Application of Stress Testing Method with Risk Measurement Using Cornish-Fisher Expansion on PEFINDO i-Grade Portfolio

Kharisma Sofiana, Abdul Hoyyi*, and Ardiana Alifatus Sa'adah

Abstract— **Stock investment is widely practiced, and diversification helps manage risk and improve returns. This analysis was conducted to apply risk analysis methods by considering extreme market conditions. Ward Clustering is chosen for its ability to produce homogeneous clusters by minimizing squared errors. The Mean-Semivariance model proposed by Markowitz focuses on downside risk, which better reflects investors' perception of risk, where portfolio weight optimization is performed using Lagrange Multipliers with the aim of minimizing risk. Risk estimation is carried out using the Cornish-Fisher VaR approach, which incorporates skewness and excess kurtosis, and is enhanced with Stress Testing to simulate potential losses during abnormal market conditions. The analysis is applied to the PEFINDO i-Grade index, which contains 30 stocks from companies with investment-grade ratings with daily closing stock prices for the period January 1, 2023, to December 31, 2024, as the basis for calculation. In clustering, the variables used are Price to Earnings Ratio (PER), Price to Book Value (PBV), and Market Capitalization. The resulting portfolio consists of 3 stocks with a weighting of 74.217% for BMRI, 19.793% for MFIN, and 5.990% for BRPT. The resulting portfolio VaR is 1.503% of the initial capital invested within one day. In extreme scenarios, the level of risk generated by Stress Testing increases, indicating that the Stress Testing method can be used to identify potential risks in extreme market conditions.**

Index Terms— **Ward Clustering, Mean-Semivariance, VaR Cornish-Fisher Expansion, Stress Testing, PEFINDO i-Grade**

I. INTRODUCTION

STOCK investment is one of the most popular investment instruments. Investment refers to the allocation of capital by investors, across various business sectors with the aim of generating profits. In the investment process, constructing an optimal portfolio is essential to minimize risk and maximize returns. The concept of an efficient portfolio through diversification as a means of reducing risk was introduced in [1]. Therefore, stock selection and risk measurement are crucial aspects of portfolio analysis. This study adopts a quantitative approach by applying Ward Clustering for stock selection, Mean-Semivariance for portfolio weighting, and Value at Risk (VaR) using the Cornish-Fisher Expansion to measure portfolio risk. Additionally, the Stress Testing method is employed to assess portfolio risk under extreme market scenarios.

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Stock selection is conducted using Ward Clustering, a hierarchical method that groups stocks based on the similarity of their characteristics, aiming to form more homogeneous clusters. Portfolio weighting is carried out using the Mean-Semivariance approach, which was developed as an improvement over the Mean-Variance model. Markowitz recommended the use of downside risk measured by Mean-Semivariance, as it is considered more relevant for investors who tend to avoid losses [2]. To measure portfolio risk, the Cornish-Fisher Expansion of VaR is used to enhance risk estimation by accounting for skewness and excess kurtosis in return distributions [3]. Since VaR is designed for normal market conditions and may fail to capture extreme events, it is complemented with Stress Testing to evaluate how the portfolio would perform under significant changes in economic or market variables [4]. The analytical methods are applied to the PEFINDO i-Grade index, which comprises 30 stocks from investment-grade-rated companies, for the purpose of portfolio optimization and risk mitigation.

II. WARD CLUSTERING

Ward Clustering works by maximizing homogeneity within clusters while minimizing the variation among objects within the same cluster. Ward Clustering is a hierarchical method based on an agglomerative approach, meaning the grouping process is performed by combining cluster pairs that best match to form a hierarchical structure in the dataset [5].

Data standardization is the process of adjusting the scale of the data being analyzed. Significant differences in scale can lead to invalid calculations in cluster analysis. In cluster analysis, data standardization is performed when the variables used have different units of measurement.

$$Z_{ik} = \frac{x_{ik} - \bar{x}_k}{s_k} \tag{1}$$

where Z_{ik} is the standardized value of observation i on variable k, x_{ik} is the original value of observation i on variable k, $\bar{x}_k$ is the mean of variable k, and s_k is the standard deviation of variable k.

Two assumptions must be fulfilled in conducting cluster analysis:

1. Sample Representativeness Assumption

   The sample must represent the population (i.e., be representative) to ensure that the clustering process is valid and reliable.

2. Non-Multicollinearity Assumption

   Multicollinearity refers to the existence of linear relationships among independent variables. Variables used in cluster analysis should be free from multicollinearity issues. Multicollinearity can affect clustering results because it can make it difficult to determine the influence of each variable analyzed. The Variance Inflation Factor (VIF) is one method used to detect the presence of multicollinearity [6].

$$VIF_k = \frac{1}{1 - R_k^2} \quad (2)$$

where R_k^2 is the coefficient determination for variable *k*. If the VIF value exceeds 10, it indicates that multicollinearity exists in that variable.

