Prediction of Domestic Value-Added Tax in Kenya Using SARIMA and Holt-Winters Methods

Nelson Kirimi Micheni, Edwine B. Atitwa and Patrick M. Kimani

Abstract—**Many countries rely on tax revenues to finance their expenditure; thus, forecasting revenue is important in fiscal planning, policy formulation, and fiscal decision-making. Tax revenue underestimation and overestimation have led to unstable economies, and for this reason, it is prudent for the country to explore scientific methods of forecasting, such as time series, since tax revenue is collected over time. Value Added Tax(VAT) is an indirect tax that is collected under domestic taxes. This study aims to predict VAT in Kenya. Specifically, it is set to identify a suitable Seasonal Autoregressive Integrated Moving Average (SARIMA) and Holt-Winters (HW) model and forecast Value Added Tax revenues in Kenya. The study used secondary data on VAT collected in Kenya between July 2014 and December 2020. The best model was selected using the Bayesian Information Criterion (BIC), Akaike's Information Criterion (AIC), and forecast accuracy measures determined using Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), and Mean Absolute Percentage Error (MAPE). The SARIMA (0,1,2)(0,1,1)[12], had the lowest BIC=1093.2 and forecasting errors (MAE=540.9, MAPE=5.04, and MASE=0.32) hence the best forecasting model. Diebold-Mariano (DM test) observed significant differences in the forecasting performances of the three models. The model confidence set (MCS) procedure retained the SARIMA model confirming that it had the highest predictive accuracy.**

Index Terms— **Diebold-Mariano (DM test), Forecasting, Holt-Winters (HW), Model Confidence Set (MCS), SARIMA, Tax Revenue, Value Added Tax (VAT)**

I. INTRODUCTION

TAXATION is a mechanism used by organizations and governments to raise money for public expenditures. This is crucial for the country's residents' access to funding for infrastructure and development initiatives such as health care and agriculture [7]. Governments now have to impose taxes and seal loopholes to make sure that each person or organization makes a fair contribution to the economy.

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Domestic taxes, which can be direct or indirect, are levied on income earned within a country. The major domestic tax types in Kenya include Pay As You Earn, Value Added Tax, Corporation Tax, Withholding Tax, Excise Duty Tax, Rental Income Tax, and Capital Gains Tax [18].

Revenue forecasting and tax analysis are significant tools that offer guidance on how to increase a country's taxes and enhance tax equality and efficiency [11]. This encourages investments, and economic expansion to raise the national income. Similarly, these tools can be used to improve budget planning and monitoring processes, which will help the government make better decisions [13] as it establishes the resource envelope that serves as the foundation for efficient medium and long-term planning. In recent years, Kenya's budget has experienced a tremendous surge; for instance, according to [11], it rose from Kshs 1.45 trillion in the fiscal year 2012-2013 to Kshs 3.02 trillion in the fiscal year 2019-2020, which constitutes a 108% rise. This increase in budget estimates has prompted discussions about the general capacity of budget absorption at the county and national levels of government, along with its capacity to generate revenue to support the expenditure plans in light of the rising levels of the national debt [12]. The Central Bank of Kenya's statistics on public debt reveal an increasing trend in the domestic and external debt levels. The Institute of Economic Affairs (IEA Kenya) claims that by preventing overly ambitious budget plans, the nation would be driven away from ongoing borrowing, as between 2014 and 2020, the revenue projections increased from 5.5% to above 19%. The IEA questioned this increase in projected government revenue, which highlights the need to boost the general predictability of public funds for better budgeting. The Kenya national treasury was informed in an article by [1] of the need for a more capable revenue forecasting system. Ofori, *et al.*[20] emphasizes the need for governments to have accurate tax revenue predictions for proper economic planning. In Kenya, the Ministry of Finance, in collaboration with the Kenya Institute for Public Policy Research and Analysis, produces revenue forecasts. The National Treasury develops the government budget estimates which are presented to the Parliament for approval. This incorporates the revenue projections proposed to fund the budget. This forms the basis for the annual revenue targets that are issued to the Kenya Revenue Authority (KRA) for implementation.

A. Value Added Tax (VAT)

VAT is an indirect tax that is imposed on the consumption of goods and services according to Kenyan tax laws and accounts for about 28% of all tax revenues in Kenya [19]. VAT is charged on all the supplies of taxable goods and services that are made or provided in Kenya and imported taxable goods or services into Kenya. It was first rolled out in January 1990, replacing sales tax due to its inherent shortcomings. The VAT Act 2013 came into operation on 2nd September 2013 and is the law that aids in enforcing this tax. It is charged by registered taxpayers only and is applicable under Section 5 of the VAT Act 2013. For it to apply to an individual, company, or partnership, one must register within 30 days if they have made supplies or expects to supply taxable supplies whose value is Kshs. Five million and above within 12 months.

Similarly, traders must be registered if they deal in both exempt and taxable supplies. All VAT taxpayers in Kenya are required to submit monthly returns using the iTax system by the 20th day of the month that follows the month in which the tax was collected. To administer this tax, supplies are divided into two categories; Exempt and Taxable. The Exempt category includes goods or services that do not attract VAT. They are outlined in the 1st schedule of the VAT Act– part 1 (Exempt goods) and part 2 (Exempt services). VAT has two types of tax rates: 0% and 16%. The 0% rate is for Zero-rated supplies that are listed in the 2nd schedule of the VAT Act, such as the Exportation of goods/services, goods supplied to Economic Processing Zones (EPZ), and privileged persons. The 16% rate is the general rate for all other Goods and Services.

