Finite-Time Anti-Synchronization for Memristive Neural Networks with Time-Varying Delays

Lian Duan, Ziyue Zhang, Ziyang Li

***Abstract*—This paper addresses the finite-time anti-synchronization issue for a type of delayed memristive neural networks. By designing a novel memoryless state-feedback controller, novel criteria on finite-time anti-synchronization of the addressed system are discovered based on drive-response framework, rigorous mathematical analysis techniques and differential inclusions theory. The established theoretical results indicate that the switching between finite-time and fixed-time anti-synchronization depends on the position of the initial functions, which are essentially different from existing switching mechanism. In addition, a simulated example is given to verify the validity of the theoretical findings.**

***Index Terms*—Memristive neural network; Finite-/fixed-time anti-synchronization; Time delay.**

I. Introduction

IN 1971, Chua firstly proposed the concept of memristors, until 2008, Stanley Williams et al. at Hewlett Packard (HP) Laboratory had achieved true memristors, called a fourth circuit component [1,2]. The size of the memristor value relies on the total amount of charge flowing into the device. Hence, Memristors are able to remember their previous experiences, and this memory property makes them promising candidates for mimicking biological synapses. In order to further expand the application range of neural networks, it usually replaces the conventional resistor in network models with the memristor, and then the conventional neural networks become the memristive neural networks (MNNs). Given the complex characteristics of memristors, the MNN models have sufficient computing power and can accommodate more information, which are of great significance for practical applications such as associative memory and information processing (see [3-7]).

Recently, the study of dynamic behaviors of NNs has became a hot research topic, such as stability [8], synchronization [9], and almost periodicity [10], [11]. In particular, synchronization related to MNNs is another meaningful topic in describing the dynamic trajectories of nodes in various neural network models, which can converge in finite horizon. Until now, there are numerous meaningful results regarding synchronization of MNNs, like exponential synchronization [12], lag complete synchronization [13], and asymptotic synchronization [14]. Remarkably, the exponential or asymptotic synchronization modes cannot guarantee that the status of the corresponding error system approaches to zero within a finite time, this greatly limits the application scope of neural networks. Actually, faster or even limited convergence time is usually required in practical applications, for instance, intercepting missiles can track targets within a limited time, even in the presence of wind deviation and signal interference [15]. Furthermore, finite-time synchronization refers to the optimal convergence time, and it also admits superior robustness and anti-interference performance [16]. For this reason, the studies on finite-time synchronization issues of MNNs have aroused the enthusiasm of many researchers and made fruitful theoretical results [17-21]. It is obvious that the above-mentioned results basically based on kinds of finite-time or fixed-time stability theorems, the switching between these two synchronization relies on the control parameters and is irrelevant to the position of the initial function. This prompts us to search for new methods to explore the finite-time synchronization problem of delayed MNNs without using the finite-time stability theory.

Meanwhile, anti-synchronization describes the behavior where the sum of relevant state variables of neural network nodes is zero. Or, to put it another way, the state variables of the nodes have the reverse signs but the identical modulus, like the chaotic behavior that often appears in neural networks. It also can be applied in network communication to transmit digital signals through continuous transformations of synchronization and anti-synchronization, which greatly improves the security and confidentiality of the communication process [22]. So, the anti-synchronization issue has become a crucial research topic by virtue of these important applications in engineering fields. Nevertheless, as we have seen, there is little literature on finite-time anti-synchronization issues MNN without using the existing finite-time stability theorems.

Motivated by above discussions, the finite-time anti-synchronization problem of a class of time-varying delayed memristive neural networks is studied in this paper. Specifically, under the drive-response framework, by using the Lyapunov stability theory and rigorous mathematical analysis techniques, a class of memoryless state-feedback controller is designed to realize the finite-time stability for the anti-synchronization error system. It is believed that this article brings some new and effective approaches for the qualitative analysis of MNNs. Finally, the established results are tested by a simulated example.

The remaining structure of this article is summarized as

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below. In Section 2, we give some necessary preliminaries. In Section 3, a type of memoryless state-feedback controller is designed to realize the finite-time anti-synchronization of the drive-response MNNs. In Section 4, a simulation numerical example is given to check the theoretical analysis. Ultimately, conclusions will be drawn in Section 5.

II. PRELIMINARIES

In this article, we consider the following MNNs

$$\begin{aligned}&\frac{d}{dt}z_\imath(t) = -d_\imath z_\imath(t)\\&+\sum_{\jmath=1}^{n} A_{\imath\jmath}\Big(\wp_\jmath(z_\jmath(t)) - z_\imath(t)\Big)\wp_\jmath(z_\jmath(t))\\&+\sum_{\jmath=1}^{n} B_{\imath\jmath}\Big(\wp_\jmath(z_\jmath(t-\iota_\jmath(t))) - z_\imath(t)\Big)\wp_\jmath(z_\jmath(t-\iota_\jmath(t)))\\&+I_\imath, \quad t\geq 0,\ \imath=1,2,...,n, \qquad (1)\end{aligned}$$

in which $z_\imath(t)$ means the voltage of the capacitor, $d_\imath > 0$ means the neuron self-inhibition, $\iota_\jmath(t)$ stands for the time-varying delay satisfying $0 \leq \iota_\jmath(t) \leq \iota, \iota = \max\limits_{\jmath=1,2,\ldots,n}\{\sup\limits_{t\geq 0}\iota_\jmath(t)\}$, $I_\imath$ is external constant input or bias, $\wp_\jmath(\cdot)$ represents the feedback function, $A_{\imath\jmath}(\wp_\jmath(z_\jmath(t)) - z_\imath(t))$, $B_{\imath\jmath}(\wp_\jmath(z_\jmath(t-\iota_\jmath(t))) - z_\imath(t))$ mean the feedback connection weight and the delayed feedback connection weight, respectively, which are carried out by memristor and their memductance dictated by the voltage applied to the memristor. Therefore, according to the mathematical model of the memductance established in [23], $A_{\imath\jmath}(z)$ and $B_{\imath\jmath}(z)$ can be described as:

