Premium Calculation of Endowment Life Insurance in the Indonesian Market Whose Benefit Follows a Ratchet-Type Equity-Indexed Annuity Under Constant and Stochastic Interest Rates

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***Abstract*—In a recent study, we addressed the problem of pricing a compound Ratchet-type equity-indexed annuity (EIA) in the Indonesian market under three interest-rate assumptions: constant, stochastic based on the Vasicek model, and stochastic based on the Cox-Ingersoll-Ross (CIR) model, where the underlying asset was modelled using a geometric Brownian motion. In the present paper, we address the calculation of the net single premium of endowment life insurance whose benefit is specified to be the effective return of the above compound Ratchet-type EIA. We assume that the mortality rate obeys the Gompertz model, estimating the parameter values using the Indonesian Mortality Table IV. Our numerical simulation reveals that the Vasicek and CIR models lead to nearly identical premiums, while the classic constant interest-rate assumption —where the interest rate is set to be the long-term mean used in the two stochastic cases— leads to an overpricing of the premium. Additionally, in the two stochastic interest-rate cases, a sensitivity analysis demonstrates that the calculated premium depends most sensitively on the interest rate's long-term mean.**

***Index Terms*—Ratchet, equity-indexed annuity, Vasicek model, Cox-Ingersoll-Ross model, geometric Brownian motion, premium, endowment life insurance, Gompertz model**

I. Introduction

In a recent study [29], we considered the pricing of a compound Ratchet-type equity-indexed annuity (EIA) in the Indonesian market under three interest-rate assumptions: constant, stochastic based on the Vasicek model, and stochastic based on the Cox-Ingersoll-Ross (CIR) model. We calculated the EIA prices under the assumption of geometric Brownian motion for the underlying asset, with the models' parameters estimated from historical Indonesian bond and stock market data. In the present paper, we aim to continue the above study by integrating the return of the aforementioned Ratchet-type EIA into the structure of life insurance policies. More specifically, we shall consider the pricing of the net single premium of endowment life insurance whose benefit is the effective return of the above Ratchet-type EIA.

The motivation for such integration arises from broader developments in life insurance product design over the past few decades. As insurance markets continue to evolve globally, actuaries and financial engineers have increasingly sought to refine conventional life products by embedding features that mirror the behaviour of financial markets. These refinements include the inclusion of equity participation, interest rate guarantees, and dynamic surplus distribution strategies [15], [23]. Capturing these complexities requires robust modelling frameworks that combine actuarial risk structures with financial stochastic processes [2], [27]. To provide context for the hybrid structure proposed in this paper, we begin by reviewing relevant developments in product design and pricing methodology, particularly in the intersection of actuarial science and quantitative finance.

Endowment insurance products continue to play an important role in life insurance portfolios due to their dual function of providing long-term savings and life protection benefits [1]. However, many traditional pricing methods for such contracts rely on overly simplified assumptions, most notably constant interest rates, which fail to capture the variability and uncertainty inherent in financial markets. Given the long-term horizon and embedded guarantees of these products, accurately reflecting interest rate dynamics is essential. Interest rates are known to fluctuate and revert to long-term means over time, especially under economic uncertainty. To address this, numerous researchers —including Bühlmann [8], Miltersen and Persson [27], as well as Bacinello and Persson [2]— have advocated the use of stochastic interest rate models that incorporate market behaviour more realistically. Models such as Vasicek [31] and Cox-Ingersoll-Ross (CIR) [11] are particularly well-suited for capturing such dynamics, as they allow interest rates to evolve over time with mean-reverting properties. These frameworks have been shown to enhance the robustness and accuracy of life insurance pricing, particularly in the presence of embedded guarantees and investment-linked features [32].

Meanwhile, financial innovation has introduced hybrid products such as unit-linked and equity-indexed annuities (EIAs), which combine insurance coverage with market-based returns. While unit-linked products transfer investment risk directly to the policyholder, EIAs offer a more conservative alternative by ensuring capital protection through minimum guarantees and participation rates. Among various EIA structures, the Ratchet-type EIA has gained notable attention for its ability to lock in annual gains and shield against downturns [15]. In our afore-cited study [29], we modelled Ratchet-type EIAs using geometric Brownian motion and considered both constant and stochastic interest rate settings, aligning with the frameworks developed in earlier studies [6],

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[18], [22], [25], [24]. Through a sensitivity analysis and data calibration to Indonesian markets [30], we highlighted the importance of model choice in EIA pricing, a theme that this paper now expands into the realm of life insurance product design.

