

# Statistical Downscaling Modeling Using Geographically and Temporally Weighted Gamma Ridge and Lasso Regression to Improve the Accuracy of Monthly Rainfall Estimation

Aan Kardiana, Aji Hamim Wigena, Anik Djuraidah, Agus Mohamad Soleh

**Abstract**— The Global Climate Model (GCM), a computer-based model comprising numerous integrated, deterministic, and numerical equations that adhere to physics, is one of the models used to predict rainfall. GCM can simulate large-scale climatic conditions. Statistical Downscaling (SD) modelling to estimate monthly rainfall in areas with monsoon rainfall patterns in Indonesia has been carried out using L1/Lasso Regulation and Principal Component Analysis (PCA), Bayes Spatio-Temporal Regression with Integrated Nested Laplace Approximations (INLA), and Spatio Temporal Generalized Linear Mixed Model. The research aims to overcome the problems faced in the utilization of GCM output data by developing the Geographically and Temporally Weighted Gamma Ridge Regression (GTWGRR) model and the Geographically and Temporally Weighted Gamma Lasso Regression (GTWGLR) model to estimate monthly rainfall in a new location in West Java Province. The study used quantitative data, namely monthly rainfall from 35 rain observation stations in West Java Province, from September 2011 to January 2012, obtained from the Indonesian Agency for Meteorological, Climatological and Geophysics (Badan Meteorologi, Klimatologi, dan Geofisika or simply BMKG) and monthly precipitation of GCM outputs using a  $3 \times 3$  domain with a resolution of  $2,5^\circ \times 2,5^\circ$ . The GTWGRR method is the best regularization method compared to the GTWGLR method in SD modelling for estimating monthly rainfall from October 2011 to January 2012. The combination of the GTWGRR method with Kriging interpolation can be used in SD modelling to estimate monthly rainfall at a new location in West Java Province. The GTWGRR and Kriging interpolation methods are useful in improving the accuracy of monthly rainfall estimates for BMKG.

**Index Terms**—GTWGR, GTWGRR, monthly rainfall, Statistical Downscaling

## I. INTRODUCTION

The Statistical Downscaling (SD) technique is a pivotal tool in climatology and employs statistical modelling to analyze the intricate relationship between global-scale and local-scale data [1]. The SD methodology is crucial for trans-

forming data derived from a grid characterized by extensive units into data represented on a grid with more refined units [2], as well as for interpolating regional-scale atmospheric predictor variables with smaller-scale variables [3].

The Global Climate Model (GCM), a computer-based model comprising numerous integrated, deterministic, and numerical equations that adhere to physics, is one of the models used to predict rainfall. Large-scale climatic conditions can be simulated by GCM [2]. The GCM constitutes a computational framework that produces extensive datasets derived from a variety of climatic variables, such as precipitation, temperature, and humidity, aimed at facilitating climate prediction, as articulated by Soleh [4].

L1/Lasso Regulation and Principal Component Analysis [5], Spatio-Temporal Bayes Regression with Integrated Nested Laplace Approximations (INLA) [6], and Spatio Temporal Generalized Linear Mixed Model [7] were used in SD modelling to estimate monthly rainfall in Indonesian regions with monsoon rainfall patterns. Apart from this approach, Geographically and Temporally Weighted Regression (GTWR) using the Weighted Least Squares (WLS) method [8] and Geographically and Temporally Weighted Gamma Regression (GTWGR) using the Maximum Likelihood Equation (MLE) method have been developed [9].

When analyzing spatially different data, local regression models like Geographically Weighted Regression (GWR) can be utilized to investigate the relationship between response and predictor variables. By employing geographic location as a weighting factor in the estimation of the model parameters that differ across various sites, the limitations of the data are effectively addressed. As noted by Foody [10], a significant advantage of Geographically Weighted Regression (GWR) lies in its capacity to investigate the non-stationary characteristics of the relationship between dependent and independent variables. Provided that the association between precipitation and spatially heterogeneous environmental factors is established, Chen et al. [11] and Xu et al. [12] introduced a GWR framework to enhance the resolution of rainfall data.

The new GWR approach manages data with geographical variety. To handle data that has temporal diversity, this approach must be devised. In order to offer distinct model parameter values for each site ( $u_i, v_i$ ) and time  $t_i$ ,

Manuscript received November 25, 2024; revised September 20, 2025.

A. Kardiana is a lecturer in the Informatics Engineering Department of YARSI University, Jakarta, 10510 Indonesia (Corresponding author, e-mail: aan.kardiana@yarsi.ac.id).

Aji H. Wigena is a Professor in the Statistics Department of IPB University, Bogor, West Java, 16680 Indonesia (e-mail: aji\_hw@apps.ipb.ac.id).

A. Djuraidah is a Professor in the Statistics Department of IPB University, Bogor, West Java, 16680 Indonesia (e-mail: anikdjuraidah@apps.ipb.ac.id).

Agus M. Soleh is a lecturer in the Statistics Department of IPB University, Bogor, West Java, 16680 Indonesia (e-mail: agusms@apps.ipb.ac.id).

Fotheringham et al. [13] transformed GWR into GTWR. Additionally, to estimate the amount of chlorophyll-a (Chl-a) in the coastal region of Zhejiang, China. Du et al. [14] converted GTWR into a geographically and temporally weighted regression spatiotemporal kriging (GTWR-STK). The GTWR-STK model produces better conjectures from the GTWR, GTWR Kriging (GTWRK), and spatiotemporal ordinary kriging (STOK) methods.

A generalized gamma linear model has been subjected to Principal Component Analysis (PCA) with L1/Lasso regularization by Soleh et al. [5]. After adding the month dummy variable, Principal Component Regression (PCR), Principal Component (PC)-Gamma, and Gamma-L1 modelling all yield the same result with comparatively slight variation, based on the least value of the Root Mean Square Error (RMSE). The best model is the generalized Gamma Linear model since it estimates rainfall that is not negative. Ali et al. [15] confirmed that rainfall data adheres to a Gamma distribution because the data has a value of zero or greater than zero. Considering the Root Mean Square Prediction Error (RMSEP) value, the study's results affirm that the 75th percentile Lasso model is superior to the 25th percentile Ridge model.

Husak et al. [16] conducted an estimation of monthly precipitation parameters across Africa utilizing the Maximum Likelihood Estimation (MLE) approach for the Gamma distribution. The concurrent analysis of shape and scale parameters facilitates the modeling of monthly precipitation data values at each specific location, thus enabling qualitative evaluations concerning the magnitude and reliability of rainfall during the seasonal period. The modeling of monthly precipitation data values at each locality is performed through the concurrent analysis of shape and scale parameters, which permits qualitative assessments regarding the volume and stability of precipitation throughout the seasonal timeframe. The Gamma distribution is a better empirical rainfall probability distribution than the Weibull distribution, according to the findings of a test of its suitability for estimating the empirical distribution model parameters using the Kolmogorov Smirnov (KS) test. Amburn et al. [17] have been forecasting rainfall using the Gamma Regression (GR) model. The findings demonstrate that the response variable's value may be accurately estimated using GR. These results are obtained based on the percentage of coverage area and the average rainfall.

Kardiana et al. [8] state that the monthly rainfall histogram for each station is generally asymmetrical and leans to the right. The frequency of the high-value monthly rainfall is low, however the frequency of the low-value monthly rainfall is very considerable. Using Gamma Regression is one method for dealing with this distribution.

The appropriateness of frequency distribution in rainfall pattern forecasting—a crucial component in predicting possible disasters—was investigated by Esberto [18]. Chi-Square and KS tests were utilized to thoroughly assess the study's monthly rainfall data from PAGASA. Using data from the previous 26–30 years, the distribution pattern was identified and assessed on over 60 probability density functions (pdf), including the Gamma distribution. For statistical inference, the best-performing pdf will be utilized. The results indicate that each location has a specific

theoretical pdf to infer rainfall events, with the gamma distribution emerging as the best pdf for a specific location and month.

According to research in the literature, there is multicollinearity and a spatial connection between grids when using GCM output data, which contains numerous predictor factors [2]; there has been no application of the GTWGR model, which is a combination of Ridge regularization with the GTWGR model and GTWGLR model, which is a combination of Lasso regularization with the GTWGR model to overcome these problems. For this reason, this study will use the GTWGR and GTWGLR models in SD to estimate monthly rainfall and monthly rainfall at new locations using a combination of the best models from GTWGR and GTWGLR with Kriging interpolation.

## II. GEOGRAPHICALLY WEIGHTED GAMMA REGRESSION (GWGR)

The GWGR model [19] is defined as follows

$$E(y_i) = \mu_i = \exp(\beta_0(u_i, v_i) + \sum_{k=1}^p \beta_k(u_i, v_i)x_{ik}) \quad (1)$$

$i = 1, 2, 3, \dots, n; k = 1, 2, \dots, p.$

where  $y_i$  is the value of the response variable at the location  $(u_i, v_i)$ ,  $\beta_0(u_i, v_i)$  is the intercept at the location  $(u_i, v_i)$ ,  $\beta_k(u_i, v_i)$ , is the regression coefficient of the  $k$ th predictor variable at the location  $(u_i, v_i)$ ,  $x_i$  is the  $k$ th predictor variable. Similar to the Geographically Weighted Regression (GWR) model, the Geographically Weighted Gamma Regression (GWGR) model uses the Maximum Likelihood Estimator (MLE) approach for parameter estimations. The difference is in the type of Gamma spread response.

The probability density function of the GWGR model is:

$$f(y_i; \alpha(u_i, v_i), \beta(u_i, v_i)) = \frac{y_i^{\alpha(u_i, v_i)-1}}{\Gamma(\alpha(u_i, v_i)) \left( \frac{\exp(X_i^T \beta(u_i, v_i))}{\alpha(u_i, v_i)} \right)^{\alpha(u_i, v_i)}} \exp \left( \frac{-y_i}{\left( \frac{\exp(X_i^T \beta(u_i, v_i))}{\alpha(u_i, v_i)} \right)} \right) \quad (2)$$

The GWGR parameters are estimated using the MLE method. Based on equation (2), the probability function is obtained:

$$L(\alpha(u_i, v_i), \beta(u_i, v_i)) = \prod_{i=1}^n \left[ \frac{y_i^{\alpha(u_i, v_i)-1}}{\Gamma(\alpha(u_i, v_i)) \left( \frac{\exp(X_i^T \beta(u_i, v_i))}{\alpha(u_i, v_i)} \right)^{\alpha(u_i, v_i)}} \exp \left( \frac{-y_i}{\left( \frac{\exp(X_i^T \beta(u_i, v_i))}{\alpha(u_i, v_i)} \right)} \right) \right] \\ = \left[ \left( \frac{1}{\Gamma(\alpha(u_i, v_i))} \right)^n \left( \frac{\exp \sum_{i=1}^n X_i^T \beta}{\alpha(u_i, v_i)^n} \right)^{-\alpha(u_i, v_i)} \right] \\ \prod_{i=1}^n y_i^{\alpha(u_i, v_i)-1} \exp \left( \sum_{i=1}^n \frac{-y_i \alpha(u_i, v_i)}{\exp(X_i^T \beta(u_i, v_i))} \right) \quad (3)$$

and the probability ln function  $l = \ln L(\alpha(u_i, v_i), \beta(u_i, v_i))$  is:

$$l = \ln L(\alpha(u_i, v_i), \beta(u_i, v_i)) \\ = \ln \left[ \left( \Gamma(\alpha(u_i, v_i)) \right)^{-n} \left( \frac{\exp \sum_{i=1}^n X_i^T \beta}{\alpha(u_i, v_i)^n} \right)^{-\alpha(u_i, v_i)} \right] \\ \prod_{i=1}^n y_i^{\alpha(u_i, v_i)-1} \exp \left( \sum_{i=1}^n \frac{-y_i \alpha(u_i, v_i)}{\exp(X_i^T \beta(u_i, v_i))} \right) \\ = -n \ln \Gamma(\alpha(u_i, v_i)) - \\ \alpha(u_i, v_i) \sum_{i=1}^n (X_i^T \beta(u_i, v_i)) + \alpha(u_i, v_i) (n \ln \alpha(u_i, v_i)) + \\ (\alpha(u_i, v_i)-1) \sum_{i=1}^n \ln y_i - \sum_{i=1}^n \frac{y_i \alpha(u_i, v_i)}{\exp(X_i^T \beta(u_i, v_i))}. \quad (4)$$

