Estimating Close Stock Prices of PT Indosat Tbk Using Newton and Cubic Spline Interpolation

Sri Purwani, Astrid Sulistya Azahra, Willen Vimelia

***Abstract*— Capital markets play an important role in the domestic and global economy. In Indonesia, their current performance is developing positively and promises to improve further. Shares or stocks are the financial instruments that are commonly chosen by investors in the capital market. While investing in stocks allows investors to earn high profits, it also puts them at risk of experiencing high losses. One effort to anticipate these losses is to estimate stock prices, which can be done mathematically by estimating the unknown prices in a range of the data through the use of interpolation. The models described herein describe the trend of the data, which allows for analysis that leads to avoiding losses and earning high profits due to stock prices. The models are constructed using Newton interpolation and cubic spline interpolation, and they are then analyzed and compared. The simulation was performed using the close stock prices data from PT Indosat Tbk, a telecommunications company, for the period 1 August to 29 September 2023. The results show that the estimation of close stock prices using cubic spline interpolation closely approaches the actual data over the entire interval, whereas Newton interpolation results in significant errors around the edges of the interval. Therefore, it can be concluded that estimating close stock prices using cubic spline interpolation produces better and more accurate results than Newton interpolation. Moreover, it is shown that the uniqueness property of cubic spline interpolation ensures that only one set of functions is produced, where each is defined on the corresponding subinterval. These results can help parties who are involved in the stock market earn high profits and avoid high losses.**

***Index Terms*—Newton interpolation, close stock prices, cubic spline, estimation**

I. Introduction

In this digital world, investment has become a main topic of discussion by various groups. Investment entails investing a sum of money in several assets to obtain profits in the future [1]. This investment can take various forms, including bonds, deposits, and stocks, which are usually called financial asset investments. These investments in financial assets or securities are usually bought and sold through the capital market. The capital market is a means of funding for companies and governments, as well as a place for investment activities for fund owners or investors.

Since 2022, the Indonesian capital market's performance has shown a trend of positive development [2]. This encourages individuals to be more actively involved in the capital market, especially in stock investment.

Stocks are one of the financial instruments that are used in trading in the capital market. They have ownership characteristics, which means that stocks are proof of a person's or an entity's ownership of a company or limited liability company [3]. Someone who is interested in investing in stocks generally tends to choose to buy stocks from state-owned enterprises or companies with positive growth, such as companies in the telecommunications industry sector. PT Indosat Tbk is one of the companies in the Indonesian telecommunications sector that buys and sells stocks. It has been a government-owned company (i.e., a state-owned company) since 1980. It first entered the capital market on 19 October 1994, where it traded its stocks.

Investing in stocks is not without the possibility of risk exposure. This is caused by fluctuations in stock prices, which can cause very high losses and very high profits. In addition, stock investment can also be influenced by a company's condition and performance, dividend risk, interest rates, inflation rates, and other factors [4]. Therefore, efforts need to be made to reduce this risk, namely by determining the development of stock values through assessments. The results of these stock assessments can help investors engage in buying and selling stock [5]. Estimating the possibility of high or low stock prices can be done through analyzing historical stock prices using a scientific approach.

Research on stock price prediction using mathematics was carried out in [6], where support vector regression was used. This study found that the prediction model that was developed using this method has predictive power, especially in terms of updating the model periodically. Furthermore, [7] conducted research on stock price prediction, where stock prices were predicted using a back-propagation neural network. The results show that this model was able to predict stock prices with a small mean squared error. Stock prediction research was also carried out in [8], where Indonesian stock prediction was analyzed using a support vector machine method. The results showed that the stock price predictions derived from this model had a sharp ratio of

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Therefore, this study estimates the close stock prices of PT Indosat Tbk using other mathematical methods, namely interpolation, with a focus on Newton and cubic spline interpolation. Interpolation is a process that is used to build a function, the plot of which goes through available data points to estimate or predict other values in the interval. Newton interpolation entails using Newton's divided difference formula for the interpolating polynomial, and it produces a smooth curve that is used for estimation. Meanwhile, a cubic spline consists of cubic polynomials that relate to each pair of consecutive data points, which also produces a smooth curve that is used for estimation [9]. This study aims to estimate and compare the close stock prices obtained using the two methods, with the more effective method then being employed to analyze the entire dataset. The appropriate and accurate estimates derived from these models will allow parties involved in the stock market to earn higher profits and prevent large losses.

II. Materials and Methods

A. Stock

In general, a company that requires long-term capital for its business interests will issue stocks. Stocks are securities that companies issue as a means of capital ownership or funds in a company [10]. This ownership proportion depends on how much capital is invested in the company. The value of a stock is usually called the "stock price." Potential investors need to pay strict attention to stock prices before investing, because they can reflect the company's performance. High stock prices not only increase a company's value but also increase market confidence in the company's current performance and future prospects [11].

There are several terms related to stock prices in the capital market, including the following:

a. "Open" or "opening price" is the first price that is provided at the opening of a stock's first trading session.
b. "High" or "highest price" is the highest price of a stock that occurs during trading on that day.
c. "Low" or "lowest price" is the lowest price of a stock that occurs during trading on that day.
d. "Close" or "closing price" is the closing price of a stock. The closing price of a day is determined at the end of the second trading session, which is at precisely 16.00.

