New Heuristic Method Based on Evolution of Random Points to the Pareto Frontier for Bi-objective Optimization Problems

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***Abstract*—We introduce a Pareto dominance-based heuristic designed to address bi-objective optimization problems by tracing the evolution from a random point to the Pareto frontier. The heuristic consists of three main steps. First, a feasible random approximation for the bi-objective problem is established. Next, this point evolves towards the optimal objectives and the Pareto frontier, leveraging trust regions around each approximation. Additionally, at specific iteration intervals, the current approximations optimizing the objectives are integrated as new approximations to the Pareto frontier, enriching the set of points along the frontier. This process iterates until convergence is reached. We would also like to propose an enhanced version of this method, which includes multiple initial points while maintaining the original method's structure. In this work, we compare the performance of the classic Weighted Sum (WS) method with both the EORP and the improved EMRP methods for solving the energy efficiency (EE) and spectral efficiency (SE) trade-off in optical code division multiple access (OCDMA) communication systems. The WS method is examined in two variants: one combined with the Hill Climbing heuristic (WS-HC) and the other with the Particle Swarm Optimization heuristic (WS-PSO).**

***Index Terms*—Bi-objective optimization, Pareto frontier, Heuristics, communication systems, Optimization, OCDMA**

I. Introduction

In this investigation, we delved into the realm of bi-objective optimization (BOO), which presents a departure from the paradigm of single objective optimization (SOO). Unlike SOO, where a singular optimal solution can typically be pinpointed, BOO poses a unique challenge as there is typically no single solution that simultaneously maximizes or minimizes both objectives. The essence of BOO lies in acknowledging that both objectives hold equal significance, leading to the pursuit of a solution set delineated by a curve representing acceptable compromises between the objectives, known as the Pareto frontier. The selection of an optimal solution within this frontier is contingent upon the specific characteristics of the problem.

While bi-objective optimization (BOO) methods often incorporate single objective optimization (SOO) techniques at certain junctures, such as weighted sum (WS), ϵ-constraint, and Chebyshev scalarizations, these methods are fraught with complexity [1] and [2]. In studies like [3], [4], and [5], emphasis was placed on analyzing resource efficiency (RE) metrics to strike a balance between energy efficiency (EE) and spectral efficiency (SE) vis-à-vis total power consumption and occupied bandwidth. In [3], the optimization of the EE-SE trade-off was framed as a multi-objective optimization (MOO) problem, wherein traditional nonlinear programming (NLP) methods, such as Lagrangian Augmented and Sequential Quadratic Programming (SQP), alongside the WS method, were employed to identify potential solutions along the Pareto frontier. Numerical analyses suggested that iterative analytical techniques rooted in NLP exhibit promise while maintaining a low computational overhead. Conversely, [3] abstained from comparing these analytical NLP-based solutions with heuristic methods, despite the latter's potential for simpler computational implementation in optimizing EE-SE within OCDMA-PON systems.

In [6], the authors introduced and characterized efficient and promising multi-objective heuristic algorithms, namely the Dominance of Random Points (DRP), the Evolution from the Dominance of Random Points (EDRP), and the Evolution of One Random Point to the Pareto frontier (EORP). Additionally, they proposed and demonstrated a theorem suggesting the equivalence between locally Pareto optimal solutions and Pareto optimal solutions for specific problems.

In this article, we provide a detailed analysis of the EORP method and introduce an enhanced version, EMRP. Additionally, we compare these methods with the classical Weighted Sum approach for identifying solutions on the Pareto front. For this purpose, we consider two variants of the WS method: one associated with Hill Climbing (WS-HC) and the other with Particle Swarm Optimization (WS-PSO), applied to the bi-objective optimization problem in optical code division multiple access (OCDMA) communications systems.

Contribution. The contribution of this work is threefold:

a) We propose and characterize efficient and promising multi-objective heuristic algorithms, including the Evolution of Multiple Random Points (EMRP), and provide detailed definitions of the EORP method. These methods apply equally to any multi-objective optimization (MOO) problem arising in optical communication systems.

b) We demonstrate Theorem 1, proposing the equivalence between locally Pareto optimal solutions and Pareto optimal solutions for bi-objective optimization (BOO).

c) We analyze the methods' performance in OCDMA networks by also evaluating the classical WS method, in-

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corporating both its variants: WS-HC and WS-PSO.