The GWGR model considers the location factors in the form of each observation point's longitude and latitude coordinates. The  $\ln$  likelihood function with location weighting is [19] as follows:

$$\begin{aligned} l &= \ln L^*(\alpha_{(u_l, v_l)}, \beta_{(u_l, v_l)}, i=1, 2, \dots, n) \\ &= \left( -\sum_{i=1}^n \ln \left( \Gamma(\alpha_{(u_l, v_l)}) \right) + \sum_{i=1}^n \alpha_{(u_l, v_l)} \ln \alpha_{(u_l, v_l)} - \right. \\ &\quad \left. \sum_{i=1}^n \alpha_{(u_l, v_l)} (x_i^T \beta_{(u_l, v_l)}) + \sum_{i=1}^n (\alpha_{(u_l, v_l)} - 1) \ln y_i - \right. \\ &\quad \left. \alpha_{(u_l, v_l)} \sum_{i=1}^n y_i \exp(-x_i^T \beta_{(u_l, v_l)}) \right) (w_{il}) \\ &= -\ln \left( \Gamma(\alpha_{(u_l, v_l)}) \right) \sum_{i=1}^n w_{il} + \alpha_{(u_l, v_l)} \ln \alpha_{(u_l, v_l)} \sum_{i=1}^n w_{il} - \\ &\quad \alpha_{(u_l, v_l)} \sum_{i=1}^n (x_i^T \beta_{(u_l, v_l)}) w_{il} + (\alpha_{(u_l, v_l)} - 1) \sum_{i=1}^n \ln y_i w_{il} - \\ &\quad \alpha_{(u_l, v_l)} \sum_{i=1}^n y_i w_{il} \exp(-x_i^T \beta_{(u_l, v_l)}). \end{aligned} \quad (5)$$

Through the utilization of the initial and subsequent partial derivatives of the logarithmic likelihood function  $l$  to the parameter to be estimated, then equalizing it to zero and determining the solution, the estimated value of the GTWR parameter is produced. This partial derivative is an implicit function because the equation formed still contains parameters, so the estimation process uses Newton Raphson (NR) iteration [20].

$$\begin{aligned} \frac{\partial l}{\partial \alpha_{(u_l, v_l)}} &= \\ &= -\frac{\Gamma'(\alpha_{(u_l, v_l)})}{\Gamma(\alpha_{(u_l, v_l)})} \sum_{i=1}^n w_{il} + \ln \alpha_{(u_l, v_l)} \sum_{i=1}^n w_{il} + \sum_{i=1}^n w_{il} - \\ &\quad \sum_{i=1}^n (x_i^T \beta_{(u_l, v_l)}) w_{il} + \sum_{i=1}^n \ln y_i w_{il} - \\ &\quad \sum_{i=1}^n y_i w_{il} \exp(-x_i^T \beta_{(u_l, v_l)}) \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial l}{\partial \beta_{(u_l, v_l)}} &= -\alpha_{(u_l, v_l)} \sum_{i=1}^n x_i w_{il} + \\ &\quad \alpha_{(u_l, v_l)} \sum_{i=1}^n y_i x_i w_{il} \exp(-x_i^T \beta_{(u_l, v_l)}) \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial^2 l}{\partial^2 \alpha_{(u_l, v_l)}} &= - \\ &= \left[ \frac{\Gamma''(\alpha_{(u_l, v_l)}) \Gamma'(\alpha_{(u_l, v_l)}) - \Gamma'(\alpha_{(u_l, v_l)})^2}{\Gamma^2(\alpha_{(u_l, v_l)})} \right] \sum_{i=1}^n w_{il} + \\ &\quad \frac{1}{\alpha_{(u_l, v_l)}} \sum_{i=1}^n w_{il} \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial^2 l}{\partial^2 \alpha_{(u_l, v_l)} \beta_{(u_l, v_l)}} &= \\ &= -\alpha_{(u_l, v_l)} \sum_{i=1}^n y_i x_i x_i^T w_{il} \exp(-x_i^T \beta_{(u_l, v_l)}) \end{aligned} \quad (9)$$

Simultaneous hypothesis testing to find out the predictor variables that simultaneously affect the model [21] are:

$$H_0 : \beta_{11}(u_i, v_i) = \beta_{12}(u_i, v_i) = \dots = \beta_{pn}(u_i, v_i) = 0$$

$$H_1 : \text{there is at least one } \beta_{ij}(u_i, v_i) \neq 0,$$

$$i = 1, 2, \dots, n; k = 1, 2, \dots, p.$$

The test statistic is calculated as:

$$G^2 = -2 \ln A = -2 \ln \left[ \frac{L(\hat{\omega})}{L(\hat{\Omega})} \right] = 2(\ln L(\hat{\Omega}) - \ln L(\hat{\omega})) \quad (10)$$

where  $L(\hat{\omega})$  is the highest value that can be obtained if  $H_0$  ( $\omega = (\beta_0, \alpha, \theta)$ ) is true, and  $L(\hat{\Omega})$  is the highest value that can be obtained if both  $H_0$  and  $H_1$  are true ( $\Omega = (\beta_0, \beta_1, \dots, \beta_p, \alpha, \theta) = (\beta, \alpha, \theta)$ ).  $H_0$  is rejected if  $G^2 > \chi^2_{(\alpha, n \times l)}$ ,  $l$  is the period/time or reject  $H_0$  if  $p\text{-value} < \alpha$ .

The impact of predictor factors on response variables was investigated by partial significance testing of regression parameters. This test is continued if the tests simultaneously decide to reject  $H_0$ . Partial hypothesis testing is:

$$H_0 : \beta_k(u_i, v_i) = 0$$

$$H_1 : \beta_k(u_i, v_i) \neq 0, k = 1, 2, \dots, p.$$

The test statistic is calculated as:

$$Z_{hit} = \frac{\hat{\beta}_k(u_i, v_i)}{SE(\hat{\beta}_k(u_i, v_i))} \quad (11)$$

where  $\hat{\beta}_{k(u_i, v_i)}$  is the parameter estimator of  $\beta_{k(u_i, v_i)}$  and  $SE(\hat{\beta}_{k(u_i, v_i)}) = \sqrt{\text{var}(\hat{\beta}_{k(u_i, v_i)})}$  is the standard error of  $\beta_{k(u_i, v_i)}$ .  $\text{var}(\hat{\beta}_{k(u_i, v_i)})$  is the  $j+1$ th diagonal element of  $(H(\hat{\gamma}))$ . Reject  $H_0$  if  $|Z_{hit}| > Z_{\alpha/2}$ .

### III. GEOGRAPHICALLY AND TEMPORALLY WEIGHTED GAMMA REGRESSION (GTWGR)

To tackle spatial and temporal non-stationary problems simultaneously, the Geographically Weighted Gamma Regression (GWGR) method was modified to create the Geographically and Temporally Weighted Gamma Regression (GTWGR) [22]. The model is:

$$E(Y_i) = \mu_i = \exp((\beta_0(u_i, v_i, t_i) + \sum_{k=1}^p \beta_k(u_i, v_i, t_i) x_{ik})) \quad (12)$$

$$i=1, 2, 3, \dots, n; k=1, 2, \dots, p.$$

By including the influence of time  $t_i$ , the data is comparable to the GTWR model. By including the influence of time  $t_i$ , the MLE approach is used to estimate the parameters  $\beta_k(u_i, v_i, t_i)$ .

The probability density function of the GTWGR model is:

$$\begin{aligned} f(y_{il}; \alpha(u_i, v_i, t_i), \beta(u_i, v_i, t_i)) &= \\ &= \frac{y_{il}^{\alpha(u_i, v_i, t_i)-1}}{\Gamma(\alpha(u_i, v_i, t_i)) \left( \frac{\exp(x_i^T \beta(u_i, v_i, t_i))}{\alpha(u_i, v_i, t_i)} \right)^{\alpha(u_i, v_i, t_i)}} \exp A \end{aligned} \quad (13)$$

with  $i = 1, 2, \dots, n$ ,  $y_{il} \geq 0$ . The GTWGR parameters are estimated using the MLE method with a likelihood function:

$$\begin{aligned} L(\alpha(u_i, v_i, t_i), \beta(u_i, v_i, t_i), i=1, 2, \dots, n, l=1, 2, \dots, L) \\ &= \prod_{l=1}^L \prod_{i=1}^n f(y_{il}) \\ &= \prod_{l=1}^L \prod_{i=1}^n \left[ \frac{1}{\Gamma(\alpha(u_i, v_i, t_i)) \left( \frac{\exp(x_i^T \beta(u_i, v_i, t_i))}{\alpha(u_i, v_i, t_i)} \right)^{\alpha(u_i, v_i, t_i)}} y_{il}^{\alpha(u_i, v_i, t_i)-1} \right. \\ &\quad \left. \exp \frac{-y_{il} \alpha(u_i, v_i, t_i)}{\exp(x_i^T \beta(u_i, v_i, t_i))} \right] \end{aligned} \quad (14)$$

and  $\ln$  probability function:

$$\begin{aligned} l &= \ln L(\alpha(u_i, v_i, t_i), \beta(u_i, v_i, t_i), i=1, 2, \dots, n) \\ &= \ln \left( \prod_{l=1}^L \prod_{i=1}^n f(y_{il}) \right) \\ &= \ln \prod_{l=1}^L \prod_{i=1}^n \left[ \frac{1}{\Gamma(\alpha(u_i, v_i, t_i)) \left( \frac{\exp(x_i^T \beta(u_i, v_i, t_i))}{\alpha(u_i, v_i, t_i)} \right)^{\alpha(u_i, v_i, t_i)}} y_{il}^{\alpha(u_i, v_i, t_i)-1} \right. \\ &\quad \left. \exp \frac{-y_{il} \alpha(u_i, v_i, t_i)}{\exp(x_i^T \beta(u_i, v_i, t_i))} \right] \end{aligned} \quad (15)$$