B. Newton Interpolation

Interpolation is the process of constructing and calculating a function, the graph of which passes through a set of data points. These points are commonly measurements that are obtained from an experiment, an observation, or a known function [12]. Therefore, interpolation becomes a useful tool for estimating function values that are not recorded or for which there are missing data.

A polynomial $P_n(x)$ of degree n can be written in the standard form as follows:

$$P_n(x) = a_0 + a_1 x + \dots + a_n x^n \tag{1}$$

Here, $a_0, a_1, \dots, a_n$ are the coefficients of the interpolation. If the coefficient a_n is not zero, then the polynomial has degree at most *n*.

Newton interpolation can be regarded as a representation of the earlier Lagrange interpolation. Lagrange interpolation is very useful in terms of developing and deriving new methods in numerical analysis. However, it is less efficient in a computational aspect due to the reformulation that is needed when raising the degree of a polynomial from lower-degree polynomials [12].

Given two points, such as (x_0, y_0) and (x_1, y_1), where x_0 and x_1 are distinct but y_0 and y_1 may be the same, a polynomial of degree at most 1 $P_1(x)$ which interpolates y_0 and y_1 at points x_0 and x_1 correspondingly, can take the following form:

$$P_1(x) = a_0 + a_1(x - x_0) \tag{2}$$

Here, a_0 and a_1 are coefficients of the polynomial. For a Newton interpolation polynomial, these coefficients can be defined as $a_0 = y_0 = f(x_0)$ and $a_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = f[x_1, x_0]$, with the form of divided difference and the interpolation property $P_1(x_i) = y_i = f\left(x_i\right), i = 0,1$ being used. Substituting these coefficients into (2) results in the following:

$$P_1(x) = f(x_0) + f[x_1, x_0](x - x_0) \tag{3}$$

With three points, such as (x_0, y_0), (x_1, y_1) and (x_2, y_2), a Newton interpolation polynomial of degree at most 2 $P_2(x)$ can take the following forms:

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_1)(x - x_0) \tag{4}$$

or

$$P_2(x) = P_1(x) + a_2(x - x_1)(x - x_0) \tag{5}$$

By substituting $x = x_2$ into (4) and applying the previous interpolation property, the value of a_2 can then be obtained as follows:

$$a_2 = \frac{f(x_2) - a_0 - a_1(x_2 - x_0)}{(x_2 - x_1)(x_2 - x_0)} \tag{6}$$

Subsequently, substituting a_0 and a_1 into (6) results in the following:

$$a_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{(x_2 - x_0)}$$

$$a_2 = \frac{f[x_2, x_1] - f[x_1, x_0]}{(x_2 - x_0)} \tag{7}$$

$$a_2 = f\left[x_2, x_1, x_0\right]$$

Thus, the Newton interpolation function of degree at most 2 can be written as follows:

$$P_2(x) = f(x_0) + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_1)(x - x_0) \tag{8}$$

In (5), $P_2(x)$ can be formed from the lower-degree polynomial $P_1(x)$. Thus, the formulation of Newton interpolation polynomials involves lower-degree polynomials. Therefore, a Newton polynomial of degree *n* can be formed from that of degree *n*-1. Similarly, its coefficients can be expressed as follows:

$$\begin{aligned} a_0 &= f[x_0] \\ a_1 &= f[x_1, x_0] \\ a_2 &= f[x_2, x_1, x_0] \\ &\vdots \\ a_n &= f[x_n, x_{n-1}, \cdots, x_0] \end{aligned}$$

Thus, the general form of the Newton interpolation is obtained, which can be expressed as follows:

$$\begin{aligned} P_n(x) = {} & f(x_0) + f[x_1, x_0](x - x_0) + ... \\ & + f[x_n, x_{n-1}, ..., x_1, x_0](x - x_0)(x - x_1)...(x - x_{n-1}) \end{aligned} \tag{9}$$

C. Uniqueness of Polynomial Interpolation

The uniqueness property inherent in polynomial interpolation is that there is only one polynomial that satisfies the interpolation condition. This is described in the following theorem.

Theorem: Let $n \geq 0$. Suppose $\{x_i\}_{i=0}^n$ is $n+1$ different numbers, and let $\{y_i\}_{i=0}^n$ be $n+1$ given numbers. Hence, among all polynomials of degree at most *n*, there is only one polynomial, $P_n(x)$, that satisfies the following:

$$P_n(x_i) = y_i, \quad i = 0, 1, \cdots, n$$

Proof: Suppose there is another polynomial $Q_n(x)$ of degree at most *n* that interpolates $\{(x_i, y_i)\}_{i=0}^n$. Denote the following:

$$R_n(x) = P_n(x) - Q_n(x). \tag{10}$$

As $P_n(x)$ and $Q_n(x)$ are both polynomials of degree at most *n*, $R_n(x)$ is a polynomial of degree at most *n*. However, according to the interpolating property of $P_n(x)$ and $Q_n(x)$, the following results:

$$\begin{aligned} R_n(x_i) &= P_n(x_i) - Q_n(x_i) \\ &= y_i - y_i = 0, \qquad \text{for } i = 0, 1, \cdots, n \end{aligned}$$

Hence, $R_n(x)$ has $n+1$ distinct roots. However, its degree is at most *n*. This is not possible, unless $R_n(x) \equiv 0$. As (10) asserts $P_n(x) = Q_n(x)$, there is only one polynomial $P_n(x)$ of degree at most *n* that satisfies the following:

$$P_n(x_i) = y_i, \quad i = 0, 1, \cdots, n.$$

D. Cubic Spline Interpolation

Cubic spline interpolation is a development of polynomial interpolation that is smooth and preserves the shape of the data [13], [14]. It connects two adjacent points with a polynomial of degree at most 3 [14].