The remainder of this paper is organized as follows: Section II provides a detailed overview of BOO problem and the EORP method, along with its key properties; Section III delves into the specifics of the EMRP method; Section IV presents a series of numerical analyzes to uncover efficient solutions along the Pareto front using realistic OCDMA networks; finally, Section V offers concluding remarks.

II. EORP - Method details

The development of BOO methods is important because they help to find optimal solutions that can provide optimal performance goals with a single algorithm. Unlike conventional optimization methods, which are designed to find the best solution for a single objective, BOO methods can evaluate two objectives at the same time, and the Pareto frontier representation of solutions can help to identify solutions that traditionally would not be possible to be found.

Usually BOO methods usually employ SOO methods in certain steps, such as WS and Chebyshev methods; however, these scalarization methods can present a high degree of complexity. In this section, we present a heuristic for MOO problems, based on Pareto dominance, that explores the EORP to the Pareto frontier to describe the promise heuristic method EMRP needed to define a BOO problem and introduce the concept of dominance. A BOO problem presents two objective functions that must be maximized simultaneously, and subject to a feasible set : $\Omega \subset \mathbb{R}^n$, $\Omega \neq \varnothing$, Ω in general, it is defined through equality and inequality constraints. Consider the following form of a BOO:

$$\max_{x\in\Omega} f(x) \quad \text{and} \quad \max_{x\in\Omega} g(x) \tag{1}$$

The concept of dominance is paramount for understanding optimality in the Pareto sense. It is essential for the evaluation and analysis of the proposed heuristic methods. Thus, the concepts of dominance and the Pareto frontier can be defined as in [6] .

Definition (Dominance): One solution x^1 dominates another solution x^2 (we denote $x^1 \succcurlyeq x^2$), with $x^1, x^2 \in \Omega$, if the following conditions are satisfied:

(i) The solution x^1 is better or equal to the solution x^2 in all objectives, i.e., $f\left(x^1\right) \geq f\left(x^2\right)$ and $g\left(x^1\right) \geq g\left(x^2\right)$;

(ii) The solution x^1 is strictly better than the solution x^2 , that is, if $f\left(x^1\right) > f\left(x^2\right)$ or $g\left(x^1\right) > g\left(x^2\right)$.

Definition 1. *(Efficient optimal solution): A solution $x^* \in \Omega$ is called efficient or optimal in the sense of Pareto, if there is no other solution $x \in \Omega$, so that $x \succcurlyeq x^*$. The set of all efficient optimal solutions in the sense of Pareto is called the Pareto frontier.*

Definition 2. *(Locally Pareto optimal): A solution vector $x^* \in \Omega$ is called locally Pareto optimal if there exist $\varepsilon > 0$ such that there is no other solution $x \in \Omega \cap \{x \in \mathbb{R}^n;\ \| \overline{x} - x \| < \varepsilon\}$, so that $x \succcurlyeq x^*$.*

Note that a Pareto optimal solution is locally Pareto optimal. The inverse is valid for convex multi-objective optimization problems. In the paper [6] it was proved that a locally Pareto optimal is the global Pareto optimal solution in the EE-SE bi-objective trade-off optimization problem.

The results found in Theorem 1 and Theorem 2 in [6] were inspired by the results of [7] and [8].

Theorem 1. *Consider the bi-objective problem 1, where $\Omega \subset \mathbb{R}^n$ is a convex subset, f is a quasiconcave function, and g is a concave function. In this case, every locally Pareto optimal solution is also globally Pareto optimal.*

Proof: Consider $\overline{\mathbf{x}} \in \Omega$ to be locally Pareto optimal. This implies the existence of $\varepsilon > 0$ such that for all $\mathbf{x} \in \Omega_{\overline{\mathbf{x}}}$, we have $\overline{\mathbf{x}} \succeq \mathbf{x}$, where $\Omega_{\overline{\mathbf{x}}} = \Omega \cap \{\mathbf{x} \in \mathbf{R}^K; \|\overline{\mathbf{x}} - \mathbf{x}\| < \varepsilon\}$. Assuming that $\overline{\mathbf{x}}$ is not a Pareto optimal solution, there must exist another solution $\mathbf{x}^* \in \Omega$ on the Pareto frontier such that

$$f(\overline{\mathbf{x}}) \leq f(\mathbf{x}^*) \text{ and } g(\overline{\mathbf{x}}) < g(\mathbf{x}^*) \tag{2}$$

Let us define $\mathbf{x}^\lambda = \lambda\mathbf{x}^* + (1-\lambda)\overline{\mathbf{x}}$, where $0 < \lambda < 1$ is selected such that $\mathbf{x}^\lambda \in \Omega_{\overline{\mathbf{x}}}$. The convexity of Ω implies that $\mathbf{x}^\lambda \in \Omega$.