The natural logarithm function of probability for estimating parameters from location [22] is:

$$\begin{aligned} l^* &= \sum_{l=1}^L \sum_{i=1}^n w_{ii^*l} \ln f(y_{ii^*l}) \\ &= \sum_{l=1}^L \sum_{i=1}^n w_{ii^*l} \left( -\ln \left( \Gamma(\alpha(u_i, v_i, t_i)) \right) \right. \\ &\quad \left. - \alpha(u_i, v_i, t_i) \ln \left( \exp(x_i^T \beta(u_i, v_i, t_i)) \right) + A \right) \end{aligned} \quad (16)$$

$$A = \alpha(u_i, v_i, t_i) \ln \alpha(u_i, v_i, t_i) + (\alpha(u_i, v_i, t_i) - 1) \ln(y_{ii^*l})$$

$$-\frac{\alpha(u_i, v_i, t_i) y_{i^*l}}{\exp(\mathbf{x}_{i^*kl}^T \boldsymbol{\beta}_{l(u_i, v_i, t_i)})}$$

By taking the first partial derivative of  $l^*$  on each parameter and then equalizing them to zero, the estimated values of the GTWGR model parameters are found:

$$\begin{aligned} \frac{\partial l^*}{\partial \alpha(u_i, v_i, t_i)} = & \left( -\frac{\Gamma'(\alpha(u_i, v_i, t_i))}{\Gamma(\alpha(u_i, v_i, t_i))} \sum_{l=1}^L \sum_{i^*=1}^n w_{ii^*l} \right. \\ & \left. + (1 + \ln \alpha(u_i, v_i, t_i)) \sum_{l=1}^L \sum_{i^*=1}^n w_{ii^*l} \right. \\ & \left. + \sum_{l=1}^L \sum_{i^*=1}^n \frac{\mathbf{x}_{i^*kl}^T \boldsymbol{\beta}_{l(u_i, v_i, t_i)} w_{ii^*l}}{\exp(\mathbf{x}_{i^*kl}^T \boldsymbol{\beta}_{l(u_i, v_i, t_i)})} \right) = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial l^*}{\partial \beta_{kl}(u_i, v_i, t_i)} = & -\alpha(u_i, v_i, t_i) \sum_{l=1}^L \sum_{i^*=1}^n \mathbf{x}_{i^*kl}^T w_{ii^*l} \\ & + \alpha(u_i, v_i, t_i) \sum_{l=1}^L \sum_{i^*=1}^n \frac{\alpha(u_i, v_i, t_i) y_{i^*l} \mathbf{x}_{i^*kl}^T w_{ii^*l}}{\exp(\mathbf{x}_{i^*kl}^T \boldsymbol{\beta}_{l(u_i, v_i, t_i)})} = 0 \end{aligned} \quad (18)$$

The first partial derivative equations (17) and (18) still contain other parameters. The parameter estimation was carried out using the NR method [20]. The collected data are in three dimensions in a spatial-temporal coordinate system. Due to the variability in units about spatial-temporal effects, the measurement of the separation between the regression point and adjacent observational data points is conducted utilizing an ellipsoidal coordinate framework.  $\lambda$  and  $\mu$  are balance factors for the impact of unit changes between place and time on spatial-temporal distance measurements, assuming that  $d_{ij}^T$  is a temporal distance function and  $d_{ij}^S$  is a spatial distance function. The temporal and spatial distance functions are combined to create the spatial-temporal distance function, specifically Huang et. al [23]:

$$(d_{ij}^{ST})^2 = \lambda(u_i - u_j)^2 + (v_i - v_j)^2 + \eta(t_i - t_j)^2 \quad (19)$$

The balance parameters for the impact of location and time unit differences on spatial-temporal distance measurements are  $\lambda$  and  $\eta$ . Suppose  $\tau = \frac{\eta}{\lambda}$ ,  $\lambda \neq 0$  then:

$$\frac{(d_{ij}^{ST})^2}{\lambda} = (u_i - u_j)^2 + (v_i - v_j)^2 + \tau(t_i - t_j)^2 \quad (20)$$

The parameter  $\tau$  serves to modulate the proportion of temporal distance to spatial distance derived from the minimum coefficient of variation by furnishing a preliminary value of  $\tau$ :

$$CV(\tau) = \sum_{i=1}^n (y_i - \hat{y}_{\neq i}(\tau))^2 \quad (21)$$

We will obtain estimators of  $\lambda$  and  $\mu$  using an iterative method.

Examining the GTWGR model parameters' relevance concurrently for the  $l$ th era is:

$$H_0 : \beta_{1l}(u_i, v_i, t_i) = \beta_{2l}(u_i, v_i, t_i) = \dots = \beta_{kl}(u_i, v_i, t_i) = 0$$

$$H_1 : \text{there is at least one } \beta_{jl}(u_i, v_i, t_i) \neq 0,$$

$$i = 1, 2, \dots, n; l = 1, 2, \dots, L.$$

The test statistic is calculated as:

$$G_l^2 = -2 \ln \Lambda = -2 \ln \left[ \frac{L(\hat{\omega})}{L(\hat{\Omega})} \right] = 2(\ln L(\hat{\Omega}) - \ln L(\hat{\omega})) \quad (22)$$

where  $L(\hat{\omega})$  is the highest value that can be obtained if  $H_0$  ( $\omega = (\beta_{0l}, \alpha, \theta)$ ) is true and  $L(\hat{\Omega})$  is the highest value that can

be obtained if both  $H_0$  and  $H_1$  are true ( $\Omega = (\beta_0, \beta_1, \dots, \beta_{pl}, \alpha, \theta) = (\boldsymbol{\beta}, \alpha, \theta)$ ) are true.  $H_0$  is rejected if  $G^2 > \chi_{(\alpha, nxl)}^2$  or rejects  $H_0$  if p-value  $< \alpha$ .

Partial testing of the GTWGR model parameters' significance [19]:

$$H_0 : \beta_{kl}(u_i, v_i, t_i) = 0$$

$$H_1 : \beta_{kl}(u_i, v_i, t_i) \neq 0, k = 1, 2, \dots, p.$$

The test statistic is:

$$Z_{hit} = \frac{\hat{\beta}_{kl}(u_i, v_i, t_i)}{SE(\hat{\beta}_{kl}(u_i, v_i, t_i))} \quad (23)$$

where  $\hat{\beta}_{kl}(u_i, v_i)$  is the parameter estimator of  $\beta_{kl}(u_i, v_i, t_i)$  and  $SE(\hat{\beta}_{kl}(u_i, v_i, t_i)) = \sqrt{\text{var}(\hat{\beta}_{kl}(u_i, v_i, t_i))}$  is the standard error of  $\beta_{kl}(u_i, v_i, t_i)$ .  $\text{var}(\hat{\beta}_{kl}(u_i, v_i, t_i))$  is  $j+1$ st diagonal element of  $\mathbf{H}(\hat{\mathbf{y}})$ . Reject  $H_0$  if  $|Z_{hit}| > Z_{\alpha/2}$ .

The adequacy of the model is ascertained through the application of the corrected Akaike Information Criterion (AICc) and the Mean Squared Error (MSE), particularly:

$$MSE = \frac{1}{n} (\sum_{i=1}^n (y_i - \hat{y}_i)^2) \quad (24)$$

$$AIC = -n \log \left( \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n} \right) + 2p \quad (25)$$

$$AICc = AIC + \frac{2p(p+1)}{n-p-1} \quad (26)$$

#### IV. RIDGE REGULATION (L2)

One method to overcome multicollinearity problems is to use Ridge regression, which was introduced by Hoerl and Kennard in 1970 by adding a penalty to the regression coefficients in the L2 norm, specifically minimizing the sum of squares of the linear regression residuals using the following criteria to estimate the  $\beta$  parameter:

$$\sum_{k=1}^p \beta_k^2 \leq c \quad (27)$$

The Lagrange equation can be used to express the estimated coefficients, specifically:

$$\hat{\boldsymbol{\beta}}^{ridge} = \arg \min_{\boldsymbol{\beta}} \left\{ \sum_{i=1}^n (y_i - \beta_0 - \sum_{k=1}^p x_{ik} \beta_k)^2 + \lambda \sum_{k=1}^p \beta_k^2 \right\} \quad (28)$$

As in linear regression, penalties can also be added to the compressed linear model to overcome multicollinearity problems. Parameter estimation in distributed linear modelling with Ridge regularization [4] is:

$$\hat{\boldsymbol{\beta}}^{ridge} = \arg \min_{\boldsymbol{\beta}} \left\{ \frac{-\log[L(\mathbf{y}; \boldsymbol{\beta})]}{n} + \lambda_1 \sum_{k=1}^p \beta_k^2 \right\} \quad (29)$$

$L(\mathbf{y}; \boldsymbol{\beta})$  is the probability function of the exponential distribution family, and  $\lambda_1 \geq 0$  is the L2 regularization penalty parameter. The coefficient estimates obtained in Ridge regression are not equivariant due to differences in the scale of the input data, so standardization of the original variables is needed [24]. Parameter estimates are obtained using the NR method. The Ridge regression coefficient estimator will decrease as the value of  $\lambda_1$  increases. The regression coefficient shrinks more until it gets close to zero as the value of  $\lambda_1$  increases.

#### V. LASSO REGULATION (L1)

In 1996, Robert Tibshirani developed the least absolute shrinkage and selection operator (Lasso) as a method to address multicollinearity and to reduce the regression coefficients of less significant predictor variables to

approximately zero or precisely zero [25]. The parameter estimators in distributed linear modelling with Lasso regularization [4] are:

$$\hat{\beta}^{lasso} = \arg \min_{\beta} \left\{ \frac{-\log[L(y;\beta)]}{n} + \lambda_2 \sum_{k=1}^p |\beta_k| \right\} \quad (30)$$

The lasso regularization can reduce overfitting and select the most helpful predictor variables from the data. The lasso regularization can also reduce data dimensionality by removing less important predictor variables, speeding up the building process, and increasing model accuracy.

By varying the Lasso model's parameter  $\lambda_2$ , the degree of decline can be used to calculate the coefficient value of the predictor variables that Lasso regularization provides. The coefficient of the derived predictor variable decreases more significantly with increasing values of  $\lambda_2$ .

However, if the value of  $\lambda_2$  is too high, then the coefficient of the predictor variable will reach zero too quickly (or close to zero) so that the model cannot correctly capture the patterns in the data. Therefore, choosing the right  $\lambda_2$  value is important so that the model can capture patterns in the data well and eliminate less essential features.

## VI. MODELING APPLICATION FOR RAINFALL IN WEST JAVA PROVINCE

### A. Data

Monthly rainfall data from 35 locations in West Java Province, compiled by the Meteorology, Climatology, and Geophysics Agency (BMKG) from January 2010 to January 2012, function as a response variable. The predictor variables are  $x_1$ , the previous month's rainfall, and  $x_2$ – $x_{10}$  are GCM output monthly precipitation data derived from the Climate Forecast System Reanalysis (CFSR) model provided by the National Centers for Environmental Prediction (NCEP) [26], at coordinates (105; –5), (107.5; –5), (110; –5), (105; –7.5), (107.5; –7.5), (110; –7.5), (105; –10), (107.5; –10), and (110; –10), respectively [27].