Given *n* data points $\{(x_i, y_i)\}$, where x_i are distinct and $i = 1, 2, \cdots, n$, a cubic spline is defined in the following form [15]:

$$S(x) = \begin{cases} S_1(x), \ x_1 \leq x \leq x_2 \\ \vdots \\ S_{n-1}(x), x_{n-1} \leq x \leq x_n \end{cases} \tag{11}$$

Here

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \tag{12}$$

is a polynomial of degree ≤ 3 that is defined on each subinterval $[x_i, x_{i+1}]$, $i = 1, 2, \cdots, n-1$.

$S(x)$ is called the natural cubic spline function that interpolates $\{(x_i, y_i)\}$ and satisfies the following conditions:

a. $S(x_i) = y_i$, where $i = 1, 2, \cdots, n$. (13)

b. $S_i(x_{i+1}) = S_{i+1}(x_{i+1})$, where $i = 1, 2, \cdots, n-2$, hence $S(x)$ is continuous on $[x_1, x_n]$

c. $S'_i(x_{i+1}) = S'_{i+1}(x_{i+1})$, where $i = 1, 2, \cdots, n-2$, hence $S'(x)$ is continuous on $[x_1, x_n]$ (14)

d. $S''_i(x_{i+1}) = S''_{i+1}(x_{i+1})$, where $i = 1, 2, \cdots, n-2$, hence $S''(x)$ is continuous on $[x_1, x_n]$

e. $S''(x_1) = S''(x_n) = 0$ (15)

Thus, combining (11) and (12) results in a form of a cubic spline that can be written as follows:

$$\begin{aligned} &S_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3, x_1 \leq x \leq x_2 \\ &S_2(x) = a_2 + b_2(x - x_2) + c_2(x - x_2)^2 + d_2(x - x_2)^3, x_2 \leq x \leq x_3 \\ &\quad \vdots \\ &S_{n-1}(x) = a_{n-1} + b_{n-1}(x - x_{n-1}) + c_{n-1}(x - x_{n-1})^2 + d_{n-1}(x - x_{n-1})^3, \\ &\qquad\qquad x_{n-1} \leq x \leq x_n \end{aligned}$$

$S(x)$ can be used in extrapolation to predict data beyond the range of the available data. This method will be discussed further in the Results and Discussion section.

E. Uniqueness of Cubic Spline

Cubic spline interpolation is unique, which means that there is only one set of functions $S_i(x)$ where each is defined on the corresponding subinterval $[x_i, x_{i+1}]$, $i=1,2,\cdots,n-1$. Each $S_i(x)$ (11) will be coupled and cannot be solved independently. A linear system will be constructed to show the uniqueness of the natural cubic spline function, in that it will have a unique solution to the linear system.

Suppose $S''(x_i)=P_i, i=1,2,\cdots,n.$ As $S_i(x)$ is cubic, $S''_i(x)$ is linear in $[x_i, x_{i+1}]$. This can be written as follows:

$$S''_i(x)=\frac{(x_{i+1}-x)P_i+(x-x_i)P_{i+1}}{h_i},\ h_i=x_{i+1}-x_i$$

The imposed condition $S''(x)$ being continuous allows for the formation of the second antiderivative of $S''(x)$ on $[x_i, x_{i+1}]$, as follows:

$$S_i(x)=\frac{(x_{i+1}-x)^3 P_i+(x-x_i)^3 P_{i+1}}{6h_i}+\lambda_i(x-x_i)+\mu_i(x_{i+1}-x) \tag{16}$$

Applying the interpolation conditions (13) and the continuity property of $S_i(x)$ results in μ_i and λ_i, as follows:

$$S_i(x_i)=y_i=\frac{1}{6}h_i^2P_i+h_i\mu_i \Rightarrow \mu_i=\frac{y_i-\frac{1}{6}h_i^2P_i}{h_i}$$

$$S_i(x_{i+1})=y_{i+1}=\frac{1}{6}h_i^2P_{i+1}+h_i\lambda_i \Rightarrow \lambda_i=\frac{y_{i+1}-\frac{1}{6}h_i^2P_{i+1}}{h_i}$$

Taking the first derivative of $S_i(x)$ (16) on $[x_i, x_{i+1}]$ results in the following equation:

$$S'_i(x)=\frac{-(x_{i+1}-x)^2 P_i+(x-x_i)^2 P_{i+1}}{2h_i}+\lambda_i-\mu_i$$

$$S'_i(x)=\frac{-(x_{i+1}-x)^2 P_i+(x-x_i)^2 P_{i+1}}{2h_i}+\frac{1}{h_i}(y_{i+1}-y_i)+\frac{1}{6}h_i(P_i-P_{i+1})$$

Moving one step backward changes the index and leads to the following:

$$S'_{i-1}(x)=\frac{-(x_i-x)^2 P_{i-1}+(x-x_{i-1})^2 P_i}{2h_{i-1}}+\frac{1}{h_{i-1}}(y_i-y_{i-1})+\frac{1}{6}h_{i-1}(P_{i-1}-P_i)$$