Utilizing (2), exploiting the quasi-concavity property of $f(\cdot)$ and the concavity of $g(\cdot)$, we derive

$$f(\overline{\mathbf{x}}) \leq \min\{f(\overline{\mathbf{x}}), f(\mathbf{x}^*)\} \leq f(\mathbf{x}^\lambda)$$

and

$$g(\overline{\mathbf{x}}) \leq \min\{f(\overline{\mathbf{x}}), f(\mathbf{x}^*)\} < g(\mathbf{x}^\lambda)$$

Since the objective function $g(\cdot)$ is concave, the inequalities above are strict. This contradicts the local optimality of $\overline{\mathbf{x}}$ in terms of Pareto optimality. Thus, $\overline{\mathbf{x}}$ belongs to the Pareto frontier.

□

The result found in the Theorem 1 motivated the construction of the EORP and EMRP heuristics.

We can summarize the EORP heuristic method with the following basic steps:

1. Choose a starting point x in the feasible region of the BOO;

$$x^f = x^g = x$$

2. After $k > 0$ iterations, approximations $\left(x^f, x^g\right)$ are included as new approximations x^i in the search for the Pareto frontier;

$$\left(x^1, \cdots, x^k, x^f, x^g\right)$$

3. The method stops when it cannot obtain better solutions in the trust regions.

The steps described show that in the first iterations, the computational cost is lower and increases as new approximations are inserted x^i for the Pareto frontier, see Figure 2. Thus, starting with a 'distant' point from the Pareto frontier should not significantly harm the method, that is, the method is not dependent on the starting point.

The evolution of the method can be seen in Figure 1. The value of x is important for the good development of the method and the spreading of the solutions in the Pareto frontier. So for a better approximation of the Pareto Frontier, it may be necessary to adjust this parameter. In algorithm 1 we present a possible choice for this parameter; we suggest

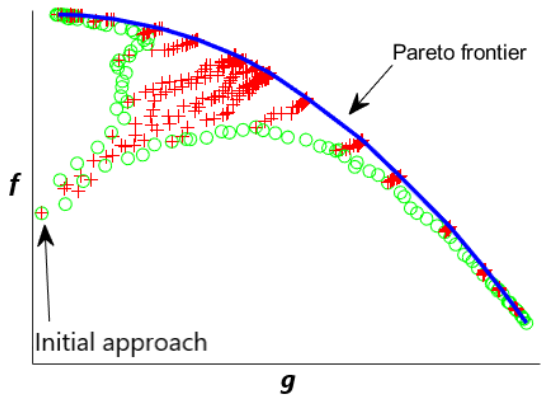

Fig. 1. Visualization of the Evolution of the EORP Method: One can observe the initialization with a random starting point and the method's evolution towards the Pareto frontier.

considering the initial k as the dimension of the set Ω, , that is, $0 < k < n$, the point increment occurs up to a pre-set limit $k_{\max}$. This value of k is increased by two every k iteration, as detailed in the algorithm. We present Algorithm 1 below, which details the EORP heuristic method for implementation.

The development of heuristic methods for BOO is important because they provide alternatives to determine fast and effective solutions to high-level optimization problems. As a result, the EORP algorithm as in [6] shown to be competitive, surpassing the classical analytical methods in terms of performance complexity trade-off. The methods compared with EORP in [6] were tested and confirmed in works [5] and [9], in these works, classic scalarization methods were analyzed, such as weighted sum and e-Constraint.

III. EMRP - Evolution of Random Points

This section introduces a method based on the EORP approach, which incorporates multiple initial points. It is expected that this approach will yield a more comprehensive coverage of the Pareto Frontier.