A review of the recent studies includes Puthran et al. [24], who evaluated and did the modeling of the Indian motorcycle business the SARIMA and Holt-Winters models. Although both models were effective, they discovered that the Holt-Winters technique outperformed the SARIMA model since it had lower MSE, MAE, and MAPE values. The SARIMA model projected Nigeria's monthly inflation rate using 120 observations from November 2003 to October 2013 by Otu et al.[21].SARIMA(1,1,1)(0,0,1)12 model best predicted the inflation rate for the first quarter of 2014. The scholar [22] examined the SARIMA, ARIMA, and Holt-Winters time series methods and found that they produced better outcomes over short forecasts but were constrained when describing the factors that influenced the relevant variables of interest. [4] used SARIMA models in the prediction of the monthly departure of tourists from Taiwan to about three destinations and found the model suitable. Similarly,[26] compared SARIMA to the structural time series models and realized that the SARIMA model generates more precise short-term forecasts than the structural time series model. Rahman *et al.* [25] forecasted monthly revenue using the additive and multiplicative seasonal models of the Holt-Winters method using monthly revenue data from the Bangladesh Bridge Authority between July 1998 and July 2016. The additive Holt-Winters method was found to be the most accurate and reliable and was used to forecast monthly revenue through January 2021. Susan, *et al.*[27] used the SARIMA model to forecast Kenya's inflation rate. The SARIMA (0,1,0)(0,0,1)4 model was found to be the most appropriate model for predicting Kenya's inflation rate, where the model's predictability was evaluated using RMSE and MAE and was found adequate. This study also proposed policies that Kenya could implement to achieve a single-digit inflation rate.

Ofori, *et al.*[20] established that the ARIMA (1,1,4) model with intervention effect had smaller MAPE, RSME, and MAD values and produced the best forecast for total domestic VAT revenue in Ghana. The author deduced the absence of literature that provides a full consensus on the method that precisely predicts revenue. Given the need for studies on the prediction of domestic revenues in Kenya, the study attempts to model domestic VAT revenues collected in the country to fill the gap in the literature.

The main objective of the study is to identify a suitable Seasonal Autoregressive Integrated Moving Average (SARIMA) and Holt-Winters (HW) model and use it in the forecasting of VAT revenues in Kenya.

II. MATERIALS AND METHODS

A. Research Design

The study used secondary data from the Kenya Revenue Authority regarding the monthly domestic VAT collections. The information is based on actual VAT data for 78 months (July 2014 to December 2020).

The population of interest is the revenue collected by the Kenya Revenue Authority. Since VAT depends on the levels of consumption and business activities in the country, data series would produce dependable time series, hence suitable for forecasting. Data analysis and visualization were done using R software which is highly accepted in the Statistics and Economics fields.

The research uses the quantitative forecasting approach which utilizes historical data and statistical models to make predictions. The analysis process involved the following steps as emphasized by [15] in the creation of time series models:

i. Plotting of the VAT trend to check seasonality in the data.
ii. Examining whether the VAT data had stationarity problems. This was confirmed using the Augmented Dickey-Fuller (ADF) and Phillip Peron (PP) tests.
iii. Initial Model estimation using the SARIMA and HW methods. This involved model identification, analysis, and testing.
iv. Estimation of model parameters.
v. Diagnostic checking and model evaluation
vi. Best model selection.
vii. Forecasting: This entails estimating future events based on current and historical data. This is done after the models pass the diagnostic tests. Performance metrics are used to evaluate and compare the performance and forecast accuracy of various models. Consider that:

Y_t is the actual observation at time t,
$\hat{Y}_t$ is the forecasted value at time t,
$e_t = Y_t - \hat{Y}_t$ is the forecast error at time t

$$Forecast\ error: e_{t+h} = Y_{t+h} - \hat{Y}_{t+h}\,; h - i \le 0 \ \ and\ h - i > 0\ then\ \varepsilon_{t+h-i} = 0 \qquad (1)$$

The Box-Jenkins Procedure summarizes the forecasting conceptual framework to be adopted by researchers. This process involves four major steps that include; Model Identification, Model Estimation, Adequacy/Diagnostic checking, and Data forecasting [2]. The steps are done repeatedly until the model adequacy is achieved.