Applying the first derivative condition (14), as follows:

$$S'_{i-1}(x_i)=S'_i(x_i)$$

results in the following linear system:

$$\frac{h_{i-1}{}^2P_i}{2h_{i-1}}+\frac{1}{h_{i-1}}(y_i-y_{i-1})+\frac{1}{6}h_{i-1}(P_{i-1}-P_i)=\frac{-h_i{}^2P_i}{2h_i}+\frac{1}{h_i}(y_{i+1}-y_i)+\frac{1}{6}h_i(P_i-P_{i+1})$$

$$\Leftrightarrow h_{i-1}P_{i-1}+2(h_{i-1}+h_i)P_i+h_iP_{i+1}=6\left(\frac{y_{i+1}-y_i}{h_i}-\frac{y_i-y_{i-1}}{h_{i-1}}\right),\ \ i=2,\cdots,n-1 \tag{17}$$

If $c_i=6\left(\frac{y_{i+1}-y_i}{h_i}-\frac{y_i-y_{i-1}}{h_{i-1}}\right)$, $n-2$ equations from (17), together with the boundary conditions of the natural cubic spline (15), lead to a tridiagonal system of linear equations.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ h_1 & 2(h_1+h_2) & h_2 & 0 & \\ 0 & h_2 & 2(h_2+h_3) & h_3 & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n-1} \\ P_n \end{bmatrix}=\begin{bmatrix} 0 \\ c_2 \\ \vdots \\ c_{n-1} \\ 0 \end{bmatrix}$$

The tridiagonal system is strictly diagonally dominant since $2(h_{i-1}+h_i)>h_{i-1}+h_i,\ \ i=2,\cdots,n-1$. As such, it has a unique solution. To show that a strictly diagonally dominant matrix has a unique solution, it needs to be shown that it is nonsingular. *M* is a strictly diagonally dominant matrix if $|m_{ii}|>\sum_{j\neq i}|m_{ij}|, \forall i$, where m_{ij} is the-*ij* element of matrix *M*.

Theorem: A strictly diagonally dominant matrix M is nonsingular.

Proof: Take a contradiction that a strictly diagonally dominant matrix M is singular. Following this, there is a vector $\vec{\varphi}\neq\vec{0}$ such that $M\vec{\varphi}=\vec{0}$ and $\vec{\varphi}$ has some elements φ_i of the largest magnitude. This is denoted as follows:

$$\sum_k m_{ik}\varphi_k=0$$

$$\Leftrightarrow m_{ii}\varphi_i=-\sum_{k\neq i}m_{ik}\varphi_k$$

$$\Leftrightarrow m_{ii}=-\sum_{k\neq i}\left(\frac{\varphi_k}{\varphi_i}\right)m_{ik}$$

$$\Leftrightarrow |m_{ii}|\leq\sum_{k\neq i}\left|\frac{\varphi_k}{\varphi_i}\right||m_{ik}|$$

$$\Leftrightarrow |m_{ii}|\leq\sum_{k\neq i}|m_{ik}| \text{ where } \left|\frac{\varphi_k}{\varphi_i}\right|\leq 1$$

This contradicts the previous assumption that M is a strictly

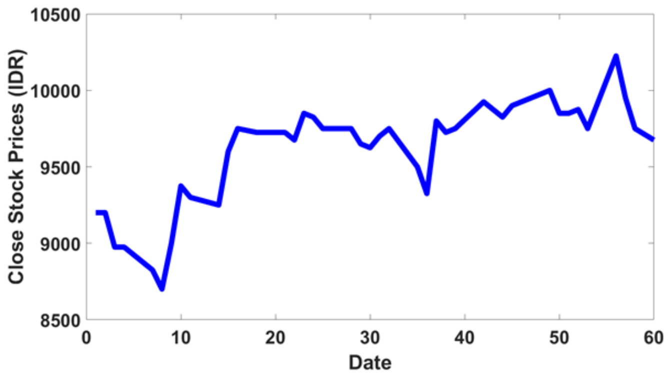

Fig. 1. Close price data for PT Indosat Tbk stocks.

diagonally dominant matrix. Therefore, the theorem that the matrix M is nonsingular is proven. This indicates that a cubic spline has a unique set of functions, where each is defined on the corresponding subinterval.

III. Results and Discussion

This research used data from one of the largest telecommunication and network service providers in Indonesia: PT Indosat Tbk. The dataset consists of the company's close stock prices from 1 August to 29 September 2023, which was obtained from the following site: http://finance. yahoo.com (see Table I). The data are sorted based on the dates on which the close stock prices occurred, which means that weekends and holidays were excluded. The close price data plot of PT Indosat Tbk stocks can be seen in Figure 1.

This figure shows that the close stock prices fluctuated regularly, starting from 1 August to 29 September 2023, with the lowest close stock price (i.e., 8,700) occurring on 8 August 2023 and the highest close stock price (i.e., 10,225) occurring on 25 September 2023 (see Table I). The rise and fall of close stock prices can be influenced by several factors, such as a company's condition, the inflation rate, the amount of profit earned by the company, and interest rates.