The EORP method, initially proposed as a heuristic for solving bi-objective optimization problems based on Pareto dominance, has proven promising. However, aiming to enhance its effectiveness and convergence capability, we introduce an improved version of EMRP, this method incorporates an evolution strategy to maximize both f and g from better approximations while directing more points toward the Pareto Frontier. Based on principles analogous to the original EORP, this strategy aims to enhance the local search for Pareto optima.

While the EORP method initializes from a single point and gradually spreads points towards the Pareto front (this approach incurs lower computational cost in the initial iterations), on the other hand, the EMRP method, in its initial iterations (requiring higher computational cost), generates random points and verifies which ones are feasible, selecting those not dominated by others to start. This higher-cost approach to initializing the method tends to create a more favorable condition for convergence and obtaining a greater number of points with better spread on the Pareto front.

In EMRP, we introduce the strategy of evolving multiple random points, and the best approximation for $\max f$ evolving for the maximum of f. Similarly, we also consider the evolution to maximize g. This search is carried out using the EORP method. For the remaining points, we apply evolution towards the Pareto Frontier.

Algorithm 1 EORP - Evolution of Random Points

Choose: randomly point $\mathbf{x}^1 \in \Omega$;
Define: $\mathbf{x}^f = \mathbf{x}^1$, $\mathbf{x}^g = \mathbf{x}^1$, $iter = 0$, $k \in \mathbb{N}$, $0 < k < n$, $k_{\max} > 0$, solution $\in \mathbb{R}^{k+2}$, solution $= [0, \cdots, 0]$ and $\overline{1} \in \mathbb{R}^{k+2}$, $\overline{1} = [1, \cdots, 1]$;
While solution $\neq \overline{1}$ **do**
    $iter = iter + 1$;
    For $i = 1$ **to** k
        IF $\exists \mathbf{x} \in \Omega \cap \left\{\mathbf{x} \in \mathbb{R}^n; \parallel \mathbf{x}^i - \mathbf{x} \parallel < \varepsilon\right\}$ with $\mathbf{x} \succeq \mathbf{x}^i$ **do**
            solution$(i) = 0$;
            $\mathbf{x}^i = \mathbf{x}$;
        Else
            solution$(i) = 1$;
        End (IF)
    End (For)
    IF $\exists \mathbf{x} \in \Omega \cap \left\{\mathbf{x} \in \mathbb{R}^n; \parallel \mathbf{x}^f - \mathbf{x} \parallel < \varepsilon\right\}$ with $f(\mathbf{x}) > f\left(\mathbf{x}^f\right)$ **do**
        solution$(n+1) = 0$;
        $\mathbf{x}^f = \mathbf{x}$;
    Else
        solution$(n+1) = 1$;
    End (IF)
    IF $\exists \mathbf{x} \in \Omega \cap \{\mathbf{x} \in \mathbb{R}^n; \parallel \mathbf{x}^g - \mathbf{x} \parallel < \varepsilon\}$ with $g(\mathbf{x}) > g\left(\mathbf{x}^g\right)$ **do**
        solution$(n+2) = 0$;
        $\mathbf{x}^g = \mathbf{x}$;
    Else
        solution$(n+2) = 1$;
    End (IF)
    IF $iter = k$ *and* $k < k_{\max}$ **do**
        $iter = 0$;
        $\mathbf{x}^{k+1} = \mathbf{x}^f$;
        $\mathbf{x}^{k+2} = \mathbf{x}^g$;
        $k = k + 2$;
        $\overline{1} = \left[\overline{1}, 1, \ 1\right]$;
        $\overline{0} = \left[\overline{0}, 0, \ 0\right]$;
    End (IF)
End (While)
Output: $\mathbf{x}^f$, $\mathbf{x}^g$, $\mathbf{x}^i$, $i = 1, \cdots, k$.

The rationale behind this approach is grounded in Theorem 1, which suggests that by incorporating this local search, we are locally seeking the Pareto optimum, under the same assumptions as the EORP method. The local search terminates when we fail to find any point that dominates the points $\mathbf{x}^i$.