In the Ward method, cluster pairs are selected based on the smallest increase in total within-cluster variance. The increase in variance for each merging step is calculated from the difference in the sum of squared Euclidean distances between data points and the cluster centroid, before and after the merging process [5].

$$SSE_A = \sum_{i=1}^{n_A} (\boldsymbol{x}_i - \bar{\boldsymbol{x}}_A)'(\boldsymbol{x}_i - \bar{\boldsymbol{x}}_A)$$
$$SSE_B = \sum_{i=1}^{n_B} (\boldsymbol{x}_i - \bar{\boldsymbol{x}}_B)'(\boldsymbol{x}_i - \bar{\boldsymbol{x}}_B) \quad (3)$$

$$SSE_{AB} = \sum_{i=1}^{n_{AB}} (\boldsymbol{x}_i - \bar{\boldsymbol{x}}_{AB})'(\boldsymbol{x}_i - \bar{\boldsymbol{x}}_{AB}) \quad (4)$$

where SSE_A is the SSE for object A, SSE_B is the SSE for object B, SSE_{AB} is the SSE for the combined object A and B after clustering, $\boldsymbol{x}_i$ represents the data observation vector, $\bar{\boldsymbol{x}}_A$ is the mean observation vector of object A, $\bar{\boldsymbol{x}}_B$ is the mean observation vector of object B, and $\bar{\boldsymbol{x}}_{AB}$ is the mean observation vector of the combined objects A and B. Objects A and B are grouped into the same cluster if $SSE_{AB} - (SSE_A + SSE_B)$ minimized.

The validation of clustering results is performed to assess the quality of the clusters using the silhouette score. The silhouette score evaluates the placement of each object by calculating the average proximity of objects to identify substantial clustering results [7].

$$SC = \frac{1}{n}\sum_{i=1}^{n} s(i) \quad (5)$$

$$s(i) = \frac{b(i) - a(i)}{\max(a(i), b(i))};$$
$$a(i) = \frac{1}{n(A)-1}\sum_{1\epsilon A, i\neq j} d(x_i, x_j)\text{; and} \quad (6)$$
$$b(i) = \min d(x_i, E)$$

where SC is the overall silhouette score, $s(i)$ is the silhouette score of the object *i*, d is the distance between objects, $a(i)$ is the average distance of object *i* to other objects in the same cluster, and $b(i)$ is the minimum average distance of object *i* to all objects in other clusters.

III. Stock Return

Stock return is one of the indicators used to assess a company's performance. Return has better statistical properties than asset prices for risk modeling, as it focuses more on the dynamics of return changes rather than price changes [8].

$$R_{t+1} = \ln(P_{t+1}) - \ln(P_t) \quad (7)$$

where R_{t+1} is the return for period $t + 1$, P_{t+1} and P_t represent the stock prices. Historical returns are used as the basis for determining expected returns and future risks. Expected return is the return that investors expect to earn in the future. If there are n (number of observations) returns, then the expected return is estimated using the sample average return.

IV. Normality Test

The assumption of data normality is fundamental in many statistical analyses, requiring that the data used in statistical modeling be distributed normally or approximately normally [9].

1. Univariate Normality Test

A formal univariate normality test can be conducted using the Kolmogorov-Smirnov test by comparing the empirical distribution function based on the sample data with the hypothesized cumulative distribution function [10].

H_0 : $F(x) = F^*(x)$ for x from $-\infty$ to $+\infty$ (data follows a normal distribution).

H_1 : $F(x) \neq F^*(x)$ for at least one x (data do not follow a normal distribution).

$$T = \sup_x |F^*(x) - S(x)| \quad (8)$$

where T is the supremum of $|F^*(x) - S(x)|$, $F^*(x)$ is the theoretical cumulative distribution function, and $S(x)$ is the empirical distribution function. Reject H_0 if $T > T_{table}$ (*two sided*) or $p - value < \alpha$.

2. Multivariate Normality Test

According to [11], the assessment of the multivariate normality assumption can be done visually by observing a Q-Q Plot between the squared Mahalanobis distance and the Chi-Square quantile, and formally by examining the correlation between them using the following steps:

a) The generalized distance is calculated using the squared Mahalanobis distance.

$$d_j^2 = (\boldsymbol{x}_j - \bar{\boldsymbol{x}})'\boldsymbol{S}^{-1}(\boldsymbol{x}_j - \bar{\boldsymbol{x}}), j = 1, 2, \dots, n \quad (9)$$

where d_j^2 is the squared Mahalanobis distance, $\boldsymbol{x}_j$ is the observation vector, $\bar{\boldsymbol{x}}$ is the mean vector of each variable, and $\boldsymbol{S}^{-1}$ is the inverse of the variance-covariance matrix.

b) The obtained distances are sorted from smallest to largest.

c) A plot is created of $\left(d_j^2, q_j = \chi_p^2\left(\frac{j-\frac{1}{2}}{n}\right)\right)$ where $q_j = \chi_p^2\left(\frac{j-\frac{1}{2}}{n}\right)$ is the percentile $100\left(j - \frac{1}{2}\right)/n$ for the Chi-Square distribution with p degrees of freedom.

d) If the plot forms a straight diagonal line, then the variables are considered to be multivariate normal distributed.

e) A formal test is conducted by calculating the correlation between d_j^2 and q_j.