### B. Method

The analysis stages carried out [28] are:

- 1) Checking multicollinearity between predictor variables using correlation values [29].
- 2) Estimating ordinary GTWGR model parameters using ten predictor variables.
- 3) Checking spatiotemporal diversity by:
  - a) Test BP for every period [30].
  - b) The boxplot method was used to see temporal variation between periods [31], [32], [33].
- 4) Carry out GTWGR and GTWGLR modelling for each response variable produced in the  $l$ th period with stages:
  - a) Estimating parameters using  $l \times n$  data in each  $l$  period.
  - b) Calculating the Euclidean distance for each  $l$  period.
  - c) Using the minimal CV value to estimate the ideal bandwidth value.
  - d) Use the Kernel Fixed Gaussian function to get the weighting matrix that yields the lowest CV value.
  - e) Estimating the optimum  $\tau$  parameter from: 1, 0.95, 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6, 0.55, 0.5, 0.45, 0.4, 0.35, 0.3, 0.25, 0.2, 0.15, 0.1, 0.095, 0.09, 0.085, 0.08, 0.07, 0.075, 0.065, 0.06, 0.055, 0.05, 0.045, 0.04, 0.035, 0.03,

0.025, 0.02, 0.015, 0.01, 0.009, 0.008, 0.007, 0.006, 0.005, 0.004, 0.003, 0.002, and 0.001.

- f) Estimating the optimum  $\lambda$  parameter from a choice of the same values as parameter  $\tau$ .
- g) Estimating the GTWGR model parameters  $\beta_r(u_i, v_i, t_i)$  at  $\tau$  and  $\lambda$ ,  $\lambda_{1.1}$  (the L2 regularization penalty coefficient utilized for ascertaining the optimal  $\tau$  parameter) and  $\lambda_{1.2}$  (the L2 regularization penalty parameter for determining the ideal  $\lambda$  parameter) is optimal for each period. The choice of parameter values for the optimum  $\lambda_{1.1}$  and  $\lambda_{1.2}$  from: 0.000001, 0.0000025, 0.000005, 0.0000075, 0.00001, 0.000025, 0.00005, 0.000075, 0.0001, 0.01, 0.05, 0.25, 0.5, 1, 2.5, 5, 10, 15, 20, 30, 50, 75, 100, 125, 150, 200, 300, 500, 1000, and 10000.
- h) Test the GTWGR model parameters simultaneously using the MLRT method:
  1.  $H_0: \beta_1 = \beta_2 = \dots = \beta_p = 0$ .
  2.  $H_1: \text{there is at least one } \beta_j \neq 0, j = 1, 2, \dots, p, i = 1, 2, \dots, n$ .
  3. Determine the set of parameters if  $H_0$  and  $H_1$  are true:  $\Omega = (\beta_0, \beta_1, \dots, \beta_p)$
  4. Determine the set of parameters if  $H_0$  is true:  $\omega = (\beta_0)$
  5. Form a maximum likelihood function if  $H_0$  and  $H_1$  are true:  $L(\hat{\Omega}) = \max_{\Omega} \prod_{i=1}^n f(y_i|\Omega)$
  6. Form a maximum likelihood function if  $H_0$  is true:  $L(\hat{\omega}) = \max \prod_{i=1}^n f(y_i|\omega)$
  7. Estimating parameters  $\Omega$  and  $\omega$  with MLE and NR method.
  8. Defining the Likelihood Function Ratio:  $\Lambda = \frac{L(\hat{\omega})}{L(\hat{\Omega})}$
  9. Determining test statistics using the MLRT method. The test statistic is calculated as:  $G^2 = -2 \ln \Lambda = -2 \ln \frac{L(\hat{\omega})}{L(\hat{\Omega})} = 2(\ln L(\hat{\Omega}) - L(\hat{\omega})) \sim \chi^2_{\alpha, 2p}$   
If  $G^2 > \chi^2_{\alpha, 2p}$  (or  $p\text{-value} < \alpha$ ), reject  $H_0$ . The quantity of predictor variables within the model is denoted by  $p$ . Partial testing is done to ascertain the importance of each parameter to the model if  $H_0$  is rejected.
- i) Calculate the AICc of each resulting response variable.
- j) Estimating the GTWGLR model parameters  $\beta_l(u_i, v_i, t_i)$  at  $\tau$  and  $\lambda$ ,  $\lambda_{2.1}$  (the L2 regularization penalty parameter for determining the ideal  $\tau$  parameter) and  $\lambda_{2.2}$  (the penalty parameter in L1 regularization in estimating the optimum  $\lambda$  parameter)) is optimal for each period.
- k) Test the GTWGLR model parameters simultaneously using the MLRT method analogue with h)1 - until h)9.
- l) Choose the model that provides the smallest AICc between the GTWGR and GTWGLR models.
- m) Use Kriging interpolation in conjunction with the chosen GTWGR or GTWGLR method to estimate the response variable's value at a new location.
- n) Create a monthly rainfall prediction map for October 2011 to January 2012.

### C. Results and Discussion

#### C1. Multicollinearity Test

To perform the multicollinearity check, the Pearson correlation coefficient between each predictor variable was calculated [29].

TABLE I  
CORRELATION BETWEEN PREDICTOR VARIABLES

|     | x1    | x2    | x3    | x4    | x5    | x6    | x7    | x8    | x9    | x10   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| x1  | -     | 0.04  | 0.09  | -0.03 | 0.04  | 0.09  | -0.03 | 0.04  | 0.09  | -0.03 |
| x2  | 0.04  | -     | 0.92  | -0.39 | 1.00  | 0.92  | -0.39 | 1.00  | 0.92  | -0.39 |
| x3  | 0.09  | 0.92  | -     | -0.24 | 0.92  | 1.00  | -0.24 | 0.92  | 1.00  | -0.24 |
| x4  | -0.03 | -0.39 | -0.24 | -     | -0.39 | -0.24 | 1.00  | -0.39 | -0.24 | 1.00  |
| x5  | 0.04  | 1.00  | 0.92  | -0.39 | -     | 0.92  | -0.39 | 1.00  | 0.92  | -0.39 |
| x6  | 0.09  | 0.92  | 1.00  | -0.24 | 0.92  | -     | -0.24 | 0.92  | 1.00  | -0.24 |
| x7  | -0.03 | -0.39 | -0.24 | 1.00  | -0.39 | -0.24 | -     | -0.39 | -0.24 | 1.00  |
| x8  | 0.04  | 1.00  | 0.92  | -0.39 | 1.00  | 0.92  | -0.39 | -     | 0.92  | -0.39 |
| x9  | 0.09  | 0.92  | 1.00  | -0.24 | 0.92  | 1.00  | -0.24 | 0.92  | -     | -0.24 |
| x10 | -0.03 | -0.39 | -0.24 | 1.00  | -0.39 | -0.24 | 1.00  | -0.39 | -0.24 | -     |

TABLE II  
PARTIAL TEST OF GLOBAL GAMMA REGRESSION MODEL PARAMETERS IN OCTOBER 2011 TO JANUARY 2012

| Variable  | October 2011 |         | November 2011 |         | December 2011 |         | January 2012 |         |
|-----------|--------------|---------|---------------|---------|---------------|---------|--------------|---------|
|           | Coefficient  | p-value | Coefficient   | p-value | Coefficient   | p-value | Coefficient  | p-value |
| Intercept | 2.66         | 0       | 3.63          | 0       | 4.50          | 0       | 5.62         | 0       |
| x1        | 0.84         | 0.01    | 0.57          | 0       | -0.10         | 0.59    | -0.06        | 0.48    |
| x2        | -3.98        | 0       | -0.29         | 0.36    | 1.65          | 0       | -2.89        | 0       |
| x3        | -1.63        | 0       | 4.69          | 0       | -4.64         | 0       | 14.69        | 0       |
| x4        | 0.967        | 0       | 1.34          | 0.88    | -0.79         | 0       | -2.42        | 0       |
| x5        | -0.94        | 0       | -7.78         | 0       | -2.35         | 0       | -2.13        | 0       |
| x6        | -0.01        | 0.81    | 3.45          | 0       | 1.27          | 0       | -9.25        | 0       |
| x7        | 0.69         | 0       | -5.50         | 0.20    | 0.99          | 0       | 0.07         | 0       |
| x8        | 1.19         | 0       | -0.20         | 0.523   | 2.28          | 0       | -0.89        | 0       |
| x9        | 1.95         | 0       | -3.02         | 0       | 0.46          | 0.01    | -3.43        | 0       |
| x10       | -3.06        | 0       | 2.26          | 0.59    | 0.05          | 0.80    | 1.02         | 0       |

In line with [2] that in utilizing GCM output data that has many predictor variables, there is a spatial correlation between grids and there is multicollinearity between predictor variables. Table I shows a perfect correlation between several predictor pairs, namely between the predictors x2 and x5, x2 and x8, x3 and x6, x3 and x9, x4 and x7, x4 and x10, x5 and x8, x6 and x9, and x7 and x10. There is also a high correlation of 0.92 between several pairs of predictors, including x2 and x3; x2 and x6; x2 and x9; x3 and x5; x3 and x8; x5 and x6; x5 and x9; x6 and x8; and x8 and x9. This result influenced the occurrence of multicollinearity, which will be handled by applying Ridge and Lasso regularization. Meanwhile, the partial test of Gamma Regression model parameters for October 2011, November 2011, December 2011, and January 2012 are described in Table II.

Table II shows that the predictor x6 is the variable that has no real effect on the Global Gamma regression model in October 2011 because it has a  $p\text{-value} > \alpha = 0.05$ . The predictor variables x2, x4, x7, x8, and x10 have no real effect on the model in November 2011. The variables x1 and x10 have no real effect on the model in December 2011 because they have a  $p\text{-value} < \alpha = 0.05$ . Likewise, the x1 variable has no real effect on the model in January 2012.

## C2. Spatial Heterogeneity Analysis

Geographically and temporally weighted gamma regression modeling can be used if the data meet the assumptions of spatial and temporal heterogeneity. First, the assumption of spatial heterogeneity will be examined and followed by the examination of temporal heterogeneity.

The spatial heterogeneity assessments conducted each month yield p-values consistently below the 0.05 threshold for significance. These results substantiate the assertion that spatial heterogeneity is present in each month. An analysis of

temporal variability was conducted utilizing box plots of residuals for each month spanning from January 2010 to December 2012 [9]. The box plots of the residuals indicate that the residuals from the global regression model exhibit a lack of homogeneity, as evidenced by the distances between the quartiles [31], [32], [33]. The outcomes of the investigation indicate that a predominant portion of geographical heterogeneity transpired within the interval from January 2010 to December 2012.

The results of examining spatial diversity using ten predictor variables for October 2011, November 2011, December 2011, and January 2012 are listed in Table III.

TABLE III  
SPATIAL DIVERSITY TEST RESULTS

| Month         | BP-value | p-value |
|---------------|----------|---------|
| October 2011  | 12.84    | 0.00163 |
| November 2011 | 25.12    | 0.00002 |
| December 2011 | 27.47    | 0.00002 |
| January 2012  | 24.98    | 0.00005 |

The monthly rainfall data reveal both regional and temporal discrepancies. To address this variability, spatial-temporal regression analysis will be employed. The modelling process proceeded with GTWGR and GTWGLR methodologies to ascertain the most effective approach for estimating monthly rainfall while mitigating multicollinearity issues.

## C3. Geographically and Temporally Weighted Gamma Regression

By identifying the ideal bandwidth that yields the lowest CV value, Ordinary Geographically and Temporally Weighted Gamma Regression (OGTWGR) modelling is then performed.