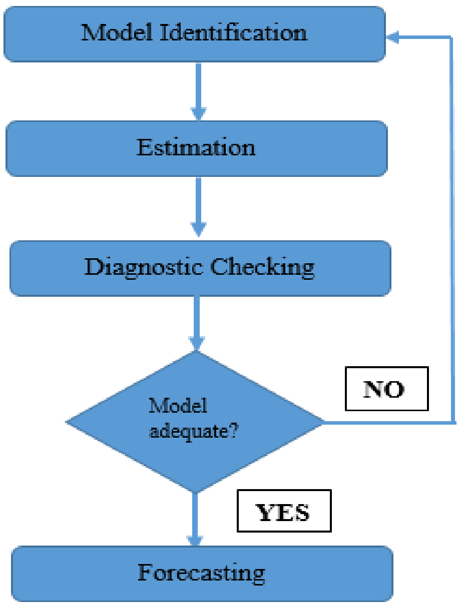

Fig. 1: Box-Jenkins Model Principles

B. Accuracy Measurement

Since forecasting is the primary objective of time series modeling, the predictive accuracy of forecasts is assessed to see which models have the fewest errors (Makridakis *et al*., [17]). To gauge the model's accuracy, Mean Absolute Error (MAE), Mean Absolute Scaled Error (MASE), Root Mean Squared error(RMSE), and Mean Absolute Percentage Error (MAPE) statistics were computed. They can be expressed as:

$$Mean\ Absolute\ Error\ (MAE) = \frac{\sum_{i=1}^{n} |\ \varepsilon_i\ |}{n} \qquad (2)$$

$$Mean\ Absolute\ Scaled\ Error\ (MASE) = mean(|a_i|) \qquad (3)$$

$$where\ (a_i) = \frac{e_i}{\frac{1}{T-1}\sum_{i=2}^{T} |\ y_t - y_{t-1}|}\ ;t = 1..T\ sample\ periods$$

$$Root\ mean\ squared\ error(RMSE) = \sqrt{\frac{\sum_{i=1}^{n} \varepsilon_i^2}{n}} \qquad (4)$$

$$Mean\ absolute\ percentage\ error(MAPE) = \frac{\sum_{i=1}^{n} |PE_i|}{n} \qquad (5)$$

$$where\ Percentage\ error\ (PE) = \frac{Y_t - F_i}{Y_t} * 100$$

Y_t *is the Actual value,* F_i *is the Estimated value and* $e_i = Y_i - \hat{Y}_i$ *is the forecast error at time i,* Y_i *is the actual observation at time i and* $\hat{Y}_i$ *is the forecasted value at time i.*

According to [3], the MAPE values of <10% are considered excellent while the range of 10% to 20% is classified as a good forecast. The Bayesian Information Criterion (BIC) and Akaike's Information Criterion (AIC) statistics were used to select the best model. They can be expressed as $AIC = -2ln(Likelihood) + 2r$ and $BIC = -2ln(Likelihood) + r\ ln(T)$. The AIC value grows in proportion to the number of model parameters (r) and is lowest for the best model [6]. The value T represents the set of all parameters.

C. SARIMA Model

The Seasonal Autoregressive Integrated Moving Average (SARIMA) model was proposed by [2, 9] to deal with non-stationary time series that exhibit seasonality. SARIMA models account for the seasonality component and occurrences of time series models. Seasonal differencing is used to remove seasonality from a time series that is not stationary.

For instance, $Z_t = X_t - X_{t-s}$ where, Z_t is the seasonally differenced series and s is the number of seasons per year, which defines a first-order seasonal difference. This is the difference between an observation and a comparable observation from the previous season [23].

Seasonal Method Autoregressive Integrated Moving Average (SARIMA) is denoted as ARIMA $(p, d, q)\ (P, D, Q)\ s$

where:

(p, d, q)-represents the non-seasonal part of the model

(P, D, Q)s-represents the seasonal part of the model

Furthermore, the seasonal AR order is P, the seasonal MA order is Q, and the seasonal differencing is D.

The SARIMA model will be expressed as follows;

$$\Phi_p(B^s)\varphi(B)\ \nabla_s^D \nabla^d Z_t = \Theta_Q\ (B^s)\theta(B)\varepsilon_t \qquad (6)$$

Considering that;

$\varphi(B)$ and $\theta(B)$ are the autoregressive and moving average polynomials of orders p and q.

$\Phi_p(B^s)$ and $\Theta_Q\ (B^s)$ the seasonal autoregressive and moving average components with orders P and Q

∇^d *and* ∇_s^D represent the ordinary and seasonal difference components, and B is the backshift operator.

$Z_t = (1 - B^d)(1 - B^s)^D m_t$ represents the product of seasonal differencing D (7)

ε_t is the non-stationary time series or the Gaussian white noise.

s is the number of periods per season

The SARIMA model is further deconstructed by [4] as follows:

$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \cdots - \varphi_p B^p$; the Non-seasonal AR of order p (8)

$\Phi_p(B^s) = 1 - \Phi_1(B^s) - \Phi_2(B^{2s}) - \Phi_3(B^{3s}) - \cdots - \Phi_p(B^{ps})$; the Seasonal AR of order P (9)

$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3 + \cdots + \theta_p B^p$; the Non-seasonal MA of order q (10)

$\Theta_Q(B^s) = 1 + \Theta_1(B^s) + \Theta_2(B^{2s}) + \Theta_3(B^{3s}) + \cdots + \Theta_Q(B^{Qs})$–Seasonal MA order Q (11)

$\nabla^d = (1 - B)^d$ (12)

$\nabla_s^D = (1 - B^s)^D$ (13)

The research will concentrate on a 12-month revenue collection time series. This implies that the seasonal period is 12 (*s=12*).