H_0: the data follow a multivariate normal distribution

H_1: the data do not follow a multivariate normal distribution

$$r_Q = \frac{\sum_{j=1}^{n}(d_j^2 - \bar{d}^2)(q_j - \bar{q})}{\sqrt{\sum_{j=1}^{n}(d_j^2 - \bar{d}^2)^2}\sqrt{\sum_{j=1}^{n}(q_j - \bar{q})^2}}. \quad (10)$$

Reject the hypothesis that the data follow a multivariate normal distribution if $r_Q < r_{table}$, based on the critical values from the Q-Q Plot Correlation Coefficients Test for Normality.

V. Mean-Semivariance

Mean-Semivariance is a method that uses downside risk (DSR) in calculating risk [2]. DSR measures a more relevant risk because it focuses on the risk that is below the benchmark (B). The benchmark represents a reference point chosen by the investor. This method does not have any distributional assumptions, making it preferable [12]. The semivariance and semicovariance formulas from [13] are as follows:

$$\Sigma_{iB}^2 = \frac{1}{T}\sum\nolimits_{t=1}^{T}\left[Min\left(R_{i,t} - B, 0\right)\right]^2 \quad (11)$$

$$\Sigma_{ijB} = \frac{1}{T}\sum\nolimits_{t=1}^{T}\left[Min\left(R_{i,t} - B, 0\right) Min\left(R_{j,t} - B, 0\right)\right] \quad (12)$$

where Σ_{iB}^2 is the semivariance of asset i, Σ_{ijB} is the semicovariance between asset i and asset j, $R_{i,t}$ is the return of asset i, $R_{j,t}$ is the return of asset j, and B is the benchmark.

Portfolio weighting with weights $\boldsymbol{w} = [w_1 \quad w_2 \quad \cdots \quad w_N]^T$ aims to minimize risk based on the semivariance of the constructed portfolio. Optimization is conducted using the Lagrange function with two multipliers, λ and β.

$$L = \boldsymbol{w^T}\boldsymbol{\Sigma}_{\boldsymbol{msv}}\boldsymbol{w} + \lambda\left(\mu_p - \boldsymbol{w^T}\boldsymbol{\mu}\right) + \beta\left(1 - \boldsymbol{w^T}\boldsymbol{1}_N\right). \quad (13)$$

The optimal value of w is determined by finding the partial derivative of L with respect to $\boldsymbol{w}$.

$$\frac{\partial L}{\partial \boldsymbol{w}} = 0$$

$$2\boldsymbol{\Sigma}_{\boldsymbol{msv}}\boldsymbol{w} - \lambda\boldsymbol{\mu} - \beta\boldsymbol{1}_N = 0$$

$$\boldsymbol{w} = \frac{1}{2}\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\left(\lambda\boldsymbol{\mu} + \beta\boldsymbol{1}_N\right),$$

multiply both side by $\boldsymbol{1}_N^T$:

$$\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{w} = \frac{1}{2}\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\left(\lambda\boldsymbol{\mu} + \beta\boldsymbol{1}_N\right)$$

$$1 = \frac{1}{2}\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\left(\lambda\boldsymbol{\mu} + \beta\boldsymbol{1}_N\right)$$

$$\beta = \frac{2 - \boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\lambda\boldsymbol{\mu}}{\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_{\boldsymbol{N}}}$$

then subtitute β:

$$\boldsymbol{w} = \frac{1}{2}\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\left(\lambda\boldsymbol{\mu} + \left(\frac{2 - \boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\lambda\boldsymbol{\mu}}{\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N}\right)\boldsymbol{1}_N\right)$$

$$\boldsymbol{w} = \frac{1}{2}\lambda\left(\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{\mu} - \frac{\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N^{\boldsymbol{T}}\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{\mu}\boldsymbol{1}_N}{\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N}\right) + \frac{\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N}{\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N}. \quad (14)$$

In the case of an efficient portfolio with minimum semivarian, there is no restriction on the portfolio mean, so $\lambda = 0$. Thus, the optimal portfolio weighting using the Mean-Semivariance is defined as follows:

$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N}{\boldsymbol{1}_N^{\boldsymbol{T}}\, \boldsymbol{\Sigma}_{\boldsymbol{msv}}^{-1}\boldsymbol{1}_N} \quad (15)$$

where $\boldsymbol{\Sigma}_{\boldsymbol{msv}}$ is the semivariance-semicovariance matrix. Based on the derived weights, the portfolio return can then be constructed as:

$$R_{pt} = \sum_{i=1}^{N} w_i R_{t,i} \text{ where } \sum_{i=1}^{N} w_i = 1 \quad (16)$$

where $R_{p,t}$ is the portfolio return at period t, $R_{t,i}$ is the return of asset i at period t, and w_i is the weight of asset i. Expected return and semivariance of the portfolio are given by:

$$\mu_p = \boldsymbol{w^T}\boldsymbol{\mu} \quad (17)$$

$$\sigma_p^2 = \boldsymbol{w^T}\boldsymbol{\Sigma}_{\mathbf{msv}}\boldsymbol{w} \quad (18)$$

where μ_p is the expected return of the portfolio, σ_p^2 is the semivariance of the portfolio return, $\boldsymbol{w}$ is the portfolio weight vector, $\boldsymbol{\mu}$ is the expected return vector of the constituent assets, and $\boldsymbol{\Sigma}_{\mathbf{msv}}$ is the semivariance-semicovariance matrix.