In addition, Figure 1 shows that the total number of close prices for PT Indosat Tbk stocks for the period under investigation comprises 42 entries (see Table I). All of these data points contribute to the construction of the model that represents the pattern of the actual data. However, the

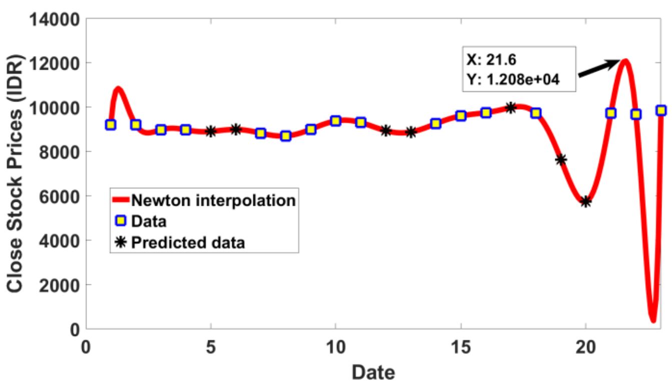

Fig. 2. Close stock price estimation results using Newton interpolation on the first 16 data.

TABLE I
CLOSE STOCK PRICE DATA OF PT INDOSAT TBK

Number	Date	Price (IDR)
1	1/8/2023	9200
2	2/8/2023	9200
3	3/8/2023	8975
4	4/8/2023	8975
5	7/8/2023	8825
6	8/8/2023	8700
7	9/8/2023	9000
8	10/8/2023	9375
9	11/8/2023	9300
10	14/8/2023	9250
11	15/8/2023	9600
12	16/8/2023	9750
13	18/8/2023	9725
14	21/8/2023	9725
15	22/8/2023	9675
16	23/8/2023	9850
17	24/8/2023	9825
18	25/8/2023	9750
19	28/8/2023	9750
20	29/8/2023	9650
21	30/8/2023	9625
22	31/8/2023	9700
23	1/9/2023	9750
24	4/9/2023	9500
25	5/9/2023	9325
26	6/9/2023	9800
27	7/9/2023	9725
28	8/9/2023	9750
29	11/9/2023	9925
30	12/9/2023	9875
31	13/9/2023	9825
32	14/9/2023	9900
33	15/9/2023	9925
34	18/9/2023	10000
35	19/9/2023	9850
36	20/9/2023	9850
37	21/9/2023	9875
38	22/9/2023	9750
39	25/9/2023	10225
40	26/9/2023	9950
41	27/9/2023	9750
42	29/9/2023	9675

missing or unavailable data will also be considered herein. Hence, we will consider the weekend and holiday data to be missing data that will be estimated using interpolation. Moreover, the resulting function can be used to predict data in the given date range.

The purpose of the interpolation was to determine to what extent the estimation results derived from the Newton and cubic spline interpolations approach the actual data values. Therefore, interpolation was initially performed on the first 16 data points (see Table I) to estimate the seven unavailable data points (see Table II). The results of

TABLE II
CLOSE STOCK PRICE PREDICTION OF PT INDOSAT TBK

No.	Date	Newton Price (IDR)	Spline Price (IDR)
1	5/8/2023	8901.8	9002.1
2	6/8/2023	8989.5	8960.5
3	12/8/2023	8934.6	9135.8
4	13/8/2023	8876.4	9086.1
5	17/8/2023	9977.5	9753.9
6	19/8/2023	7635.7	9731.4
7	20/8/2023	5739.2	9747.6

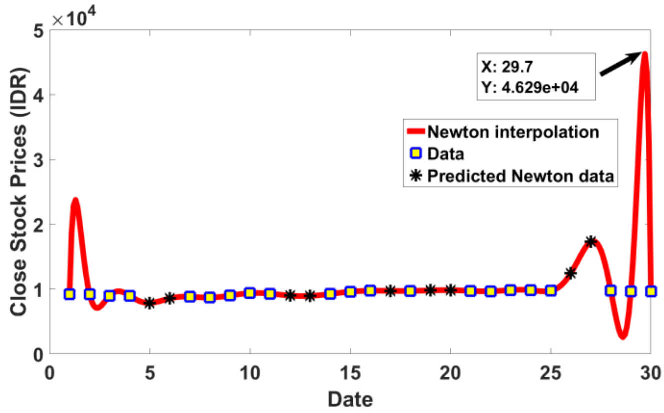

Fig. 3. Close stock price estimation results using Newton interpolation on the first 21 data.

estimating the close stock prices using Newton interpolation can be seen in Table II, and its plot is shown in Figure 2. The square represents the actual close stock price, while the star denotes the estimates of the close stock prices. The results show that the first five estimated data points follow the pattern of the actual data, but the last two estimated data points differ significantly from their adjacent data.

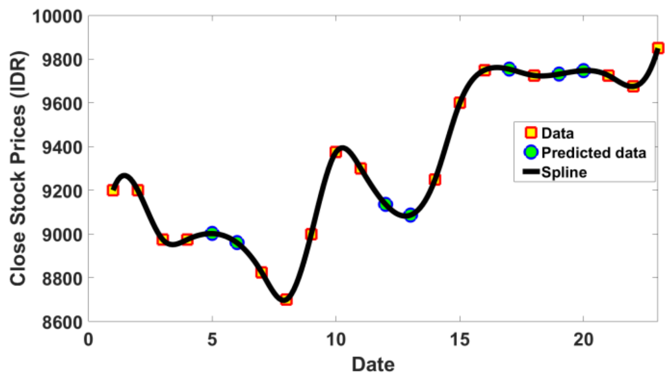

Fig. 4. Close stock price estimation results using spline on the first 16 data.