As a result, the SARIMA model will be;

$\Phi_p(B^{12})\,\varphi(B)\nabla_{12}^D\nabla^d Z_t = \Theta_Q\,(B^{12})\theta(B)\varepsilon_t$ (14)

D. Holt-Winters Model

Exponential smoothing is a method of revising a given prediction in light of more recent experience. The Holt-Winters method, which is based on exponential smoothing, includes trend smoothing, seasonal smoothing, and overall smoothing.

The coefficients α, β, γ are the three smoothing parameters, and p stands for the number of observations made during each seasonal cycle. Holt-Winters methods are classified as additive or multiplicative [28].

When a time series has a linear trend with an additive seasonal pattern, the additive Holt-Winters method is used (Hyndman *et al.* [9]-[10]). It may be appropriate for modeling some tax heads. However, as economic performance changes, the tax trend usually changes in a multiplicative manner. The estimate L_t represents the series level, b_t the trend, S_t the seasonal component, α the smoothing parameter for the level, β the smoothing parameter for the trend component, γ the smoothing parameter for the seasonality component while F_{t+m} will be the forecast for the m periods ahead, and t the index-denoting period in this method.

The multiplicative model is represented by the equations below:

$F_{t+m} = (L_t + b_t m)S_{t-s+m}$ for $m = 1, \ldots \ldots, M$ (15)

where;

$L_t = \alpha \frac{Y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1})$

$b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$

$S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)\,S_{t-s}$

and additive Holt-Winters model is represented as:

$F_{t+m} = L_t + b_t m + S_{t-s+m}$ for $m = 1, \ldots \ldots, M$ (16)

where;

$L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1})$

$b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$

$S_t = \gamma(Y_t - L_t) + (1-\gamma)\,S_{t-s}$

The suitable Holt-Winters smoothing factors (alpha=α, beta=β, and gamma=γ) for the best-fit model were determined on the least MAE, MAPE, and MASE values.

III. RESULTS AND DISCUSSION

A. VAT Trend

Between July 2014 and December 2020, the total VAT collection revealed an increasing and fluctuating trend. The highest amounts of VAT collections were noticed at the beginning of every year. This was highly attributed to increased consumption of vatable products as the year ends due to festivities. This is shown in Figure 2.

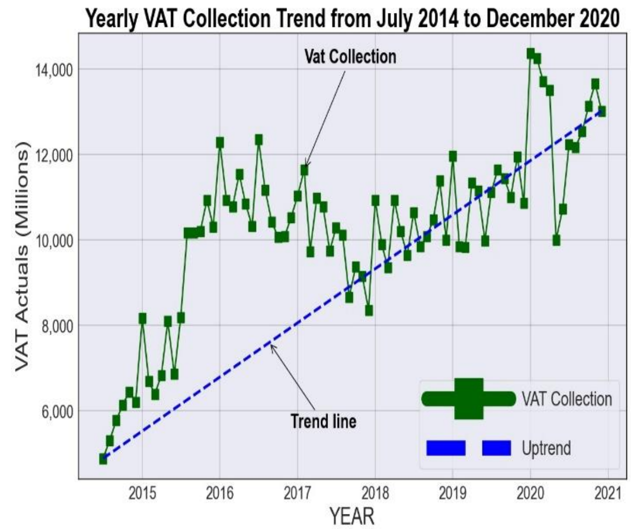

Fig. 2: Time plot of VAT from July 2014 to December 2020

Figure 2 above indicates the presence of variability and seasonality hence the need for the stationarity test.

B. Test of Stationarity

It is recommended to test for stationarity before using a variable with time series data for modeling to avoid estimates of misleading relations. To reduce data variability and seasonality, differencing was done ($y' = y_t - y_{t-1}$), as shown in Figure 3.

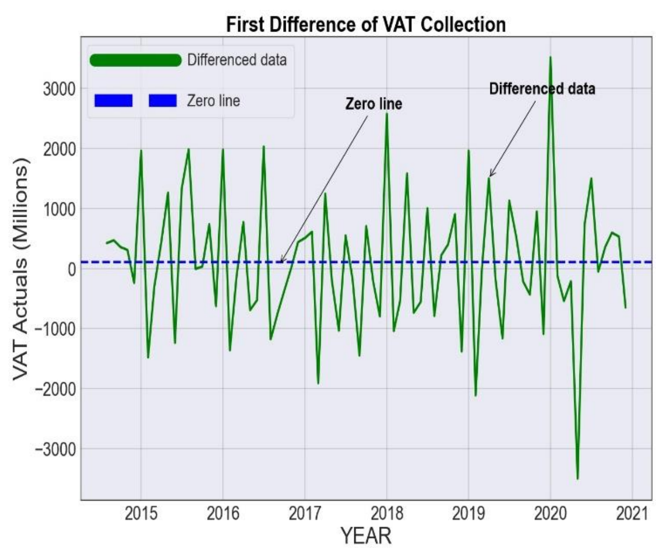

Fig. 3: VAT Trend at First Difference

The stationarity was also confirmed using the Augmented Dickey-Fuller (ADF) and Phillip Peron (PP) tests. The assumption in the Null hypothesis is that the time series has a unit root and is non-stationary.

