the phenomenon of value chain climbing, there are still significant gaps. First, and most critically, most studies treat vertical competition stemming from supplier entry as an exogenous factor and seldom explore its endogenous link to suppliers' value chain climbing through learning. For example, many studies assume that suppliers' entry and competition are given, without in-depth analysis of how suppliers actively change the competitive landscape through learning and market entry strategies. Second, the treatment of stochastic demand in multi-stage outsourcing decisions is insufficient, with most research focusing on single-stage decisions and neglecting its dynamic interplay with value chain climbing. Third, existing studies mostly focus on binary decisions (full outsourcing vs. full in-house production), while neglecting mixed strategies that combine both approaches, which are more reflective of real-world supply chain management practices.

Building on this background, we examine the outsourcing strategies of a buyer and a supplier in a two-stage process: a learning stage (Stage 1) and a competing stage (Stage 2). The buyer may outsource to the supplier in either or both stages. We develop a two-stage Stackelberg game model with stochastic demand dependent on retail prices. Our numerical study investigates key factors influencing outsourcing strategies and profits, including different stochastic demand types and the impact of

Manuscript received April 25, 2025; revised September 29, 2025.

This work was supported by the Zhejiang Province Philosophy and Social Science Planning Projects [grant number 25NDJC033YBM]; "Applied Economics" program, the "161 Project" for Advantageous and Distinctive Discipline Development in Ningbo's Higher Education Institutions.

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product/demand information.

Our study makes several key contributions: First, we construct a novel two-stage game-theoretic model capturing the dynamic interplay of learning, potential competition, and stochastic demand. Second, we endogenize vertical competition by explicitly modeling how suppliers' learning and market entry strategies dynamically reshape the competitive landscape with buyers. Third, we explore mixed outsourcing strategies alongside traditional binary options, providing a more realistic and comprehensive view. Our findings offer significant theoretical insights into outsourcing dynamics and practical guidance for firms navigating complex supplier relationships.

II. LITERATURE REVIEW

We first summarize several key environmental factors that catalyze value chain climbing. First, increasing disposable income in developing countries has created large domestic markets, providing a foundation for suppliers before global expansion [5, 6]. Second, trade liberalization and globalization enable emerging competitors to adopt strategies akin to "surrounding cities from the countryside" [7]. Third, technological advances such as e-commerce, 5G, and blockchain have lowered entry barriers, facilitating suppliers' efforts to climb the value chain. Although many suppliers have attempted to introduce branded products by learning technology, few have succeeded. Leading branded buyers often use restrictive outsourcing agreements to relegate suppliers to subordinate roles and distance them from customer-facing activities [1, 8]. However, outsourcing is projected to grow, favoring companies with strategic approaches. Deng [9] identifies four conditions influencing technological upgrading strategies: technical resources (firm-level), technological intensity (industry-level), and technological protection (regional-level), which align with the strategy tripod perspective. Unlike other studies, Niu [10] investigates production timing decisions based on market acceptance uncertainty in a co-opetitive supply chain where a manufacturer functions as both an OEM's upstream contract manufacturer and downstream competitor. Case studies [11] and empirical investigations [1] reveal the implications of suppliers climbing value chains, cautioning Western companies to be wary of their suppliers in developing countries to avoid vertical competition. From the buyer's perspective, Wan and Wu [12] developed an analytical model to study how value chain climbing affects value distribution in buyer-supplier relationships, identifying three outsourcing strategies: accommodation, squeeze, and dump. However, their analysis overlooks operational factors like demand and inventory, and the uncertain operating environment can lead to strategy failure.

We also address cooperation and competition in supply chains under various conditions. Typically, such coexistence manifests as vertical cooperation and horizontal competition [13-15]. Horizontal competition may occur upstream or downstream. Upstream competition often refers to the competition between suppliers to a retailer or other downstream buyer, which is also called the supplier selection problem [16, 17]. Downstream competition often refers to the competition between retailers, closely related to channel decisions [18, 19]. In recent years, the coexistence

of vertical cooperation and competition has garnered significant theoretical and practical attention. As outsourcing constitutes a form of vertical cooperation in the supply chain, the impact of a single - tier competition on outsourcing strategy has been extensively examined, such as horizontal competition among downstream buyers or upstream suppliers. For instance, Wang et al. [20] investigate the outsourcing strategy problem of two OEMs whose products each consist of two components. Specifically, each OEM produces a distinct component and determines whether to outsource the other component to the rival OEM or a third - party supplier. Niu et al. [21] examine how two competing firms outsource product manufacturing to two different manufacturers, while the first manufacturer produces a critical component that is required by every product and the second manufacturer provides additional services that enhance product value. Arya 2008 make show that when two competitive firms outsource to a unique external supplier, they may also pay a premium to the supplier. Some scholars have discussed important factors affecting the cooperation in outsourcing strategies [22, 23]. For example, Deng et al. [24] investigate the outsourcing contract in a supply chain with two competing OEMs and a common CM whose production cost of the CM decreases by the learning-by-doing effect. Liu and Tyagi [25] examine the effects of upward channel decentralization between two competing firms that outsource the production to upstream suppliers. Surprisingly, the suppliers do not provide cost advantages to these firms, demonstrating that when the downstream firms' production positioning is endogenous, upward channel decentralization with a peculiar outsourcing strategy is still profitable.

A well - designed contract is essential for firms to maximize benefits and minimize outsourcing risks. Most relevant literature focuses on the quantity-based outsourcing contracts. Recently, many scholars have paid attention to price competition in outsourcing strategies. Niu et al. [17] investigate price competition between an OEM and its ODM in three different forms: simultaneous pricing game, OEM-price-first game, and ODM-price-first game. Feng and Lu [26] consider outsourcing contracts in a two-tier supply chain consisting of two competing manufacturers and an upstream supplier based on both quantity competition and price competition. Each manufacturer can produce in - house or outsource to the supplier, and the supplier can be negotiated to act as either an exclusive supplier or a common supplier. Their analysis reveals that wholesale-price contracts always weaken the competition among manufacturers. Chen et al. [27] also investigate the outsourcing strategies of an OEM and a CM in the Cournot competition setting, where either the OEM or the CM determines the wholesale price of the outsourcing product. Most closely related to our paper, Shi [28] explores the CM's encroachment strategy and quality decisions, such as building a private brand to compete with the OEM and deciding the quality level and selling price of the private brand.

To summarize, the existing literature rarely examines how suppliers' value chain climbing impacts outsourcing contracts under operational uncertainty, and it usually treats vertical competition as an exogenous rather than endogenous

factor. Consequently, the dynamic decision - making process of buyers and suppliers—facing both the supplier's growing capabilities and demand uncertainty — remains largely unexplored. To address these limitations, we develop a two - stage outsourcing contract model in a supply chain with a buyer and a supplier. The decision process is divided into two stages: a learning stage where the supplier receives outsourcing requirements, and a competing stage where the supplier may launch its own branded product to compete with the buyer if outsourcing occurred in the first stage. In Stage 1, the supplier sets the outsourcing price, and the buyer decides on retail price and outsourcing/in - house quantities, with demand linearly related to the buyer's price. In Stage 2, if outsourcing occurred, the supplier enters the market and engages in price competition with the buyer, whose stochastic demands are linearly related to both parties' retail prices.

III. MODELLING

We model a supply chain comprising of a buyer (she, subscribed as b) who sells the product to the market and a supplier (he, subscribed as s) that can produce the product for the buyer with the production cost c_s per unit, and provide the outsourcing contract. Both buyer and supplier are occupied with complete and perfect information. This simplified assumption is applicable to mature industries (such as consumer electronics), where the buyer and the supplier establish cost transparency through long - term cooperation. The annual cost auditing mechanism between Dell and Flex is a typical case [11].

We adopt the wholesale price contract for its operational feasibility and theoretical applicability [10,13], wherein the dominant supplier—with bargaining primacy from asset-specific investments like proprietary processes or specialized equipment (e.g., semiconductor photolithography systems)—sets wholesale prices initially. This first-mover dynamic reflects actual power asymmetries in Electronics Manufacturing Services (EMS) industries, as observed in Apple-Samsung relationships and TSMC's capability advantage. Subsequently, the buyer has three choices: (1) accept the contract provided by the supplier and only outsource the production process to the supplier; (2) reject the contract and only produce in-house with the production cost of c_b per unit, $c_s < c_b < p$, (3) or adopt the mixed strategy where outsourcing and in-house production coexist. If the buyer outsources production to the supplier, the supplier may learn the buyer's technology and marketing knowledge, potentially launching its own product to compete with the buyer. This is termed value chain climbing. For simplicity, we assume the buyer's and supplier's products are substitutable. The assumption of substitutability is a common starting point in models of competition, which allows us to analyze the impact of the supplier's potential entry into the market on the buyer's outsourcing decision.

We divide the timeline into two stages: Stage 1 is the learning stage, the supplier does not have the ability to provide products to the market, he needs to improve corresponding capabilities from value chain climbing. Stage 2 is the competition stage: the supplier is able to provide products to the market and competes with the buyer if the

outsourcing relationship exists in stage 1.

We conceptualize the supplier's and buyer's sequential decisions in Figure 1. The whole decision process includes two stages: In stage 1, the supplier decides the wholesale price w at first, then the buyer decides outsourcing quantity q_{out1} and in-house production quantity q_{in1} as well as retail price p_{b1} simultaneously. After these decisions, the demand in stage 1, $D_1(p_{b1}, \epsilon_1)$, is realized. Normally, $D_1(p_{b1}, \epsilon_1)$ can be characterized as additive model or multiplicative model [29]. In this paper, we apply the additive model for all demand function owing to its capacity to preserve the intrinsic attributes of price elasticity whilst safeguarding the viability of analytical solutions. That is, $D_1(p_{b1}, \epsilon_1) := L_1(p_{b1}) + \epsilon_1$ where $L_1(p_{b1})$ is the deterministic component of the demand, $dL_1/dp_{b1} \leq 0$. Normally, ϵ_1 is the stochastic component of the demand with known probability density function (pdf) $f_1(\cdot)$ and cumulative distribution function (cdf) $F_1(\cdot)$. In stage 2, the the buyer decides the retail price p_{b2} and outsourcing quantity q_{out2} while the supplier decides his retail price p_s and production quantity q_s (if the supplier does not enter the market, p_s and q_s are all equal to 0) at the same time. Then the stochastic demands, $D_{b2}(p_{b2}, p_s, \epsilon_{b2})$ and $D_{s2}(p_{b2}, p_s, \epsilon_{s2})$, are realized.

$$D_{b2}(p_{b2}, p_s, \epsilon_{b2}) := L_{b2}(p_{b2}, p_s) + \epsilon_{b2}$$

$$D_{s2}(p_{b2}, p_s, \epsilon_{s2}, q_{out1}) := (L_{s2}(p_{b2}, p_s) + \epsilon_{s2})I_{q_{out1}}$$

where $L_{b2}(p_{b2}, p_s)$ is the deterministic part of demand faced by buyer in stage 2 , and $L_{s2}(p_{b2}, p_s)$ is the deterministic part of demand faced by supplier in stage 2. ϵ_{b2} and ϵ_{s2} are independent stochastic components with $\epsilon_{b2} \sim f_{b2}(\cdot)(F_{b2}(\cdot))$, $\epsilon_{s2} \sim f_{s2}(\cdot)(F_{s2}(\cdot))$, and

We assume that $L_{b2}(p_{b2}, p_s)$ is decreasing in p_{b2} and increasing in p_s , while $L_{s2}(p_{b2}, p_s)$ is decreasing in p_s and increasing in p_{b2} to capture the economics of price competition between the buyer and supplier. The failure rate of stochastic components ϵ_1 , ϵ_{b2} and ϵ_{s2} are

$$\begin{aligned} r_1(\cdot) &:= f_1(\cdot)/(1 - F_1(\cdot)), \\ r_{b2}(\cdot) &:= f_{b2}(\cdot)/(1 - F_{b2}(\cdot)), \\ r_{s2}(\cdot) &:= f_{s2}(\cdot)/(1 - F_{s2}(\cdot)), \end{aligned}$$

In stage 1, we use the newsvendor setting to describe the expected profit of the buyer:

$$\begin{aligned} \pi_b^1(w, p_{b1}, q_{out1}, q_{in1}) \\ = p_{b1}E[\min(q_{out1} + q_{in1}, D_1(p_{b1}, \epsilon_1))] - wq_{out1} - c_bq_{in1} \end{aligned} \quad (1)$$

The expected profit of the supplier at this stage is:

$$\pi_s^1(w, q_{out1}) = (w - c_s)q_{out1} \quad (2)$$

Stage 2 contains the vertical competition between the buyer and the supplier. At this stage, the buyer decides her price p_{b2} and outsourcing quantity q_{out2} . q_{out2} satisfies the stochastic demand $D_{b2}(p_{b2}, p_s, \epsilon_{b2})$ (generally, buyer will outsource only for $w < c_b$). We assume that when $w = c_b$, the buyer will outsource at this stage to simplify the model, indicating the situation that outsourcing does not exist at all). At the same time, the supplier decides his quantity price p_s and quantity q_s . q_s satisfies the stochastic demand $D_{s2}(p_{b2}, p_s, \epsilon_{s2}, q_{out1})$. We assume that if the buyer does not outsource to the supplier in stage 1, the supplier will not enter the market in stage 2 in this

paper($D_{s2}(p_{b2}, p_s, \epsilon_{s2}, 0) = 0$). Note that the supplier may also upgrade the value chain and enter the market by self-investment meant in stage 1. In this case, the relationship between buyers and suppliers becomes more complicated. In conclusion, we write the buyer's expected profit in stage 2 as:

$$\pi_b^2(w, p_{b2}, p_s, q_{out}) = p_{b2}E[\min(q_{out}2, D_{b2}(p_{b2}, p_s, \epsilon_{b2}))] - wq_{out2} \quad (3)$$

The supplier's expected profit in stage 2 as:

$$\pi_s^2(w, p_{b2}, p_s, q_{out1}, q_{out2}, q_s) = (w - c_s)q_{out2} + p_sE[\min(q_s, D_{s2}(p_{b2}, p_s, \epsilon_{s2}, q_{out1}))] - c_sq_s \quad (4)$$

Above all, the total expected profit of buyer is $\Pi_b(w, p_{b1}, p_{b2}, p_s, q_{out1}, q_{in1}, q_{out2}) = \pi_b^1(w, p_{b1}, q_{out1}, q_{in1}) + \pi_b^2(w, p_{b2}, p_s, q_{out2})$; the total expected profit of supplier is $\Pi_s(w, p_{b2}, p_s, q_{out1}, q_{out2}, q_s) = \pi_s^1(w, q_{out1}) + \pi_s^2(w, p_{b2}, p_s, q_{out1}, q_{out2}, q_s)$.