Even though Newton interpolation produces a smooth function, however, it greatly fluctuates at the edges of the interval causing the last 2 prediction data far apart from their neighbouring data (see Figure 2). Futhermore, performing

TABLE III
CLOSE STOCK PRICE ESTIMATION USING CUBIC SPLINE

Number	Date	Price (IDR)
1	5/8/2023	9002.1
2	6/8/2023	8960.5
3	12/8/2023	9135.8
4	13/8/2023	9086.2
5	17/8/2023	9752.0
6	19/8/2023	9740.7
7	20/8/2023	9762.2
8	26/8/2023	9749.1
9	27/8/2023	9774.8
10	2/9/2023	9773.3
11	3/9/2023	9716.1
12	9/9/2023	9844.6
13	10/9/2023	9910.5
14	16/9/2023	9986.0
15	17/9/2023	1004.4
16	23/9/2023	9888.7
17	24/9/2023	1015.5
18	28/9/2023	9683.6

Newton interpolation on the first 21 data in Table I, results in a function whose values magnify significantly at the right end interval, that is almost 4 times of those in Figure 2 (cf., Figures 2 and 3). It reflects the Runge's phenomenon that usually occurs for interpolation with high degree polynomial on evenly spaced interpolation points. This unavoidably produces unreliable prediction data around that range (see Figure 3). Thus, it can be said that estimating close stock prices using Newton interpolation is not accurate enough. Therefore, an estimation is then carried out using cubic spline to obtain better estimation results. The results of estimating close stock prices using cubic spline on the first 16 data in Table I can be seen in Table II and its plot is displayed in Figure 4.

The square denotes the actual close stock price, while the circle represents the results of the close stock price estimations using a cubic spline. Note that different scales are used in the y-axes of Figures 2 and 4, in that the y-axis in Figure 2 has a scale of 2,000, while that in Figure 4 has a scale of 200 (i.e., one tenth smaller). This affects the way that the results are displayed. For example, the same error size will appear larger in Figure 4 than in Figure 2.

Figure 4 also shows that the plot of the close stock price estimation derived using the cubic spline produces a smooth

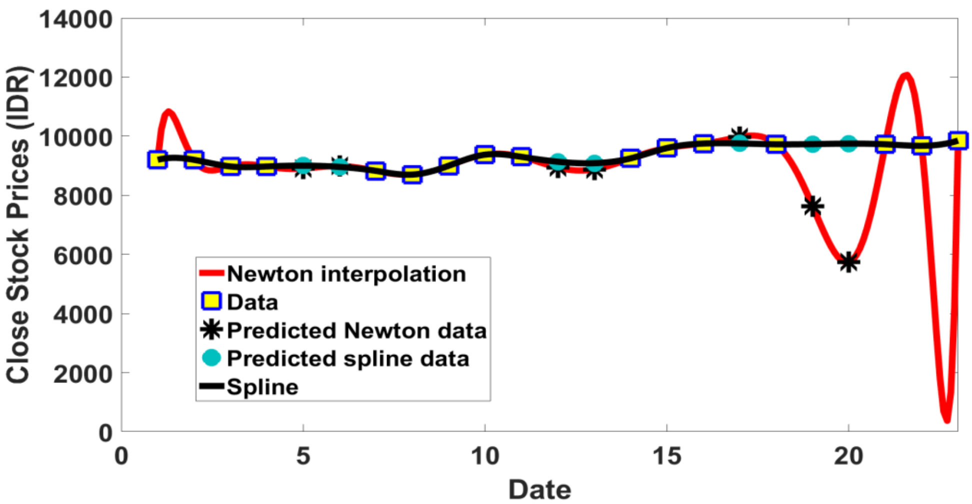

Fig. 5. Close stock price estimation results using cubic spline and Newton interpolation on the first 16 data.

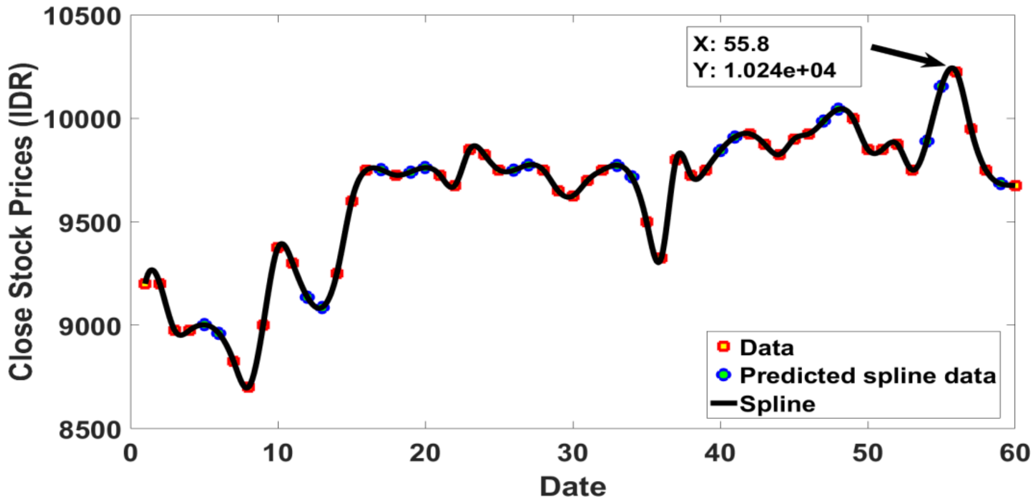

Fig. 6. Close stock price estimation results using cubic spline on the whole 42 data.

function. Moreover, since a cubic spline is a piecewise polynomial of degree at most 3, it can address Runge's problem that was discussed previously (encountered when Newton interpolation was used). Therefore, the cubic spline estimation resulted in smaller errors throughout the interval. This means that the close stock prices estimated using the cubic spline interpolation produced more accurate estimation results than those derived from the Newton interpolation.