Remark 1. *In the bi-objective problem (1), we observe that as we increase the value of g, the value of f decreases. Therefore, the co-funding behavior between f and g indicates that the Pareto frontier of the problem begins at the maximum point of f and ends at the maximum point of g, as discussed in [3].*

As described in Observation 1, we are aware of the initial and final points of the Pareto frontier, namely, it begins

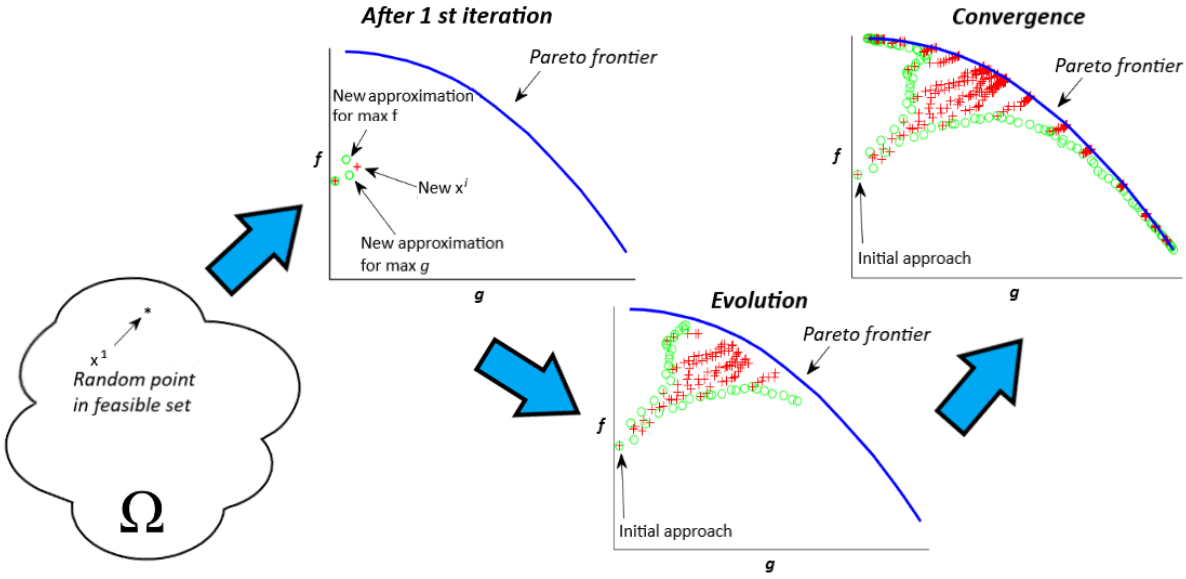

Fig. 2. Visualization of the Evolution of the EORP Method: Starting from a randomly chosen feasible point as the initial point, the method evolves and spreads points towards the Pareto frontier until convergence.

at $\max f$ and ends at $\max g$. Therefore, to enhance the accuracy of the Pareto frontier approximation, we incorporate a search for these points into the EMRP heuristic method. This involves utilizing the best approximations for $\max g$ and $\max f$, denoted as $\overline{\mathbf{x}}^g$ and $\overline{\mathbf{x}}^f$ respectively, among the vectors $\overline{\mathbf{x}}^i$. Algorithm 2 outlines the pseudocode for the EMRP method.

Theorem 2. *Let x^* be a limit point of the EORP or EMRP Algorithm, then x^* is on the Pareto frontier.*

Proof: Let $\mathbf{x}^*$ Algorithm limit 1 or 2, then $\mathbf{x}^* \in \Omega$, for $\varepsilon > 0$ defined in Algorithm no exist $\mathbf{x} \in \Omega \cap \{\mathbf{x} \in \mathbf{R}^K; \|\mathbf{x}^* - \mathbf{x}\| < \varepsilon\}$ with $\mathbf{x} \succeq \mathbf{x}^*$. So $\mathbf{x}^*$ is locally Pareto optimal, it follows from Theorem 1 that $\mathbf{x}^*$ is also Pareto optimal.

□

IV. Numerical Results

This section presents numerical tests for the EORP, EMRP, WS-PSO, and WS-HC methods, focusing on the EE-SE trade-off optimization problem in optical networks. This problem has been extensively studied and is important in analyzing optical networks. Several authors have proposed formulations for this problem, such as [10], [11], [12], and [5]. Additionally, works like [9] introduce techniques for solving the problem.