VI. CORNISH-FISHER EXPANSION VALUE AT RISK

VaR (Value at Risk) is a tool for risk management that tells us the worst expected loss of portfolio with a certain confidence level and for a given period of time [14]. The Cornish-Fisher expansion in the context of VaR is a semi-parametric approach used to estimate quantiles of a non-normal distribution by incorporating standard normal quantiles, skewness, and excess kurtosis of the sample [15]. This method provides a simple relationship between skewness and excess kurtosis with VaR, thereby facilitating portfolio risk measurement. This method is designed to address non-normality of variables by incorporating skewness and excess kurtosis [3].

$$z_{cf} = q_{(1-\alpha)} + \frac{\left(q_{(1-\alpha)}^2 - 1\right)\gamma_1}{6} + \frac{\left(q_{(1-\alpha)}^3 - 3q_{(1-\alpha)}\right)\gamma_2}{24} - \frac{\left(2q_{(1-\alpha)}^3 - 5q_{(1-\alpha)}\right)\gamma_1^2}{36} \quad (19)$$

where z_{cf} is Cornish-Fisher quantile, $q_{(1-\alpha)}$ is standard normal quantile, γ_1 is skewness, and γ_2 is excess kurtosis. The VaR formula using the Cornish-Fisher is given by:

$$VaR_\alpha = V_0 \times \left(\mu_p + \sigma_p\, z_{cf}\right) \times \sqrt{hp} \quad (20)$$

where V_0 is the initial investment, μ_p is the expected portfolio return, σ_p is the volatility of portfolio return, z_{cf} is the Cornish-Fisher quantile, and hp is the investment holding period.

VII. STRESS TESTING

Stress Testing is designed to complement Value at Risk (VaR) in anticipating extreme events. Stress Testing is a useful tool for financial risk managers because it gives us a clear idea of the vulnerability of a defined portfolio [16]. The selection of extreme scenarios is subjective and depends on the stress tester's assessment and experience [17]. The types of scenario analysis are categorized as follows [18]:

1. Historical Scenarios of Crisis: scenarios are formed using historical data of extreme events that have occurred as a basis for Stress Testing.
2. Stylized Scenarios: scenarios are formed by simulating market movements in interest rates, exchange rates, stock prices, and commodity prices against the portfolio.
3. Hypothetical Events: scenarios are formed through a reflection process by considering the consequences of certain hypothetical situations.

The basis of Stress Testing is to recalculate VaR estimates with higher volatility. In the G-30 Best Practices Report, it is recommended to conduct stress simulations that reflect adverse moves of historical and future events [19]. Historical scenarios of simulation-based Stress Testing are:

1. Stress Testing Using Monte-Carlo Simulation

The Monte Carlo simulation is a parametric approach that requires input parameters based on the historical distribution of data under extreme conditions.

$$P_t = P_{t-1}\, e^{\sigma\varepsilon\sqrt{t}} \quad (21)$$

where P_t is the simulated price, P_{t-1} is the current stock price, e is Euler number (2,71828), ε is a standard normally distributed random variable, σ is the volatility, and if VaR is estimated for one day, then t value is equal to one.

The formula in Equation (21) cannot be applied to a portfolio case, as it is only valid for a single asset. Therefore, the simulation process becomes more complex by transforming the uncorrelated standard normal random variable $\boldsymbol{\varepsilon}$ into a correlated random variable $\boldsymbol{Z}$ using a Cholesky matrix $(\boldsymbol{A})$:

$$\begin{bmatrix} Z_1 \\ Z_2 \\ \cdots \\ Z_n \end{bmatrix}_{n\times 1} = [\boldsymbol{A}]_{n\times n} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{bmatrix}_{n\times 1} . \tag{22}$$

Extreme simulated prices are then obtained by transforming the current prices using the modified Equation (21), where ε is replaced with Z.

Based on the resulting simulated prices, the portfolio value is calculated by multiplying the number of shares by the simulated prices. The distribution of portfolio profits and losses is computed using Equation (23):

$$P\&L = V_i - V_0 \tag{23}$$

where $P\&L$ represent profit or loss, V_i is the portfolio value from the $i-th$ mulation, and V_0 is the current portfolio value. If the portfolio's simulated $P\&L$ results are sorted in ascending order, the VaR estimate is derived from the α-th percentile of this distribution.

2. Stress Testing Using Historical Simulation

The historical simulation method involves constructing a scenario under the assumption that past events may recur, thereby requiring a revaluation of both individual asset values and the overall portfolio. The practical implementation steps of this method are as follows:

a) Selecting a period corresponding to the extreme scenario to be analyzed.
b) Calculating historical returns for each scenario constructed.
c) Simulating prices using historical simulation.

$$P_i = P_t\, e^{R_i} \tag{24}$$

where P_i is the i-th simulated price, P_t is the current price, e is Euler's number (2,71828), and R_i is the i-th return. From this point, the process is identical to that described for the Monte Carlo Simulation. [16].