We organise our work as follows. In the upcoming section II, we first review the models involved in the calculation of our endowment insurance's premium, which include the three different interest-rate models (constant, Vasicek, and CIR), the geometric Brownian motion model, the Gompertz model, and a model for our EIA itself. In the subsequent section III, we discuss a formula to calculate the premium of our insurance, along with an estimate provided by the trapezoidal method. In section IV, we present the results of our numerical simulation, in which we calculate the premium of our insurance using the three interest-rate models after estimating the models' parameters using datasets provided by the Jakarta Stock Exchange Composite index, the Indonesian bond yields, and the Indonesian Mortality Table IV. In section V, we complement the results with a sensitivity analysis of the calculated premiums with respect to the models' parameters in the two stochastic interest-rate cases. In the final section VI, we present our conclusions and suggest a number of ways to extend our study.

II. The models

The calculation of an insurance's premium is built upon three key quantities: the interest rate, the asset value, and the mortality rate. In this study, the asset price is associated with the return of an EIA, and is represented as a stochastic process: a geometric Brownian motion. Our study is conducted under two different interest rate conditions: constant and stochastic. Although a constant interest rate assumption is frequently deemed appropriate for short-term financial products, such as those having maturities under one year, using it for longer-term options like EIAs, which generally have maturities between one and ten years, could lead to considerable pricing errors. Accordingly, we consider three interest-rate models: a constant rate, a stochastic rate governed by the Vasicek model, and an additional stochastic rate governed by the Cox-Ingersoll-Ross (CIR) model. On the other hand, the mortality rate is assumed not to be stochastic, but to be characterised by the Gompertz law, which describes an exponential rise in mortality risks as age advances. This formulation allows for the calculation of survival probabilities while ensuring analytical manageability within the premium calculation. Table I presents a summary of the parameters utilised in the Vasicek model, CIR model, geometric Brownian motion, Gompertz law, and the EIA framework involved in our study. To maintain alignment with Indonesian market conditions, the models' parameters are estimated utilising historical data from Indonesian government bond yields and the Jakarta Stock Exchange Composite index, as outlined in the numerical simulation section (Section IV).

TABLE I
Summary of parameters.

Parameters	Description	Unit
κ	the interest rate's mean-reversion rate: the speed at which the interest rate reverts to its long-term mean	time^{-1}
θ	the interest rate's long-term mean: the long-term average level to which the interest rate reverts	$\% \cdot \text{time}^{-1}$
σ	the interest rate's volatility: the standard deviation of the interest rate changes	$\% \cdot \sqrt{\text{time}}$
η	the asset price's drift rate: the expected rate of return of the asset	time^{-1}
ψ	the asset price's volatility: the standard deviation of the asset's returns	$\% \cdot \sqrt{\text{time}}$
n	the insurance's maturity time	time
x	the policyholder's entry age	time
B	the baseline mortality rate	time^{-1}
C	the exponential growth factor of the mortality rate	time^{-1}
f	the guaranteed minimum interest rate	%
c	the guaranteed maximum interest rate	%
β	the participation rate	%
I	the initial invested amount	currency

A. The interest rate models

Let us now review our three different interest-rate models: the constant, Vasicek, and Cox-Ingersoll-Ross (CIR) models involved in our present study, which were also involved in our previous study [29].

1) The constant interest rate model: In the constant interest rate model, the short rate $r(t)$ is treated as fixed throughout the considered period. Although this assumption may lack realism, it provides a useful benchmark for comparison with stochastic models, where

$$P(t,T) = \mathrm{e}^{-r(T-t)}; \tag{1}$$

see [7, equation (1.7)]

2) The Vasicek model: As noted in our previous work [29], while the value of the interest rate $r(t)$ in the Vasicek model can be negative, albeit with a very small probability, the Vasicek model allows for mean-reversion in interest rates, making the model widely employed. The model is given by the stochastic differential equation

$$\mathrm{d}r(t) = \kappa\left[\theta - r(t)\right]\mathrm{d}t + \sigma\,\mathrm{d}W_r(t),$$

which leads to

$$r(t) = \theta + \mathrm{e}^{-\kappa t}\left(r(0) - \theta + \int_0^t \sigma \mathrm{e}^{\kappa u}\,\mathrm{d}W(u)\right),$$

where κ, θ, and σ are positive parameters, and $\{W_r(t)\}$ is the standard Brownian motion correlated with $\{W_s(t)\}$ with correlation coefficient ρ. It can be shown [28] that the zero-coupon bond prize at time t for maturity time T is given by