A. Defining the Bi-objective EE-SE Optimization Problem

In OCDMA systems, every information bit can be spread spectrally over N time intervals using spreading sequences, where each interval is known as a chip-time, denoted as T_c. The signal-to-noise-plus-interference ratio (SINR) for the i-th user can be expressed as follows, according to [9]:

$$\gamma_i = \frac{F_i\, p_i G_{ii}}{\sum_{j\neq i}^{K} p_j G_{ij} + \sigma^2}, \tag{3}$$

where p_i denotes the transmission power of the i-th user, K represents the total number of users in the system, G_{ii} signifies the attenuation of the signal for the i-th user, reflecting the attenuation along the optical code path. Additionally, G_{ij} denotes the attenuation between the j-th transmitting node and the i-th receiving node, while σ^2 represents the power of the white Gaussian additive noise (AWGN) at the input of the i-th receiver. Also, let us consider the spectral efficiency (η_{SE}) figure of merit for the OCDMA system performance measure.

$$\eta_{SE} = \frac{1}{r_c}\sum_{i=1}^{K} w_i \log_2(1+\theta_i\gamma_i) \qquad \left[\frac{\text{bits}}{\text{s.Hz}}\right], \tag{4}$$

where r_c represents the optical bandwidth occupied by the system and the gap θ_i is typically defined as presented in [9], [13], and [3].

The energy efficiency (EE) of OCDMA networks involves determining the suitable transmission power allocated to each user, which can be defined as:

$$\eta_{EE} = \frac{\sum_{i=1}^{K} w_i \log_2(1+\theta_i\gamma_i)}{\iota \sum_{i}^{K} p_i + P_{\text{C}}} \qquad \left[\frac{\text{bit}}{\text{Joule}}\right] \tag{5}$$

where P_{C} represents the circuit power. By combining equations (4) and (5), the trade-off optimization problem between energy efficiency (EE) and spectral efficiency (SE) can be formulated as a bi-objective optimization (BOO) problem, as stated in [5]:

$$\begin{aligned} &\text{maximize} \quad \eta_{SE} \quad \text{and} \quad \text{maximize} \quad \eta_{EE} \\ &\text{s.t.} \quad (\text{C}.1) \qquad \gamma_i \geq \gamma_i^*, \quad \forall i \\ &\qquad\quad (\text{C}.2) \qquad r_i \geq r_{i,\min}^{\text{serv}} \\ &\qquad\quad (\text{C}.3) \qquad p_{\min} \leq p_i \leq p_{\max} \end{aligned} \tag{6}$$

For the numerical evaluations conducted in this section, realistic OCDMA networks, as described in [9], [13], [14], and [15], were deployed within the OCDMA-PON system. We considered groups of users denoted by $i = 1, 2, \ldots K_v$ and classes of service denoted by $v = 1, 2, \ldots, \mathcal{C}$, following the approach outlined in [9] and [6]. Consequently, the total number of active OCDMA network users is $K = \Sigma_v^{\mathcal{C}} K_v$. The bi-objective EE-SE optimization problem satisfies the conditions of Theorem 1. We will carefully consider the