TABLE IV  
OPTIMUM  $\tau$ , OPTIMUM  $\lambda$ , DEVIANS, AND RMSE VALUES OF THE OGTWGR MODEL FROM OCTOBER 2011 TO JANUARY 2012

| Month         | $\tau$ | $\lambda$ | Devians | RMSE   | AICc      | p-value |
|---------------|--------|-----------|---------|--------|-----------|---------|
| October 2011  | 1      | 1         | 296.28  | 132.55 | 20089.56  | 0.0000  |
| November 2011 | 1      | 1         | 733.41  | 264.26 | 42097.33  | 0.0000  |
| December 2011 | 0.3    | 1         | 725.57  | 255.03 | 71131.76  | 0.0000  |
| January 2012  | 0.55   | 1         | 688.09  | 264.97 | 108025.66 | 0.0000  |

TABLE V  
THE OPTIMAL VALUES ( $\lambda_{1,1}$ ,  $\tau$ ,  $\lambda_{1,2}$ ,  $\lambda$ ), AICc, DEVIANS, AND P-VALUE OF THE GTWGRR MODEL FROM OCTOBER 2011 TO JANUARY 2012

| Month         | $\lambda_{1,1}$ | $\tau$ | $\lambda_{1,2}$ | $\lambda$ | AICc      | Devians | p-value |
|---------------|-----------------|--------|-----------------|-----------|-----------|---------|---------|
| October 2011  | 0.00001         | 0.95   | 0.01            | 0.55      | 19899.33  | 104.03  | 0.0051  |
| November 2011 | 0.0000025       | 1      | 0.0000025       | 1         | 41748.66  | 217.53  | 0.0000  |
| December 2011 | 0.000075        | 0.3    | 0.05            | 0.8       | 70540.78  | 169.43  | 0.0457  |
| January 2012  | 0.000005        | 0.55   | 0.05            | 0.035     | 107470.02 | 373.36  | 0.0000  |

TABLE VI  
THE OPTIMAL VALUES ( $\lambda_{2,1}$ ,  $\tau$ ,  $\lambda_{2,2}$ ,  $\lambda$ ), AICc, DEVIANS, AND P-VALUE OF THE GTWGLR MODEL FROM OCTOBER 2011 TO JANUARY 2012

| Month         | $\lambda_{2,1}$ | $\tau$ | $\lambda_{2,2}$ | $\lambda$ | AICc      | Devians | p-value |
|---------------|-----------------|--------|-----------------|-----------|-----------|---------|---------|
| October 2011  | 0.0000025       | 1      | 0.05            | 0.65      | 19904.12  | 111.55  | 0.0011  |
| November 2011 | 0.00001         | 1      | 0.00001         | 0.02      | 41747.70  | 217.80  | 0.0000  |
| December 2011 | 0.0001          | 0.3    | 0.25            | 0.35      | 70633.20  | 279.64  | 0.0000  |
| January 2012  | 0.05            | 0.75   | 0.25            | 0.075     | 107739.15 | 642.91  | 0.0000  |

The optimum  $\tau$ , optimum  $\lambda$ , Devians, and RMSE values of the OGTWGR model for October 2011, November 2011, December 2011, and January 2012 are listed in Table IV.

Based on Table IV, the devians for October 2011, November 2011, December 2011, and January 2012 have a p-value = 0.0000 which is smaller than  $\alpha = 0.05$ . This result that all the predictor variables ( $x_1$ ,  $x_2$ , ...,  $x_{10}$ ) have a significant effect on the OGTWGR model.

#### C4. Geographically and Temporally Weighted Gamma Ridge Regression (GTWGRR)

The optimal values of  $\lambda_{1,1}$ ,  $\tau$ ,  $\lambda_{1,2}$ , and  $\lambda$ , along with AICc, Devians, and p-values of GTWGRR and GTWGLR were conducted. The optimal values ( $\lambda_{1,1}$ ,  $\tau$ ,  $\lambda_{1,2}$ , and  $\lambda$ ), AICc, Devians, and p-values for October 2011, November 2011, December 2011, and January 2012, the results of the GTWGRR model are listed in Table V, and the optimal values ( $\lambda_{2,1}$ ,  $\tau$ ,  $\lambda_{2,2}$  and  $\lambda$ ), AICc, Devians, and p-values f for October 2011, November 2011, December 2011, and January

2012, the results of the GTWGLR model are listed in Table VI.

Both models (Table V and Table VI) showed the p-value of Devians for each Month smaller than  $\alpha = 0.05$  for each month, indicating that all the predictor variables have a significant effect on the response variable.

Analysis of the average AICc values and standard deviations for the GTWGRR model (Table V) and the GTWGLR model (Table VI) reveals significant findings. Over four months, the GTWGRR method achieved an average AICc of 59914.69 with a standard deviation of 37884.28. In contrast, the GTWGLR method had a higher average AICc of 60006.04 and a standard deviation of 38004.02. This indicates that the GTWGRR method is the better option, showing both the lowest average AICc and the smallest standard deviation. Table VII provides an overview of the minimum, average, and maximum parameter estimation values for the GTWGRR model in October 2011, November 2011, December 2011, and January 2012.

TABLE VII  
SUMMARY OF THE RESULTS OF PARAMETER ESTIMATION ON GTWGRR MODELS IN OCTOBER 2011, NOVEMBER 2011, DECEMBER 2011, AND JANUARY 2012

| Month         | Statistics | b1     | b2     | b3     | b4     | b5     | b6     | b7     | b8     | b9     | b10    |
|---------------|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| October 2011  | Minimum    | 0.228  | -0.435 | -0.359 | -0.028 | -0.471 | -0.337 | -0.025 | -0.234 | -0.323 | -0.041 |
|               | Average    | 0.315  | -0.237 | -0.234 | 0.093  | -0.299 | -0.177 | 0.079  | -0.142 | -0.198 | 0.085  |
|               | Maximum    | 0.422  | -0.118 | -0.116 | 0.223  | -0.190 | -0.045 | 0.199  | 0.000  | -0.073 | 0.187  |
| November 2011 | Minimum    | 0.346  | -0.622 | -0.176 | -0.207 | -0.533 | -0.334 | -0.083 | -0.610 | -0.458 | 0.161  |
|               | Average    | 0.465  | -0.478 | -0.038 | -0.087 | -0.341 | -0.152 | 0.053  | -0.437 | -0.208 | 0.300  |
|               | Maximum    | 0.597  | -0.352 | 0.258  | 0.066  | -0.080 | -0.012 | 0.229  | -0.285 | -0.073 | 0.457  |
| December 2011 | Minimum    | -0.129 | -0.374 | -0.821 | -0.010 | -0.739 | -0.622 | -0.219 | -0.341 | -0.444 | -0.457 |
|               | Average    | 0.061  | -0.130 | -0.465 | 0.196  | -0.505 | -0.362 | 0.201  | 0.034  | -0.132 | -0.284 |
|               | Maximum    | 0.242  | 0.178  | 0.115  | 0.604  | 0.172  | 0.120  | 0.421  | 0.286  | 0.103  | 0.113  |
| January 2012  | Minimum    | -0.546 | -2.540 | -2.083 | 0.911  | 0.179  | -1.448 | -1.840 | -1.478 | -3.579 | -0.615 |
|               | Average    | -0.240 | 1.037  | -0.638 | 1.536  | 0.879  | -0.192 | -1.205 | -0.353 | -2.320 | -0.060 |
|               | Maximum    | 0.261  | 3.232  | 1.446  | 2.506  | 2.238  | 0.559  | -0.792 | 1.480  | -1.688 | 0.313  |

TABLE VIII  
GTWGRR MODEL AT MALABAR AND WANGUN WATI STATIONS IN OCTOBER 2011, NOVEMBER 2011, AND JANUARY 2012

| Period        | Station     | GTWGRR Model                                                                                                              |
|---------------|-------------|---------------------------------------------------------------------------------------------------------------------------|
| October 2011  | Malabar     | $\hat{\mu} = \exp(0.6887 + 0.2949x_1 - 0.3202x_2 - 0.2681x_3 + 0.0639x_4 - 0.4405x_5 + \dots - 0.0726x_9 + 0.1361x_{10})$ |
|               | Wangun Wati | $\hat{\mu} = \exp(0.3742 - 0.2510x_1 - 0.2457x_2 + 0.1974x_3 - 0.3607x_4 - 0.2207x_5 + \dots - 0.2344x_9 + 0.0572x_{10})$ |
| November 2011 | Malabar     | $\hat{\mu} = \exp(0.7246 + 0.4863x_1 - 0.4558x_2 + 0.0236x_3 - 0.0637x_4 - 0.3159x_5 + \dots - 0.3215x_9 + 0.2940x_{10})$ |
|               | Wangun Wati | $\hat{\mu} = \exp(0.6546 + 0.4707x_1 - 0.4768x_2 - 0.1763x_3 - 0.1591x_4 - 0.329x_5 + \dots - 0.0848x_9 + 0.2617x_{10})$  |
| January 2012  | Malabar     | $\hat{\mu} = \exp(1.8088 + 0.2453x_1 - 0.918x_2 + 0.9093x_3 + 1.7809x_4 + 1.7312x_5 + \dots - 3.0005x_9 - 0.5105x_{10})$  |
|               | Wangun Wati | $\hat{\mu} = \exp(2.0551 - 0.4907x_1 + 0.4948x_2 - 0.6662x_3 + 2.2252x_4 + 0.9662x_5 + \dots - 1.8463x_9 - 0.2422x_{10})$ |

Table VII illustrates that in October 2011, the variables x2, x3, x5, x6, and x9 demonstrated negative values across their minimum, average, and maximum thresholds. Variables x4, x7, and x10 revealed negative values solely at the minimum threshold. The x1 variable presented positive values at the minimum, average, and maximum levels, whereas the x8 variable showed positive values exclusively at the maximum level. In November 2011, the x1 and x10 variables displayed negative values across their minimum, average, and maximum thresholds. In contrast, the variables x2, x5, x6, x8, and x9 exhibited positive values at the minimum, average, and maximum thresholds. Variables x3 and x4 reflected negative values at both the minimum and average, while Variable x7 showed negative values only at the minimum threshold. As of December 2011, variables x1, x4, x7, and x8 displayed negative values at the minimum, with the remaining variables reflecting positive values. Variables x2, x3, x5, x6, x9, and x10 registered positive values at the maximum level, whereas the other variables remained negative. In January 2012, the variables x1, x3, x6, x8, and

x10 showed negative values for both the minimum and average levels. The variables x4 and x5 maintained positive values at the minimum, average, and maximum thresholds. The x2 variable recorded a negative value at its minimum threshold.

Several examples of GTWGRR models formed in October 2011, November 2011, and January 2012 at Malabar and Wangun Wati stations are in Table VIII.

Based on the GTWGRR model in November 2011 in Table VIII, at Malabar station: if the other predictors are fixed then every one unit increase in predictor x1 will increase monthly rainfall by  $e^{0.4863} = 1.63$  times. If the other predictors are fixed, then every one-unit increase in predictor x2 will reduce monthly rainfall by  $e^{-0.4558} = 0.63$  times. At Wangun Wati station: if the other predictors are fixed then every one unit increase in predictor x1 will increase monthly rainfall by  $e^{0.4707} = 1.61$  times. If the other predictors are fixed, then every one-unit increase in predictor x2 will reduce monthly rainfall by  $e^{-0.4768} = 0.62$  times.