In this game, the supplier prioritizes choosing appropriate w to induce the buyer's behavior to maximize his total expected profit. Given w , the buyer will choose her optimal price and outsourcing/production quantities in stage 1: p_{b1} , q_{out1} and q_{in1} . Then, whether the supplier can enter the market in stage 2 is known to all, and the buyer and the supplier simultaneously make their price and quantity decisions in stage 2. The objective functions of the buyer and supplier are maximizing their respective total expected profits.

Observing Equations (1) to (4), given other decision variables, all quantity decision, q_{out1} , q_{in1} , q_{out2} and q_s , can be obtained by solving several separate newsvendor models, so we can rewrite q_{out1} , q_{in1} , q_{out2} and q_s as functions containing other decision variables and simplify the analysis process. **Lemma 1** shows the quantity decisions of the buyer and the supplier at two stages given the decision variables (and q_{out1}) in stage 1. Without loss of generality, we set that $q_{out1} = 0$ and $p_s = 0$ if $w = c_b$.

Lemma 1. (1) Given w and p_{b1} , the quantity decisions in stage 1 are : $q_{out1}(w, p_{b1}) = F_1^{-1}\left(\frac{p_{b1}-w}{p_{b1}}\right) + L_1(p_{b1})$, $q_{in1}(p_{b1}) = 0, w \in [0, c_b]$; or $q_{out1}(c_b, p_{b1}) = 0, q_{in1}(p_{b1}) = F_1^{-1}\left(\frac{p_{b1}-c_b}{p_{b1}}\right) + L_1(p_{b1}), w = c_b$.

(2) Let

$$I_{w,p_{b1}} := \begin{cases} 1, & \text{if } q_{out1}(w, p_{b1}) = 1 \\ 0, & \text{if } q_{out1}(w, p_{b1}) = 0 \end{cases}$$

Given $I_{w,p_{b1}}, w, p_{b2}$ and p_s , the quantity decisions in stage 2 are:

1) If $I_{w,p_{b1}} = 0$, then $q_{out2}(I_{w,p_{b1}}, w, p_{b2}, p_s) = q_{out2}(0, w, p_{b2}, 0) = F_{b2}^{-1}\left(\frac{p_{b2}-w}{p_{b2}}\right) + L_{b2}(p_{b2}, 0)$,

$q_s(I_{w,p_{b1}}, w, p_{b2}, p_s) = q_s(0, w, p_{b2}, 0) = 0$;

2) If $I_{w,p_{b1}} = 1$, then $q_{out2}(I_{w,p_{b1}}, w, p_{b2}, p_s) = q_{out2}(1, w, p_{b2}, p_s) = F_{b2}^{-1}\left(\frac{p_{b2}-w}{p_{b2}}\right) + L_{b2}(p_{b2}, p_s)$,

$q_s(I_{w,p_{b1}}, w, p_{b2}, p_s) = q_s(1, w, p_{b2}, p_s) = F_{s2}^{-1}\left(\frac{p_s-c_s}{p_s}\right) + L_{s2}(p_{b2}, p_s)$.

Now, there are price decision variables left: w, p_{b1}, p_{b2} and p_s , and we can rewrite a multi-stage game model to

explicit the the outsourcing strategy decisions of the buyer and supplier. To ensure compactness of the decision variables, we assume that $\max\{p_{b1}, p_{b2}, p_s\} \leq p^{max} < +\infty$, where p^{max} is sufficiently large that they would not impact any of the choices, as pointed out by Cachon and Netessine ([12]), "Therefore, the transformed game [with compact sets] behaves just like the original game with an unbounded strategy space." Without loss of generality, $L_1(p_{b1}) \geq 0, \forall p_{b1} \in [0, p^{max}]$, and $L_i(p_{b2}, p_s) \geq 0, \forall i \in \{b2, s2\}, \forall (p_{b2}, p_s) \in [0, p^{max}] \times [0, p^{max}]$.

We rewrite $\pi_b^2(w, p_{b2}, p_s, q_{out2})$ and $\pi_s^2(w, p_{b2}, p_s, q_{out1}, q_{out2}, q_s)$ as $\pi_b^2(I_{w,p_{b1}}, w, p_{b2}, p_s)$ and $\pi_s^2(I_{w,p_{b1}}, w, p_{b2}, p_s)$ by plugging $q_{out2}(I_{w,p_{b1}}, w, p_{b2}, p_s)$ and $q_s(I_{w,p_{b1}}, w, p_{b2}, p_s)$ into the equations (3)-(4), respectively. Similarly, we rewrite $\pi_b^1(w, p_{b1}, q_{out1}, q_{in1})$ and $\pi_s^1(w, q_{out1})$ as $\pi_b^1(w, p_{b1})$ and $\pi_s^1(w, p_{b1})$ by plugging $q_{out1}(w, p_{b1})$ and $q_{in1}(p_{b1})$ into equations (1)-(2), respectively. By backward induction, given w and p_{b1} , the buyer and supplier's simultaneous decisions in stage 2 are represented by the following system:

$$\begin{cases} \max_{w \leq p_{b2} \leq p^{max}} \pi_b^2(I_{w,p_{b1}}, w, p_{b2}, p_s) \\ \max_{p_s \in \mathcal{D}_1} \pi_s^2(I_{w,p_{b1}}, w, p_{b2}, p_s) \end{cases} \quad (5)$$

where

$$\mathcal{D}_1 = \begin{cases} 0, & \text{if } I_{w,p_{b1}} = 0 \\ [c_s, p^{max}], & \text{if } I_{w,p_{b1}} = 1 \end{cases}$$

Define $(p_{b2}^*(I_{w,p_{b1}}, w), p_s^*(I_{w,p_{b1}}, w))$ as the optimal solution to the problem (5). Then, we write the buyer's total expected profit as $\Pi_b(I_{w,p_{b1}}, w, p_{b1}) := \pi_b^1(c_b, p_{b1}) + \pi_b^2(0, w, p_{b2}^*(0, w), 0)$ for $I_{w,p_{b1}} = 0$; or $\Pi_b(I_{w,p_{b1}}, w, p_{b1}) := \pi_b^1(w, p_{b1}) + \pi_b^2(1, w, p_{b2}^*(1, w), p_s^*(1, w))$ for $I_{w,p_{b1}} = 1$. In stage 1, the buyer first decides p_{b1} given w , which can be expressed as:

$$\max_{w \leq p_{b1} \leq p^{max}} \Pi_b(I_{w,p_{b1}}, w, p_{b1}) \quad (6)$$

Define $p_{b1}^*(w)$ as the optimal solution to the problem (6). The supplier's total expected profit is $\Pi_s(I_{w,p_{b1}^*(w)}, w) := \pi_s^1(w, q_{out1}(w, p_{b1}^*(w)))$

$$+ \pi_s^2(w, p_{b2}^*(w, p_{b1}^*(w)), p_s^*(I_{w,p_{b1}^*(w)}, w))$$

and the optimization problem to the supplier in stage 1 is

$$\max_{0 \leq w \leq c_b} \Pi_s(I_{w,p_{b1}^*(w)}, w) \quad (7)$$

Note that when $w = c_b$, it is equivalent to the buyer's producing in-house at two stages, so that the supplier cannot benefit from value chain climbing in stage 2.

IV. EQUILIBRIUM ANALYSIS

In this section, we analyze the existence of the Nash equilibria by solving the problems (5), (6) and (7) sequentially.

We first solve the problem (5). According to Lemma 1, we need to discuss the following two cases separately: $I_{w,p_{b1}} = 0$, and $I_{w,p_{b1}} = 1$. The case $I_{w,p_{b1}} = 0$ means that the buyer owns all market in two stages by producing in-house in stage 1, and the case $I_{w,p_{b1}} = 1$ implies that the

buyer focuses on the cost advantage and ignore the value chain climbing of the supplier. In the following, we refer to the case $I_{w,p_{b1}} = 0$ as the market maintaining case, and the case $I_{w,p_{b1}} = 1$ as the cost advantage seeking case.

Let $\pi_b^{d2}(w, p_{b2}, p_s) := (p_{b2} - w)L_{b2}(p_{b2}, p_s)$, $q_{out2}^v(w, p_{b2}) := F_1^{-1}\left(\frac{p_{b2}-w}{p_{b1}}\right), \forall w \in [0, c_b]$; and $\pi_s^{d2}(w, p_{b2}, p_s) := (w - c_s)L_{b2}(p_{b2}, p_s) + (p_s - c_s)L_{s2}(p_{b2}, p_s)$, $q_s^v(p_s) := F_2^{-1}\left(\frac{p_s-c_s}{p_s}\right)$. **Theorem 1** shows the quasi-concavity of problem system (5) and the properties of optimal solutions $p_{b2}^*(w, p_{b1})$ and $p_s^*(w, p_{b1})$.

Theorem 1. (i) In marketing maintaining case ($I_{w,p_{b1}} = 0$), $p_s^*(0, w) = 0$, assume (A1) $\frac{\partial^2 \pi_b^{d2}(c_b, p_{b2}, 0)}{\partial p_{b2}^2} < 0$, $\frac{\partial^3 \pi_b^{d2}(c_b, p_{b2}, 0)}{\partial p_{b2}^3} \leq 0$ and (B1) ϵ_{b2} have an increasing failure rate (IFR) distribution with $r_{b2}(\phi) \leq 3, \forall \phi \in [0, 1]$, then $\pi_b^{d2}(0, w, p_{b2}, 0)$ is quasi-concave in p_{b2} and $p_{b2}^*(0, w)$ is the unique solution of equation (8)

$$E[\min(q_{out2}^v(w, p_{b2}), \epsilon_{b2})] + \frac{\partial \pi_b^{d2}(w, p_{b2}, 0)}{\partial p_{b2}} = 0 \quad (8)$$

Otherwise, $p_{b2}^*(0, w) \in (w, p^{max})$ satisfying equation (8).

(ii) In cost advantage seeking case ($I_{w,p_{b1}} = 1$), assume (A2) $\frac{\partial^2 \pi_b^{d2}(w, p_{b2}, p_s)}{\partial p_{b2}^2} < 0$, $\frac{\partial^2 \pi_s^{d2}(w, p_{b2}, p_s)}{\partial p_s^2} < 0$, $\frac{\partial^3 \pi_b^{d2}(w, p_{b2}, p_s)}{\partial p_{b2}^3} \leq 0$, $\frac{\partial^3 \pi_s^{d2}(w, p_{b2}, p_s)}{\partial p_s^3} \leq 0$ and (B2) $\epsilon_{b2}, \epsilon_{s2}$ have IFR distributions with $r_{b2}(\phi) \leq 3$ and $r_{s2}(\phi) \leq 3, \forall \phi \in [0, 1]$, then $\pi_b^{d2}(1, w, p_{b2}, p_s)$ is quasiconcave in p_{b2} and $\pi_s^{d2}(1, w, p_{b2}, p_s)$ is quasiconcave in p_s , so the optimal solution to the problem (5), $(p_{b2}^*(1, w), p_s^*(1, w))$, is the unique solution of (9)-(10)

$$E[\min(q_{out2}^v(w, p_{b2}), \epsilon_{b2})] + \frac{\partial \pi_b^{d2}(w, p_{b2}, p_s)}{\partial p_{b2}} = 0 \quad (9)$$

$$E[\min(q_s^v(p_s), \epsilon_{s2})] + \frac{\partial \pi_s^{d2}(w, p_{b2}, p_s)}{\partial p_s} = 0 \quad (10)$$

Otherwise, $(p_{b2}^*(1, w), p_s^*(1, w))$ can be determined by searching all interior points in the domain $(w, p^{max}) \times (0, p^{max})$ satisfying equations (9) and (10).

In **Theorem 1**(i) and (ii), the conditions (A1) and (A2) are standard for the existences of unique solution for the deterministic cases of problem (5); Conditions (B1) and (B2) implies the effective stochastic demand components, $D_{b2}(p_{b2}, p_s, \epsilon_{b2})$ and $D_{s2}(p_{b2}, p_s, \epsilon_{b2})$ given $I_{q_{out1}} = 1(I_{w,p_{b1}} = 1)$, have IFR distributions. Furthermore, ϵ_{b2} and ϵ_{s2} may be dependent in cost advantage seeking case, which relaxes the conventional assumption that stochastic demands are independent of each other. Of course, if the conditions of quasi-concavity shown in Theorem 1 do not hold, because the maxima of problem (5) are interior (which we have shown in the Proof of Theorem 1), $p_{b2}^*(0, w)$ or $(p_{b2}^*(1, w), p_s^*(1, w))$ can be determined by exhaustively searching over all points satisfying equation (8) or equation set (9)-(10).

Next, we solve the problem (6). **Theorem 2** shows the conditions of quasi-concavity of $\Pi_b(I_{w,p_{b1}}, w, p_{b1})$ in p_{b1} and the properties of the optimal solution $p_{b1}^*(w)$. Let $\pi_b^{d1}(w, p_{b1}) := (p_{b1} - w)L_1(p_{b1}), \forall w \in [0, c_b]$ be the

deterministic part of the buyer's profit in stage 1, $q_{out1}^v(w, p_{b1}) := F_1^{-1}\left(\frac{p_{b1}-w}{p_{b1}}\right) \forall w \in [0, c_b]$ or $q_{in1}^v(p_{b1}) := F_1^{-1}\left(\frac{p_{b1}-c_b}{p_{b1}}\right)$ is the stochastic part of the outsourcing/in-house production quantity in stage 1 given w and p_{b1} .