TABLE I
TESTING FOR STATIONARITY

	P-values		
Level	**ADF Test**	**PP Test**	**Decision**
Time series at level	0.3299	2.2E-16	Not stationary and Stationary
First Difference	0.01	0.001152	Both stationary
Log First Difference	0.01	0.001142	Both stationary

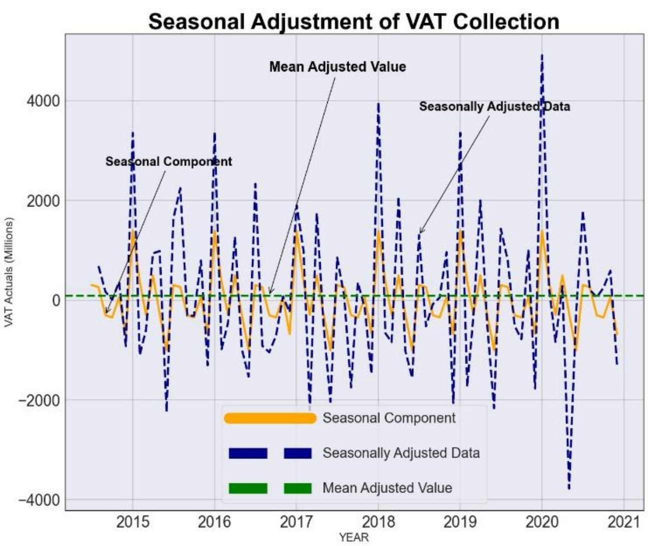

Fig. 4: Detrended and Deseasonalized data

When transformed and differenced once, the Augmented Dickey-Fuller (ADF) and Phillip Peron (PP) tests resulted in $(ADF = -5.671, p = .01)$ and $F(1,74) = 11.44, p = .0012)$ respectively. A p-value of less than 5% indicates that the series is stationary since the null hypothesis is rejected as shown in Table 1.

The detrended and deseasonalized data is represented in Figure 4. This allowed the isolation of trends and seasonal patterns hence making more accurate forecasts.

C. Diagnostic Checks

The diagnostic checks were carried out using the residuals from the models. The study used ACF plots of the residuals, Histograms, and p-values from the Ljung-Box statistic.

The ACF from the SARIMA and Holt-Winters residuals was very near the zero line, with most spikes falling within the significant zone. This demonstrates the independence of the residuals [8]. This is also confirmed by the Ljung-Box test $(\chi^2_{13} = 9.918, p = .7006)$, which suggests the independence of residuals. The histograms' generally bell-shaped appearance denotes the normality of the data as shown in Figure 5. The SARIMA and Holt-Winters diagnostic checks confirmed that the residuals were uncorrelated, emanated from a well-specified model, and could be utilized to forecast.

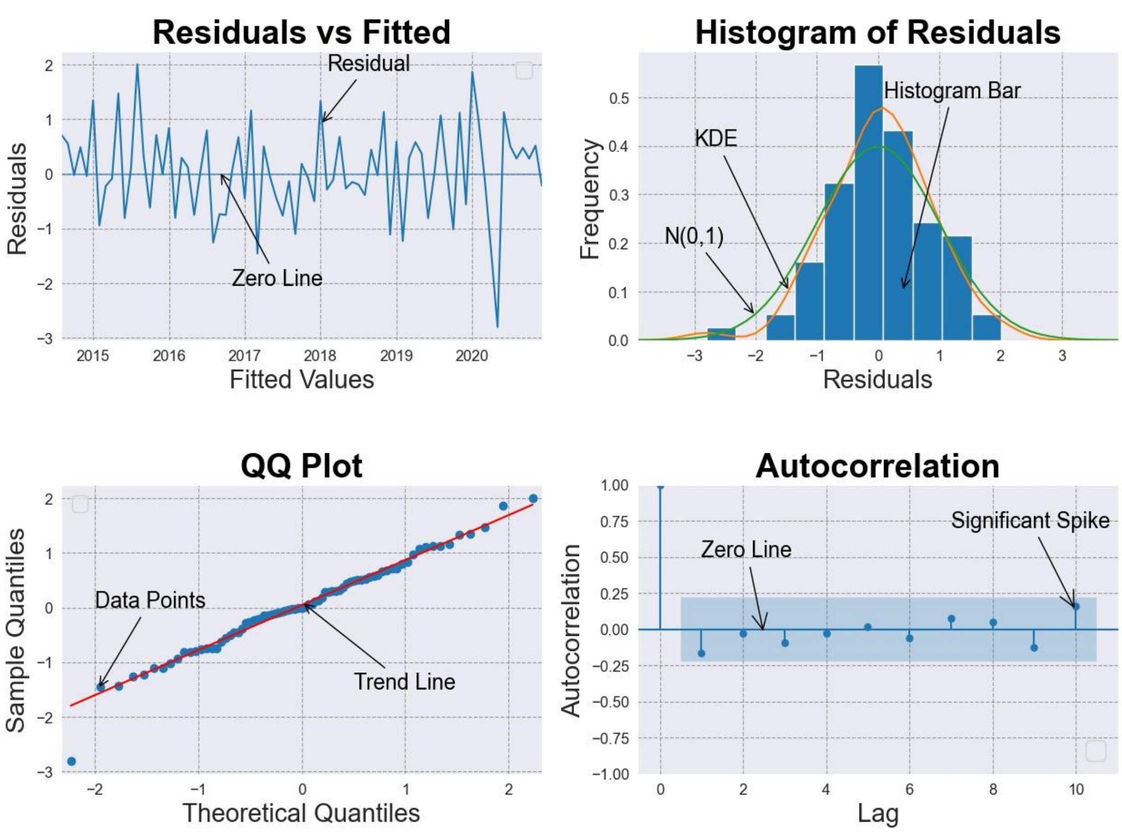

Fig. 5: Model diagnostics

D. Holt-Winters Model

This model is typically applied when a time series data exhibits seasonality and a linear trend. The model divides time series data into three components: trend, level value, and seasonal, with weights ranging from 0 to 1 to enable model fitting and prediction. The Holt-Winters model finds the optimal values of the smoothing factors (alpha=α, beta=β, and gamma=γ).