$$A_{\imath\jmath}(z) \doteq A_{\imath\jmath}\Big(\wp_\jmath(z_\jmath(t)) - z_\imath(t)\Big) = \begin{cases} \dot{\Theta}_{\imath\jmath}, & \frac{d\wp_\jmath(z_\jmath(t))}{dt} - \frac{dz_\imath(t)}{dt} < 0,\\ \text{unchange}, & \frac{d\wp_\jmath(z_\jmath(t))}{dt} - \frac{dz_\imath(t)}{dt} = 0,\\ \ddot{\Theta}_{\imath\jmath}, & \frac{d\wp_\jmath(z_\jmath(t))}{dt} - \frac{dz_\imath(t)}{dt} > 0,\end{cases}$$

and

$$B_{\imath\jmath}(z) \doteq B_{\imath\jmath}\Big(\wp_\jmath(z_\jmath(t-\iota_\jmath(t))) - z_\imath(t)\Big) = \begin{cases} \dot{\Pi}_{\imath\jmath}, & \frac{d\wp_\jmath(z_\jmath(t-\iota_\jmath(t)))}{dt} - \frac{dz_\imath(t)}{dt} < 0,\\ \text{unchange}, & \frac{d\wp_\jmath(z_\jmath(t-\iota_\jmath(t)))}{dt} - \frac{dz_\imath(t)}{dt} = 0,\\ \ddot{\Pi}_{\imath\jmath}, & \frac{d\wp_\jmath(z_\jmath(t-\iota_\jmath(t)))}{dt} - \frac{dz_\imath(t)}{dt} > 0.\end{cases}$$

The initial functions of system (1) are

$$z_\imath(\hbar) = \overline{\varphi}_\imath(\hbar), \hbar \in [-\iota, 0],$$

and

$$\overline{\varphi}_\imath(\hbar) \in \mathcal{C}([-\iota,0],\Re), \imath = 1,2,...,n.$$

Let MNNs (1) be the drive system and consider the following response MNNs

$$\begin{aligned}&\frac{d}{dt}w_\imath(t) = -d_\imath w_\imath(t)\\&+\sum_{\jmath=1}^{n} A_{\imath\jmath}\Big(\wp_\jmath(w_\jmath(t)) - w_\imath(t)\Big)\wp_\jmath(w_\jmath(t))\\&+\sum_{\jmath=1}^{n} B_{\imath\jmath}\Big(\wp_\jmath(w_\jmath(t-\iota_\jmath(t))) - w_\imath(t)\Big)\wp_\jmath(w_\jmath(t-\iota_\jmath(t)))\\&+I_\imath + \overline{\mu}_\imath(t), \quad t\geq 0,\ \imath=1,2,...,n, \qquad (2)\end{aligned}$$

where $w_\imath(t)$ means the state of the response system, $\overline{\mu}_\imath(t)$ denotes the control input for reaching the anti-synchronization target, and other system parameters admit the same meaning as system (1). The initial data of system (2) is given by

$$w_\imath(\hbar) = \hat{\varphi}_\imath(\hbar), \hbar \in [-\iota, 0],$$

and

$$\hat{\varphi}_\imath(\hbar) \in \mathcal{C}([-\iota,0],\Re), \imath = 1,2,...,n.$$

Next, we consider the anti-synchronization error $e_\imath(t) = z_\imath(t) + w_\imath(t)$ with initial functions $e_\imath(\hbar) = \bar{\varphi}_\imath(\hbar) + \hat{\varphi}_\imath(\hbar)$, $\hbar \in [-\iota, 0]$, $e_\imath(\hbar) \in \mathcal{C}([-\iota,0],\Re), \imath = 1,2,...,n$. Adding (1) and (2) yields that

$$\begin{aligned}&\frac{d}{dt}e_\imath(t) = -d_\imath e_\imath(t) + \sum_{\jmath=1}^{n} A_{\imath\jmath}(z)\wp_\jmath(z_\jmath(t))\\&+\sum_{\jmath=1}^{n} A_{\imath\jmath}(w)\wp_\jmath(w_\jmath(t)) + \sum_{\jmath=1}^{n} B_{\imath\jmath}(z)\wp_\jmath(z_\jmath(t-\iota_\jmath(t)))\\&+\sum_{\jmath=1}^{n} B_{\imath\jmath}(w)\wp_\jmath(w_\jmath((t-\iota_\jmath(t))) + 2I_\imath\\&+\overline{\mu}_\imath(t), \quad a.a.\ t\geq 0,\ \imath=1,2,...,n. \qquad (3)\end{aligned}$$

Definition 2.1 The drive-response systems (1) and (2) are said to be finite-time anti-synchronized if, for a appropriate designed feedback controller $\overline{\mu}_\imath(t)$, there is a time $T(\bar{\varphi}_\imath, \hat{\varphi}_\imath) \geq 0$ such that

$$\lim_{t\to T(\bar{\varphi}_\imath,\hat{\varphi}_\imath)} |e_\imath(t)| = 0,$$

and

$$|e_\imath(t)| \equiv 0, \quad t \geq T(\bar{\varphi}_\imath, \hat{\varphi}_\imath), \quad \imath = 1,2,...,n,$$

where $T(\bar{\varphi}_\imath, \hat{\varphi}_\imath)$ is called the settling time of anti-synchronization, which relies on the initial functions. Furthermore, if $T(\bar{\varphi}_\imath, \hat{\varphi}_\imath)$ is uniformly bounded, in other words, there is a number $T_{\max} > 0$ such that $T(\bar{\varphi}_\imath, \hat{\varphi}_\imath) \leq T_{\max}$, then systems (1) and (2) are said to be fixed-time anti-synchronized. From the above definition, one can find that the finite-time anti-synchronization of systems (1) and (2) is equivalent to the finite-time convergence of $e_\imath(t)$ described by (3) to zero. To this end, the following technical lemma and assumption on the activation functions are needed.