$$P(t,T) = A(t,T)\,\mathrm{e}^{-r(t)\,B(t,T)}, \tag{2}$$

where

$$A(t,T) = \exp\left(\left(B(t,T) - (T-t)\right)\left(\theta - \frac{\sigma^2}{2\kappa^2}\right) - \frac{\sigma^2}{4\kappa}\left(B(t,T)\right)^2\right),$$

$$B(t,T) = \frac{1-\mathrm{e}^{-\kappa(T-t)}}{\kappa}.$$

3) The CIR model: The CIR model is an enhancement of the Vasicek model that modifies the volatility term to be proportional to the square root of the short rate, thereby preventing negative rates. The model is represented by the stochastic differential equation

$$\mathrm{d}r(t) = \kappa\,[\theta - r(t)]\,\mathrm{d}t + \sigma\sqrt{r(t)}\,\mathrm{d}W_r(t),$$

which leads to

$$r(t) = \theta + \mathrm{e}^{-\kappa t}\left(r(0) - \theta + \int_0^t \sigma \mathrm{e}^{\kappa u}\sqrt{r(u)}\,\mathrm{d}W(u) \right),$$

where κ, θ, and σ are positive parameters, and $\{W_r(t)\}$ is the standard Brownian motion correlated with $\{W_s(t)\}$ with correlation coefficient ρ. It can be shown [11] that the zero-coupon bond prize at time t for maturity time T is given by

$$P(t,T) = A(t,T)\mathrm{e}^{-r(t)\,B(t,T)}, \tag{3}$$

where

$$\begin{aligned} A(t,T) &= \left(\frac{2\gamma \mathrm{e}^{(\gamma+\kappa)(T-t)/2}}{C(T,t)}\right)^{2\kappa\theta/\sigma^2}, \\ B(t,T) &= 2\frac{\mathrm{e}^{\gamma(T-t)}-1}{C(t,T)}, \\ C(t,T) &= 2\gamma + (\kappa+\gamma)\left(\mathrm{e}^{\gamma(T-t)}-1\right), \end{aligned}$$

with $\gamma = \sqrt{\kappa^2 + 2\sigma^2}$.

B. The geometric Brownian motion

As in our previous work [29], we assume that the equity index level S_t used to determine the asset price benefits of the policy follows a geometric Brownian motion. The motion is defined by the stochastic differential equation

$$\mathrm{d}S_t = \eta S_t\,\mathrm{d}t + \psi S_t\,\mathrm{d}W(t),$$

where η and ψ are positive parameters [26]. This equation captures both the growth trend and random fluctuations in the equity market. The geometric Brownian motion is widely used in financial modelling due to its tractability and consistency with the log-normal distribution of rate return.

C. The Gompertz Model

Hereafter, let us assume that our time is measured in years. As noted by Dickson et al. [12], several continuous models are commonly used to describe mortality rates, including the de Moivre model, the Gompertz model, and the Makeham model. In this paper, we use the Gompertz model to represent the mortality rate of an Indonesian male at every age.

The mortality rate of a person aged $x > 0$ years according to the Gompertz model is given by

$$\mu_x = BC^x, \tag{4}$$

where $0 < B < 1$ and $C > 1$. The associated survival function is given by

$$\begin{aligned} S_x(t) &= \exp\left(-\int_0^t \mu_{x+s}\,\mathrm{d}s\right) \\ &= \exp\left(-\frac{BC^x}{\ln C}\left(C^t - 1\right)\right). \end{aligned} \tag{5}$$

D. The EIA

Equity-indexed annuities (EIAs) offer investment-linked returns with built-in protections, including a guaranteed minimum interest rate to prevent losses during market downturns and a maximum cap to limit insurer liability during strong market growth. They also feature a participation ratio that determines the portion of index returns credited to the policyholder. In Ratchet-type EIAs [22], [3], [14], interest is calculated annually and compounded over the contract term, allowing funds to grow cumulatively. Ratchet-type EIAs can be classified as simple or compound, the latter carrying forward prior gains to enhance future compounding. The present study focuses on a compound Ratchet-type EIA.