In order to obtain a reliable comparison of the results obtained from the Newton and cubic spline interpolations on the first 16 data points, they were plotted together in one image (Figure 5), which shows that estimating close stock prices using Newton and cubic spline interpolation produces smooth functions that are almost the same in the interval [2,17]. However, the Newton interpolation resulted in significant changes of the function in the interval [18,23], which may represent large errors, such as those seen in the last two predicted data points in that range (see Table II).

Conversely, the cubic spline preserved the shape of the whole dataset strictly and smoothly, in that it does not fluctuate significantly. This means that using the cubic spline interpolation to estimate close stock prices produced values that were very close to the actual prices throughout the interval. Therefore, the cubic spline interpolation produced a more accurate and reliable measurement compared to the Newton interpolation. For this reason, only the cubic spline was used to analyze all of the 42 data points, the plot of which is shown in Figure 6, with the 18 predicted data points being displayed in Table III. This illustrates that the prediction data matched the actual data more closely. Therefore, a deeper understanding of how a cubic spline behaves when interpolating a different number of consecutive interpolation points is needed. That is dependent on whether it produces the same graph and, hence, the same prediction data on the common interval.

The cubic spline was then performed on the first 21 data points (Table I), as well as the entire dataset of 42 points. Figure 7 illustrates the plot of this analysis, and the prediction data are displayed in Table IV. It can be seen that the prediction data derived from the cubic spline interpolation of the first 21 data points are precisely located above those obtained from the analysis of the 42 data points. This is confirmed by the values of their differences (i.e., errors) that are shown in Table IV. Almost all of the corresponding prediction data are the same, with only two of these differing by very small margins (see Table IV).

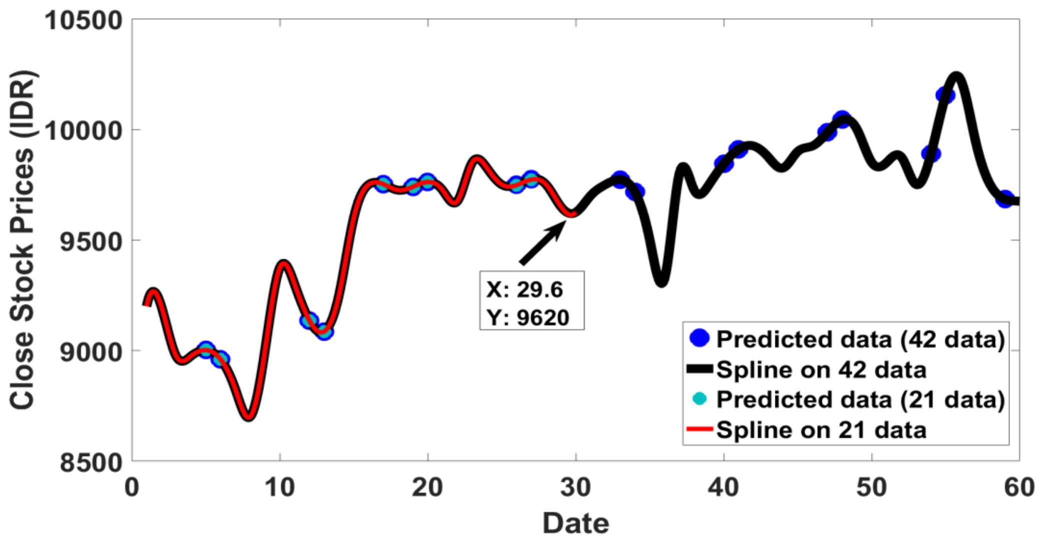

Fig. 7. Close stock price estimation results using cubic spline on the first 21 data and the whole 42 data.

TABLE IV
CLOSE STOCK PRICE PREDICTION OF PT INDOSAT TBK

Date	Spline Price (IDR) on 42 Data	Spline Price (IDR) on 21 Data	Error
5/8/2023	9002.1	9002.1	0
6/8/2023	8960.5	8960.5	0
12/8/2023	9135.8	9135.8	-0
13/8/2023	9086.2	9086.2	-0
17/8/2023	9752.0	9752.0	0
19/8/2023	9740.7	9740.7	-0
20/8/2023	9762.2	9762.2	-0
26/8/2023	9749.1	9748.0	1.1
27/8/2023	9774.8	9772.9	1.9

Furthermore, the values of both resulting functions are almost the same in the common interval [1,30], the maximum value of their absolute differences of which is 6.5166 with a relative error of 6.7741E-04 at x = 29.6 (see the arrow in Figure 7).