VIII. Data and Method

The data used in this study consists of variables employed in the clustering process and variables used in determining weights to Stress Testing. The variables used for clustering include the Price to Earnings Ratio (PER), Price to Book Value (PBV), and Market Capitalization of 30 companies listed in the PEFINDO i-Grade index as of December 2024. Meanwhile, the variables used in determining stock weights for the optimal portfolio and during the Stress Testing process comprise the daily closing prices of the 30 stocks listed in the PEFINDO i-Grade index, as well as the daily closing prices of the Indonesia Composite Index (IHSG), spanning from January 2, 2023, to December 30, 2024. The data utilized in this research are secondary data obtained from several sources:

1. Information on stocks listed in the PEFINDO i-Grade index was retrieved from [20] under the Index section.
2. Data on the Price to Earnings Ratio (PER), Price to Book Value (PBV), and Market Capitalization of each company in the PEFINDO i-Grade index were obtained from the Stock Screener feature on [21].
3. Daily closing price data were obtained from [22].

The data analysis process in this study involves the following steps:

1. Grouping stocks using Ward Clustering, followed by selecting portfolio constituents from each cluster based on their expected return.
2. Constructing the optimal portfolio using the Mean-Semivariance method with the IHSG return as the benchmark to determine the optimal weights.
3. Estimating the maximum potential risk of the constructed stock portfolio using the Cornish-Fisher Expansion VaR.
4. Developing extreme scenarios based on daily IHSG prices and estimating the maximum potential portfolio risk under these stressed conditions.

Data were processed using software Python Google Colab.

IX. Result and Discussion

This section outlines the stock selection process using Ward Clustering, portfolio weighting through the Mean-Semivariance approach, and risk measurement using the Cornish-Fisher VaR method, complemented by Stress Testing under extreme market conditions.

A. *Stocks Selection Using Ward Clustering*

This section explains stock selection using Ward Clustering. Stocks that make up the portfolio are selected from representatives of each cluster based on their expected return. The variables used in the clustering consist of PER, PBV, and Market Capitalization of 30 stocks included in the PEFINDO i-Grade, which have different scales, so standardization was required. Multicollinearity detection was carried out using the VIF. The VIF values for PER, PBV, and Market Capitalization are 2.38432, 3.01531, and 1.43488, respectively. Since all VIF values are below 10, it can be concluded that there is no multicollinearity, and the assumptions for cluster analysis are satisfied.

Ward Clustering was applied, starting with each observation as its own cluster and then successively merging clusters based on the smallest increase in within-cluster sum of squares (SSE), until a single cluster is formed. The optimal number of clusters was determined using the silhouette score.

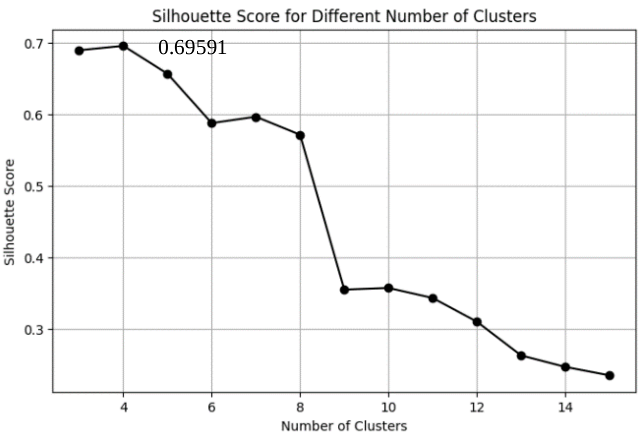

Fig 1. Silhouette Score Plot

The optimal number of clusters for the PEFINDO i-Grade stock data clustering is 4 clusters that have the highest silhouette score.