Theorem 2. Assume (A) $\frac{\partial^2 \pi_b^{d1}(w, p_{b1})}{\partial p_{b1}^2} < 0$, $\frac{\partial^3 \pi_b^{d1}(w, p_{b1})}{\partial p_{b1}^3} \leq 0$ and (B) ϵ_1 have an IFR distribution with $r_1(\phi) \leq 3, \forall \phi \in [0, 1]$, then $\pi_b^{d1}(w, p_{b1})$ is quasi-concave in p_{b1} and $p_{b1}^*(w)$ is the unique solution of equation (11) or equation (12).

Market maintaining case:

$$E[\min(q_{in1}^v(p_{b1}), \epsilon_1)] + \frac{\partial \pi_b^{d1}(c_b, p_{b1})}{\partial p_{b1}} = 0 \quad (11)$$

Cost advantage seeking case:

$$E[\min(q_{out1}^v(w, p_{b1}), \epsilon_1)] + \frac{\partial \pi_b^{d1}(w, p_{b1})}{\partial p_{b1}} = 0, w \in [0, c_b] \quad (12)$$

Otherwise, $p_{b1}^*(w) \in (w, p^{max})$ satisfying equation (11) or $p_{b1}^*(c_b) \in (c_b, p^{max})$ satisfying equation (12).

Similar to **Theorem 1**, the condition (A) in **Theorem 2** ensure the existences of unique solution for the deterministic cases of problem (6); Condition (B) in **Theorem 2** implies the stochastic demand component, $D_1(p_{b1}, \epsilon_1)$ has IFR distribution. Also, if the conditions of quasi-concavity shown in Theorem 2 do not hold, $p_{b1}^*(w)$ is interior and can be determined by exhaustively searching over all points satisfying equation (11). Actually, given $w \in [c_s, c_b]$, problem (5) and problem (6) are independent and can be solved simultaneously.

After obtaining $p_{b1}^*(w)$, we proceed to solve problem (7). We define $\tilde{W} := \{w \mid \Pi_b(0, w, p_{b1}^*(w)) \geq \Pi_b(1, w, p_{b1}^*(w)), w \in [0, c_b]\}$ as the region of w under which the buyer produces in-house in stage 1 (market maintaining case, $I_{w,p_{b1}^*(w)} = 0$); and $\hat{W} := \{w \mid \Pi_b(1, w, p_{b1}^*(w)) \geq \Pi_b(0, w, p_{b1}^*(w)), w \in [0, c_b]\}$ as the region of w under which the buyer outsources in stage 1 (cost advantage seeking case, $I_{w,p_{b1}^*(w)} = 1$). Note that in cost advantage seeking case, $I_{w,p_{b1}^*(w)} = 1, w \neq c_b$.

In market maintaining case, we define $\tilde{L}_{b2}(w) := L_{b2}(p_{b2}^*(0, w), p_s^*(0, w))$, which is the deterministic demand of buyer in stage 2. Then, let $\tilde{q}_{out2}^v(w) := F_2^{-1}\left(\frac{p_{b2}^*(0,w)-w}{p_{b2}^*(0,w)}\right)$ is the stochastic part of outsourcing quantity in stage 2.

$$\Pi_s(0, w) = (w - c_s)[\tilde{L}_{b2}(w) + \tilde{q}_{out2}^v(w)], w \in \tilde{W} \quad (13)$$

In cost advantage seeking case, we define $\hat{L}_1(w) :=$

$L_1(p_{b1}^*(w)), \hat{L}_{b2}(w) := L_{b2}(p_{b2}^*(1, w), p_s^*(1, w))$ and $\hat{L}_{s2}(w) := L_{s2}(p_{b2}^*(1, w), p_s^*(1, w))$. $\hat{L}_i(w), \forall i \in \{1, b2, s2\}$ are deterministic demands of buyer/supplier in stage 1/2.

Then, let $\hat{q}_{out1}^v(w) := F_1^{-1}\left(\frac{p_{b1}^*(w)-w}{p_{b1}^*(w)}\right), \hat{q}_{out2}^v(w) :=$

$F_2^{-1}\left(\frac{p_{b2}^*(1,w)-w}{p_{b2}^*(1,w)}\right)$ and $\hat{q}_s^v(w) := F_2^{-1}\left(\frac{p_s^*(1,w)-c_s}{p_s^*(1,w)}\right)$ as

stochastic parts of quantities. Then,

$$\Pi_s(1, w) = (w - c_s)[\hat{L}_1(w) + \hat{L}_{b2}(w) + \hat{q}_{out1}^v(w) + \hat{q}_{out2}^v(w)] + (p_s^*(1, w) - c_s)\hat{L}_{s2}(w)$$

$$+p_s^*(1, w)E[\min(\hat{q}_s^v(w), \epsilon_{s2})] - c_s \hat{q}_s^v(w), w \in \hat{W} \quad (14)$$

Ignoring $w \in \tilde{W}$ or $w \in \hat{W}$, **Theorem 3** shows the conditions of concavity of $\Pi_s(I_{w, p_{b1}^*(w)}, w)$ in w and the properties of w^* .

Theorem 3. Ignoring $w \in \tilde{W}$ or $w \in \hat{W}$, $\Pi_s(0, w)$ and $\Pi_s(1, w)$ are concave on $[c_s, c_b]$ if all the following conditions hold: (A) $\tilde{L}_{b2}(w)$ and $\tilde{L}_i(w)$ are non-increasing and concave in $w \in [0, c_b]$, $\forall i \in \{1, b_2\}$; (B) $p_{b1}^*(w), p_{b2}^*(0, w)$ and $p_{b2}^*(1, w)$ are decreasing and concave in $w \in [0, c_s]$; (C) ϵ_1 and ϵ_{b2} are IFR distributions with $dr_i(\phi)/d\phi \geq (r_i(\phi))^2, \forall \phi \in [0, 1], \forall i \in \{1, b_2\}$ and (D) $J(w) = \frac{dp_s^*(w)}{dw}(\tilde{L}_{s2}(w) + E[\min(\hat{q}_s^v(w), \epsilon_{s2})]) + (p_s^*(w) - c_s) \frac{d\tilde{L}_{s2}(w)}{dw}$ is non-increasing in w and $J(w) \geq 0$ when $w \in [0, c_b]$. Then, $w^* \in (c_s, c_b]$.

Based on **Theorem 3**, **Lemma 2** shows the properties of w^* .

Lemma 2. If all conditions showed in Theorem 3 hold, then $w^* \in (c_s, c_b]$ can be determined as:

- (1) If $w_0 \in (c_s, c_b]$ satisfying $\left. \frac{d\Pi_s(0, w)}{dw} \right|_{w=w_0} = 0$ and $\Pi_s(0, w_0) \geq \Pi_s(1, w), \forall w \in (c_s, c_b)$, then $w^* = w_0$;
- (2) If $w_1 \in (c_s, c_b)$ satisfying $\left. \frac{d\Pi_s(1, w)}{dw} \right|_{w=w_1} = 0$ and $\Pi_s(1, w_1) \geq \Pi_s(0, w), \forall w \in (c_s, c_b]$, then $w^* = w_1$;
- (3) If $\lim_{w \rightarrow c_b} \frac{d\Pi_s(1, w)}{dw} \geq 0$ and $\exists w_2 \in (c_s, c_b)$ such that $\Pi_s(1, w_2) = \max_{c_s \leq w \leq c_b} \Pi_s(0, w)$, then $c_s < w_2 < w^* < c_b$;
- (4) Otherwise, $w^* = c_b$.

If the conditions of concavity shown in **Theorem 3** cannot be satisfied, w^* can be searched by comparing $\Pi_s(0), \Pi_s(c_b), \lim_{w \rightarrow c_b} \Pi_s(w), \Pi_s(w^{t0})$ and $\Pi_s(w^{t1})$, where

$$w^{t0} := \arg \max_{w \in W^{t0}} \Pi_s(0, w),$$

$$W^{t0} := \left\{ w \mid \frac{d\Pi_s(0, w)}{dw} = 0, \frac{d^2 \Pi_s(0, w)}{dw^2} \leq 0, w \in [c_s, c_b] \right\};$$

$$w^{t1} := \arg \max_{w \in W^{t1}} \Pi_s(1, w),$$

$$W^{t1} := \left\{ w \mid \frac{d\Pi_s(1, w)}{dw} = 0, \frac{d^2 \Pi_s(1, w)}{dw^2} \leq 0, w \in [0, c_b] \right\}$$

That is, the optimal solution to problem (7) may be the boundary point (0 or c_b), local maximum (w^{t0} or w^{t1}), or be infinitely close to c_b from the left.

V. THE PROPERTIES OF THE NASH EQUILIBRIA UNDER ADDITIVE STOCHASTIC DEMAND WITH LINEAR DETERMINISTIC PART

In this section, we consider a special case that the deterministic parts of all demand are linear, which is widely applied in the literature on the interfaces of operations and marketing ([10]). That is, in stage 1, $L_1(p_{b1}) = a_1 - b_1 p_{b1}$. In Stage 2, if $p_s = 0 (I_{w, p_{b1}} = 0)$, then $L_{b2}(p_{b2}, 0) = a_3 - b_3 p_{b2}$; if $p_s > 0 (I_{w, p_{b1}} = 1)$, then $L_{b2}(p_{b2}, p_s) = a_2 - b_2 p_{b2} + \theta p_s$ and $L_{s2}(p_{b2}, p_s) = a_2 - b_2 p_s + \theta p_{b2}$, where θ refers to the intensity of price competition. Without generality, we assume that all parameters ($a_i, b_i, i \in \{1, 2, 3\}$ and θ are positive and $\theta \leq b_2$). We check the properties of

the optimal retail prices and order/production quantities under different equilibrium, then we compare the price competitive case ($\theta > 0$) to the noncompetitive case ($\theta = 0$).

Proposition 1. In market maintaining case, if $b_1 > \frac{1}{f_1(\phi)}$ and $b_3 > \frac{1}{f_{b2}(\phi)}, \forall \phi \in [0, 1]$, then (1) $p_{b1}^*(c_b)$ is increasing in c_b and $p_{b2}^*(0, w)$ is increasing in w . Specifically, $\frac{dp_{b1}^*(c_b)}{dc_b} < 1$ and $\frac{dp_{b2}^*(0, w)}{dw} < 1$; (2) $q_{in1}(p_{b1}^*(c_b))$ is decreasing in c_b and $q_{out2}(0, w, p_{b2}^*(0, w), 0)$ is decreasing in w .

The market maintaining case means the buyer owns the whole market in two stages. If the price elasticity of demand in all stages, b_1 or b_3 , exceeds a specific limit, then the optimal retail prices of the buyer increase as the increase in the unit cost c_b and w , the optimal production/outsourcing quantities changes in an opposite way. That is, if there is a lack of competition in the market and the price elasticity of demand is high enough in each stage, then as the unit cost of the buyer increases, the increase in retail price with the reduced market demand can bring higher profits than maintaining or reducing the retail price with increased market demand.

In cost advantage seeking case, Proposition 2 shows how the optimal retail prices and outsourcing/production quantities change with the outsourcing price.

Proposition 2. In cost advantage seeking case, if $b_1 > \frac{1}{f_1(\phi)}$ and $b_2 > \max\left\{\frac{1}{f_{b2}(\phi)}, \frac{1}{f_{s2}(\phi)}\right\}, \forall \phi \in [0, 1]$, then (1) $p_{b1}^*(w), p_{b2}^*(1, w)$ and $p_s^*(1, w)$ are increasing in w and $\frac{dp_{b2}^*(1, w)}{dw} < 1$; (2) $q_{out1}(w, p_{b1}^*(w))$ is decreasing in w .

We still find that if the price elasticity of demand at both two stages are high enough, with the increase in supplier's outsourcing price, buyer's retail prices increase and the buyer's outsourcing quantity in stage 1 decreases. Meanwhile, the supplier's retail prices in stage two also increases. Differentiating $q_{out2}^*(1, w, p_{b2}^*(1, w), p_s^*(1, w))$ and $q_s^*(1, w, p_{b2}^*(1, w), p_s^*(1, w))$ with w , we have

$$\begin{aligned} & \frac{dq_{out2}^*(1, w, p_{b2}^*(1, w), p_s^*(1, w))}{dw} \\ &= \left(\frac{w}{(p_{b2}^*(1, w))^2} \frac{1}{f_{b2}\left(\frac{p_{b2}^*(1, w) - c_s}{p_{b2}^*(1, w)}\right)} - b_2 \right) \frac{dp_{b2}^*(1, w)}{dw} \\ & - \frac{1}{p_{b2}^*(1, w)} \frac{1}{f_{b2}\left(\frac{p_{b2}^*(1, w) - c_s}{p_{b2}^*(1, w)}\right)} + \theta \frac{dp_s^*(1, w)}{dw}, \\ & \frac{dq_s^*(1, w, p_{b2}^*(1, w), p_s^*(1, w))}{dw} \\ &= \left(\frac{c_s}{(p_s^*(1, w))^2} \frac{1}{f_{s2}\left(\frac{p_s^*(1, w) - c_s}{p_s^*(1, w)}\right)} - b_2 \right) \frac{dp_s^*(1, w)}{dw} \\ & + \theta \frac{dp_{b2}^*(1, w)}{dw} \end{aligned}$$

Given $b_2 > \max\left\{\frac{1}{f_{b2}(\phi)}, \frac{1}{f_{s2}(\phi)}\right\}$, $\forall \phi \in [0,1]$, it can be shown that only the last term on the right of both equations, $\theta \frac{dp_s^*(1,w)}{dw}$ and $\theta \frac{dp_{b2}^*(1,w)}{dw}$, are positive. Actually, when θ is relatively low, the last term of both two equations have a much smaller impact than other terms, $q_{out2}^*(1,w, p_{b2}^*(1,w), p_s^*(1,w))$ and $q_s^*(1,w, p_{b2}^*(1,w), p_s^*(1,w))$ are decreasing in w , indicating that higher retail prices will be accompanied by lower market demand, this is the most likely scenario. However, when θ is high enough, $q_s^*(1,w, p_{b2}^*(1,w), p_s^*(1,w))$ may be increasing in w , referring to the special case that the higher retail price and the higher market demand coexist for the supplier, which shows the huge benefits of climbing the value chain.