As noted in our previous work [29], the expected value of the effective annual return of the EIA contract at the end of year t is given by

$$\begin{aligned} E\left(\tilde{P}_t\right) &= (1+f)\,N(z_1) + (1-\alpha)\,[N(z_2) - N(z_1)] \\ &\quad + \beta \mathrm{e}^{\eta}\,[N(z_3) - N(z_4)] \\ &\quad + (1+c)\,(1 - N(z_2)), \end{aligned} \tag{6}$$

where

$$\begin{aligned} z_1 &= \frac{\ln(1+f/\beta) - \left(\eta - \psi^2/2\right)}{\psi}, \\ z_2 &= \frac{\ln(1+c/\beta) - \left(\eta - \psi^2/2\right)}{\psi}, \\ z_3 &= \frac{\ln(1+c/\beta) - \left(\eta + \psi^2/2\right)}{\psi}, \\ z_4 &= \frac{\ln(1+f/\beta) - \left(\eta + \psi^2/2\right)}{\psi}, \end{aligned}$$

and N is the cumulative distribution function of the standard normal distribution.

Assuming an initial investment of I, the investment value of the EIA at the contract's maturity time T is given by

$$P_{\text{cr}} = I\prod_{t=1}^{T}\tilde{P}_t,$$

as noted by Hsieh and Chiu [16]. Consequently, the value of the compound Ratchet-type EIA at the maturity time T is given by

$$V(T) = E(P_{\text{cr}}) = I\cdot E\left(\prod_{t=1}^{T}\tilde{P}_t\right) = I\cdot\left(E\left(\tilde{P}_t\right)\right)^T, \tag{7}$$

where $E\left(\tilde{P}_t\right)$ is given by equation (6).

III. The premium calculation

A life insurance is an agreement in which a policyholder pays premiums in return for a payout from the insurer upon the death of the insured. Life insurances are generally classified into four categories: whole life, term life, pure endowment, and endowment insurances. As detailed in Dickson et al. [12], a life insurance's benefit payments may be formulated in either a discrete or a continuous framework. In the continuous framework, benefits are disbursed immediately after death, whereas in the discrete framework, they are distributed at the conclusion of the policy year in which death occurs. Dickson et al. [12] also noted that premiums can be established through the equivalence principle or the percentile premium principle while maintaining a constant interest rate. The former presumes that the insurer expects neither profit nor loss when the contract begins, whereas the latter limits the potential losses for the insurer. The present study applies the equivalence principle in the setting of endowment life insurance that includes ongoing benefit distributions.

Endowment life insurance involves two types of benefits: a death benefit and a survival benefit. The death benefit is payable only if the insured, aged x years, passes away within n years, whereas the survival benefit is payable only if the insured lives beyond n years. As formulated by Dickson et al. [12], the present value of a benefit of 1 of an n-year continuous endowment life insurance is given by the random variable

$$Z = \begin{cases} v^{T_x} = \mathrm{e}^{-\delta T_x}, & \text{if } T_x \leqslant n; \\ v^n = \mathrm{e}^{-\delta n}, & \text{if } T_x > n, \end{cases}$$

where δ is the so-called force of interest. Consequently, the expected present value of the benefit of 1 of the same insurance is given by

$$\bar{A}_{x:\overline{n}|} = E(Z) = \int_0^n \mathrm{e}^{-\delta t}\, S_x(t)\, \mu_{x+t}\, \mathrm{d}t + \mathrm{e}^{-\delta n}\, S_x(n).$$

Since we consider the EIA's effective return to be the benefit of the endowment life insurance and incorporate both constant and stochastic interest rates, the expected present value of the above benefit in our setting is given by

$$\mathcal{P} = \int_0^n P(0,t)\, V(t)\, S_x(t)\, \mu_{x+t}\, \mathrm{d}t + P(0,n)\, V(n)\, S_x(n), \tag{8}$$

which also represents the net single premium that the policyholder is required to pay, according to the equivalence principle.

Since the definite integral in equation (8) is not computable in an exact manner, we shall estimate its value using the trapezoidal method [9]. More specifically, we divide the interval of integration $[0, n]$ into m subintervals of equal length $\Delta t = n/m$ using the points $t_i = i\Delta t$ for every $i \in \{0, \ldots, m\}$, so that the premium in equation (8) can be estimated as

$$\mathcal{P} \approx \frac{n}{2m}\Bigg[P(0,0)V(0)S_x(0)\mu_x + 2\sum_{i=1}^{n-1} P(0,t_i)V(t_i)S_x(t_i)\mu_{x+t_i} + P(0,n)V(n)S_x(n)\mu_{x+n}\Bigg] + P(0,n)V(n)S_x(n).$$