The resultant additive Holt-Winters model for VAT (Y_t), level mean estimate (L_t), trend estimate (b_t), and seasonal factors (S_t) are as follows:

$$ln(Y_t)=(13347.44 - 102.61t) + s_t + e_t \quad (17)$$

$$b_t = 0.0471(\mathrm{L_t} - \mathrm{L_{t-1}}) + (1 - 0.0471)b_{t-1}$$

$$S_t = 1 + (Y_t - L_t) + (1 - 1)\, S_{t-1} \approx 1 + (Y_t - L_t)$$

The resulting multiplicative Holt-Winters model, on the other hand, can be fitted as indicated below;

$$Y_t = (13500.37 - 101.21t) + sn_t + e_t \quad (18)$$

$$L_t = 0.678(\frac{Y_t}{S_{t-s}}) + (1 - 0.678)(L_{t-1} + b_{t-1})$$

$$b_t = 0.0504\,(L_t - L_{t-1}) + (1 - 0.0504)b_{t-1}$$

$$S_t = 1\left(\frac{Y_t}{L_t}\right) + (1 - 1)\, S_{t-s} \approx 1\left(\frac{Y_t}{L_t}\right)$$

The above resultant values are shown in Table 2.

TABLE II
HOLT-WINTERS SMOOTHING FACTORS

		Additive-Holt-Winters	**Multiplicative-Holt-Winters**
Factors	alpha(α)	0.7099	0.678
	beta(β)	0.0471	0.0504
	gamma(γ)	1	1
	Level Mean(a)	13347.44	13500.37
	Trend(b)	102.61	101.21
Smoothing Parameters	$\hat{s}_1$	1982.51	1.20
	$\hat{s}_2$	526.39	1.06
	$\hat{s}_3$	-544.15	0.96
	$\hat{s}_4$	-271.05	0.97
	$\hat{s}_5$	-1620.22	0.84
	$\hat{s}_6$	-1491.26	0.85
	$\hat{s}_7$	-395.42	0.94
	$\hat{s}_8$	-340.12	0.94
	$\hat{s}_9$	-234.94	0.95
	$\hat{s}_{10}$	11.05	0.98
	$\hat{s}_{11}$	564.34	1.04
	$\hat{s}_{12}$	-333.28	0.96

E. SARIMA Model Selection

AIC, AICc, and BIC were used to examine three different SARIMA models. SARIMA(0,1,2)(0,1,1)$_{12}$ had the lowest AIC, AICc, and BIC values and was chosen as the best-fit model for the VAT data. This implies that less information is lost when predicting using this model. Table 3 shows the findings of the analysis.

TABLE III
SARIMA***(p, d, q)(P,D,Q)***$_{12}$ MODELS

Models	**AIC**	**AICc**	**BIC**
ARIMA(0,1,2)(0,1,1)[12]	1084.52	1085.19	1093.22
ARIMA(2,1,0)(0,1,1)[12]	1084.68	1085.34	1093.37
ARIMA(3,1,0)(0,1,1)[12]	1086.36	1087.38	1097.24

F. Forecasting

The observed VAT collections between July 2014 and December 2020 were used to predict the likely collection in the next 12 months, from Jan 2021 to December 2021. Figures 6, 7, and 8 below illustrate how the SARIMA, additive, and multiplicative Holt-Winters model projections were visualized.