Now, we check how the intensity of price competition θ affects the outsourcing decision. Firstly, we define the optimal retail price as the function of w and θ based on **Theorem 1(ii)** given $I_{w, p_{b1}} = 1: (p_{b2}^*(w, \theta), p_s^*(w, \theta))$ is the solution to the following equations:

$$E[\min(q_{out2}^v(w, p_{b2}), \epsilon_{b2})] + a_2 - 2b_2 p_{b2} + \theta p_s$$

Obviously, $J_1(w, \theta)$ is the expected profit of the outsourcing orders earned by the supplier from the buyer.

We further define $q_s^v(w, \theta) := F_{s2}^{-1}\left(\frac{p_s^*(w, \theta) - c_s}{p_s^*(w, \theta)}\right)$ and $\hat{L}_{s2}(w, \theta) := a_2 - b_2 p_s^*(w, \theta) + \theta p_{b2}^*(w, \theta)$. Then, we rewrite the $\Pi_s(1, w), 0 \leq w < c_b$ as

$$\begin{aligned} \Pi_s(w, \theta) = & J_1(w, \theta) + (p_s^*(w, \theta) - c_s) \hat{L}_{s2}(w, \theta) \\ & + p_s^*(w, \theta) E[\min(\hat{q}_s^v(w, \theta), \epsilon_{s2})] \\ & - c_s \hat{q}_s^v(w, \theta) \end{aligned} \quad (18)$$

Proposition 3 shows a lower bound of θ that ensure the outsourcing relationship exists at both two stages.

Proposition 3. Assume (1) $b_2 > \max\left\{\frac{1}{f_{b2}(\phi)}, \frac{1}{f_{s2}(\phi)}\right\}$, $\forall \phi \in [0,1]$ and (2) $\frac{\partial p_s^*(w, \theta)}{\partial \theta} / \frac{\partial p_{b2}^*(w, \theta)}{\partial \theta} \geq \frac{b_2}{\theta}$, if $\exists w^c \in [0, c_b)$, there exists $\underline{\theta}(w^c) = \min\left\{\theta | J_1(w^c, \theta) \geq \max_{c_s \leq w \leq c_b} \Pi_s(0, w), 0 \leq \theta < b_2\right\}$, then when $\theta \in [\underline{\theta}(w^c), b_2]$, $w^* = \arg \max_{0 \leq w < c_b} \Pi_s(w, \theta)$.

From **Proposition 3**, we know that when (1) the price elasticity of demand in stage 2 is high enough, and (2) $p_s^*(w, \theta)$ is more sensitive than $p_{b2}^*(w, \theta)$ with respect to θ for any $w \in [0, c_b)$, then there exists a lower bound of θ that ensure $w^* < c_b$. The managerial insight from this Proposition is: Given the outsourcing price w , when the price elasticity of the products is high enough, and the supplier is more sensitive to the intensity of price competition, once the intensity of price competition exceeds a minimum extent, the outsourcing relationship exists at both two stages. For the buyer, on the one hand, she has a certain advantage in the face of high-intensity price competition after the supplier enters the market; on the other hand, the cost reduction by outsourcing improves her net profit, so she is inclined to outsource. For the supplier, maintaining the outsourcing relationship with the buyer by providing a lower outsourcing price w will at least allow him to profit more at

$$+ b_2 w = 0 \quad (15)$$

$$\begin{aligned} & E[\min(q_s^v(p_s), \epsilon_{s2})] + (w - c_s)\theta + a_2 - 2b_2 p_s + \theta p_{b2} \\ & + b_2 c_s = 0 \end{aligned} \quad (16)$$

Lemma 3. If $b_2 > \max\left\{\frac{1}{f_{b2}(\phi)}, \frac{1}{f_{s2}(\phi)}\right\}$, $\forall \phi \in [0,1]$, then $p_{b2}^*(w, \theta)$ and $p_s^*(w, \theta)$ are increasing in θ when $w \in [c_s, c_b)$.

That is, the optimal retail prices in stage 2 are both increasing in θ , indicating higher price competition intensity induces the buyer and supplier increase their retail prices when supplier enters the market in stage 2. Based on $p_{b2}^*(w, \theta)$, let $q_{out2}^v(w, \theta) := F_{b2}^{-1}\left(\frac{p_{b2}^*(w, \theta) - w}{p_{b2}^*(w, \theta)}\right)$, $\hat{L}_{b2}(w, \theta) := a_2 - b_2 p_{b2}^*(w, \theta) + \theta p_s^*(w, \theta)$. Then,

$$\begin{aligned} J_1(w, \theta) = & (w - c_s)[\hat{L}_1(w) + \hat{L}_{b2}(w, \theta) + q_{out1}^v(w) \\ & + q_{out2}^v(w, \theta)] \end{aligned} \quad (17)$$

the end market by value chain climbing.

VI. NUMERICAL STUDY

We present numerical examples to exhibit our analyses conducted in this paper. Specifically, we carry out two experiments: a baseline experiment and a nonlinear experiment. In the baseline experiment, the deterministic component of the demand function is in linear form as $Q = a - bp$, $a, b > 0$. In contrast, in the nonlinear experiment, the deterministic component of a demand function is expressed in power form as $Q = ap^{-b}$, $a, b > 0$. The stochastic components of demand functions, denoted as ϵ_{b2} and ϵ_{s2} , are independent and identically distributed and are shared across both experiments. For ease of calculation, we consider two demand distributions: low demand distribution $F_L(\cdot)(f_L(\cdot))$ and high demand distribution $F_H(\cdot)(f_H(\cdot))$, where the probability density functions f_L and f_H are defined as:

$$f_L(x) = \begin{cases} 1.5, & 0 < x \leq 0.5 \\ 0.5, & 0.5 < x < 1 \end{cases}$$

$$f_H(x) = \begin{cases} 0.5, & 0 < x \leq 0.5 \\ 1.5, & 0.5 < x < 1 \end{cases}$$

In distribution $f_L(\cdot)$, stochastic demand is more likely to fall within the low demand interval $(0, 0.5]$. In contrast, distribution $f_H(\cdot)$ is more likely to fall within the high demand interval $(0.5, 1)$. Based on the above two types of probability distribution types, we define the following four scenarios:

Type 1: $\epsilon_1 \sim F_1(\cdot) = F_H(\cdot)$ and $\epsilon_{b2}(\epsilon_{s2}) \sim F_2(\cdot) = F_L(\cdot)$;

Type 2: $\epsilon_1 \sim F_1(\cdot) = F_L(\cdot)$ and $\epsilon_{b2}(\epsilon_{s2}) \sim F_2(\cdot) = F_H(\cdot)$;

Type 3: $\epsilon_1 \sim F_1(\cdot) = F_H(\cdot)$ and $\epsilon_{b2}(\epsilon_{s2}) \sim F_2(\cdot) = F_H(\cdot)$;

Type 4: $\epsilon_1 \sim F_1(\cdot) = F_L(\cdot)$ and $\epsilon_{b2}(\epsilon_{s2}) \sim F_2(\cdot) = F_L(\cdot)$;

Since w^* in Problem (7) cannot be solved analytically, we design **Algorithm 1** to find the approximate optimal solutions $(\hat{w}^*, \hat{p}_{b1}^*, \hat{p}_{b2}^*, \hat{p}_s^*, \hat{q}_{out1}^*, \hat{q}_{in1}^*, \hat{q}_{out2}^*, \hat{q}_s^*)$ and optimal values $(\hat{\Pi}_b^*, \hat{\Pi}_s^*)$. All the equilibrium information

mentioned hereinafter are referred to $\tilde{w}^*, \tilde{p}_{b1}^*, \tilde{p}_{b2}^*, \tilde{p}_s^*, \tilde{q}_{out1}^*, \tilde{q}_{in1}^*, \tilde{q}_{out2}^*, \tilde{q}_s^*, \tilde{\Pi}_b^*$ and $\tilde{\Pi}_s^*$.

The design idea of the **Algorithm 1** is as follows: 1) In Steps 1-2, we initialize the optimal solutions and values. 2) In Steps 3 – 20, we enumerate $w^{\text{tem}} \in [0, c_b)$ with a step size of 0.005 to find if a better solutions for Problem (5) to Problem (7). After the loop, we obtain the $\tilde{w}^*, \tilde{p}_{b1}^*, \tilde{p}_{b2}^*, \tilde{p}_s^*, \tilde{q}_{out1}^*, \tilde{q}_{in1}^*, \tilde{q}_{out2}^*, \tilde{q}_s^*, \tilde{\Pi}_b^*, \tilde{\Pi}_s^*$ when $w < c_b$. 3) In Steps 21-27, we compute $\Pi_b(c_b, p_{b1}^*(c_b), p_{b2}^*(0, c_b), 0, 0, q_{in1}(p_{b1}^*(c_b)), q_{out2}(0, c_b, p_{b2}^*(0, c_b), 0))$ and $\Pi_s(c_b, p_{b2}^*(0, c_b), 0, 0, q_{out2}(0, c_b, p_{b2}^*(0, c_b), 0), 0)$ and check whether the optimal solutions and optimal values should be updated.

We first examine the approximate optimal solutions ($\tilde{w}^*, \tilde{p}_{b1}^*, \tilde{p}_{b2}^*, \tilde{p}_s^*, \tilde{q}_{out1}^*, \tilde{q}_{in1}^*, \tilde{q}_{out2}^*, \tilde{q}_s^*$) and values ($\tilde{\Pi}_b^*, \tilde{\Pi}_s^*$) in the baseline experiment, then conduct a sensitivity analysis to investigate the influence of relevant parameters on the outsourcing decisions and profits within the supply chain. Subsequently, we proceed to explore the approximate optimal solutions and values for both parties in the nonlinear experiment, similarly performing a sensitivity analysis to assess how the relevant parameters affect the outsourcing decisions and profits. Finally, we conduct cross-model performance comparisons between the two cases, focusing specifically on the comparative impacts of the common influencing factors.

A. Baseline Experiment: Strategies and Sensitivity Analysis

Assume that in the baseline experiment, the deterministic components of the demand functions are expressed as: $L_1(p_{b1}) = a_1 - b_1 p_{b1}$, $L_{b2}(p_{b2}, p_s) = a_2 - b_2 p_{b2} + \theta p_s$, and $L_{s2}(p_{b2}, p_s) = a_2 - b_2 p_s + \theta p_{b2}$, or $L_{b2}(p_{b2}, 0) = a_3 - b_3 p_{b2}$. Let $c_s = 0.1, c_b = 0.2, a_1 = a_3 = 1, a_2 = 0.6, b = b_1 = b_2 = b_3 = 0.1$, and $\theta = 0.01$. The approximate solutions and values of Problem (5) to Problem (7) are solved by MATLAB R2016a (the 'fmincon' function is used to solve the nonlinear optimization problem in Algorithm 1). The approximate optimal solutions and values under different stochastic demand types are shown in Table I.

From Table I, we can see that: (1) Based on parameters $c_b, c_s, a_1, a_2, a_3, b$ and θ , the buyer always outsources in stage 1 ($\tilde{q}_{in1} = 0$) and the approximate optimal wholesale price $\tilde{w}$ approaches c_b from left for each stochastic demand type. (2) A higher retail price and lower outsourcing quantity are offered by the buyer in stage 1 when $\epsilon_1 \sim F_H(\cdot)$ compared to $\epsilon_1 \sim F_L(\cdot)$, and the buyer and supplier also make the similar price and quantity decisions in stage 2. This phenomenon indicates that the buyer and supplier apply a high price to gain more profit when stochastic demands are relatively high and set a lower price to increase sales when the stochastic demands are relatively low. (3) In stage 2, $\tilde{p}_{b2}^*$ is slightly higher than $\tilde{p}_s^*$ and $\tilde{q}_{out2}^*$ is slightly lower than $\tilde{q}_s^*$. (4) When ϵ_1 and ϵ_{b2} follow the same distribution (Type 3 and Type 4), the buyer will decrease the retail price and outsourcing quantities from stage 1 to stage 2 ($\tilde{p}_{b1}^* > \tilde{p}_{b2}^*$ and $\tilde{q}_{out1}^* > \tilde{q}_{out2}^*$). Meanwhile, the sum of quantities at the

market in stage 2 is large than the quantity in stage 1 ($\tilde{q}_{out2}^* + \tilde{q}_s^* > \tilde{q}_{out1}^*$) for each stochastic demand type. Compared with the market monopolized by the buyer in stage 1, the competition brought by the entry of the supplier in stage 2 enables consumers to buy more products at lower prices.

In sensitivity analysis, we explore the factors that may affect outsourcing decisions under each stochastic demand types. Normally, the ratio of production costs between the buyer and the supplier (c_b/c_s) is a key drive of the outsourcing behavior. In this part, we fix c_s and let c_b take two levels: $c_b = 0.2(c_b/c_s = 2)$ for low level with label CB1 and $c_b = 0.8(c_b/c_s = 8)$ for high level with label CB2. In addition, we also explore the impact of price elasticity of deterministic demand b and the intensity of price competition θ . Let $a_1 = a_3 = 1$ and $a_2 = 0.6, b = 0.1$ is the low level of the price elasticity with label B1, $b = 0.5$ is the high level of the price elasticity with label B2. Similarly, $\theta = 0.01$ is the low level of intensity of price competition between the supplier and buyer with THETA1, and $\theta = 0.09$ is the high level of intensity of price competition between the supplier and buyer with label THETA2. Run the Algorithm 1 under different combination of variable parameters (c_b, b and θ) and stochastic demand type, Table II shows the equilibrium type (E.T.) and approximate total expected profits of buyer and supplier ($\tilde{\Pi}_b^*$ and $\tilde{\Pi}_s^*$), where O_1 refers to the equilibrium type that the buyer outsources in stage 1 with $c_s < \tilde{w}^* < c_b$.