Lemma 2.1 (see Lemma 3 in [24]) Assuming that $e_\imath(t)$ is a solution of system (3), if it is a differential function on $\Re$. Then, the upper right Dini derivative $D^+|e_\imath(t)|$ of $|e_\imath(t)|$ is

$$D^+|e_\imath(t)| = \begin{cases} \frac{e_\imath(t)}{|e_\imath(t)|}\dot{e}_\imath(t), & \text{if } e_\imath(t) \neq 0,\\ \dot{e}_\imath(t), & \text{if } e_\imath(t)=0,\ \dot{e}_\imath(t) > 0,\\ -\dot{e}_\imath(t), & \text{if } e_\imath(t)=0,\ \dot{e}_\imath(t) < 0,\\ 0, & \text{if } e_\imath(t)=0,\ \dot{e}_\imath(t) = 0.\end{cases}$$

Assumption 2.2 For all $\imath = 1,2,...,n$, there exist nonnegative constants $\hat{\wp}_\imath$ and $l_\imath$ such that

$$|\wp_\imath(ß)| \leq \hat{\wp}_\imath, \quad |\wp_\imath(ß) + \wp_\imath(ç)| \leq l_\imath|ß + ç|, \text{ for all } ß, ç \in \Re.$$

Remark 2.1 There are some activation functions used in engineering fields satisfy Assumption 2.2, such as $\sin ß$ and $\arctan ß$.

III. Main results

In this part, we shall discuss the finite-time anti-synchronization problem for systems (1) and (2). We first design a memoryless state-feedback controller as follows

$$\overline{\mu}_\imath(t)=-sign(e_\imath(t))(\lambda_\imath|e_\imath(t)|+\beta_\imath|e_\imath(t)|^\theta+q_\imath),\ \ 0<\theta<1, \tag{4}$$

where $\lambda_\imath,\beta_\imath,q_\imath$ are positive constants to be determined later which are used to anti-synchronize the drive-response systems (1) and (2). To present concisely, here we let

$$\overline{A}_{\imath\jmath}=\max\{\dot{\Theta}_{\imath\jmath},\ddot{\Theta}_{\imath\jmath}\},\quad \underline{A}_{\imath\jmath}=\min\{\dot{\Theta}_{\imath\jmath},\ddot{\Theta}_{\imath\jmath}\},$$

$$\overline{B}_{\imath\jmath}=\max\{\dot{\Pi}_{\imath\jmath},\ddot{\Pi}_{\imath\jmath}\},\quad \underline{B}_{\imath\jmath}=\min\{\dot{\Pi}_{\imath\jmath},\ddot{\Pi}_{\imath\jmath}\}.$$

$$\hat{A}_{\imath\jmath}=\max\{|\dot{\Theta}_{\imath\jmath}|,|\ddot{\Theta}_{\imath\jmath}|\},\quad \hat{B}_{\imath\jmath}=\max\{|\dot{\Pi}_{\imath\jmath}|,|\ddot{\Pi}_{\imath\jmath}|\},$$

Theorem 3.1 Assume that Assumption 2.2 holds, if the error system (3) is controlled with the following memoryless state-feedback controller,

$$\overline{\mu}_\imath(t)=-sign(e_\imath(t))(\lambda_\imath|e_\imath(t)|+\beta_\imath|e_\imath(t)|^\theta+q_\imath),\ \ 0<\theta<1, \tag{5}$$

in which $\lambda_\imath>0$, $\beta_\imath>0$, $q_\imath>0$, and they satisfy

$$\lambda_\imath>-d_\imath+\sum_{\jmath=1}^n\hat{A}_{\imath\jmath}l_\jmath+\sum_{\jmath=1}^n\hat{B}_{\imath\jmath}l_\jmath\cdot 2^{\frac{\theta}{1-\theta}}, \tag{6}$$

$$\begin{aligned}q_\imath>&\sum_{\jmath=1}^n|\overline{A}_{\imath\jmath}-\underline{A}_{\imath\jmath}|\hat{\wp}_\jmath+\sum_{\jmath=1}^n|\overline{B}_{\imath\jmath}-\underline{B}_{\imath\jmath}|\hat{\wp}_\jmath\\&+\sum_{\jmath=1}^n\hat{B}_{\imath\jmath}l_\jmath\cdot 2^{\frac{\theta}{1-\theta}}\cdot((1-\theta)\beta_{\min}\iota)^{\frac{1}{1-\theta}}\\&+2|I_\imath|,\qquad \imath=1,2,...,n,\end{aligned} \tag{7}$$

then the response system (2) can be anti-synchronized with the drive system (1) in a finite time.

To prove this theorem, two lemmas need to be established.

Lemma 3.2 Under the conditions of Theorem 3.1, then for every $e_\imath(t)$ of system (3) with $\sup\limits_{-\iota\le\hbar\le 0}(\max\limits_{1\le\imath\le n}|e_\imath(\hbar)|)>1$, it would finite-timely cross the hyperplane with

$$\sup_{-\iota\le\hbar\le 0}(\max_{1\le\imath\le n}|e_\imath(\hbar)|)=1.$$

Proof. Since $0<\theta<1$, it can be concluded from (6) and (7) that

$$\lambda_\imath>-d_\imath+\sum_{\jmath=1}^n\hat{A}_{\imath\jmath}l_\jmath+\sum_{\jmath=1}^n\hat{B}_{\imath\jmath}l_\jmath \tag{8}$$

and

$$q_\imath>\sum_{\jmath=1}^n|\overline{A}_{\imath\jmath}-\underline{A}_{\imath\jmath}|\hat{\wp}_\jmath+\sum_{\jmath=1}^n|\overline{B}_{\imath\jmath}-\underline{B}_{\imath\jmath}|\hat{\wp}_\jmath+2|I_\imath|. \tag{9}$$

Continuity argument ensures that there is a sufficiently small $\epsilon>0$ satisfying

$$\epsilon-\lambda_\imath-d_\imath+\sum_{\jmath=1}^n\hat{A}_{\imath\jmath}l_\jmath+e^{\epsilon\iota}\sum_{\jmath=1}^n\hat{B}_{\imath\jmath}l_\jmath<0. \tag{10}$$