Algorithm 2 EMRP – Evolution of Multiple Random Points

Choose: randomly points $\mathbf{x}^n \in \Omega$, $n = 1, ..., N$;
Define: k=0; $\varepsilon > 0$.
For $n = 1$ **to** N
  If $\mathbf{x}^n \succeq \mathbf{x}^m$, $\forall m \neq n$ **do**
    k=k+1;
    $\overline{\mathbf{x}}^i = \mathbf{x}^n$;
  End(If)
End(For)
Choose: $\overline{\mathbf{x}}^f$ and $\overline{\mathbf{x}}^g$ in $\{\overline{\mathbf{x}}^i, i = 1, ..., k\}$
Define: $solution \in \mathbb{R}^{k+2}$, $solution = [0, ..., 0]$ and $\mathbf{1} \in \mathbb{R}^{k+2}$, $\mathbf{1} = [1, ..., 1]$.
While $solution \neq \mathbf{1}$ **do**
  For $i = 1$ **to** k
    If exist $\mathbf{x} \in \Omega \cap \{\mathbf{x} \in \mathbb{R}^K; \|\overline{\mathbf{x}}^i - \mathbf{x}\| < \varepsilon\}$ with $\mathbf{x} \succeq \overline{\mathbf{x}}^i$ **do**
      $solution(i) = 0$;
      $\overline{\mathbf{x}}^i = \mathbf{x}$;
    Else
      $solution(i) = 1$;
    End(IF)
  End(For)
  If exist $\mathbf{x} \in \Omega \cap \{\mathbf{x} \in \mathbb{R}^K; \|\overline{\mathbf{x}}^f - \mathbf{x}\| < \varepsilon\}$ with $f(\mathbf{x}) > f(\overline{\mathbf{x}}^f)$ **do**
    $solution(k+1) = 0$;
    $\overline{\mathbf{x}}^f = \mathbf{x}$;
  Else
    $solution(k+1) = 1$;
  End(IF)
  If exist $\mathbf{x} \in \Omega \cap \{\mathbf{x} \in \mathbb{R}^K; \|\overline{\mathbf{x}}^g - \mathbf{x}\| < \varepsilon\}$ with $g(\mathbf{x}) > g(\overline{\mathbf{x}}^g)$ **do**
    $solution(k+2) = 0$;
    $\overline{\mathbf{x}}^g = \mathbf{x}$;
  Else
    $solution(k+2) = 1$;
  End(IF) **IF** $iter = k$ *and* $k < k_{\max}$ **do**
    $iter = 0$;
    $\mathbf{x}^{k+1} = \mathbf{x}^f$;
    $\mathbf{x}^{k+2} = \mathbf{x}^g$;
    $k = k + 2$;
    $\overline{1} = \left[\overline{1}, 1, \ 1\right]$;
    $\overline{0} = \left[\overline{0}, 0, \ 0\right]$;
  End (IF)
End(While)
Output: $\overline{\mathbf{x}}^f$, $\overline{\mathbf{x}}^g$, $\overline{\mathbf{x}}^i$, $i = 1, ..., k$.

following properties of the functions involved in the problem (6): $\mathcal{X}$ is a convex set, $\eta_{SE}(\cdot)$ is a continuous and concave function, and $\eta_{EE}(\cdot)$ is a continuous and quasi-concave fractional function [16].

B. Weighted Sum Method

The Weighted Sum (WS) method transforms the bi-objective problem, which involves the objective functions (η_{EE} and η_{SE}), into a single-objective optimization problem by constructing a convex combination of the two objectives. This re-scaling approach combines the objectives into a single scalar function. Consequently, the original bi-objective EE-SE trade-off problem (6) can be reformulated using the WS method as the following single-objective optimization (SOO) problem:

$$\begin{aligned} \underset{\mathbf{x}\in\mathbb{R}^K}{\text{maximize}} \quad & (1-\lambda)\eta_{EE} + \lambda \cdot r_c \frac{\eta_{SE}}{\iota \sum_i^K p_i^{\max} + P_{\mathrm{C}}} \\ \text{s.t.} \quad (\mathrm{C}.1) \quad & \gamma_i \geq \gamma_i^*, \quad \forall i \\ (\mathrm{C}.2) \quad & r_i \geq r_{i,\min}^{\mathrm{serv}} \\ (\mathrm{C}.3) \quad & p_{\min} \leq p_i \leq p_{\max} \end{aligned} \tag{7}$$

The weight factor $\lambda \in [0, 1]$, with EE defined in eq. (7) and SE in eq. (4). Since EE is measured in [bits/Joule] and SE in [bits/s.Hz], it is unsuitable to directly add EE and SE due to the unit difference. In optical systems, where the bandwidth $W \approx r_c$ is generally larger than the total transmission power P_T, a direct sum of EE and SE would prioritize EE, distorting the balance between the two objectives. To correct this, we divide η_{SE} by $\iota K p_{\max} + P_{\mathrm{C}}$, ensuring both EE and SE are on the same scale for optimization.

The WS scalarization method for solving the EE-SE trade-off optimization problem is outlined in Algorithm 3. One of the drawbacks of the WS method is that it often leads to repeated solutions, as highlighted in [4], [3], and [1]. Furthermore, as noted in [3], the global solutions to the problem (7), when considering $\lambda \in [0, 1]$, are guaranteed to lie on the Pareto frontier of the original bi-objective optimization problem. However, this approach may fail to capture certain sections of the Pareto frontier, particularly in non-convex regions, limiting its effectiveness in fully exploring the trade-offs between EE and SE (6).