From Table II, we find that $\tilde{w}^* = 0.1950$ when $c_b = 0.2$ and $\tilde{w}^* = 0.7950$ when $c_b = 0.8$, which means the $\tilde{w}^*$ is the maximum value when the loop in Algorithm 1 (Steps 3-20) ends. Therefore, $\tilde{w}^*$ approaches c_b from left under each combination of parameters and stochastic demand type. For the buyer, there are two possible reasons to outsource in stage 1, regardless of the supplier's value chain climbing in stage 2: 1) Given $w < c_b$, the buyer can indirectly adjust her market demand and unit profit in each stage by appropriately adjusting the retail prices because we apply the additive model to describe the stochastic demands in this paper. Even if the supplier enters the market in stage 2, the buyer can also adjust p_{b2} to obtain a reasonable product market demand $D_{b2}(p_{b2}, p_s, \epsilon_{b2})$ and unit profit $p_{b2} - w$ to ensure global profit maximization. Unlike the price-based outsourcing contract, the buyer cannot adjust the market demand and the unit profit in the quantitative outsourcing contract, in which the product's retail price is fixed and the stochastic demand in each stage is independent of the retail price. So the loss of market share derived from value chain climbing in stage 2 may largely hurt the buyer's profit, and she may choose to produce in-house in stage 1. 2) For the deterministic demand part $L_i(P, \epsilon_i)$, where $i \in \{1, b2, s2\}, P \in \{\{p_{b1}\}, \{p_{b2}, p_s\}\}$, the price elasticity of buyer and supplier under linear form is fixed ($-b$), which may induce the buyer refuse to change the outsourcing strategy, and the value of b in the experiment can ensure that the buyer obtain the enough benefit when outsourcing in both stages.

Based on the data in Table II, we further explore whether the different levels of $b, c_b/c_s$, and θ have a significant effect on the total expected profit of the buyer and supplier for each stochastic demand type. We use the 3-Way

ANOVA without replication, the confidence level is 0.05 , the analysis results are shown in Table III.

From Table III, we know that for the mean responses of $\tilde{I}_b^*$ and $\tilde{I}_s^*$ in each stochastic demand type are significantly different for the levels B1 and B2 of the factor , but not for the levels CB1 and CB2 of the factor c_b/c_s , nor the levels of THETA1 and THETA2 of the factor θ . Next, we perform multiple comparison tests for each combination of three grouping variables, b , c_b/c_s , and θ regarding to $\tilde{I}_b^*$ and $\tilde{I}_s^*$, respectively. The results are shown in Figure 2 and 3. In each sub-figure of these two figures, the group with the maximum mean response of $\tilde{I}_b^*$ or $\tilde{I}_s^*$ is in blue, and we named it as benchmark group. Those groups with a significant population margin mean from benchmark group are shown in red, while those with a non-significant population margin mean are in grey.

In Figure 2, we show that, for all stochastic demand types, the mean responses of $\tilde{I}_b^*$ are higher under the combinations with level B1 than those under the combinations with level B2, indicating that the low price demand elasticity increases the buyer's total expected profit. The maximum mean response of $\tilde{I}_b^*$ appears under the combination of B1, CB1, and THETA 2. Meanwhile, the two combinations of B2 and THETA1 are the buyer's worst decision-making environment, for the $\tilde{I}_b^*$ under these two combinations are significantly lower than that under the combination of B1, CB1, and THETA2. In Figure 3, we find that the mean responses of $\tilde{I}_s^*$ under the combinations of B 1 are higher than B 2, but not significantly so.

B. Nonlinear Experiment: Strategies and Sensitivity Analysis

In this subsection, we analyze the outsourcing strategies and profits of the buyer and supplier under the stochastic demand with nonlinear deterministic component expressed in power function, i.e., $L_1(p_{b1}) = p_{b1}^{-0.5}$, $L_{b2}(p_{b2}, p_s) = p_{b2}^{-0.5} + \theta p_s$, and $L_{s2}(p_{b2}, p_s) = p_s^{-0.5} + \theta p_{b2}$ or $L_{b2}(p_{b2}, 0) = p_{b2}^{-0.5}$. To avoid scenarios where $\tilde{p}_{b1}^*$ approaches to infinity and $\tilde{q}_{out1}^*$ approaches to 0 in cost advantage seeking case, or $\tilde{p}_{b1}^*$ approaches to infinity and $\tilde{q}_{out2}^*$ approaches to 0 in market maintaining case, we introduce the parameter P^{max} as the upper bound of p_{b1} and p_{b2} . Let $c_s = 0.1$, $c_b = 0.2$, $P^{max} = 2$ and $\theta = 0.01$, the approximate solutions and values of Problem (5) to Problem (7) are solved by MATLAB R2016a (the 'fmincon' function is used to solve the nonlinear optimization problem in Algorithm 1). The approximate optimal solutions and values are shown in Table IV.

From Table IV, we can see that: (1) Based on parameters c_b , c_s , P^{max} and θ , the buyer always outsources in stage 1 ($\tilde{q}_{in1}^* = 0$) and the approximate optimal wholesale price $\tilde{w}$ approaches c_b from left for each stochastic demand type. (2) In stage 1, the buyer sets a sufficiently high retail price to maintain a high margin profit, regardless of the demand distribution. In stage 2, the buyer and supplier set higher retail prices when $\epsilon_{b2}(\epsilon_{s2}) \sim F_H(\cdot)$ compared to $\epsilon_{b2}(\epsilon_{s2}) \sim F_L(\cdot)$. This phenomenon indicates that the buyer and supplier apply a high price to gain more profit when stochastic demands are relatively high and set a lower price to increase sales when the stochastic demands are relatively

low. (3) In stage 2, $\tilde{p}_{b2}^*$ is slightly higher than $\tilde{p}_s^*$ and $\tilde{q}_{out2}^*$ is slightly lower than $\tilde{q}_s^*$. (4) When ϵ_1 and ϵ_{b2} follow the same distribution (Type 3 and Type 4), the sum of quantities at the market in stage 2 is large than the quantity in stage 1 ($\tilde{q}_{out2}^* + \tilde{q}_s^* > \tilde{q}_{out1}^*$) for each stochastic demand type. Compared with the market monopolized by the buyer in stage 1, the competition brought by the entry of the supplier in stage 2 enables consumers to buy more products at lower prices.

In sensitivity analysis, we explore the impact of P^{max} , the ratio of production costs between the buyer and supplier ($c_b/c_s = 2$) and the intensity of price competition (θ) on the outsourcing behavior of the buyer and supplier. Let $P^{max} = 2$ be the low level with label PMAX1 and $P^{max} = 10$ be the high level with label PMAX2. $c_b/c_s = 2$ is the low level with label CB1 and $c_b/c_s = 8$ is the high level with label CB2; $\theta = 0.01$ is the low level with label THETA1 and $\theta = 0.09$ is the high level with label THETA2. Run the Algorithm 1 under different combination of variable parameters (c_b , $P^{max} = 10$ and θ) and stochastic demand type, Table V show the equilibrium type (E.T.) and approximate total expected profits of buyer and supplier ($\tilde{I}_b^*$ and $\tilde{I}_s^*$), where O_1 refers to the equilibrium type that the buyer outsources in stage 1 with $c_s < \tilde{w}^* < c_b$, and O_2 refers to the equilibrium type that the buyer outsources in stage 1 with $\tilde{w}^* = c_b$.

We find that $\tilde{w}^* = 0.1950$ when $c_b = 0.2$ and $\tilde{w}^* = 0.7950$ when $c_b = 0.8$ if the equilibrium type is O_1 . Compared to the case of linear deterministic demand part, when $P^{max} = 2$ (PMAX1, low level) and $\theta = 0.09$ (THETA2, high level) and stochastic demand type is 'Type 1' ($\epsilon_1 \sim F_1(\cdot) = F_H(\cdot)$ and $\epsilon_{b2}(\epsilon_{s2}) \sim F_2(\cdot) = F_L(\cdot)$), the equilibrium type with nonlinear deterministic demand part is O_2 (supplier outsources at $\tilde{w}^* = c_s$): On the one hand, the lower price ceiling makes the buyer unwilling to outsource unless the outsourcing price is low in stage 1. Furthermore, when stochastic demand in the second stage is relatively low, and the price competition between the buyer and supplier is fierce, only the cost advantage of outsourcing can enable the buyer to obtain reasonable profit margins and competitive advantages. On the other hand, the low stochastic demand in stage 2 limits the supplier into the market because of the limited benefits provided by outsourcing. Also, $\tilde{p}_{b1}^* = P^{max}$ indicating that the buyer sells product at the highest price in a monopoly market when the deterministic demand part is in power function form.

Based on the data in Table V, we further explore whether the different levels of P^{max} , c_b/c_s , and θ have a significant effect on the total expected profit of the buyer and supplier for each stochastic demand type. We use the 3-Way ANOVA without replication, and the confidence level is 0.05 . The analysis results are shown in Table VI.

From Table VI, we know that the mean responses of $\tilde{I}_b^*$ and $\tilde{I}_s^*$ in each stochastic demand type are significantly different for the levels PMAX1 and PMAX2 of the factor P^{max} , but not for the levels CB1 and CB2 of the factor c_b/c_s . For the buyer, the mean responses of $\tilde{I}_b^*$ are significantly different for the levels THETA1 and THETA2 of θ only in stochastic demand Type 1 and Type 4 . For the supplier, the levels of θ do not affect the mean response of $\tilde{I}_s^*$.

Next, we perform multiple comparison tests for each combination of three grouping variables, p^{max} , c_b/c_s , and θ regarding to $\tilde{\Pi}_b^*$ and $\tilde{\Pi}_s^*$, respectively. The results are shown in Figure 4 and 5. In each subfigure of these two figures, the group with the maximum mean response of $\tilde{\Pi}_b^*$ or $\tilde{\Pi}_s^*$ is in blue, and we named it as benchmark group. Those groups with a significant population margin mean from benchmark group are shown in red or pink (the mean responses of groups in red are also significantly different from those in pink), while those with a non-significant population margin mean are in grey.

In Figure 4, we show that, for all stochastic demand types, the mean responses of $\tilde{\Pi}_b^*$ are higher under the combinations with level B1 than those under the combinations with level PMAX2, indicating that the high price ceiling in stage 1 increases the buyer's total expected profit. The maximum mean response of $\tilde{\Pi}_b^*$ appears under the combination of PMAX2, CB2, and THETA2. Meanwhile, the two combinations of PMAX1 and THETA1 are the buyer's worst decision-making environment, for the $\tilde{\Pi}_b^*$ under these two combinations are significantly lower than under the combination of PMAX2, CB2, and THETA2 except in Type 2. Specifically, in Figure 4(a) and Figure 4(d), the eight groups are divided into three clusters, and the mean responses of each cluster are significantly different, indicating that the high stochastic demand in stage 2 (Type 1 and Type 4) makes the impact of the combinations of p^{max} , c_b/c_s , and θ more differentiated on $\tilde{\Pi}_b^*$. In Figure 5, we find that the mean responses of $\tilde{\Pi}_s^*$ under the combinations of PMAX2 are higher than PMAX1, but not significantly so.

C. Cross-Model Performance Comparisons

In this subsection, we synthesize the core insights from the baseline experiment and nonlinear experiment. We examine the comparative impacts of the common factors in both experiments—production cost ratio (c_b/c_s), competition intensity (θ), and stochastic demand types—on the profits of supply chain members. We omit the impact on outsourcing decisions because, under all existing conditions, the buyer always chooses to outsource.

We first compare the impact of the production cost ratio (c_b/c_s) on baseline and nonlinear experiments. Using data from Table II, we compute the profit change rates for the buyer and the supplier under each stochastic distribution combination (Type 1 to Type 4) during the shift from CB1 (low level, $c_b/c_s=2$) to CB2 (high level, $c_b/c_s=8$) across the four factor combinations (B1+THETA1, B1+THETA2, B2+THETA1, B2+THETA2) in the baseline experiment, then we calculate their means and standard deviations. Similarly, we compute the profit change rates for the buyer and the supplier under each stochastic distribution combination (Type 1 to Type 4) during the shift from CB1 (low level, $c_b/c_s=2$) to CB2 (high level, $c_b/c_s=8$) across the four factor combinations (PMAX1+THETA1, PMAX1+THETA2, PMAX2+THETA1, PMAX2+THETA2) in the nonlinear experiment, then compute their means and standard deviations. Figure 6 displays these statistics as a bar chart with error bars.

From the upper chart of Figure 6, we observe that buyer's profit change rates are predominantly negative (-30.8% to 2.9%) in the baseline experiment, indicating a general profit decrease as the production cost ratio (c_b/c_s) rises. This demonstrates that higher cost ratios significantly adversely affect buyer's profits when the deterministic component of the stochastic demand is linear. In contrast, buyer's profit change rate in the nonlinear experiment shows a more complex pattern. Though still generally declining, the rates span a wider range (-2.0% to 6.0%), indicating that cost ratio increases do not uniformly impact the buyer's profit change rate and other factors also influence the outcome, leading to a mix of positive and negative profit changes.

The lower chart of Figure 6 displays the supplier's profit change rates, which remain consistently positive (45.4% to 68.4%), showing that supplier profits increase with higher cost ratios. In the Nonlinear Experiment, supplier's profit change rates are also positive but show greater increases (52.0% to 70.3%) compared to Baseline experiment, demonstrating a more pronounced positive effect from the cost ratio increase. Overall, these patterns reveal the complex impact of production cost ratio changes on supply chain profit distribution. While Baseline experiment proves detrimental to the buyer's profit but beneficial to the supplier's, the nonlinear experiment shows more complex impact on the buyer's profit combined with significantly enhanced supplier's benefit. These findings are crucial for understanding how cost ratio changes affect all supply chain participants.

We then compare the impact of competition intensity (θ) on baseline and nonlinear experiments. Using data from Table V, we compute the profit change rates for the buyer and the supplier under each stochastic distribution combination (Type 1 to Type 4) during the shift from THETA1 (low level, $\theta=0.01$) to THETA2 (high level, $\theta=0.09$) across the four factor combinations (B1+CB1, B1+CB2, B2+CB1, B2+CB2) in the baseline experiment, then calculate their means and standard deviations. Similarly, we compute the profit change rates for the buyer and the supplier under each stochastic distribution combination (Type 1 to Type 4) during the shift from THETA1 (low level, $\theta=0.01$) to THETA2 (high level, $\theta=0.09$) across the four factor combinations (PMAX1+CB1, PMAX1+CB2, PMAX2+CB1, PMAX2+CB2) in the nonlinear experiment, then calculate their means and standard deviations. Figure 7 displays these statistics as a bar chart with error bars.