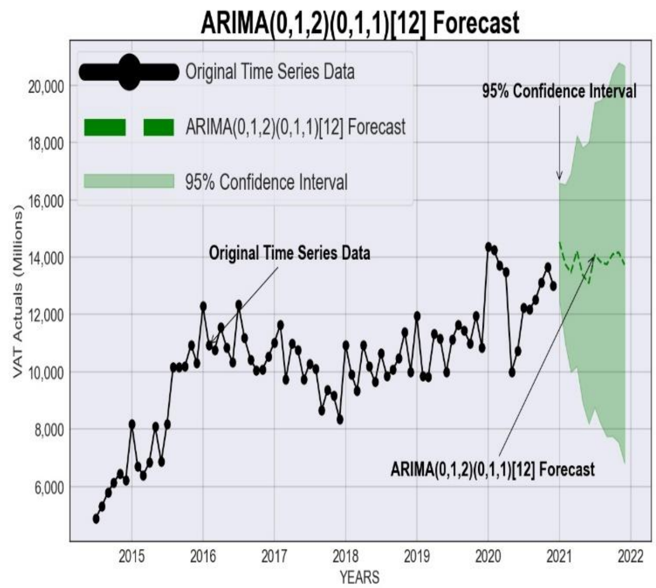

Fig. 6: SARIMA VAT Forecast

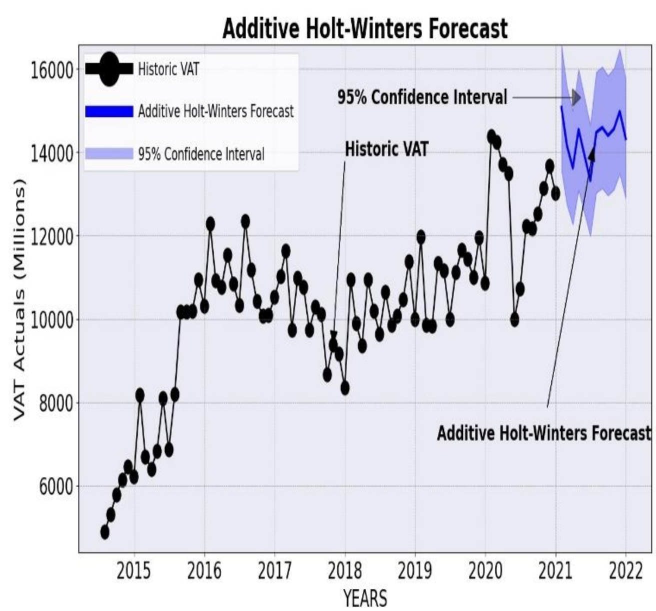

Fig. 7: Additive HW VAT Forecast

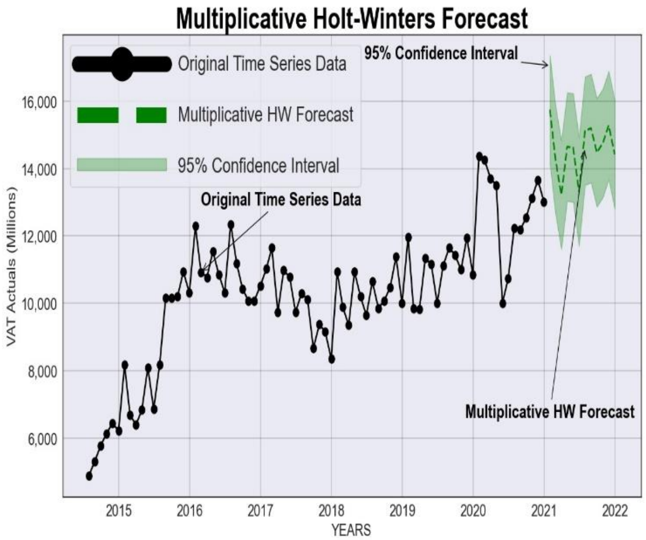

Fig. 8: Multiplicative HW VAT Forecast

The pattern of the forecasted part (January 2021 to December 2021) was almost similar to the actual. This shows that the models were good at forecasting domestic VAT data.
The blue line in Figures 6,7 and 8 is the plot of the actual VAT collections from July 2014 to December 2020. The green line is the plot of the predicted value of the VAT collections from January 2021 to December 2021.

G. Model Accuracy

The resultant prediction models were tested to deduce the model with the least prediction errors (MAE, MAPE, and MASE). The SARIMA $(0,1,2)(0,1,1)_{12}$ model exhibited the least errors, followed by the additive and multiplicative Holt-Winters models, respectively (Table 4). In all the models, MAPE values were less than 10% signifying excellent model forecasts.

TABLE IV
HOLT-WINTERS AND SARIMA MODEL ERROR METRICS

Models	SARIMA $(0,1,2)(0,1,1)_{12}$	Additive Holt-Winters Method	Multiplicative Holt-Winters Method
MAE	541.26	723.63	836.37
MAPE	5.03	6.84	7.83
MASE	0.32	0.43	0.50

H. Prediction versus Training Data

The VAT collections for 12 months were predicted using the 3 test models and compared with the actual training data from the revenue agency. These collections were aggregated to fit the financial calendar of the country, which depicts (Quarter 3(Q3)-January-March, Quarter 4(Q4)-April-June, Quarter 1(Q1)-July-September, and Quarter 2 (Q2): October to December). The resultant values are shown in Table 5.

TABLE V
VAT PREDICTED AND ACTUAL VALUES IN KSH. MILLIONS

Period	Additive HW Values	Multiplicative HW Values	Sarima Values	Actual Values
Q3	49,787.92	51,649.08	49,884.96	45,626.85
Q4	49,052.95	47,888.63	50,937.75	46,360.55
Q1	55,724.60	55,520.99	54,814.57	50,181.64
Q2	61,112.49	62,514.54	57,327.28	50,543.96
RMSE	2189.72	2464.54	1780.35	
MAE	1,913.75	2,083.09	1,687.63	

Source: Authors Computation

The comparison above was used to verify the most suitable prediction model. This was deduced by picking the model with the least RMSE and MAE. The SARIMA model produced the best prediction with the smallest RMSE and MAE of 1780.35 and 1687.63, respectively. This was followed by the additive HW (RSME=2189.7, MAE=1913.8) and Multiplicative HW (RSME=2464.5, MAE=2083.1).