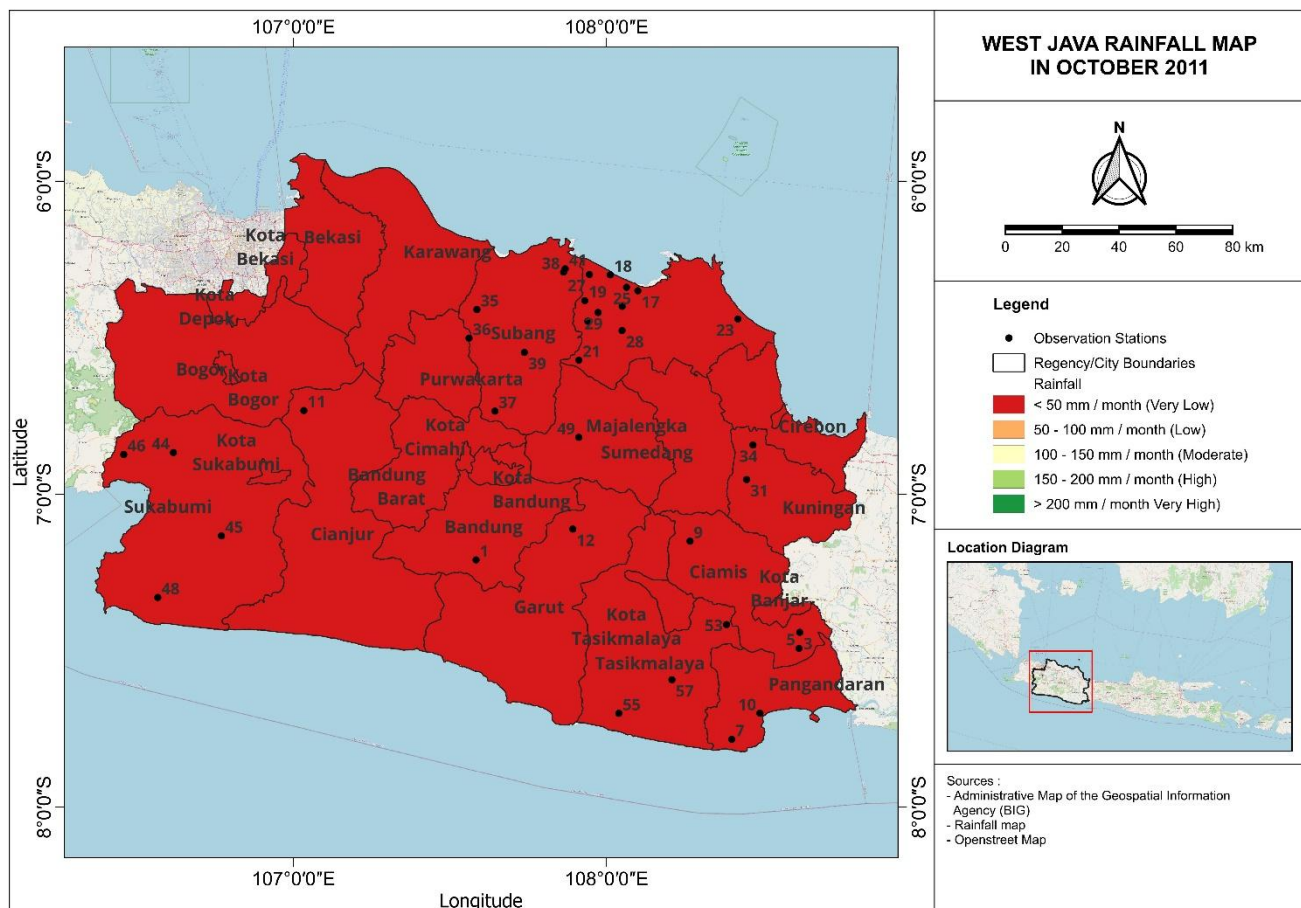


Fig. 1. Map of monthly rainfall predictions for October 2011.



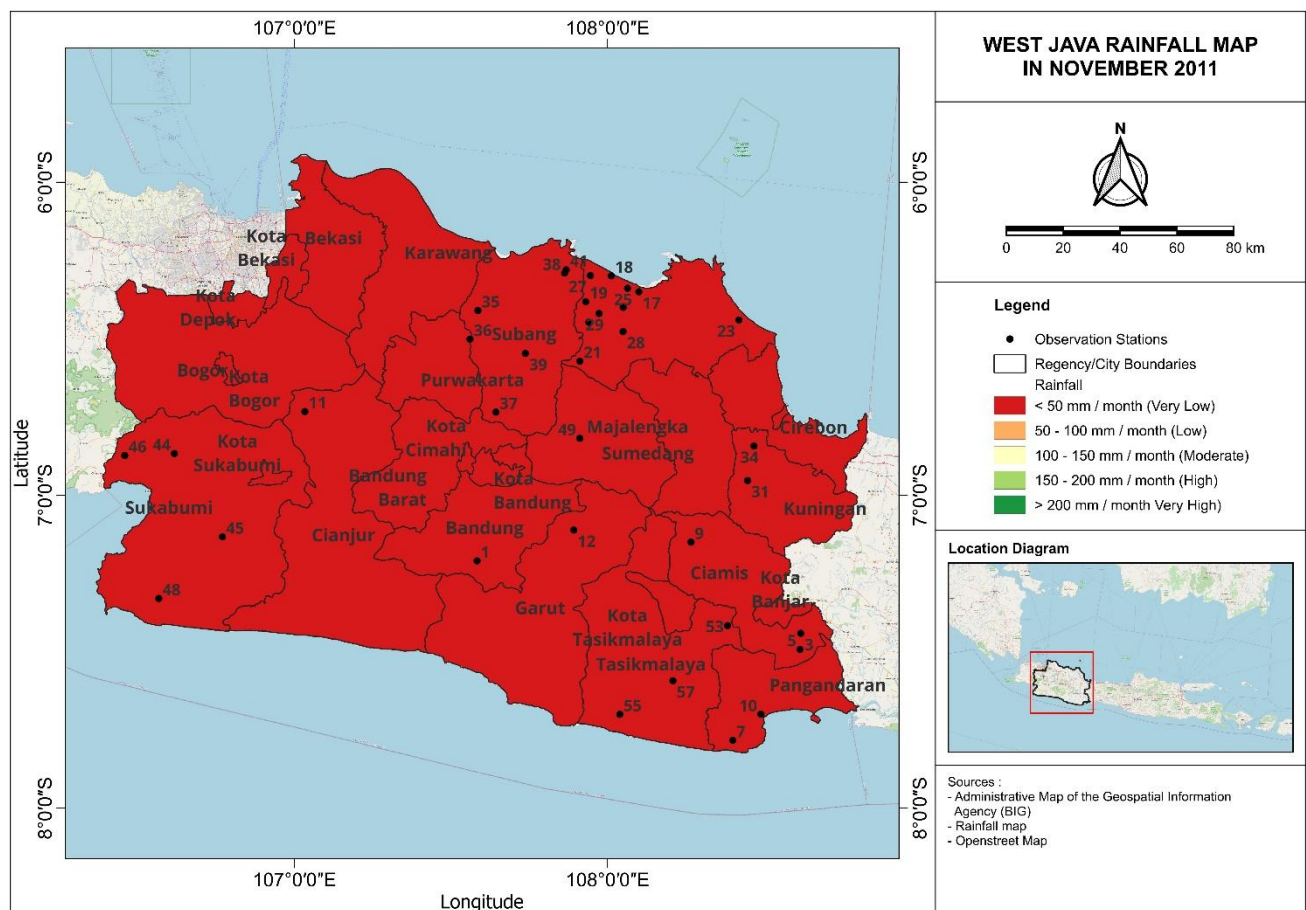


Fig. 2. Map of monthly rainfall predictions for November 2011.

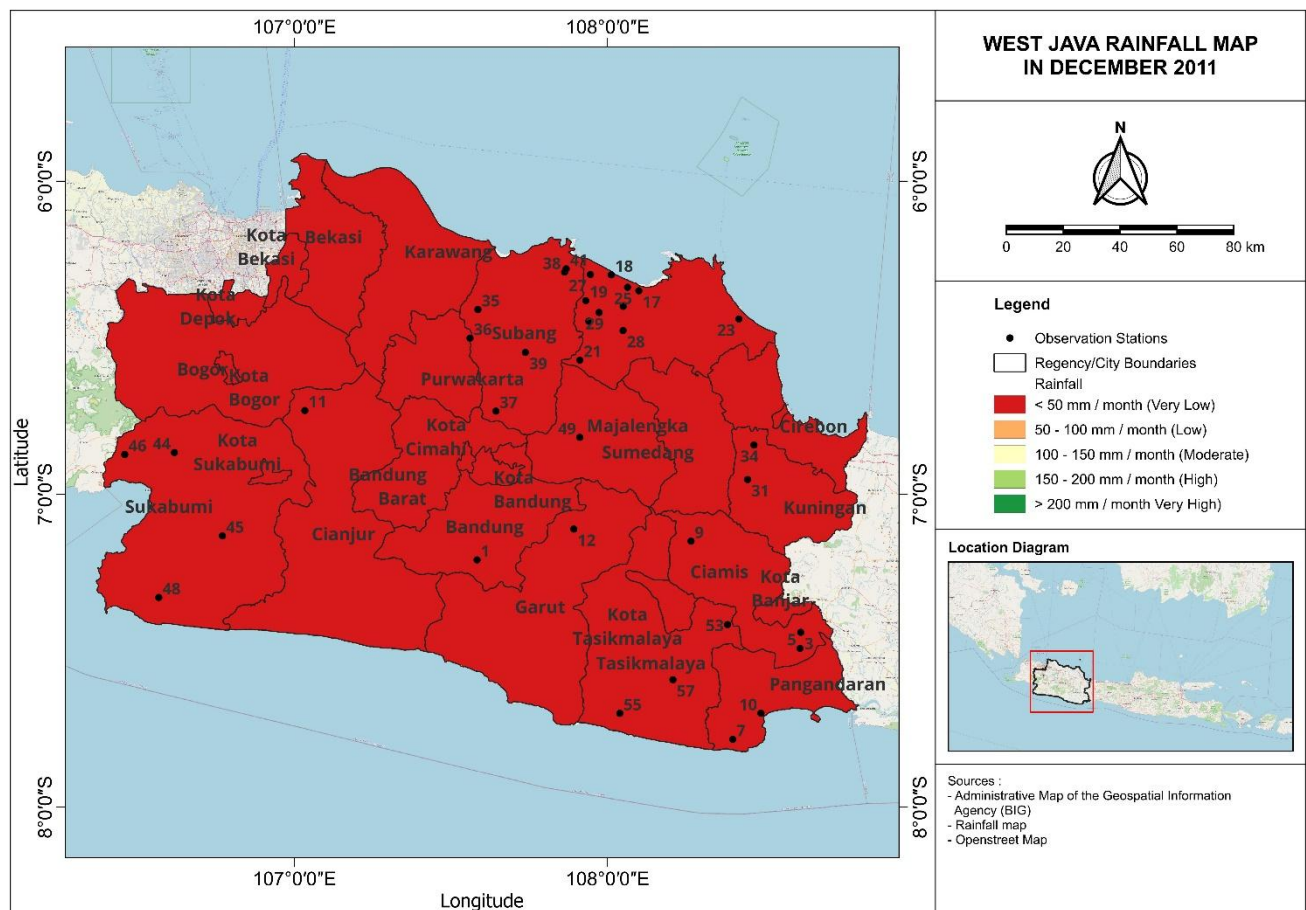


Fig. 3. Map of monthly rainfall predictions for December 2011.