At x = 29.6, the cubic spline interpolation of the first 21 data points for the interval [29,30] is given as follows:

$$S_{20}(x) = 9650 - 95.4142x + 37.5x^2 + 32.9142x^3.$$

Conversely, that for the interpolation of the entire dataset for the interval [29,30] is presented as follows:

$$S_{20}(x) = 9650 - 84.8533x + 54.5915x^2 + 5.2617x^3.$$

Both functions represent two cubic splines for the interval [29,30]. Even though their formulation is different, their values are almost the same. This is confirmed by the graphs shown in Figure 7 for both of the common intervals [29,30] and [1,30]. The uniqueness of the cubic spline interpolation is illustrated by only one set of functions $S_i(x)$ being provided, where each is defined on the corresponding subinterval $[x_i, x_{i+1}]$, where $i = 1, 2, \cdots, n-1$. Although the corresponding pairs of the cubic splines that were produced by the different numbers of consecutive data points are different, their graphs are the same for their common interval.

Furthermore, the results from the cubic spline interpolation were used to predict data beyond the range of

TABLE V
CLOSE STOCK PRICE PREDICTION OF PT INDOSAT TBK USING EXTRAPOLATION

No.	Date	x	Spline Price (IDR) on 42 Data
1	30/9/2023	61	9648.6
2	1/10/2023	62	9528.7
3	2/10/2023	63	9239.6

the available data, which is called extrapolation. These results are shown in Figure 8. Therefore, extrapolation, as a result of interpolation, can be used to estimate future data. However, as the extrapolation function is rapidly decreasing, it can be used only to predict a limited range of data, as shown in Table V.

The results of the close stock price prediction illustrated in Figures 6 and 8 provide valuable information for investors, as they can use it to analyze the future prospects of their stock investments. This information is not only useful for identifying potential profits, but also for hedging investments against possible market risks. In addition, stock price predictions provide opportunities for market participants, whether they are speculators or arbitrageurs, to profit from fluctuations in trading stock. The more accurate the prediction is, the greater the potential profit that can be achieved. Therefore, stock price prediction is an important key in reducing investment risk and maximizing investors' profit potential. Cubic spline interpolation is one of the tools that can be used, as this study shows that it can be considered to predict close stock prices with a high degree of accuracy. Therefore, integrating this technique into investment analysis allows investors to gain confidence in making better investment decisions.

IV. CONCLUSION

The results of predicting PT Indosat Tbk's close stock prices for the period 1 August to 29 September 2023 using Newton and cubic spline interpolations indicate that both methods produce a smooth function. However, Newton interpolation results in significant changes, especially at the edges of an interval, which causes large errors to occur,

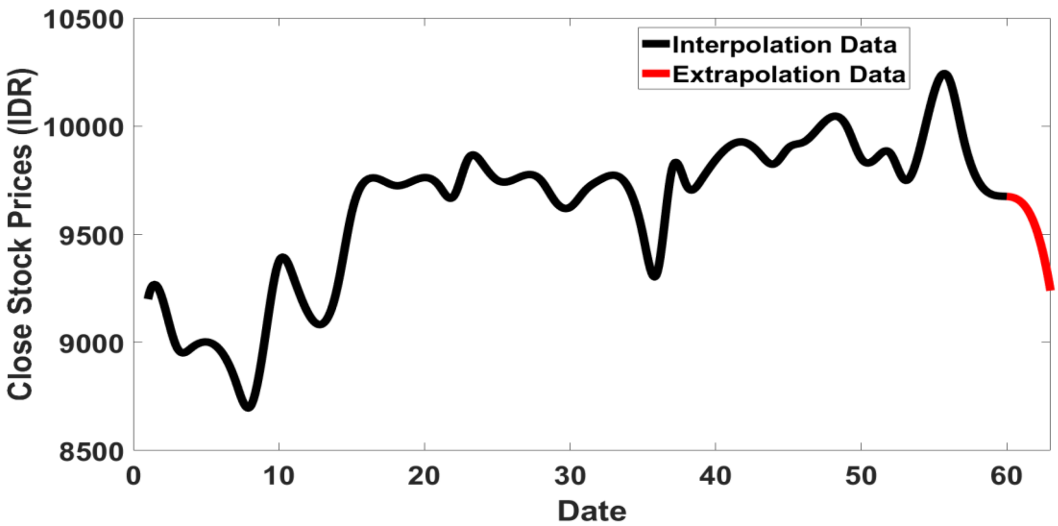

Fig. 8. Prediction of close stock prices beyond the range of the given data using cubic spline extrapolation on the whole 42 data.

which means that its estimates are less accurate. However, cubic spline interpolation does not show significant changes throughout the interval. Hence, it can predict missing or unavailable data, and the resulting function can predict data in the given interval range. Therefore, the cubic spline provides more effective and accurate estimation results than those derived from the Newton interpolation. The uniqueness property of the cubic spline ensures that it produces only one set of functions, where each is defined on the corresponding subinterval. However, implementing cubic spline interpolation on a different number of consecutive data points produces almost identical prediction data and function values over their common intervals.

Extrapolation using the results of interpolation produces data beyond the range of the given data. Hence, it can be used to predict future data, but only a limited range of data can be estimated due to it decreasing rapidly. Therefore, cubic spline interpolation can be used as a means to estimate close stock price data, especially PT Indosat Tbk stock, in the future.

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