Denote

$$\Lambda(e(t))=\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}e^{\epsilon\hbar}|e_\imath(\hbar)|\right),\quad t\ge 0.$$

Obviously, $e^{\epsilon t}|e_\imath(t)|\le\Lambda(e(t))$, $\imath=1,2,...,n$, and we will discuss the following two situations:

Situation A. $e^{\epsilon t}|e_\imath(t)|<\Lambda(e(t))$, $\imath=1,2,...,n$. We obtain from the property of continuous functions that there is a sufficiently small $\zeta>0$, such that

$$e^{\epsilon\hbar}|e_\imath(\hbar)|<\Lambda(e(t)),\ \text{and}\ \Lambda(e(\hbar))<\Lambda(e(t)),$$

$$\text{for}\ \hbar\in(t,t+\zeta),\ \imath=1,2,...,n.$$

Situation B. If there exist a index $\imath_0$ and a time $t_0\ge 0$ satisfying $e^{\epsilon t_0}|e_{\imath_0}(t_0)|=\Lambda(e(t_0))$, we have from lemma 2.1 that

$$\begin{aligned}&D^+(e^{\epsilon t}|e_{\imath_0}(t)|)\Big|_{t=t_0}=\epsilon e^{\epsilon t_0}|e_{\imath_0}(t_0)|\\&+e^{\epsilon t_0}sign(e_{\imath_0}(t_0))\Big[-d_{\imath_0}e_{\imath_0}(t_0)\\&+\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\wp_\jmath(z_\jmath(t_0))+\sum_{\jmath=1}^nA_{\imath_0\jmath}(w)\wp_\jmath(w_\jmath(t_0))\\&+\sum_{\jmath=1}^nB_{\imath_0\jmath}(z)\wp_\jmath(z_\jmath(t_0-\iota_\jmath(t_0)))\\&+\sum_{\jmath=1}^nB_{\imath_0\jmath}(w)\wp_\jmath(w_\jmath(t_0-\iota_\jmath(t_0)))+2I_{\imath_0}\\&-sign(e_{\imath_0}(t_0))(\lambda_{\imath_0}|e_{\imath_0}(t_0)|+\beta_{\imath_0}|e_{\imath_0}(t_0)|^\theta\\&+q_{\imath_0})\Big].\end{aligned} \tag{11}$$

Observe that

$$\begin{aligned}&\Big|\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\wp_\jmath(z_\jmath(t_0))+\sum_{\jmath=1}^nA_{\imath_0\jmath}(w)\wp_\jmath(w_\jmath(t_0))\Big|\\&=\Big|\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\wp_\jmath(z_\jmath(t_0))+\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\wp_\jmath(w_\jmath(t_0))\\&+\sum_{\jmath=1}^nA_{\imath_0\jmath}(w)\wp_\jmath(w_\jmath(t_0))-\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\wp_\jmath(w_\jmath(t_0))\Big|\\&=\Big|\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\Big(\wp_\jmath(z_\jmath(t_0))+\wp_\jmath(w_\jmath(t_0))\Big)\\&+\Big(\sum_{\jmath=1}^nA_{\imath_0\jmath}(w)-\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\Big)\wp_\jmath(w_\jmath(t_0))\Big|\\&\le\Big|\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\Big(\wp_\jmath(z_\jmath(t_0))+\wp_\jmath(w_\jmath(t_0))\Big)\Big|\\&+\Big|\Big(\sum_{\jmath=1}^nA_{\imath_0\jmath}(w)-\sum_{\jmath=1}^nA_{\imath_0\jmath}(z)\Big)\wp_\jmath(w_\jmath(t_0))\Big|\\&\le\sum_{\jmath=1}^n\hat{A}_{\imath_0\jmath}l_\jmath|e_\jmath(t_0)|+\sum_{\jmath=1}^n|\overline{A}_{\imath_0\jmath}-\underline{A}_{\imath_0\jmath}|\hat{\wp}_\jmath.\end{aligned} \tag{12}$$

A similar manner as (12) produces

$$\begin{aligned}&\Big|\sum_{\jmath=1}^nB_{\imath_0\jmath}(z)\wp_\jmath(z_\jmath(t_0-\iota_\jmath(t_0)))\\&+\sum_{\jmath=1}^nB_{\imath_0\jmath}(w)\wp_\jmath(w_\jmath(t_0-\iota_\jmath(t_0)))\Big|\\&\le\sum_{\jmath=1}^n\hat{B}_{\imath_0\jmath}l_\jmath|e_\jmath(t_0-\iota_\jmath(t_0))|\end{aligned}$$

$$+\sum_{\jmath=1}^{n}|\overline{B}_{\imath_0\jmath}-\underline{B}_{\imath_0\jmath}|\hat{\wp}_{\jmath}. \quad (13)$$

Moreover, it is easy to see from

$$e^{\epsilon(t_0-\iota_\jmath(t_0))}|e_\jmath(t_0-\iota_\jmath(t_0))|\le e^{\epsilon t_0}|e_{\imath_0}(t_0)|,$$

that

$$|e_\jmath(t_0-\iota_\jmath(t_0))|\le e^{\epsilon\iota}|e_{\imath_0}(t_0)|,\ \jmath=1,2,...,n. \quad (14)$$