From the upper chart of Figure 7, we observe that the buyer's profit change rates in the baseline experiment show significant fluctuations (35.6% to 170.4%), indicating considerable variation in buyer's profit growth under lower competition intensity. Notably, the buyer experienced the highest profit increase in Type1 and endured a relatively lower growth in Type 4. The standard deviations reveal profit change rate volatility, demonstrating different stochastic demand scenario (Type 1, Type 2, Type 3 and Type 4) varying sensitivity to competition intensity changes. In nonlinear experiment, buyer's profit change rates span a

broader range (46.1% to 205.5%), suggesting that under higher competition intensity, buyer's profit change rates are generally higher with greater volatility—particularly for Type 4 in which buyer's profit change rate reaches 205.5%. This indicates more complex and variable impacts of competition intensity on buyer's profit when the deterministic component of stochastic demand is nonlinear.

From the lower chart of Figure 7, we observe that the supplier's profit change rates in the baseline experiment vary (95.7% to 177.8%), showing considerable differences in profit growth under lower competition intensity. Specifically, the supplier achieved the greatest increase in Type 1 and showed a relatively smaller growth in Type 2. The relatively high standard deviations indicate significant profit change rate volatility, reflecting the supplier's diverse responses to competition intensity changes. In the nonlinear experiment, the supplier's profit change rates show greater consistency across types (96.4% to 182.9%). Although the standard deviations remain high, volatility is somewhat reduced compared to the baseline experiment. This indicates that when the deterministic component of stochastic demand is nonlinear, increased competition intensity significantly impacts the supplier's profit while diminishing differences among stochastic demand distribution scenarios. Overall, Figure 7 analysis reveals that increased competition intensity notably affects supply chain profit change rates for both parties. Nonlinear experiment typically show higher profit change rates and greater volatility, indicating profound and complex profit effects from competition intensity changes.

VII. CONCLUSION

In this paper, we develop a two-stage game-theoretic model based on price competition to examine the vertical co-competition resulting from value chain climbing in a supply chain under uncertain demand. In Stage 1, the supplier sets the outsourcing price, followed by the buyer's decisions on retail price and outsourcing/in-house production quantities. In Stage 2, if outsourcing occurred in Stage 1, the supplier enters the market to compete with the buyer, and both parties simultaneously make price and quantity decisions. Otherwise, only the buyer decides on retail price and outsourcing quantity. In both stages, stochastic demand is a function of retail prices. The supplier may set the outsourcing price at or below its production cost to gain market entry and climb the value chain.

Our equilibrium analysis identifies two key scenarios: the market maintaining case, where the buyer fully outsources in Stage 1, and the cost advantage seeking case, where the buyer produces entirely in-house. We derive the fundamental properties of each equilibrium for additive stochastic demand with a linear deterministic component. Specifically, we explore the optimal retail prices and order/production quantities under different equilibria and compare the competitive and non-competitive cases. Results indicate that when the price elasticity of demand is sufficiently high, the optimal prices in both stages increase with the supplier's cost advantage, while the buyer's outsourcing and in-house quantities decrease. Moreover, under certain conditions, a lower bound of price competition

intensity ensures that full outsourcing remains optimal in Stage 1.

The numerical study further illustrates, under linear and nonlinear deterministic demand, the optimal outsourcing strategies and their impact on total expected profits. Results show that buyers tend to outsource in the presence of price competition, as value chain climbing by suppliers can effectively expand the market. This finding underscores the importance of balancing cost advantages with the potential risks of vertical competition. Specifically, buyers are more likely to outsource when facing high price elasticity of demand and intense price competition, as these factors can mitigate the impact of potential competition from suppliers. Conversely, when demand is less elastic or price competition is weaker, buyers may prefer in-house production to avoid direct competition. Additionally, the study highlights the significant role of demand uncertainty in shaping outsourcing decisions. In highly uncertain environments, buyers may opt for outsourcing to leverage the supplier's economies of scale and flexibility, while in stable demand conditions, in-house production may be preferred to maintain control over quality and supply chain oversight. Our study provides valuable insights into strategic decision-making in supply chains, highlighting the critical role of price competition and demand uncertainty in shaping outsourcing strategies. These findings offer practical guidance for firms navigating complex supply chain dynamics, particularly in balancing the benefits of cost reduction with the risks of supplier competition.

The numerical study demonstrates that buyer consistently maintains outsourcing strategies irrespective of supplier's value chain climbing risk, the various stochastic demand types and other factors. This persistence stems from the buyer's ability to dynamically adjust retail prices in response to stochastic demand, offsetting the competition from the supplier. In baseline experiment (the deterministic component of the stochastic demand is linear), a higher degree of price elasticity is positively correlated with increased buyer profitability, whereas the level of competition intensity does not exert a statistically significant influence on the buyer's profit. Conversely, in the nonlinear experiment (the deterministic component of the stochastic demand is nonlinear), increased competitive intensity leads to heightened profit volatility for the buyer, characterized by greater fluctuations in realized profits. Furthermore, production cost disparities asymmetrically affect the buyer and supplier: the supplier consistently gains from rising cost ratios in both two experiments, whereas the buyer faces profit erosion in baseline experiment. These findings collectively validate that strategic outsourcing endures under competition primarily through adaptive pricing.

This study offers insights into the strategic interactions between outsourcing and value chain climbing but has limitations that point to future research directions. Firstly, the study employs a simplified demand model with linear and nonlinear deterministic components, which might not fully capture real-world complexities like nonlinear relationships and dynamic customer preferences. Future research could be enhanced by using more sophisticated demand models to better reflect market realities. Secondly, the study centers on binary decision-making (full

outsourcing vs. full in-house production), possibly ignoring hybrid strategies that merge both approaches. Future research could look into these mixed strategies to present a more realistic view of supply chain management. By tackling these limitations, future research can build on the current findings to offer practical advice for firms dealing with complex supply chains.

AUTHORS CONTRIBUTIONS

All authors contributed to the design and writing of the study. Dr. Lin and Dr. Wang were responsible for the conceptualization and methodology development. Dr. Wang also provided financial support for the research. Dr. Zhang conducted data collection and software validation. All authors participated in the formal analysis. Dr. Lin supervised the project and overall research process and revised the manuscript. Finally, all authors reviewed and approved the final manuscript.

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Algorithm 1: Find the approximate optimal solutions and values of Problem (5) to Problem (7)

Input: $a_1, b_1, a_2, b_2, a_3, b_3, \theta, c_b, c_s$

Output: $\tilde{w}^*, \tilde{p}_{b1}^*, \tilde{p}_{b2}^*, \tilde{p}_s^*, \tilde{q}_{out1}^*, \tilde{q}_{in1}^*, \tilde{q}_{out2}^*, \tilde{q}_s^*, \tilde{\Pi}_b^*, \tilde{\Pi}_s^*$

1. $\tilde{p}_{b1}^* \leftarrow 0, \tilde{p}_{b2}^* \leftarrow 0, \tilde{p}_s^* \leftarrow 0, \tilde{q}_{out1}^* \leftarrow 0, \tilde{q}_{in1}^* \leftarrow 0, \tilde{q}_{out2}^* \leftarrow 0, \tilde{q}_s^* \leftarrow 0$
2. $\Pi_b^* \leftarrow 0, \Pi_s^* \leftarrow 0$
3. **for** $w=0:0.005:c_b - 0.0001$ **do**
4. $p_{b2}^{mar} \leftarrow p_{b2}^*(0, w)$ *via Theorem 1(i)*
5. $(p_{b2}^{out}, p_s^{out}) \leftarrow (p_{b2}^*(1, w), p_s^*(1, w))$ *via Theorem 1(ii)*
6. $p_{b1}^{mar} \leftarrow p_{b1}^*(c_b), p_{b1}^{out} \leftarrow p_{b1}^*(w)$ *via Theorem 2*
7. $q_{in1}^{mar} \leftarrow q_{in1}(p_{b1}^{mar}), q_{out1}^{out} \leftarrow q_{out1}(w, p_{b1}^{out})$ *via Lemma 1(1)*
8. $q_{out2}^{mar} \leftarrow q_{out}(0, w, p_{b2}^{mar}, 0), q_{out2}^{out} \leftarrow q_{out2}(1, w, p_{b2}^{out}, p_s^{out}), q_s^{out} \leftarrow q_s(1, w, p_{b2}^{out}, p_s^{out})$ *via Lemma 1(2)*
9. $\Pi_b^{mar} \leftarrow \Pi_b(w, p_{b1}^{mar}, p_{b2}^{mar}, 0, 0, q_{in1}^{mar}, q_{out2}^{mar}), \Pi_s^{mar} \leftarrow \Pi_s(w, p_{b2}^{mar}, 0, 0, q_{out2}^{mar}, 0)$
10. $\Pi_b^{out} \leftarrow \Pi_b(w, p_{b1}^{out}, p_{b2}^{out}, p_s^{out}, q_{out1}^{out}, 0, q_{out2}^{out}), \Pi_s^{out} \leftarrow \Pi_s(w, p_{b2}^{out}, p_s^{out}, q_{out1}^{out}, q_{out2}^{out}, q_s^{out})$
11. **if** $(\Pi_s^{mar} \geq \tilde{\Pi}_s^* \text{ or } \Pi_s^{out} \geq \tilde{\Pi}_s^*)$ **then**
12. **if** $(\Pi_s^{mar} > \Pi_s^{out})$ **then**
13. $\tilde{w}^* \leftarrow w, \tilde{p}_{b1}^* \leftarrow p_{b1}^{mar}, \tilde{p}_{b2}^* \leftarrow p_{b2}^{mar}, \tilde{p}_s^* \leftarrow 0, \tilde{q}_{out1}^* \leftarrow 0, \tilde{q}_{in1}^* \leftarrow q_{in1}^{mar}, \tilde{q}_{out2}^* \leftarrow q_{out2}^{mar}, \tilde{q}_s^* \leftarrow 0$
14. $\tilde{\Pi}_b^* \leftarrow \Pi_b^{mar}, \tilde{\Pi}_s^* \leftarrow \Pi_s^{mar}$
15. **else**
16. $\tilde{w}^* \leftarrow w, \tilde{p}_{b1}^* \leftarrow p_{b1}^{out}, \tilde{p}_{b2}^* \leftarrow p_{b2}^{out}, \tilde{p}_s^* \leftarrow p_s^{out}, \tilde{q}_{out1}^* \leftarrow q_{out1}^{out}, \tilde{q}_{in1}^* \leftarrow 0, \tilde{q}_{out2}^* \leftarrow q_{out2}^{out}, \tilde{q}_s^* \leftarrow q_s^{out}$
17. $\tilde{\Pi}_b^* \leftarrow \Pi_b^{out}, \tilde{\Pi}_s^* \leftarrow \Pi_s^{out}$
18. **end if**
19. **end if**
20. **end for**
21. $w \leftarrow c_b, p_{b2}^{mar} \leftarrow p_{b2}^*(0, c_b)$ *via Theorem 1(i),* $p_{b1}^{mar} \leftarrow p_{b1}^*(c_b)$ *via Theorem 2*
22. $q_{in1}^{mar} \leftarrow q_{in1}(p_{b1}^{mar})$ *via Lemma 1(1),* $q_{out2}^{mar} \leftarrow q_{out2}(0, w, p_{b2}^{mar}, 0)$ *via Lemma 1(2).*
23. $\Pi_b^{mar} \leftarrow \Pi_b(c_b, p_{b1}^{mar}, p_{b2}^{mar}, 0, 0, q_{in1}^{mar}, q_{out2}^{mar}), \Pi_s^{mar} \leftarrow \Pi_s(c_b, p_{b2}^{mar}, 0, 0, q_{out2}^{mar}, 0)$
24. **if** $(\Pi_s^{mar} \geq \tilde{\Pi}_s^*)$ **then**
25. $\tilde{w}^* \leftarrow c_b, \tilde{p}_{b1}^* \leftarrow p_{b1}^{mar}, \tilde{p}_{b2}^* \leftarrow p_{b2}^{mar}, \tilde{p}_s^* \leftarrow 0, \tilde{q}_{out1}^* \leftarrow 0, \tilde{q}_{in1}^* \leftarrow q_{in1}^{mar}, \tilde{q}_{out2}^* \leftarrow q_{out2}^{mar}, \tilde{q}_s^* \leftarrow 0$
26. $\tilde{\Pi}_b^* \leftarrow \Pi_b^{mar}, \tilde{\Pi}_s^* \leftarrow \Pi_s^{mar}$
27. **end if**

Table I. The approximate optimal solutions and values in the baseline experiment

	$\tilde{w}^*$	$\tilde{p}_{b1}^*$	$\tilde{p}_{b2}^*$	$\tilde{p}_s^*$	$\tilde{q}_{out1}^*$	$\tilde{q}_{in1}^*$	$\tilde{q}_{out2}^*$	$\tilde{q}_s^*$	$\tilde{\Pi}_b^*$	$\tilde{\Pi}_s^*$
Type 1	0.1950	8.1741	5.2250	5.1891	1.1667	0	1.0548	1.0948	8.8121	2.7375
Type 2	0.1950	6.9210	6.5464	6.5067	1.2515	0	0.9906	1.0046	8.4707	4.2750
Type 3	0.1950	8.1741	6.5464	6.5067	1.1667	0	0.9906	1.0046	10.3543	4.2669
Type 4	0.1950	6.9210	5.2250	5.1891	1.2515	0	1.0548	1.0948	6.9285	2.7455

Table II. The sensitivity analysis in the baseline experiment

b		B1				B2			
c_b/c_s		CB1		CB2		CB1		CB2	
θ		THETA1	THETA2	THETA1	THETA2	THETA1	THETA2	THETA1	THETA2
Type 1	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\check{H}_b^*$	8.8121	14.0906	7.6781	13.0615	1.4233	1.5056	0.6865	0.7315
	$\check{H}_s^*$	2.7375	8.0931	3.8819	9.9360	0.5559	0.6530	1.1557	1.3122
Type 2	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\check{H}_b^*$	8.4707	16.7901	7.3913	15.9056	1.3338	1.4698	0.6738	0.7729
	$\check{H}_s^*$	4.2750	12.6865	5.4612	14.6953	0.8488	1.0001	1.4484	1.6736
Type 3	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\check{H}_b^*$	10.3543	18.6736	9.2129	17.7272	1.7020	1.8380	0.8240	0.9232
	$\check{H}_s^*$	4.2669	12.6785	5.4867	14.7208	0.8552	1.0064	1.6243	1.8495
Type 4	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\check{H}_b^*$	6.9285	12.2070	5.8565	11.2400	1.0550	1.1374	0.5362	0.5812
	$\check{H}_s^*$	2.7455	8.1011	3.8564	9.9104	0.5495	0.6466	0.9798	1.1362