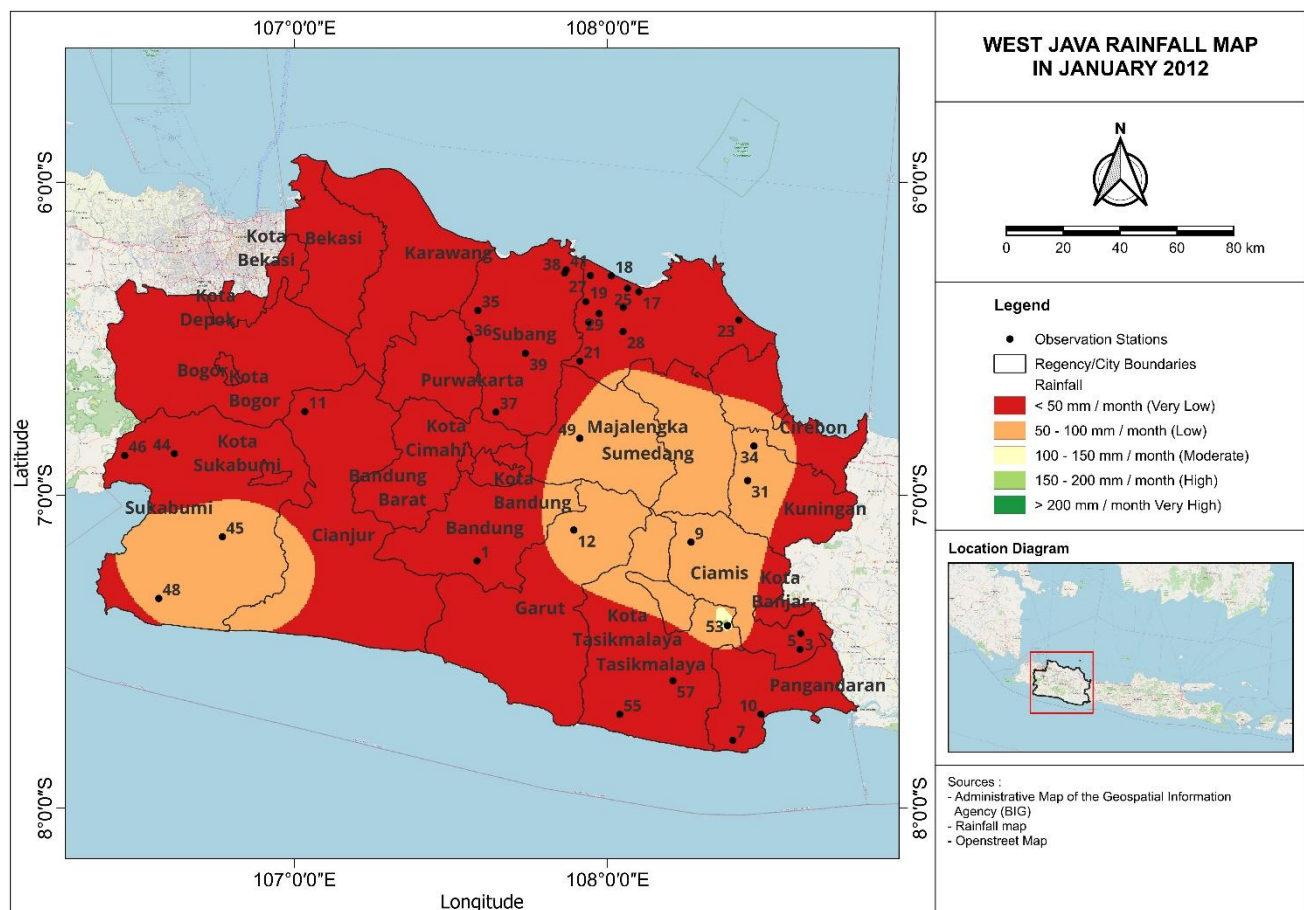


Fig. 4. Map of monthly rainfall predictions for January 2012.

#### C5. Estimating Data Values at New Locations

Monthly rainfall prediction maps for October 2011 to January 2012 using a combination of the GTWGR method and Kriging interpolation are in Fig. 1 to Fig. 4.

The integration of the GTWGR approach with kriging interpolation facilitated the calculation of monthly rainfall values across 18,038 grid cells.

The calculation results showed that the average monthly rainfall in October 2011 was 8.44 mm/month. As shown in Fig. 1, 53% of areas in West Java had below-average monthly rainfall in October 2011. The minimum monthly rainfall, 8.12 mm/month, was located at the coordinate (107.5182, -6.7019), whereas the maximum, 10.22 mm/month, was occurred at the coordinate (108.7623; -7.6953).

The average monthly rainfall in November 2011 (Fig. 2) was 11.75 mm, covering 63% area of West Java, as presented in Fig. 2. The minimum monthly rainfall of 8 mm was located at coordinate (107,8011;-6.7838), while the maximum, 15.27 mm/month, was detected at coordinate (107.7338;-76.3761).

In December 2011 (Fig. 3), a minimum monthly rainfall of 17.63 mm/month was achieved in the cell (106.6154;-6.8658). A total of 53.13% of cells reduced the average monthly rainfall. Reaching 43.46 mm/month rainfall per month is achieved in the cell (108.2099; -7.5904).

In January 2012 (Fig. 4), monthly rainfall below the average of 46.52 mm/month occurred in approximately 28.83 % of West Java area in January 2012, as shown in Fig. 4. The lowest and highest rainfall, 18.54 mm/month and 175.235 mm/month, were observed at the coordinates (108.6186;-7.4399) and (108.3805;-7.4126), respectively. A significant 75.83% of the cells reported monthly rainfall of less than 50

mm/month, while 24.07% recorded between 50 mm/month and 100 mm/month, and 0.1% exceeded 200 mm/month.

The data derived from the integration of the Kriging interpolation technique and the GTWGR model for the interval from October 2011 to January 2012 indicates that November experienced the least monthly rainfall at 8 mm/month, whereas January 2012 recorded the peak at 175.24 mm/month. This finding aligns with the mean monthly rainfall statistic as well. Analysis of the rainfall estimates from October 2011 to January 2012, as depicted in Fig. 1 to Fig. 4, reveals a predominance of monthly rainfall values falling below 50 mm/month. This contrasts with the actual rainfall data, which indicates that various locations recorded monthly rainfall exceeding 50 mm/month. The actual average monthly rainfall was at its minimum in October 2011 at 85.03 mm/month and reached its maximum in January 2012 at 239.32 mm/month.

Fig. 1 to Fig. 4 illustrate that the monthly rainfall forecasts for October 2011, November 2011, December 2011, and January 2012 predominantly yield values less than 50 mm/month, showing minimal variability across most locations. This suggests that the amalgamation of the Kriging interpolation method with the GTWGR model is not optimal to produce the differences in monthly rainfall predictions across various sites.

The rainfall predictions for January 2012 indicate values ranging between 50-100 mm/month in the southwest region, particularly at the Lengkong station in Sukabumi Regency, and in the southeast at the Jati Mulya station in Sumedang Regency, Leles station in Garut Regency, Kahuripan station in Tasikmalaya Regency, Panjalu station in Ciamis Regency,

as well as the Cigugur and Mandi Rancang stations in Kuningan Regency, whereas the preceding months were predominantly characterized by rainfall below 50 mm/month.

Commencing in January 2012, the projected monthly rainfall values began to exhibit variability. For researchers seeking to conduct a more comprehensive analysis, it is advisable to utilize a dataset spanning a longer timeframe (e.g. 24 months) to discern underlying patterns more effectively. This study was limited to a four-month analysis due to constraints on time and resources.

TABLE IX

RMSEP VALUE OF RAINFALL ESTIMATES FOR OCTOBER 2011 TO JANUARY 2012 AT NEW LOCATIONS

| Month         | RMSEP  |
|---------------|--------|
| October 2011  | 2.498  |
| November 2011 | 6.166  |
| December 2011 | 5.037  |
| January 2012  | 17.066 |

Table IX shows the RMSEP value of rainfall estimates for October 2011, November 2011, December 2011, and January 2012 at new locations. The smallest RMSEP value was 2.498 in October 2011, while the most considerable RMSEP value was 17.066 achieved in January 2012. In general, the RMSEP value is below average at 7.692.

## VII. CONCLUSION

The GTWGR method outperforms the GTWGLR method in SD modelling for estimating monthly rainfall for October 2011, November 2011, December 2011, and January 2012. This is because it provides the smallest average value and standard deviation of AICc across all four months. Combining the GTWGR with Kriging interpolation demonstrates the potential for estimating monthly rainfall in other locations across West Java Province.

## AUTHORS' CONTRIBUTIONS

All authors contributed to the design, implementation, and analysis of the research. Aan Kardiaana prepared the manuscript in consultation with Aji Hamim Wigena, Anik Djuraidah, and Agus Mohamad Soleh.