From (11), (12), (13), and (14), one deduces that

$$\begin{aligned}
&D^+(e^{\epsilon t}|e_{\imath_0}(t)|)\Big|_{t=t_0}\\
&\le e^{\epsilon t_0}\Big[\epsilon|e_{\imath_0}(t_0)|-d_{\imath_0}|e_{\imath_0}(t_0)|+\sum_{\jmath=1}^{n}\hat{A}_{\imath_0\jmath}l_\jmath|e_\jmath(t_0)|\\
&+\sum_{\jmath=1}^{n}|\overline{A}_{\imath_0\jmath}-\underline{A}_{\imath_0\jmath}|\hat{\wp}_\jmath+\sum_{\jmath=1}^{n}\hat{B}_{\imath_0\jmath}l_\jmath e^{\epsilon\iota}|e_{\imath_0}(t_0)|\\
&+\sum_{\jmath=1}^{n}|\overline{B}_{\imath_0\jmath}-\underline{B}_{\imath_0\jmath}|\hat{\wp}_\jmath+2|I_{\imath_0}|-\lambda_{\imath_0}|e_{\imath_0}(t_0)|\\
&-\beta_{\imath_0}|e_{\imath_0}(t_0)|^\theta-q_{\imath_0}\Big]\\
&\le e^{\epsilon t_0}\Big[\Big(\epsilon-d_{\imath_0}+\sum_{\jmath=1}^{n}\hat{A}_{\imath_0\jmath}l_\jmath+\sum_{\jmath=1}^{n}\hat{B}_{\imath_0\jmath}l_\jmath e^{\epsilon\iota}\\
&-\lambda_{\imath_0}\Big)|e_{\imath_0}(t_0)|+\sum_{\jmath=1}^{n}|\overline{A}_{\imath_0\jmath}-\underline{A}_{\imath_0\jmath}|\hat{\wp}_\jmath\\
&+\sum_{\jmath=1}^{n}|\overline{B}_{\imath_0\jmath}-\underline{B}_{\imath_0\jmath}|\hat{\wp}_\jmath+2|I_{\imath_0}|-q_{\imath_0}\Big]\\
&<0. \qquad (15)
\end{aligned}$$

Therefore, there exists a $\zeta>0$, such that $e^{\epsilon\hbar}|e_{\imath_0}(\hbar)|<e^{\epsilon t_0}|e_{\imath_0}(t_0)|$, and $\Lambda(e(\hbar))<\Lambda(e(t_0))$, $\hbar\in(t_0,t_0+\zeta)$.

We draw a conclusion from the above two situations that $\Lambda(e(t))$ is decreasing, and $\Lambda(e(t))\le\Lambda(e(0))$, $t\ge0$, and thus

$$e^{\epsilon(t-\iota)}\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)\le\Lambda(e(t))\le\Lambda(e(0)),$$

which means that

$$\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)\le\frac{\Lambda(e(0))}{e^{\epsilon(t-\iota)}}.$$

Accordingly, $\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)$ will be less than 1 as t increases. Let the first time satisfy $\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)=1$ as T_1, then we obtain

$$T_1\le\frac{\ln\Lambda(e(0))}{\epsilon}+\iota,$$

which shows that each anti-error function $e_\imath(t)$ would cross the hyperplane $\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)=1$, and the time it takes is no more than T_1. Lemma 3.2 proof completed.

Lemma 3.3 Under the conditions of Theorem 3.1. Then for every $e_\imath(t)$ of system (3) with $\sup_{-\iota\le\hbar\le0}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|\right)\le1$ would fixed-timely flow to 0.

Proof. Let

$$\begin{aligned}
&\Xi(e(t))=\sup_{t-\iota\le\hbar\le t}\left(\max_{1\le\imath\le n}\frac{|e_\imath(\hbar)|^{1-\theta}}{1-\theta}+\beta_{\min}\hbar\right),\\
&\beta_{\min}=\min_{1\le\imath\le n}\{\beta_\imath\},\quad t\ge0. \qquad (16)
\end{aligned}$$

It is obvious that

$$\frac{|e_\imath(t)|^{1-\theta}}{1-\theta}+\beta_{\min}t\le\Xi(e(t)),\quad \imath=1,2,...,n,$$

and if there is an index $\imath_1$ and a time $t_1\ge0$, such that $\frac{|e_{\imath_1}(t_1)|^{1-\theta}}{1-\theta}+\beta_{\min}t_1=\Xi(e(t_1))$, it then follows that

$$\begin{aligned}
&D^+\Big(\frac{|e_{\imath_1}(t)|^{1-\theta}}{1-\theta}+\beta_{\min}t\Big)\Big|_{t=t_1}\\
&=|e_{\imath_1}(t_1)|^{-\theta}\times sign(e_{\imath_1}(t_1))\Big[-d_{\imath_1}e_{\imath_1}(t_1)\\
&+\sum_{\jmath=1}^{n}A_{\imath_1\jmath}(z)\wp_\jmath(z_\jmath(t_1))+\sum_{\jmath=1}^{n}A_{\imath_1\jmath}(w)\wp_\jmath(w_\jmath(t_1))\\
&+\sum_{\jmath=1}^{n}B_{\imath_1\jmath}(z)\wp_\jmath(z_\jmath(t_1-\iota_\jmath(t_1)))\\
&+\sum_{\jmath=1}^{n}B_{\imath_1\jmath}(w)\wp_\jmath(w_\jmath(t_1-\iota_\jmath(t_1)))\\
&+2I_{\imath_1}-sign(e_{\imath_1}(t_1))(\lambda_{\imath_1}|e_{\imath_1}(t_1)|\\
&+\beta_{\imath_1}|e_{\imath_1}(t_1)|^\theta+q_{\imath_1})\Big]+\beta_{\min}\\
&\le|e_{\imath_1}(t_1)|^{-\theta}\times\Big(-d_{\imath_1}|e_{\imath_1}(t_1)|\\
&+\sum_{\jmath=1}^{n}\hat{A}_{\imath_1\jmath}l_\jmath|e_\jmath(t_1)|+\sum_{\jmath=1}^{n}|\overline{A}_{\imath_1\jmath}-\underline{A}_{\imath_1\jmath}|\hat{\wp}_\jmath\\
&+\sum_{\jmath=1}^{n}\hat{B}_{\imath_1\jmath}l_\jmath|e_\jmath(t_1-\iota_\jmath(t_1))|+\sum_{\jmath=1}^{n}|\overline{B}_{\imath_1\jmath}-\underline{B}_{\imath_1\jmath}|\hat{\wp}_\jmath\\
&+2|I_{\imath_1}|-\lambda_{\imath_1}|e_{\imath_1}(t_1)|-\beta_{\imath_1}|e_{\imath_1}(t_1)|^\theta-q_{\imath_1}\Big)\\
&+\beta_{\min}. \qquad (17)
\end{aligned}$$