Table III. Three-Way ANOVA result in the baseline experiment

Type 1: $\check{H}_b^*$						Type 1: $\check{H}_s^*$				
Source	b	c_b/c_s	θ	Error	Total	b	c_b/c_s	θ	Error	Total
Sum Sq.	193.0210	1.6877	14.5495	13.9397	223.1880	54.9776	2.2537	17.0035	16.0542	90.2889
d.f.	1	1	1	4	7	1	1	1	4	7
Mean Sq.	193.0210	1.6877	14.5495	3.4820		54.9776	2.2537	17.0035	4.0135	
F	55.4300	0.4800	4.1800			13.7000	0.5600	4.2400		
p -value	0.0017**	0.5247	0.1104			0.0208*	0.4953	0.1087		
Type 2: $\check{H}_b^*$						Type 2: $\check{H}_s^*$				
Source	b	c_b/c_s	θ	Error	Total	b	c_b/c_s	θ	Error	Total
Sum Sq.	245.4030	1.3790	36.4140	34.4990	317.6950	129.1843	2.4948	40.5968	37.9128	210.1888
d.f.	1	1	1	4	7	1	1	1	4	7
Mean Sq.	245.4032	1.3792	36.4142	8.6247		129.1843	2.4948	40.5968	9.4782	
F	28.4536	0.1599	4.2221			13.6296	0.2632	4.2832		
p -value	0.0059**	0.7097	0.1091			0.0210*	0.6350	0.1073		
Type 3: $\check{H}_b^*$						Type 3: $\check{H}_s^*$				
Source	b	c_b/c_s	θ	Error	Total	b	c_b/c_s	θ	Error	Total
Sum Sq.	321.0705	1.8826	36.4171	34.4599	393.8301	126.5442	2.9699	40.5995	37.7894	207.9029
d.f.	1	1	1	4	7	1	1	1	4	7
Mean Sq.	321.0705	1.8826	36.4171	8.6150		126.5442	2.9699	40.5995	9.4473	
F	37.2689	0.2185	4.2272			13.3947	0.3144	4.2975		
p -value	0.0036**	0.6645	0.1089			0.0216*	0.6049	0.1069		
Type 4: $\check{H}_b^*$						Type 4: $\check{H}_s^*$				
Source	b	c_b/c_s	θ	Error	Total	b	c_b/c_s	θ	Error	Total
Sum Sq.	135.4856	1.2123	14.5509	13.9920	165.2407	56.7192	1.8431	17.0029	16.1809	91.7462
d.f.	1	1	1	4	7	1	1	1	4	7
Mean Sq.	135.4856	1.2123	14.5509	3.4980		56.7192	1.8431	17.0029	4.0452	
F	38.7324	0.3466	4.1598			14.0212	0.4556	4.2032		
p -value	0.0034**	0.5877	0.1110			0.0200*	0.5367	0.1097		

Table IV. The approximate optimal solutions and values in the nonlinear experiment

	$\check{w}^*$	$\check{p}_{b1}^*$	$\check{p}_{b2}^*$	$\check{p}_s^*$	$\check{q}_{out1}^*$	$\check{q}_{in1}^*$	$\check{q}_{out2}^*$	$\check{q}_s^*$	$\check{H}_b^*$	$\check{H}_s^*$
Type 1	0.1950	1.9999	1.0482	1.0269	1.4350	0	1.2106	1.4000	2.7427	1.0975
Type 2	0.1950	1.9999	1.3282	1.2872	1.3050	0	1.3667	1.4276	2.6951	1.5301
Type 3	0.1950	1.9999	1.3282	1.2872	1.4350	0	1.3667	1.4276	3.1824	1.5424
Type 4	0.1950	1.9999	1.0482	1.0269	1.3050	0	1.2106	1.4000	2.2554	1.0851

Table V. The sensitivity analysis in the nonlinear experiment

p^{max}		PMA1				PMA2			
c_b/c_s		CB1		CB2		CB1		CB2	
θ		THETA1	THETA2	THETA1	THETA2	THETA1	THETA2	THETA1	THETA2
Type 1	E.T.	O_1	O_2	O_1	O_2	O_1	O_1	O_1	O_1
	$\tilde{\Pi}_b^*$	2.7427	8.6540	2.1709	8.6540	9.6565	18.0171	11.3983	20.6259
	$\tilde{\Pi}_s^*$	1.0975	3.1038	2.4059	3.1038	2.9092	11.4232	5.0717	15.5672
Type 2	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\tilde{\Pi}_b^*$	2.6951	2.9175	2.2664	2.4959	9.4426	23.3249	11.4322	26.4205
	$\tilde{\Pi}_s^*$	1.5301	1.7746	2.9327	3.3856	5.1862	19.2665	7.5978	24.1238
Type 3	E.T.	O_1	O_1	O_1	O_1	O_1	O_1	O_1	O_1
	$\tilde{\Pi}_b^*$	3.1824	3.4048	2.5847	2.8142	11.9400	25.8224	13.8900	28.8784
	$\tilde{\Pi}_s^*$	1.5424	1.7870	3.1644	3.6173	5.1886	19.2690	7.6715	24.1975
Type 4	E.T.	O_1	O_2	O_1	O_2	O_1	O_1	O_1	O_1
	$\tilde{\Pi}_b^*$	2.2554	8.1573	1.8526	8.1573	7.1590	15.5197	8.9404	18.1680
	$\tilde{\Pi}_s^*$	1.0851	3.1038	2.1742	3.1038	2.9067	11.4208	4.9980	15.4935

Table VI. Three-Way ANOVA results in the nonlinear experiment

Type 1: $\tilde{\Pi}_b^*$						Type 1: $\tilde{\Pi}_s^*$					
Source	p^{max}	c_b/c_s	θ	Error	Total	Source	p^{max}	c_b/c_s	θ	Error	Total
Sum Sq.	175.5550	1.7850	112.3720	6.6710	296.3820	Sum Sq.	79.7603	7.2483	58.9356	37.7650	183.7093
d.f.	1	1	1	4	7	d.f.	1	1	1	4	7
Mean Sq.	175.5550	1.7850	112.3720	1.6680		Mean Sq.	79.7603	7.2483	58.9356	9.4413	
F	105.2700	1.0700	67.3800			F	8.4481	0.7677	6.2423		
p-value	0.0005***	0.3593	0.0012**			p-value	0.0438*	0.4304	0.0669		
Type 2: $\tilde{\Pi}_b^*$						Type 2: $\tilde{\Pi}_s^*$					
Source	p^{max}	c_b/c_s	θ	Error	Total	Source	p^{max}	c_b/c_s	θ	Error	Total
Sum Sq.	453.6870	2.2418	107.4761	105.6624	669.0673	Sum Sq.	270.8779	13.2162	122.4902	115.5875	522.1718
d.f.	1	1	1	4	7	d.f.	1	1	1	4	7
Mean Sq.	453.6870	2.2418	107.4761	26.4156		Mean Sq.	270.8779	13.2162	122.4902	28.8969	
F	17.1750	0.0849	4.0687			F	9.3740	0.4574	4.2389		
p-value	0.0143*	0.7853	0.1139			p-value	0.0376*	0.5359	0.1086		
Type 3: $\tilde{\Pi}_b^*$						Type 3: $\tilde{\Pi}_s^*$					
Source	p^{max}	c_b/c_s	θ	Error	Total	Source	p^{max}	c_b/c_s	θ	Error	Total
Sum Sq.	587.2970	1.8219	107.4776	106.0562	802.6527	Sum Sq.	266.9841	14.7525	122.4918	115.2832	519.5115
d.f.	1	1	1	4	7	d.f.	1	1	1	4	7
Mean Sq.	587.2970	1.8219	107.4776	26.5141		Mean Sq.	266.9841	14.7525	122.4918	28.8208	
F	22.1504	0.0687	4.0536			F	9.2636	0.5119	4.2501		
p-value	0.0093**	0.8062	0.1144			p-value	0.0383*	0.5139	0.1083		
Type 4: $\tilde{\Pi}_b^*$						Type 4: $\tilde{\Pi}_s^*$					
Source	p^{max}	c_b/c_s	θ	Error	Total	Source	p^{max}	c_b/c_s	θ	Error	Total
Sum Sq.	107.7813	2.0266	110.9700	6.7681	227.5460	Sum Sq.	80.3411	6.5759	60.2687	36.7430	183.9287
d.f.	1	1	1	4	7	d.f.	1	1	1	4	7
Mean Sq.	107.7813	2.0266	110.9700	1.6920		Mean Sq.	80.3411	6.5759	60.2687	9.1858	
F	63.6992	1.1977	65.5838			F	8.7463	0.7159	6.5611		
p-value	0.0013**	0.3353	0.0013**			p-value	0.0417*	0.4451	0.0625		

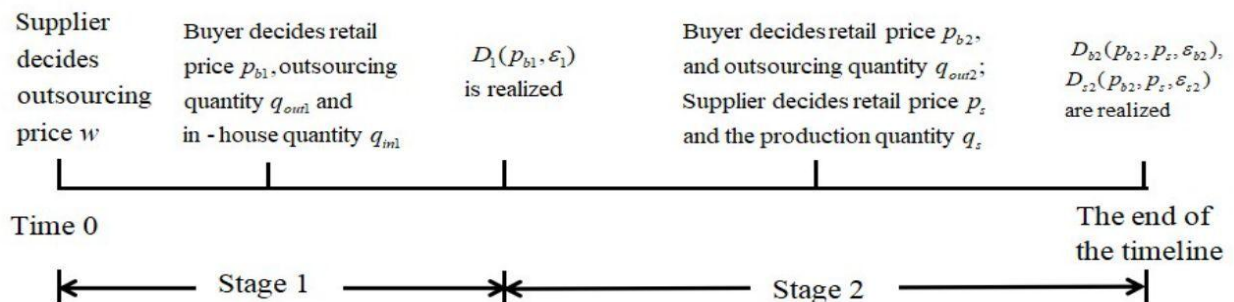
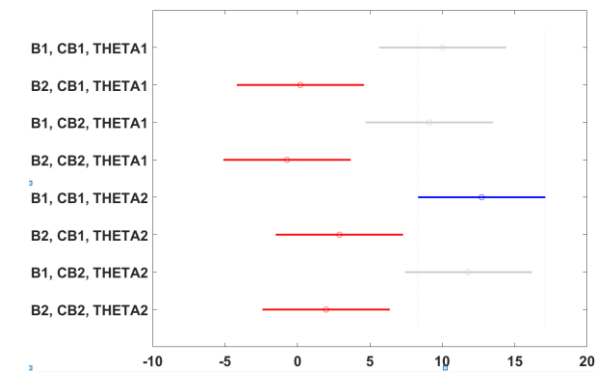
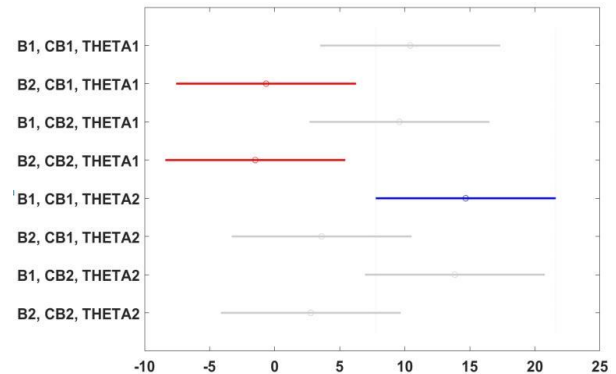


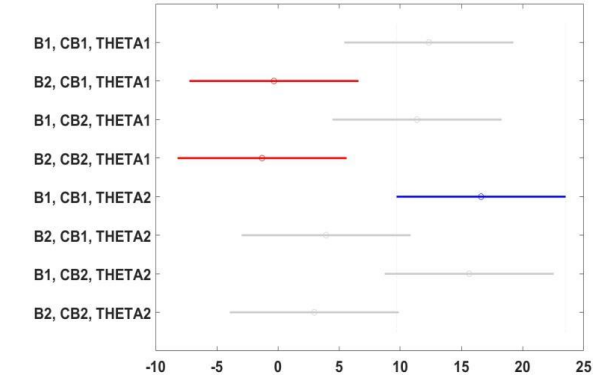
Figure 1. Sequence of decisions



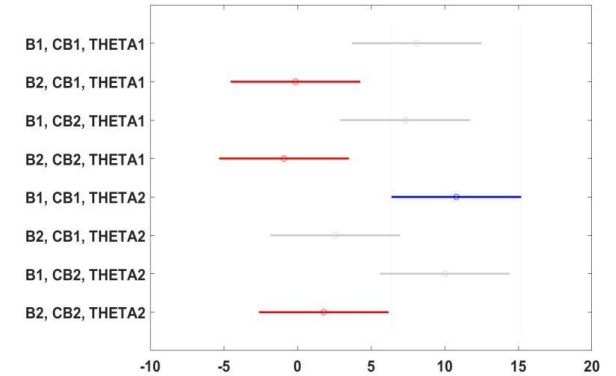
(a) The comparison of $\tilde{\Pi}_b^*$ in Type 1: 4 groups have population marginal means significantly different from B1, CB1,THETA2



(b) The comparison of $\tilde{\Pi}_b^*$ in Type 2: 2 groups have population marginal means significantly different from B1, CB1,THETA2