IV. Numerical simulation

Let us now describe our numerical simulation conducted to apply the previously discussed models to the premium calculation of endowment life insurance policy. Once again, we carry out the calculation in three distinct interest-rate cases: constant, stochastic according to the Vasicek model, and stochastic according to the CIR model. Our numerical simulation is a continuation of that which was conducted in our previous work [29, section V]. We consider a 35-year-old male individual who enrolls in a continuous 10-year endowment life insurance policy whose benefit is specified to be the effective return of a compound Ratchet-type EIA. Accordingly, $x = 35$ and $n = 10$. As in our previous work [29, section V], we suppose that the individual provides an initial capital investment of $I = 100$, that the interest rates feature a minimum of $f = 6\%$ and a maximum of $c = 11\%$, and that the participation rate is $\beta = 90\%$.

TABLE II
THE ESTIMATED VALUES OF PARAMETERS.

	$\hat{\kappa}$	$\hat{\theta}$	$\hat{\sigma}$
Vasicek	0.9261	0.0711	0.0107
CIR	0.9253	0.0711	0.0396

	$\hat{\psi}$	$\hat{\eta}$
EIA	0.1478	0.0529

	$\hat{B}$	$\hat{C}$
Gompertz	$9.7045\cdot 10^{-5}$	1.0824

Applying the formulae provided in our previous work [29, subsection IV-C] to the 10-year daily dataset of the Jakarta Stock Exchange Composite index from March 13th, 2014 to March 13th, 2024 [20], we obtain the estimates $\hat{\psi} \approx 0.1478$ and $\hat{\eta} \approx 0.0529$ for the parameters ψ and η involved in our EIA. Similarly, applying the formulae provided in our previous work [29, subsections IV-A and IV-B] the 10-year daily dataset of the Indonesian bond yields from March 13th, 2014 to March 13th, 2024 [21], we obtain the estimates $\hat{\kappa} \approx 0.9261$, $\hat{\theta} \approx 0.0711$, and $\hat{\sigma} \approx 0.0107$ for the parameters κ, θ, and σ involved in our Vasicek model, and the estimates $\hat{\kappa} \approx 0.9253$, $\hat{\theta} \approx 0.0711$, and $\hat{\sigma} \approx 0.0396$ for the parameters κ, θ, and σ involved in our CIR model. Finally, the estimation of the parameters B and C involved in our Gompertz model is carried out using the Indonesian Mortality Table IV. The table provides the probabilities q_x of a person currently aged x years dying within one year, for various values of x. Using these values, we perform a linear regression to estimate the values of B and C from the linearisable equation

$$q_x = 1 - S_x(1) = 1 - \exp\left(-\frac{BC^x}{\ln C}(C-1)\right).$$

The regression is performed on RStudio, using the built-in function `lm`. The results are the estimates $\hat{B} \approx 9.7045\cdot 10^{-5}$ and $\hat{C} \approx 1.0824$. The above estimates of parameter values are summarised in Table II. In the constant interest-rate case, the interest rate used is the mean of the values of $\hat{\theta}$ obtained in the Vasicek and CIR cases.

Using the parameter values summarised in Table II and applying equations (2), (4), (5), and (7), we calculate the insurance's premium in the aforementioned three distinct interest-rate cases, where the trapezoidal method is implemented using $m = 1000$ subintervals. The results are shown in Table III. We observe that the highest premium is obtained in the constant interest-rate case. This is not surprising, since the removal of downward interest rate fluctuations leads to more stable projected cash flows, thereby increasing the premium. On the other hand, the premium calculated

TABLE III
PREMIUM UNDER THREE DIFFERENT INTEREST-RATE ASSUMPTIONS.

Interest-rate assumption	Premium
Constant	104.0853
Vasicek	100.6706
CIR	100.6654

TABLE IV
THE SENSITIVITY INDICES OF THE PREMIUM PRICE WITH RESPECT TO κ, θ, σ, η, ψ, x, AND n IN THE TWO STOCHASTIC INTEREST RATE CASES.

Sensitivity index	Interest rate assumption	
	Vasicek	CIR
$\Upsilon_{\kappa}^{\mathcal{P}}$	0.0090	0.0086
$\Upsilon_{\theta}^{\mathcal{P}}$	-0.6270	-0.5995
$\Upsilon_{\sigma}^{\mathcal{P}}$	0.0011	-0.0010
$\Upsilon_{\eta}^{\mathcal{P}}$	0.0000	0.0000
$\Upsilon_{\psi}^{\mathcal{P}}$	0.0000	0.0000
$\Upsilon_{x}^{\mathcal{P}}$	0.0000	0.0000
$\Upsilon_{n}^{\mathcal{P}}$	0.0317	0.0629

using the CIR model is found to be nearly identical to that calculated using the Vasicek model. This indicates that the additional volatility incorporated into the CIR model exerts little influence on the premium value under the prevailing market conditions, highlighting the dominant effect of the mean-reverting behaviour already captured by the Vasicek model.