Observe that

$$\begin{aligned}
&\frac{|e_\jmath(t_1-\iota_\jmath(t_1))|^{1-\theta}}{1-\theta}+\beta_{\min}(t_1-\iota_\jmath(t_1))\\
&\le\frac{|e_{\imath_1}(t_1)|^{1-\theta}}{1-\theta}+\beta_{\min}t_1.
\end{aligned}$$

A simple calculation produces

$$\begin{aligned}
&|e_\jmath(t_1-\iota_\jmath(t_1))|\\
&\le\Big(|e_{\imath_1}(t_1)|^{1-\theta}+(1-\theta)\beta_{\min}\iota\Big)^{\frac{1}{1-\theta}}\\
&=|e_{\imath_1}(t_1)|\Big(1+\frac{(1-\theta)\beta_{\min}\iota}{|e_{\imath_1}(t_1)|^{1-\theta}}\Big)^{\frac{1}{1-\theta}}\\
&\le|e_{\imath_1}(t_1)|\times2^{\frac{\theta}{1-\theta}}\Big(1+\frac{((1-\theta)\beta_{\min}\iota)^{\frac{1}{1-\theta}}}{|e_{\imath_1}(t_1)|}\Big)\\
&=2^{\frac{\theta}{1-\theta}}\Big(|e_{\imath_1}(t_1)|+((1-\theta)\beta_{\min}\iota)^{\frac{1}{1-\theta}}\Big),
\end{aligned}$$

which, together with (17), results in

$$
\begin{aligned}
& D^{+}\Big(\frac{|e_{\imath_1}(t)|^{1-\theta}}{1-\theta}+\beta_{\min}t\Big)\Big|_{t=t_1} \\
\le\ & |e_{\imath_1}(t_1)|^{-\theta}\Big[-d_{\imath_1}|e_{\imath_1}(t_1)| \\
& +\sum_{\jmath=1}^{n}\hat{A}_{\imath_1\jmath}l_\jmath|e_\jmath(t_1)|+\sum_{\jmath=1}^{n}|\overline{A}_{\imath_1\jmath}-\underline{A}_{\imath_1\jmath}|\hat{\wp}_\jmath \\
& +\sum_{\jmath=1}^{n}\hat{B}_{\imath_1\jmath}l_\jmath 2^{\frac{\theta}{1-\theta}}(|e_{\imath_1}(t_1)|+((1-\theta)\beta_{\min}\iota)^{\frac{1}{1-\theta}}) \\
& +\sum_{\jmath=1}^{n}|\overline{B}_{\imath_1\jmath}-\underline{B}_{\imath_1\jmath}|\hat{\wp}_\jmath+2|I_{\imath_1}| \\
& -\lambda_{\imath_1}|e_{\imath_1}(t_1)|-\beta_{\imath_1}|e_{\imath_1}(t_1)|^{\theta}-q_{\imath_1}\Big]+\beta_{\min} \\
\le\ & |e_{\imath_1}(t_1)|^{-\theta}\Big[\Big(-d_{\imath_1}+\sum_{\jmath=1}^{n}\hat{A}_{\imath_1\jmath}l_\jmath+\sum_{\jmath=1}^{n}\hat{B}_{\imath_1\jmath}l_\jmath 2^{\frac{\theta}{1-\theta}} \\
& -\lambda_{\imath_1}\Big)|e_{\imath_1}(t_1)|+\sum_{\jmath=1}^{n}|\overline{A}_{\imath_1\jmath}-\underline{A}_{\imath_1\jmath}|\hat{\wp}_\jmath \\
& +\sum_{\jmath=1}^{n}\hat{B}_{\imath_1\jmath}l_\jmath 2^{\frac{\theta}{1-\theta}}((1-\theta)\beta_{\min}\iota)^{\frac{1}{1-\theta}} \\
& +\sum_{\jmath=1}^{n}|\overline{B}_{\imath_1\jmath}-\underline{B}_{\imath_1\jmath}|\hat{\wp}_\jmath+2|I_{\imath_1}|-\beta_{\imath_1}|e_{\imath_1}(t_1)|^{\theta}-q_{\imath_1}\Big] \\
& +\beta_{\min} \\
<\ & -|e_{\imath_1}(t_1)|^{-\theta}\times\beta_{\imath_1}|e_{\imath_1}(t_1)|^{\theta}+\beta_{\min} \\
\le\ & 0. \qquad (18)
\end{aligned}
$$

This implies that there is a $\chi>0$, such that

$$
\frac{|e_{\imath_1}(\hbar)|^{1-\theta}}{1-\theta}+\beta_{\min}\hbar\le\frac{|e_{\imath_1}(t_1)|^{1-\theta}}{1-\theta}+\beta_{\min}t_1,
$$

$$\text{for } \hbar\in(t_1,t_1+\chi).$$

Hence, from the above discussion, we conclude that

$$
\begin{aligned}
& \max_{1\le\imath\le n}\frac{|e_\imath(t)|^{1-\theta}}{1-\theta}+\beta_{\min}t\le\Xi(e(t))\le\Xi(e(0)) \\
& \quad\le\sup_{-\iota\le\hbar\le0}\left(\max_{1\le\imath\le n}\frac{|e_\imath(\hbar)|^{1-\theta}}{1-\theta}\right),\quad t\ge0,
\end{aligned}
$$

that is,

$$
\begin{aligned}
\left(\max_{1\le\imath\le n}|e_\imath(t)|\right)^{1-\theta}\le\ & \sup_{-\iota\le\hbar\le0}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|^{1-\theta}\right) \\
& -\beta_{\min}(1-\theta)t,\quad t\ge0,
\end{aligned}
$$