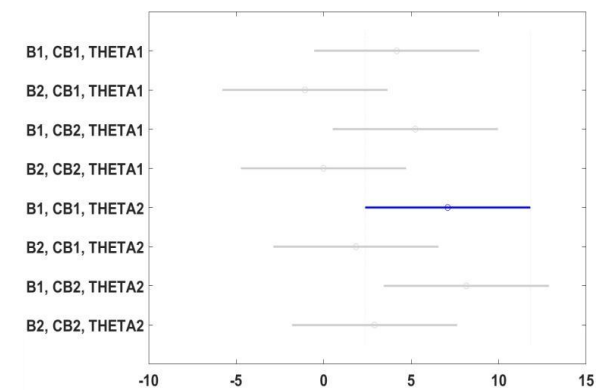


(c) The comparison of $\tilde{\Pi}_b^*$ in Type 3: 2 groups have population marginal means significantly different from B1, CB1,THETA2

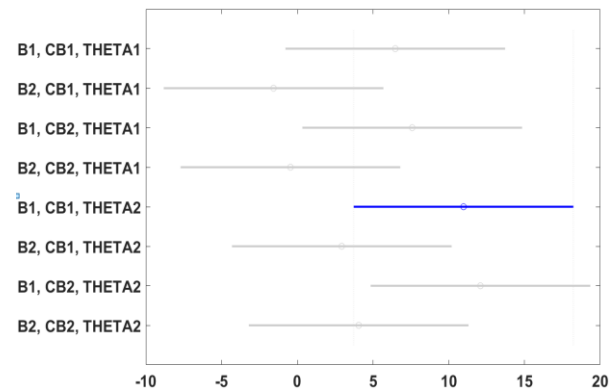


(d) The comparison of $\tilde{\Pi}_b^*$ in Type 4: 3 groups have population marginal means significantly different from B1, CB1,THETA2

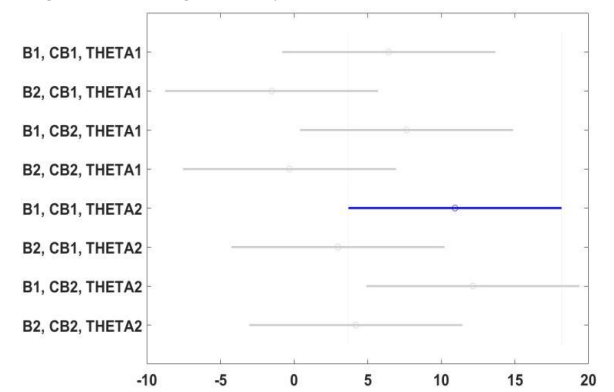
Figure 2. Multiple comparison tests for each combination of three grouping variables regarding to $\tilde{\Pi}_b^*$: Baseline scenario



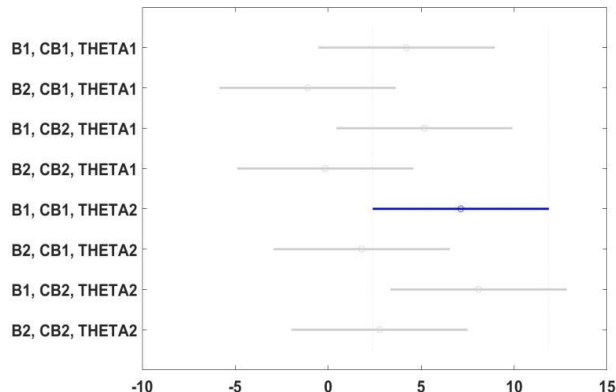
(a) The comparison of $\tilde{\Pi}_s^*$ in Type 1: No groups have population marginal means significantly different from B1, CB1,THETA2



(b) The comparison of $\tilde{\Pi}_s^*$ in Type 2: No groups have population marginal means significantly different from B1, CB1,THETA2

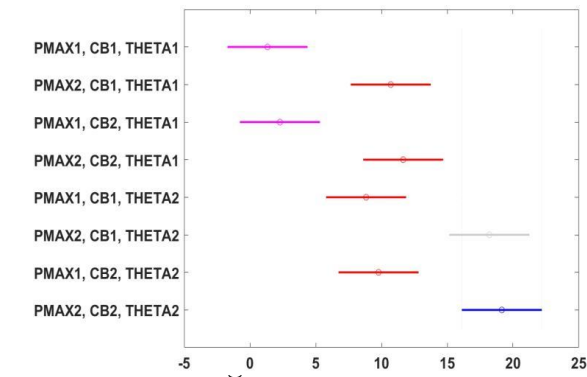


(c) The comparison of $\tilde{\Pi}_s^*$ in Type 3: No groups have population marginal means significantly different from B1, CB1,THETA2

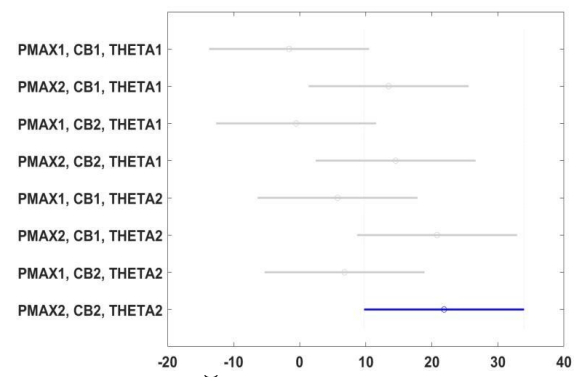


(d) The comparison of $\tilde{\Pi}_s^*$ in Type 4: No groups have population marginal means significantly different from B1, CB1,THETA2

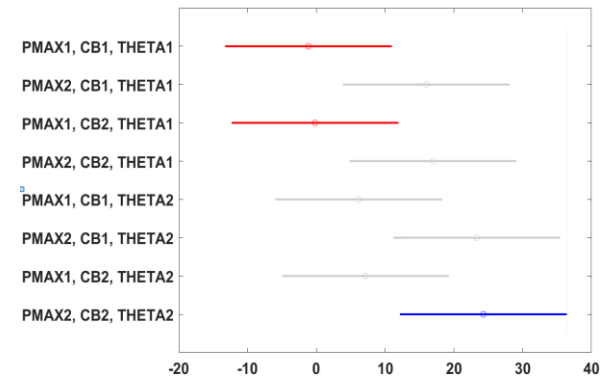
Figure 3. Multiple comparison tests for each combination of three grouping variables regarding to $\tilde{\Pi}_s^*$: Baseline scenario



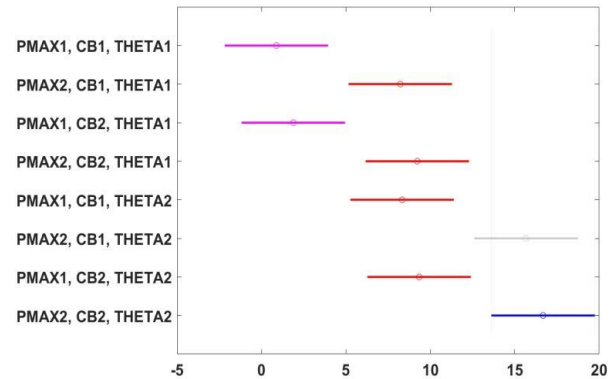
(a) The comparison of $\tilde{\Pi}_b^*$ in Type 1: 6 groups have population marginal means significantly different from PMAX2, CB2, THETA2



(b) The comparison of $\tilde{\Pi}_b^*$ in Type 2: No groups have population marginal means significantly different from PMAX2, CB2, THETA2

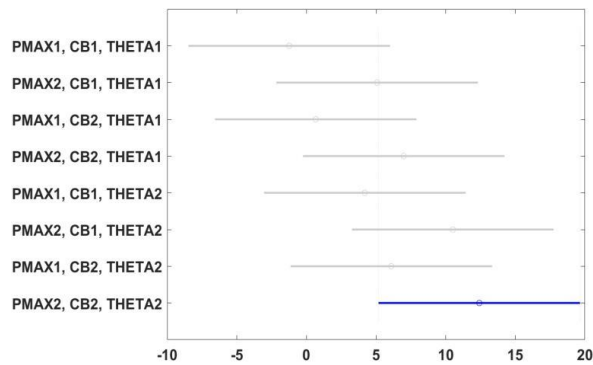


(c) The comparison of $\tilde{\Pi}_b^*$ in Type 3: 2 groups have population marginal means significantly different from PMAX2, CB2, THETA2

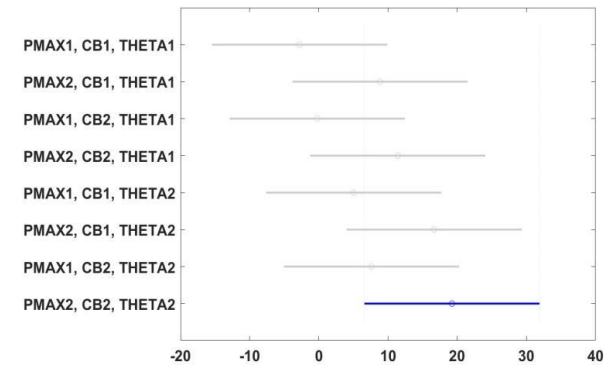


(d) The comparison of $\tilde{\Pi}_b^*$ in Type 4: 6 groups have population marginal means significantly different from PMAX2, CB2, THETA2

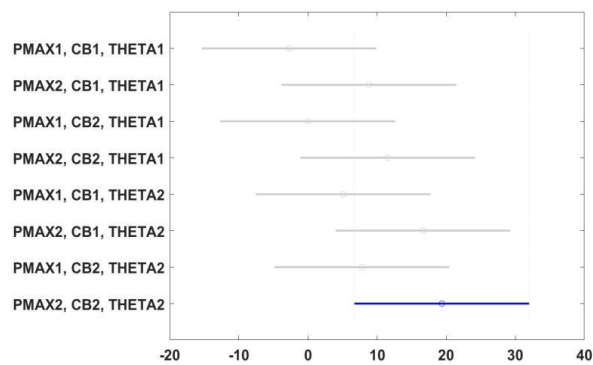
Figure 4. Multiple comparison tests for each combination of three grouping variables regarding to $\tilde{\Pi}_b^*$: Nonlinear deterministic demand part



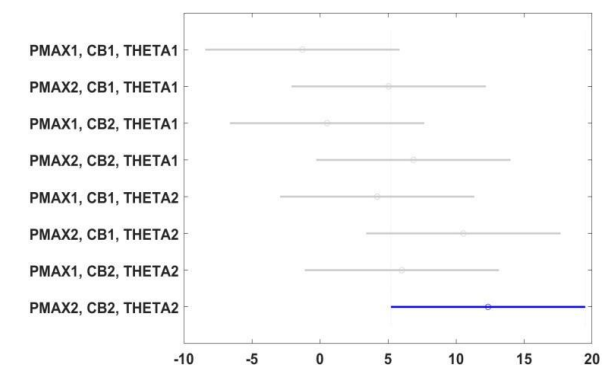
(a) The comparison of $\tilde{\Pi}_s^*$ in Type 1: No groups have population marginal means significantly different from PMAX2, CB2, THETA2



(b) The comparison of $\tilde{\Pi}_s^*$ in Type 2: No groups have population marginal means significantly different from PMAX2, CB2, THETA2



(c) The comparison of $\tilde{\Pi}_s^*$ in Type 3: No groups have population marginal means significantly different from PMAX2, CB2, THETA2



(d) The comparison of $\tilde{\Pi}_s^*$ in Type 4: No groups have population marginal means significantly different from PMAX2, CB2, THETA2

Figure 5. Multiple comparison tests for each combination of three grouping variables regarding to $\tilde{\Pi}_s^*$: Nonlinear deterministic demand part

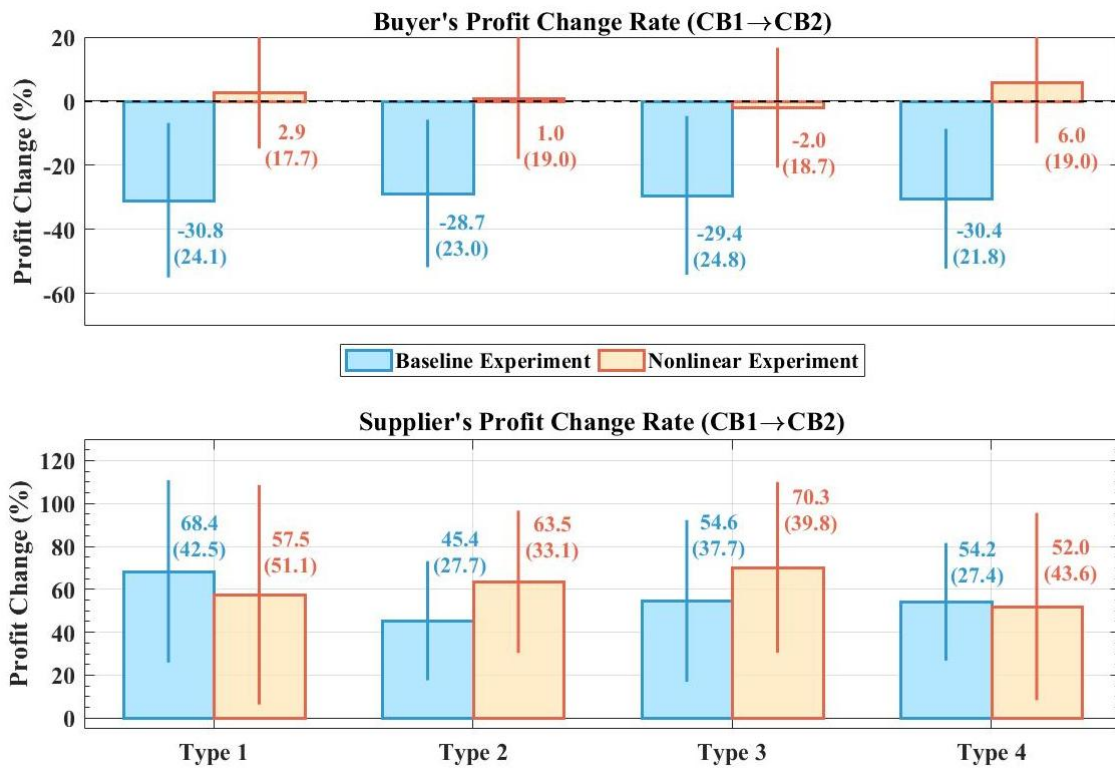


Figure 6. Cross-model comparisons: the impact of the production cost ratio (c_b/c_s) change



Figure 7. Cross-model comparisons: the impact of the competitive intensity (θ) change