V. SENSITIVITY ANALYSIS

Let us now analyse the sensitivity of the premium values obtained in section IV to the models' parameters in the two stochastic interest-rate cases. To quantify the sensitivity of the premium $\mathcal{P}$ with respect to a parameter p, we employ the sensitivity index of $\mathcal{P}$ with respect to p [10]:

$$\Upsilon_p^{\mathcal{P}} = \frac{\partial \mathcal{P}}{\partial p} \cdot \frac{p}{\mathcal{P}} \approx \frac{\Delta \mathcal{P}/\mathcal{P}}{\Delta p/p},$$

This index provides an estimate for the ratio of a relative change in $\mathcal{P}$ with respect to a relative change in the parameter p. In the case $\Upsilon_p^{\mathcal{P}} > 0$, a 1% increase in the parameter p results in an increase of $\Upsilon_p^{\mathcal{P}}\%$ in the premium $\mathcal{P}$. In the case $\Upsilon_p^{\mathcal{P}} < 0$, a 1% increase in p yields a decrease of $\Upsilon_p^{\mathcal{P}}\%$ in the premium $\mathcal{P}$.

In both Vasicek and CIR cases, the partial derivative $\partial\mathcal{P}/\partial p$ is can be computed analytically by using formula (7), allowing the sensitivity index $\Upsilon_p^{\mathcal{P}}$ to be evaluated in an exact manner for each parameter $p \in \{\kappa, \theta, \sigma, \eta, \psi, x, n\}$. The results, evaluated using the parameter values employed in our numerical simulations (Section IV), are summarised in Table IV.

As apparent from Table IV, the premium $\mathcal{P}$ is found to be most sensitive to the interest rate's long-term mean θ in both stochastic interest-rate models. Specifically, a 1% increase in θ results in reductions of 0.6270% and 0.5995% in the premium $\mathcal{P}$ in the Vasicek and CIR cases, respectively. By contrast, limited sensitivity of the premium $\mathcal{P}$ is observed with respect to the mean-reversion rate κ and the interest rate's volatility σ, while no sensitivity of the premium $\mathcal{P}$ is detected with respect to the asset price's drift η and volatility ψ. Such insensitivity, which implies that changes in these parameters exert minimal or no influence on the premium, is likely due to the structural characteristics of the Ratchet design, which provides downside protection through guaranteed minimum returns.

VI. CONCLUSIONS AND FUTURE RESEARCH

We have discussed the premium pricing of endowment life insurance whose benefit is specified to be the effective return of a compound Ratchet-type equity indexed annuity (EIA). We modelled the asset price using geometric Brownian motion, the mortality rate using the Gompertz law, and the interest rate using two well-known stochastic models: the Vasicek and Cox-Ingersoll-Ross (CIR) models. After estimating the models' parameters using historical data of Indonesian government bond yields, the Jakarta Stock Exchange Composite index, and the Indonesian Mortality Table IV, we discovered through numerical simulations that the premium values obtained using the two interest-rate models are almost identical, indicating that the added complexity of the CIR model did not yield significant influence under the given market conditions. By contrast, the classic assumption of a constant interest rate —the long-term mean used in our two stochastic models— led to a mispricing —in our case an overpricing— which is unsurprising since such an assumption removes possible declines in interest rates, thereby stabilising the anticipated cash flows. Complementarily, a sensitivity analysis revealed that the premium was most affected by changes in the long-term mean of the interest rate, with a 1% increase in the long-term mean leading to an approximately 0.6% decrease in the premium. By contrast, the other parameters exert minimal or negligible influence on the premium value.

As we pointed out in our previous work [29], natural ways to extend our study include the use of more sophisticated models for the time-evolution of both the interest rate and the asset price. The Hull-White model [19], the Dothan model [13], or the Black-Derman-Toy model [4] constitute examples for the former, while regime-switching models [5] and Lévy processes [17] constitute examples for the latter. In addition, our work can be extended by replacing the modest Gompertz model for the mortality rate with alternative models such as the Weibull model, the inverse-Weibull model, the Gompertz-Makeham model, or stochastic mortality-rate models.

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