and hence, we have that $\max\limits_{1\le\imath\le n}|e_\imath(t)|$ would flow to 0 as time t increases. Denote T_2 as the time such that $\max\limits_{1\le\imath\le n}|e_\imath(T_2)|=0$, then we have

$$
\begin{aligned}
T_2\le\ & \frac{1}{\beta_{\min}(1-\theta)}\Big\{\sup_{-\iota\le\hbar\le0}\left(\max_{1\le\imath\le n}|e_\imath(\hbar)|^{1-\theta}\right) \\
& -\left(\max_{1\le\imath\le n}|e_\imath(T_2)|\right)^{1-\theta}\Big\} \\
\le\ & \frac{1}{\beta_{\min}(1-\theta)}(1-0) \\
=\ & \frac{1}{\beta_{\min}(1-\theta)}, \qquad (19)
\end{aligned}
$$

which means that for each $|e_\imath(t)|$ it takes no longer than $\frac{1}{\beta_{\min}(1-\theta)}$ from 1 to 0, $\imath=1,2,...,n$. This proof is finished. With the above two lemmas in hand, we can give the proof of the main results.

Proof of Theorem 3.1. For each solution $e_\imath(t)$ of anti-error system (3), the finite-time anti-synchronization of systems (1)-(2) can be realized according to the location of the initial error function. Specially speaking,

Situation A: $\sup\limits_{-\iota\le\hbar\le0}(\max\limits_{1\le\imath\le n}|e_\imath(\hbar)|)\le1$.

In the situation, we know from Lemma 3.3 that every anti-error function $e_\imath(t)$ would flow to 0, and the required time will not exceed $\frac{1}{\beta_{\min}(1-\theta)}$, $\imath=1,2,...,n$. Namely, the drive-response systems (1) and (2) achieve fixed-time anti-synchronization under the designed controller.

Situation B: $\sup\limits_{-\iota\le\hbar\le0}(\max\limits_{1\le\imath\le n}|e_\imath(\hbar)|)>1$.

In the situation, we can find from Lemma 3.2 that $|e_\imath(t)|$ would flow to 1 within a finite-time, which is less than $\frac{\ln\Lambda(e(0))}{\epsilon}+\iota$.

To sum up, as time t increases, every $e_\imath(t)$ would reach 0 in finite time T_{all} at last, and $T_{all}\le\frac{\ln\Lambda(e(0))}{\epsilon}+\iota+\frac{1}{\beta_{\min}(1-\theta)}$.

IV. A NUMERICAL EXAMPLE

In this section, we will provide a simulated example to explain the validity of the proposed theoretical results.

Example 4.1. Consider the following drive MNNs with time delays

$$
\begin{cases}
\dot{z}_1(t)= & -z_1(t)+A_{11}(z)\wp_1(z_1(t))+A_{12}(z)\wp_2(z_2(t)) \\
& +B_{11}(z)\wp_1(z_1(t-0.2)) \\
& +B_{12}(z)\wp_2(z_2(t-0.3))+0.4, \\
\dot{z}_2(t)= & -2z_2(t)+A_{21}(z)\wp_1(z_1(t)) \\
& +A_{22}(z)\wp_2(z_2(t))+B_{21}(z)\wp_1(z_1(t-0.2)) \\
& +B_{22}(z)\wp_2(z_2(t-0.3))+0.6,
\end{cases} \tag{20}
$$

where

$$
\overline{A}=(\ddot{\Theta}_{\imath\jmath})_{2\times2}=\begin{pmatrix}-1.1 & 0.33\\ 1.2 & -0.82\end{pmatrix},
$$

$$
\underline{A}=(\dot{\Theta}_{\imath\jmath})_{2\times2}=\begin{pmatrix}1.8 & -0.92\\ 2.1 & -0.6\end{pmatrix},
$$

$$
\overline{B}=(\ddot{\Pi}_{\imath\jmath})_{2\times2}=\begin{pmatrix}0.47 & -1.92\\ -1.92 & 0.31\end{pmatrix},
$$

$$
\underline{B}=(\dot{\Pi}_{\imath\jmath})_{2\times2}=\begin{pmatrix}0.96 & -3.11\\ -2.96 & 0.17\end{pmatrix},
$$

the initial functions of system (20) are given as $z_1(\hbar)=3.6, z_2(\hbar)=-4.8, \hbar\in[-0.3,0]$.

The corresponding response MNNs with time delays are presented as

$$
\begin{cases}
\dot{w}_1(t)= & -w_1(t)+A_{11}(w)\wp_1(w_1(t)) \\
& +A_{12}(w)\wp_2(w_2(t)) \\
& +B_{11}(w)\wp_1(w_1(t-0.2)) \\
& +B_{12}(w)\wp_2(w_2(t-0.3)) \\
& +0.4+\overline{\mu}_1(t), \\
\dot{w}_2(t)= & -2w_2(t)+A_{21}(w)\wp_1(w_1(t)) \\
& +A_{22}(w)\wp_2(w_2(t)) \\
& +B_{21}(w)\wp_1(w_1(t-0.2)) \\
& +B_{22}(w)\wp_2(w_2(t-0.3)) \\
& +0.6+\overline{\mu}_2(t),
\end{cases} \tag{21}
$$

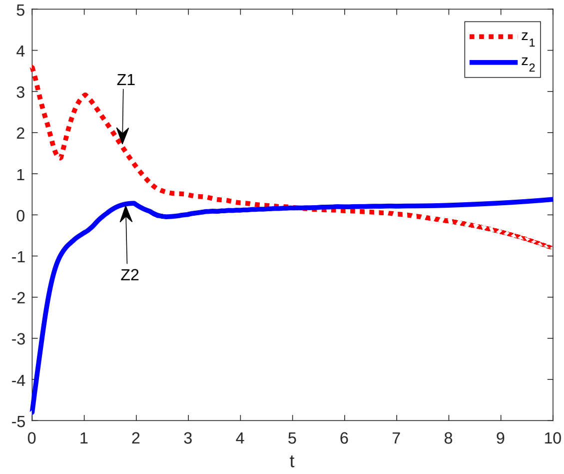

Fig. 1. States of $z_\imath(t), \imath = 1, 2$.