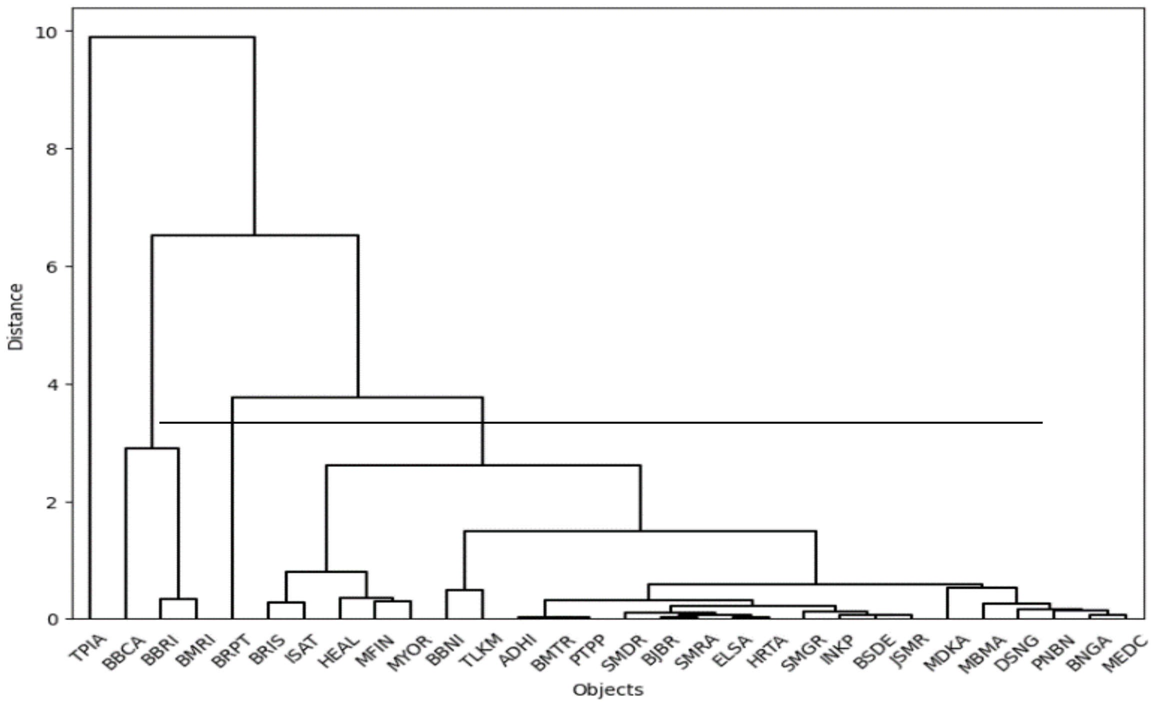

Fig 2. Clustering Dendrogram

TABLE I
CULSTER MEMBERSHIP

Cluster	Count	Cluster Members
1	3	BBCA, BBRI, BMRI
2	25	ADHI, BBNI, BJBR, BMTR, BNGA, BRIS, BSDE, DSNG, ELSA, HEAL, HRTA, INKP, ISAT, JSMR, MBMA, MDKA, MEDC, MFIN, MYOR, PNBN, PTPP, SMDR, SMGR, SMRA, TLKM
3	1	BRPT
4	1	TPIA

TABLE II
CULSTER MEMBERSHIP

Cluster	PER (times)	PBV (times)	Market Capitalization (IDR)
1	13.887	2.770	773,205,455,638,220
2	9.355	1.217	43,673,124,241,475
3	339.610	1.380	86,247,440,600,480
4	-556.660	15.810	631,534,000,000,000

The characteristics of each resulting cluster were examined through the average value of each variable within the respective clusters in Table II. TPIA was excluded due to having a negative PER alongside the highest PBV. This condition indicates that the company has not yet been able to generate sufficient profits to support its stock price, which is considered overvalued. The selection of stocks for portfolio construction was based on the highest positive expected return from each cluster. A positive expected return reflects the anticipated gain from an investment. The higher the expected return, the greater the potential profit that can be expected. The portfolio consists of BMRI (PT Bank Mandiri (Persero) Tbk) from Cluster 1 with an expected return of 0.00030, MFIN (PT Mandala Multifinance Tbk) from Cluster 2 with an expected return of 0.00283, and BRPT (PT Barito Pacific Tbk) from Cluster 3 with an expected return of 0.00039.

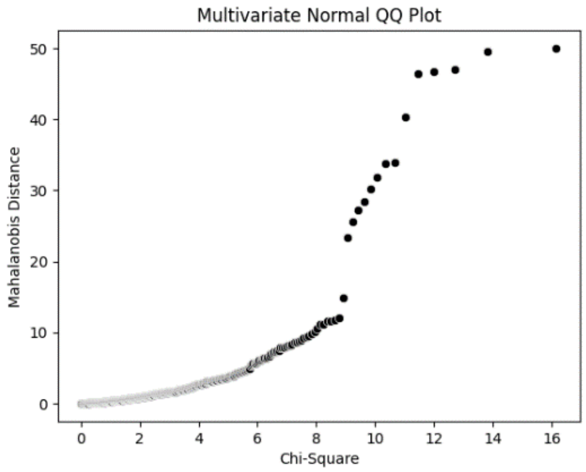

Fig 3. Multivariate Normal Q-Q Plot

B. Optimum Portfolio Using Mean-Semivariance

This section explains the optimum weighting for each of the portfolio's constituent stocks using Mean-Semivariance. Previously, the assumption of multivariate normality was tested to determine whether the portfolio's constituent stock return is multivariately normally distributed. Meanwhile, if the data is not normally distributed, the portfolio optimization method can be done using Mean-Semivariance.

Based on Figure 3, the plot does not follow a straight diagonal line. At a significance level of α = 5%, the correlation between the squared Mahalanobis distances and the Chi-Square quantiles was 0.82764, which is lower than the critical value. Therefore, it is concluded that the portfolio stock returns do not follow a multivariate normal distribution.

The construction of the optimal portfolio using the Mean-Semivariance approach begins with the formation of the semivariance-semicovariance matrix. The benchmark used in this analysis is the return of the Indonesia Composite Index (IHSG).