## REFERENCES

- [1] R. E. Benestad, C. D., and I. Hanssen-Bauer, *Empirical-statistical downscaling in climate modeling*. Singapore: World Scientific Publishing Co. Pte. Ltd., 2008. doi: 10.1029/2004EO420002.
- [2] A. H. Wigena, "Statistical Downscaling Modeling with Regression Projection Pursuit for Monthly Rainfall Forecasting (Monthly Rainfall Case in Indramayu) (Pemodelan Statistical Downscaling Dengan Regresi Projection Pursuit Untuk Peramalan Curah Hujan Bulanan. Kasus Curah hujan bulanan di Indramayu)," Ph.D. dissertation, Dept. Statistics, IPB Univ., Bogor, Indonesia, 2006, pp. 14-29.
- [3] R. L. Wilby and T. M. L. Wigley, "Downscaling general circulation model output: a review of methods and limitations," *Prog. Phys. Geogr. Earth Environ.*, vol. 214, pp. 530-548, Dec. 1997, doi: 10.1177/030913339702100403.
- [4] A. M. Soleh, "Linear Modeling of Amplified Gamma and Pareto Distribution with L1 Regulation in Statistical Downscaling for Estimating Monthly Rainfall; Application to Rainfall Modeling in Indramayu Regency (Pemodelan Linier Sebaran Gamma dan Pareto Terampat dengan Regularisasi L1 Pada Statistical Downscaling untuk Pendugaan Curah Hujan Bulanan. Aplikasi Pada Pemodelan Curah Hujan di Kabupaten Indramayu)," Ph.D. dissertation, Dept. Statistics, IPB Univ., Bogor, Indonesia, 2015, pp. 21-28.
- [5] A. M. Soleh, A. H. Wigena, A. Djuraidah, and A. Saefuddin, "Statistical Downscaling to Predict Monthly Rainfall Using Generalized Linear Model with Gamma Distribution," *Inform. Pertan.*, vol. 24, no. 2, p. 215, 2015, doi: 10.21082/ip.v24n2.2015.p215-222.
- [6] A. Djuraidah, R. N. Rachmawati, A. H. Wigena, and I. W. Mangku, "Extreme data analysis using spatio-temporal bayes regression with INLA in statistical downscaling model," *Int. J. Innov. Comput. Inf. Control*, vol. 17, no. 1, pp. 259-273, 2021, doi: 10.24507/ijicic.17.01.259.
- [7] M. Hayati, A. H. Wigena, A. Djuraidah, and A. Kurnia, "A new approach to statistical downscaling using tweedie compound poisson gamma response and lasso regularization," *Commun. Math. Biol. Neurosci.*, vol. 2021, pp. 1-16, 2021, doi: 10.28919/cmbn/5936.
- [8] A. Kardiaana, A. H. Wigena, A. Djuraidah, and A. M. Soleh, "Geographically and temporally weighted regression modeling in statistical downscaling modeling for the estimation of monthly rainfall," *J. Math. Comput. Sci.*, pp. 1-20, 2022, doi: 10.28919/jmcs/7427.
- [9] A. Kardiaana, A. H. Wigena, A. Djuraidah, and A. M. Soleh, "Statistical Downscaling Modeling for Monthly Rainfall Estimation Using Geographical and Temporal Weighted Gamma Regression," *Asian J. Math. Comput. Res.*, vol. 29, no. 2, pp. 43-55, 2022, doi: 10.56557/ajomcor/2022/v29i27923.
- [10] G. . Foody, "Geographical weighting as a further refinement to regression modelling: An example focused on the NDVI-rainfall relationship," *Remote Sens. Environ.*, vol. 88, no. 3, pp. 283-293, Dec. 2003, doi: 10.1016/j.rse.2003.08.004.
- [11] F. Chen, Y. Liu, Q. Liu, and X. Li, "Spatial downscaling of TRMM 3B43 precipitation considering spatial heterogeneity," *Int. J. Remote Sens.*, vol. 35, no. 9, pp. 3074-3093, 2014, doi: 10.1080/01431161.2014.902550.
- [12] S. Xu, C. Wu, L. Wang, A. Gonsamo, Y. Shen, and Z. Niu, "A new satellite-based monthly precipitation downscaling algorithm with non-stationary relationship between precipitation and land surface characteristics," *Remote Sens. Environ.*, vol. 162, pp. 119-140, Jun. 2015, doi: 10.1016/j.rse.2015.02.024.
- [13] A. S. Fotheringham, R. Crespo, and J. Yao, "Geographical and Temporal Weighted Regression (GTWR)," *Geogr. Anal.*, vol. 47, no. 4, pp. 431-452, Oct. 2015, doi: 10.1111/gean.12071.
- [14] Z. Du, S. Wu, M.-P. Kwan, C. Zhang, F. Zhang, and R. Liu, "A spatiotemporal regression-kriging model for space-time interpolation: a case study of chlorophyll-a prediction in the coastal areas of Zhejiang, China," *Int. J. Geogr. Inf. Sci.*, vol. 32, no. 10, pp. 1927-1947, Oct. 2018, doi: 10.1080/13658816.2018.1471607.
- [15] I. Ali, A. Djuraidah, and A. M. Soleh, "Gamma response regression with percentile lasso and ridge to estimate extreme rainfall," in *ICMCE*, 2016.
- [16] G. J. Husak, J. Michaelsen, and C. Funk, "Use of the gamma distribution to represent monthly rainfall in Africa for drought monitoring applications," *Int. J. Climatol.*, vol. 30, no. 2, pp. 349-367, Jun. 2015, doi: 10.1002/joc.1441.
- [17] S. A. Amburn, A. S. I. D. Lang, and M. A. Buonaiuto, "Precipitation forecasting with gamma distribution models for gridded precipitation events in Eastern Oklahoma and Northwestern Arkansas," *Weather Forecast.*, vol. 30, no. 2, pp. 349-367, 2014, doi: http://dx.doi.org/10.1175/WAF-D-14-00054.s1.
- [18] M. D. P. Esberto, "Probability Distribution Fitting of Rainfall Patterns in Philippine Regions for Effective Risk Management," *Environ. Ecol. Res.*, vol. 6, no. 3, pp. 178-186, 2018, doi: 10.13189/eer.2018.060305.
- [19] D. E. Puteri, "Geographically Weighted Gamma Regression Modeling (Case Study: River Pollution in the City of Surabaya in 2013) (Pemodelan Geographically Weighted Gamma Regression (Studi Kasus: Pencemaran Sungai di Kota Surabaya Tahun 2013)," M.S. thesis, Dept. Statistics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia, 2016, pp. 19-28.
- [20] A. C. Cameron and P. K. Trivedi, *Regression Analysis of Count Data*. New York: Cambridge University Press, 1998, pp. 19-21.
- [21] D. W. Hosmer and S. Lemeshow, "Applied Logistic Regression," *Wiley series in probability and statistics. Texts and references section*, 2000, pp. 36-42.
- [22] A. S. Pradana, "Analysis of the Effect of Climate Factors on Covid-19 Recovery Rates in East Java Province Using Geographically and Temporally Weighted Gamma Regression (Analisis Pengaruh Faktor Iklim Terhadap Tingkat Kesembuhan Covid-19 Di Provinsi Jawa Timur Menggunakan Metode Geographically and Temporally Weighted Gamma Regression)," Final Project, Dept. Statistics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia, 2021, pp. 13-25.
- [23] B. Huang, B. Wu, and M. Barry, "Geographically and temporally weighted regression for modeling spatio-temporal variation in house prices," *Int. J. Geogr. Inf. Sci.*, vol. 24, no. 3, pp. 383-401, Mar. 2010, doi: 10.1080/13658810802672469.
- [24] J. Friedman, T. Hastie, R. Tibshirani, *The Elements of Statistical Learning, Data Mining, Inference, and Prediction*, Second Edition,

- Springer, 2008, pp. 21-28.
- [25] R. Tibshirani, "Regression Shrinkage and Selection via the Lasso," *J. R. Stat. Soc. Ser. B*, vol. 58, no. 1, pp. 267-288, 1996, [Online]. Available: [jstor.org/stable/2346178](https://www.jstor.org/stable/2346178)
  - [26] H. Y. Saha S, Liu Q, Meng J, Liu H, Chelliah M, Ek M, Higgins W, Xue Y, Yang R, "NCEP Climate Forecast System Reanalysis (CFSR) monthly products, January 1979 to December 2010. (Research Data Archive at the 16 National Center for Atmospheric Research, Computational and Information Systems Laboratory)," 2010. [Online]. Available: <https://rda.ucar.edu/datasets/ds093.2/>
  - [27] N. Fadhli, A. H. Wigena, and A. Djuraidah, "Determination of General Circulation Model Grid Resolution to Improve Accuracy of Rainfall Prediction in West Java," *Int. J. Sci. Res. Publ.*, vol. 9, no. 7, p. p91106, Jul. 2019, doi: 10.29322/IJSRP.9.07.2019.p91106.
  - [28] A. Kardiana, "Statistical Downscaling Modeling in Rainfall Estimation Using Penalized Temporal Geographically Weighted Gamma Regression (Pemodelan Statistical Downscaling pada pendugaan Curah Hujan Buanan Menggunakan Regresi Gamma Terboboti Geografis Temporal Terpenalti)," Ph.D. dissertation, Dept. Statistics, IPB Univ., Bogor, Indonesia, 2024, pp. 51-58.
  - [29] J. Zhao, C. Zhu, and K. Zhao, "Spatial and Temporal Characteristics of the Impact of TOD Built Environment on Rail Transit Riders' Travels," *IAENG International Journal of Applied Mathematics*, vol. 55, no. 1, pp. 65-73, 2025.
  - [30] N. A. Alya, Q. Almaulidiah, B. R. Farouk, D. Rantini, A. Ramadan, and F. Othman, "Comparison of Geographically Weighted Regression (GWR) and Mixed Geographically Weighted Regression (MGWR) Models on the Poverty Levels in Central Java in 2023," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 12, pp. 2746-2757, 2024.
  - [31] M. Sholihin, A. M. Soleh, and A. Djuraidah, "Geographically and Temporally Weighted Regression (GTWR) for Modeling Economic Growth using R," *IJCSN-International J. Comput. Sci. Netw.*, vol. 6, no. 6, pp. 800-805, 2017.
  - [32] S. Haryanto, M. N. Aidi, and A. Djuraidah, "Modelling of GRDP the Construction Sector in Java Island Using Robust Geographically and Temporally Weighted Regression (RGTR)," *Int. J. Sci. Res. Sci. Eng. Technol.*, pp. 165-174, 2019, doi: 10.32628/ijsrset196141.
  - [33] A. Asianingrum, A. H., Djuraidah, "Robust Mixed Geographically and Temporally Weighted Regression to Modeling the Percentage of Poverty Population in Java in 2012-2018," *Int. J. Sci. Basic Appl. Res. Int. J. Sci. Basic Appl. Res.*, vol. 53, no. 2, pp. 186-197, 2020, [Online]. <http://gssrr.org/index.php?journal=JournalOfBasicAndApplied>.

**Aan Kardiana** is a member of the Computational Intelligent Research Group of the Faculty of Information Technology, YARSI University, and a member of YARSI E-Health Research Center (YEHRC). His research interests are statistics, data mining, and e-Health. He has won research grants and published several papers in national proceedings and international journals. His educational background are as follows: (1) Ph.D. degree from IPB University, Bogor, Indonesia, in September 2017-November 2024; (2) M.S. degree from Bogor Agricultural University, Bogor, Indonesia, in September 1996-December 1999; (3) B.S. degree from Bandung Institute of Technology, Bandung, Indonesia, in September 1984-October 1991.

**Aji Hamim Wigena** is a Professor in the Statistics Department of IPB University, Bogor, Indonesia. He holds a Ph.D. from the Statistics Department of IPB University, M.S. degree from Asian Institute of Technology, Thailand, and B.S. degree from the Statistics Department of Bogor Agricultural University, Bogor, Indonesia. His research interests are Statistical Downscaling, Statistical Modelling, Theory and Applied Statistics. He teaches Non-parametric statistics, Computational Statistics, and Statistical Modelling.

**Anik Djuraidah** is a Professor in the Statistics Department of IPB University, where she received all of her degrees. Her research interests are Statistical Downscaling, Statistical Modelling, Theory, and Applied Statistics. She has taught Statistics and Data Analysis, Statistical Theory, Statistical Method, Spatial Regression, and Spatial Analysis courses since 2007.

**Agus Mohamad Soleh** is a lecturer in the Statistics Department of IPB University. He holds Ph.D. from the Statistics Department of IPB University, Bogor, Indonesia, M.S. degree from Bandung Institute of Technology, Bandung, Indonesia, and B.S. degree from the Statistics Department of Bogor Agricultural University, Bogor, Indonesia. He is an Associate Professor in the Statistics Department of IPB University. His research interests are Machine Learning, Computational Statistics, and Statistical Modelling. He teaches Statistics and Data Analysis, Statistical Programming, and Computational Statistics.