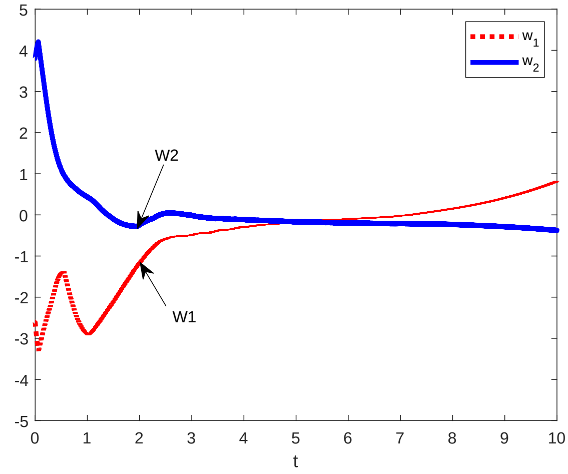

Fig. 2. States of $w_\imath(t), \imath = 1, 2$.

the initial functions of system (21) are presented by as $w_1(\hbar) = -2.6, w_2(\hbar) = 3.8, \hbar \in [-0.3, 0]$.

Let $\wp_\imath(z_\imath(t)) = \sin(z_\imath(t)), \wp_\imath(w_\imath(t)) = \sin(w_\imath(t)), \imath = 1, 2$, and $\overline{\mu}_1(t)$, $\overline{\mu}_2(t)$ are designed as

$$\overline{\mu}_1(t) = -sign(e_1(t))(10.1|e_1(t)| + 1.2|e_1(t)|^{\frac{1}{2}} + 9.8),$$
$$\overline{\mu}_2(t) = -sign(e_2(t))(8.5|e_2(t)| + |e_2(t)|^{\frac{1}{2}} + 8.2).$$

Obviously, $l_\imath = 1, \hat{\wp}_\imath = 1, \imath = 1, 2$, and it can be calculated that

$$\lambda_1 = 10.1 > 9.86 = -d_1 + \sum_{\jmath=1}^{2} \hat{A}_{1\jmath} l_\jmath + \sum_{\jmath=1}^{2} \hat{B}_{1\jmath} l_\jmath \times 2,$$

$$\lambda_2 = 8.5 > 7.46 = -d_2 + \sum_{\jmath=1}^{2} \hat{A}_{2\jmath} l_\jmath + \sum_{\jmath=1}^{2} \hat{B}_{2\jmath} l_\jmath \times 2,$$

$$q_1 = 9.8 > 6.8 \approx \sum_{\jmath=1}^{2} |\overline{A}_{1\jmath} - \underline{A}_{1\jmath}| \hat{\wp}_\jmath$$
$$+ \sum_{\jmath=1}^{2} |\overline{B}_{1\jmath} - \underline{B}_{1\jmath}| \hat{\wp}_\jmath + \sum_{\jmath=1}^{2} \hat{B}_{1\jmath} l_\jmath \times 0.5 \times 0.3^2 + 2|I_1|,$$
$$q_2 = 8.2 > 3.65 \approx \sum_{\jmath=1}^{2} |\overline{A}_{2\jmath} - \underline{A}_{2\jmath}| \hat{\wp}_\jmath$$
$$+ \sum_{\jmath=1}^{2} |\overline{B}_{2\jmath} - \underline{B}_{2\jmath}| \hat{\wp}_\jmath + \sum_{\jmath=1}^{2} \hat{B}_{2\jmath} l_\jmath \times 0.5 \times 0.3^2 + 2|I_2|.$$

This means that all assumptions of Theorem 3.1 are valid, and it can be deduced that the finite-time anti-synchronization between systems (20)-(21) is achieved. In the simulation, the time evolution of synchronization curves of drive-response systems are presented in Figs. 1-3, we can know from Fig. 3 that a clear convergence of the trajectory of the anti-error dynamics to the origin in finite-time. These show that the drive system (20) anti-synchronizes with its response system (21) in finite-time.

Remark 4.1 In such as [17-21], some conditions for finite-time synchronization of relevant MNNS are established using finite-time stability theorems, but in this article, new criteria

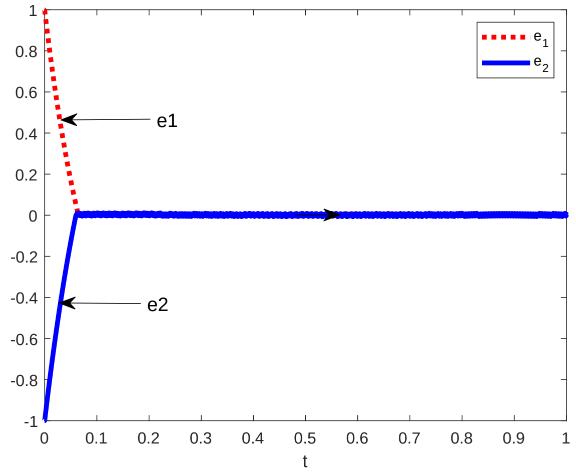

Fig. 3. States of $e_\imath(t), \imath = 1, 2$.

for finite-time anti-synchronization of systems (1)-(2) are obtained using rigorous analytical techniques without utilizing various finite-time stability theorems. On the other hand, the designed controller (4) does not depend on time delays and only rely on the current states, which means that such type of controller can be validated and realized easily in practice.

V. Conclusion

In this paper, inspired by [25,26], we have reinvestigated finite-time anti-synchronization of the MNNs with time-varying delays. By using a novel analytical method and designing a novel memoryless state-feedback controller, we establish some new criteria realizing the finite-time anti-synchronization for considered MNNs. A numerical example has also been given to demonstrate the effectiveness of our results. To the best of our knowledge, this is the first paper to study the finite-time anti-synchronization for delayed MNNs without the help of finite-time stability theory, which further enriches the synchronization theory of MNNs.

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