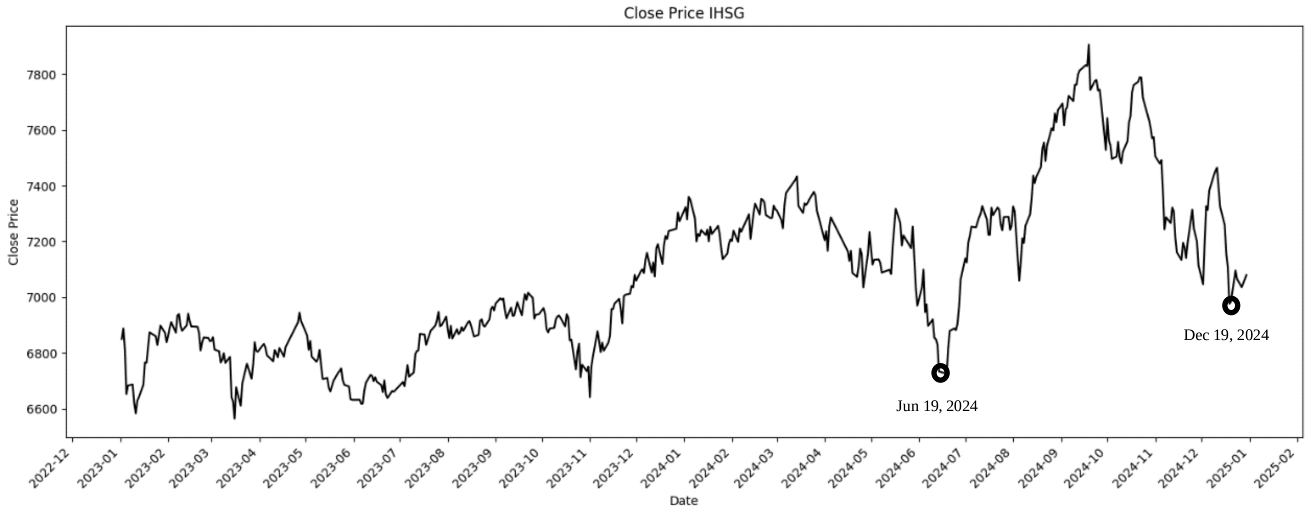

Fig 4. Historical IHSG Prices

TABLE III
CORNISH-FISHER CALCULATION COMPONENTS

Statistics	Portfolio Return
Expected Return	0.00081
Semivariance	0.00009
Semideviation	0.00926
Excess Kurtosis	1.98019
Skewness	-0.37873

$$\boldsymbol{\Sigma}_{msv} = \begin{bmatrix} \Sigma^2_{BMRI,B} & \Sigma_{BMRI,MFIN,B} & \Sigma_{BMRI,BRPTB} \\ \Sigma_{BMRI,MFIN,B} & \Sigma^2_{MFIN,B} & \Sigma_{MFIN,BRPT,B} \\ \Sigma_{BMRI,BRPT,B} & \Sigma_{MFIN,BRPT,B} & \Sigma^2_{BRPT,B} \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{msv} = \begin{bmatrix} 0.00010 & 0.00003 & 0.00006 \\ 0.00003 & 0.00032 & 0.00005 \\ 0.00006 & 0.00005 & 0.00054 \end{bmatrix}.$$

The inverse of the semivariance-semicovariance matrix ($\boldsymbol{\Sigma}^{-1}_{msv}$) is presented as follows:

$$\boldsymbol{\Sigma}^{-1}_{msv} = \begin{bmatrix} 10501.692 & -753.256 & -1098.001 \\ -753.256 & 3248.904 & -188.685 \\ -1098.001 & -188.685 & 1984.877 \end{bmatrix}.$$

$\boldsymbol{\Sigma}^{-1}_{msv}$ is then used to determine the optimal weights for the portfolio stocks:

$$\boldsymbol{w} = \frac{\begin{bmatrix} 10501.692 & -753.256 & -1098.001 \\ -753.256 & 3248.904 & -188.685 \\ -1098.001 & -188.685 & 1984.877 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 10501.692 & -753.256 & -1098.001 \\ -753.256 & 3248.904 & -188.685 \\ -1098.001 & -188.685 & 1984.877 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$$\boldsymbol{w} = \begin{bmatrix} w_{BMRI} \\ w_{MFIN} \\ w_{BRPT} \end{bmatrix} = \begin{bmatrix} 0.74217 \\ 0.19793 \\ 0.05990 \end{bmatrix}.$$

The resulting optimal weights for BMRI, MFIN, and BRPT are 74.217%, 19.793%, and 5.990%, respectively.

C. Portfolio Risk Using Cornsh-Fisher Expansion VaR

This section explains the maximum potential risk of the constructed portfolio using the Cornish-Fisher VaR. Prior to this, a normality test was conducted on the portfolio returns to assess whether the returns follow a normal distribution. If the return distribution does not significantly deviate from normality, the Cornish-Fisher Expansion VaR can be applied. Based on the weights, the portfolio return was calculated using Equation (16). Checking the univariate normality assumption on the portfolio return obtained a test statistic value of 0.06677 with p-value of 0.02764. At 5% significance level, it can be concluded that the portfolio return is not normally distributed.