I. Model Evaluation

The Diebold-Mariano test was used to test the predictive accuracy of the three competing models.

The DM test is expressed as:

$DM = \frac{\bar{d}}{\sqrt{\frac{\hat{V}(d)}{T}}}$ such that $\bar{d}$ is the mean of the loss differentials, $\hat{V}(d)$ the variance of the loss differentials and T the number of observations.

Hypothesis for Diebold-Mariano (DW) Test

Taking Model 1=SARIMA, Model 2=Additive HW, and Model 3=Multiplicative HW, below is the DM test hypothesis for each pair of models.

1. SARIMA vs Additive HW (DW Test 1)

Null Hypothesis (Ho): The two models have equal predictive accuracy ($H_0: E[Loss(e_{1,t}) - Loss(e_{2,t})] = 0$)
Alternative Hypothesis (H_a): The two models have no equal predictive accuracy ($H_a: E[Loss(e_{1,t}) - Loss(e_{2,t})] \neq 0$)

2. SARIMA vs Multiplicative HW (Test 2)

Null Hypothesis (H_0): The two models have equal predictive accuracy ($H_0: E[Loss(e_{1,t}) - Loss(e_{3,t})] = 0$)
Alternative Hypothesis (H_b): The two models have no equal predictive accuracy ($H_a: E[Loss(e_{1,t}) - Loss(e_{3,t})] \neq 0$)

3. Additive HW vs Multiplicative HW (Test 3)

Null Hypothesis (H_0): The two models have equal predictive accuracy ($H_0: E[Loss(e_{2,t}) - Loss(e_{3,t})] = 0$)
Alternative Hypothesis (H_b): The two models have no equal predictive accuracy ($H_a: E[Loss(e_{2,t}) - Loss(e_{3,t})] \neq 0$)

The results of the analysis are provided in Table 6 as follows:

TABLE VI
DIEBOLD-MARIANO (DW) TEST RESULTS

Model Pair	DM Value	P-value
DM Test 1:SARIMA Vs Additive HW	-2.99	0.01
DM Test 2:SARIMA Vs Multiplicative HW	-2.75	0.02
DM Test 3:Additive HW Vs Multiplicative HW	-2.01	0.07

The test statistics for DM Tests for the 3 model pairs were: (DM=-2.99, p=.01), (DM=-2.75, p=.02), and (DM=-2.01, p=.07) respectively. Given that the p-value is less than .05 for tests 1 and 2, we reject the null hypothesis of equal predictive accuracy [5]. The p-value for DM test 3 is greater than .05 signifying no difference in predictive accuracies of the two models.

The model confidence set (MCS) procedure was applied to the 3 model pairs to eliminate the inferior models. The method is used to identify the statistically indistinguishable models. Using the MCS procedure at a 5% level of significance, Model 1 (SARIMA) was retained (p=.045) in the final model set. Model 2 (Additive HW) and Model 3 (Multiplicative HW) were excluded from the model set signifying that they had lower predictive accuracies than SARIMA.

The study results are consistent with findings by [16], which recommended the use of both the SARIMA and Holt-Winters models in forecasting Personal income taxes and VAT in South Africa. However, the study found the Holt-Winters model to outperform the SARIMA model in forecasting Corporate Income Tax (CIT) and Total Tax Revenue (TTAXR). The study findings are also consistent with [14], who found time series models to effectively predict VAT collection in Kenya using data from the financial year 2009/2010 to 2015/2016 and used the ARIMA model.

Despite of ARIMA model outperforming the Holt-Linear model in the forecasting of domestic VAT in Ghana, the resultant model was ARIMA (1,1,4) which is different from the results of this study (Makridakis *et al*., [17]).

IV. CONCLUSION

The estimates and predictive abilities of the SARIMA and Holt-Winters time series forecasting models were compared using historical data on value-added tax receipts collected between July 2014 and December 2020. The diagnostic checks on the data from the ACF, PACF, and the Ljung-Box statistics were satisfactory. Similarly, the tests for normality were met. This implied that data was fit for the application of the SARIMA and Holt-Winters models for forecasting. The SARIMA $(0,1,2)(0,1,1)_{12}$ model outperformed the additive and multiplicative Holt-Winters approaches in terms of predicting accuracy based on the size of forecasting errors (MAE=540.9, MAPE=5.04, and MASE=0.32). Similarly, the model confidence set (MCS) procedure retained the SARIMA model implying that it had a higher predictive accuracy (p=.045) than the additive and multiplicative Holt-Winters models at a 5% significance level. Using these results, the government can generate realistic projections for VAT. This could aid in reducing fiscal deficits. In general, both Holt-Winters and SARIMA models were sufficiently accurate to estimate Kenya's value-added tax receipts.

This research project has potential limitations. Some exogenous factors affect the economy of a country. These include political unrest, global economic instability, terrorism, and government policies. Moreover, the world faced the COVID-19 crisis, which affected the economy and revenue collection. In Kenya, the first case was reported on 12th March 2020. These could affect the accuracy of the VAT model in Kenya since they cause structural breaks.

These structural discontinuities may be taken into account in future studies. The macroeconomic variables and exogenous factors such as the inflation rate, currency value, political instability, and unemployment levels can be incorporated.

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