The portfolio risk using the Cornish-Fisher VaR requires several descriptive statistical components of the portfolio return in Table III. Portfolio risk in the analysis was calculated at the 95% confidence interval or significance level $\alpha = 5\%$, so the $q_{95\%}$ value is -1.645 [18]. The Cornish-Fisher quantile (z_{cf}) calculated using Equation (19) is -1,70985. Assuming an initial investment of IDR 10,000,000 and an investment period of 1 day, the maximum potential loss an investor may experience is IDR 150,298, equivalent to 1.503% of the initial capital invested.

TABLE IV
NORMALITY TEST STATISTICS OF STOCK PRICES

Scenario	Period	Stock	T	p-value
Scenario 1	22/03/2024-19/06/2024	BMRI	0.12784	0.35678
		MFIN	0.11779	0.45703
		BRPT	0.21626	0.01563
Scenario 2	10/10/2024-19/12/2024	BMRI	0.14952	0.19312
		MFIN	0.43074	0.00000
		BRPT	0.12499	0.38368

D. Portfolio Risk in Extreme Conditions Using Stress Testing

The selection of extreme scenarios is carried out by analyzing extreme events using the historical scenario and identifying factors influencing the level of risk based on market risk, which is driven by stock price fluctuations. Parameters to identify extreme events were determined using the composite stock index, under the assumption that the IHSG represents the movement of most individual stocks. Extreme scenarios were determined based on periods of continuous IHSG price decline leading to its lowest point presented in Figure 4. Fifty periods prior to the lowest point were selected to capture the downward trend, assuming this was sufficient to represent the decline. Therefore, the extreme scenarios analyzed occurred during March 22, 2024 – June 19, 2024, and October 10, 2024 – December 19, 2024.

Based on the normality test results across all crisis scenarios presented in Table IV, none of the closing prices of the selected portfolio stocks followed a normal distribution. This study employed a simulation - based Stress Testing,

TABLE V
STRESS TESTING VAR CALCULATION RESULTS

Scenario	VaR Cornish-Fisher Expansion		VaR Historical Simulation	
	VaR (IDR)	VaR (%)	VaR (IDR)	VaR (%)
Scenario 1	270,426	2.704	337,734	3.387
Scenario 2	219,676	2.197	317,843	3.187

TABLE VI
INVESTMENT REALIZATION

Stock	Weight (%)	Fund Allocation (IDR)	Stock Price (IDR)		Stock Quantity	
			30/12/24	2/01/25	Lot	Shares
BMRI	74.22	7,421,706	5,700	5,850	13	1300
MFIN	19.79	1,979,276	3,350	3,480	6	600
BRPT	5.99	599,018	920	940	6	600

utilizing the historical simulation method exclusively to better capture the non-normal characteristics of the data. Risk evaluation under extreme conditions was carried out using Stress Testing through both the Cornish-Fisher VaR and historical simulation. Stress Testing with the Cornish-Fisher VaR was calculated based on portfolio returns using actual prices adjusted to the chosen extreme periods, while the historical simulation approach used simulated stock prices for each scenario.

Based on Table V, the VaR values obtained through historical simulation were higher than those using actual prices in both extreme scenarios. This difference implies that the historical simulation method is more conservative, as it yields a higher estimated risk. Portfolio risk under extreme conditions was higher than the risk calculated over the entire study period, which was 1.503%.

An investment realization assessment was conducted to determine whether the potential loss from an investment made on December 30, 2024, over one day would exceed the estimated VaR. On January 2, 2025, the stock prices of BMRI, MFIN, and BRPT were IDR 5,850, IDR 3,480, and IDR 940. Based on these prices, the closing prices of all three stocks increased compared to the previous period. Based on Table VI, the portfolio value on December 30, 2024, was IDR 9,972,000, and it increased to IDR 10,257,000 on January 2, 2025. Thus, an investor who allocated IDR 9,972,000 to the portfolio gained a capital return of IDR 285,000 or 2.858% of the initial investment.

X. CONCLUSION

The stocks from the PEFINDO i-Grade index selected for the portfolio are BMRI (PT Bank Mandiri (Persero) Tbk) with a weight of 74.217%, MFIN (PT Mandala Multifinance Tbk) with a weight of 19.793%, and BRPT (PT Barito Pacific Tbk) with a weight of 5.990%. The maximum potential loss for an investor allocating IDR 10.000.000 to this portfolio at a 95% confidence level is IDR 150,298 for the following day, equivalent to 1.503% of the initial investment. Portfolio risk under extreme conditions indicates a higher potential loss. The Cornish-Fisher VaR values under Scenario 1 and Scenario 2 are 2.704% and 2.197%, respectively, while the historical simulation VaR values under the same scenarios are 3.387% and 3.187%. The level of risk resulting from Stress Testing reflects that under extreme market conditions, the potential for loss increases significantly.

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