Algorithm 3 WS-PSO or WS-HC

Choose: $\delta \in (0, 1)$, randomly initialize $\mathbf{x}^0$ within the bounds of $p_{\min} \leq \mathbf{x} \leq p_{\max}$.
Define: $\lambda = 0$ and $k = 0$;
While $\lambda \leq 1$ **do**
  Solve the problem (7) using either the PSO or HC method;
  $\lambda \leftarrow \lambda + \delta$;
  $k = k + 1$;
  Save $\mathbf{x}^k$;
End(while)
Output: $\mathbf{x}^* = \mathbf{x}^k$;

To solve the subproblem resulting from the WS approach, we consider employing either Hill Climbing or Particle Swarm Optimization methods, each offering distinct strategies for exploring the solution space. Below, we briefly describe these methods:

The HC algorithm is a greedy local search optimization method that begins with an initial solution called the current node and iterative searches for better solutions within its neighborhood. A step size controls the distance between the current solution and its neighbors. The algorithm updates the current solution if a superior neighboring solution is found. It repeats the process until either a better solution is no longer found or a pre-defined stopping condition, such as a maximum number of iterations, is met. Further details on the HC algorithm can be found in [17], [18] and [19].

A pseudo-code for the HC algorithm adapted to the WS is described in Algorithm 4. The cost function F deployed

in the Algorithm 4 can be defined as:

$$F(\mathbf{p}) = (1-\lambda)\eta_{EE} + \lambda \cdot r_c \frac{\eta_{SE}}{\iota \sum_i^K p_i^{\max} + P_{\mathrm{C}}} + \quad (8)$$
$$+\rho\{\sum_i [\max(0, \gamma_i - \gamma_i^*)]^2 + \sum_i [\max(0, r_i - r_{i,\min}^{\mathrm{serv}})]^2\}.$$

where ρ is the penalty parameter associated with violating the problem restrictions (C.1) and (C.2). The cost function (8) with constant penalty parameter requires a high but limited value to inhibit restrictions' violation that makes up the problem (7).

Algorithm 4 HC

Choose: Initial approximation $\mathbf{x}$, such that $p_{\min} \leq \mathbf{x}_i \leq p_{\max}$.
Define: $solution$=0; It=0; Max-It > 0.
While $solution$=0 and It $\leq$ Max-It
    It=It+1;
    Generate neighbours;
    if any neighboring point has a higher value of F than $\mathbf{x}$ **then**
        $\mathbf{x}$ is updated to the neighboring point with the best value for F;
    else
        $solution$=1;
    End(if)
End(while)
Output: x.

The PSO algorithm, on the other hand, is a population-based stochastic optimization technique inspired by the social behavior of animals, such as birds flocking or fish schooling. It initializes with a population of random solutions, called particles, which move through the search space to find better positions based on their individual experience (personal best) and the experience of the group (global best). The algorithm updates each particle's position and velocity until a termination condition is satisfied. For more details on PSO, refer to [20], [21] and [22].

At each iteration of Algorithm (3), the PSO algorithm is used to maximize the function (8) within the bounds $[p_{\min}, p_{\max}]$. The method implemented for the tests follows the approach presented in [22].

C. Hypervolume Evaluation metric

To evaluate the performance of the methods, the hypervolume evaluation metric (HvEM) will be used, as proposed in the study [23], where the authors analyzed various evolutionary methods based on specific performance metrics. This metric will be used as a quantitative criterion to compare the algorithms, enabling a detailed analysis of the results obtained regarding the trade-off between EE and SE.

To define HvEM, we start by considering the set of non-dominated solutions S that approximate the Pareto front generated by a specific method, along with a reference point R dominated by the points in S. The hypervolume $\mathcal{H}(S, R)$ is then calculated as the sum of the volumes of hypercubes v_i, each representing the space dominated by a solution $S_i \in S$, with R serving as the limits, as shown in Figure 3 for the EE-SE trade-off. In our scenario, we define the reference point R as $(\min \eta_{SE}, \min \eta_{EE})$ on the $\eta_{EE} \times \eta_{SE}$ plane. It is important to note that R does not necessarily need to be a feasible point for the problem 6; it simply acts as a reference point for calculating HvEM.

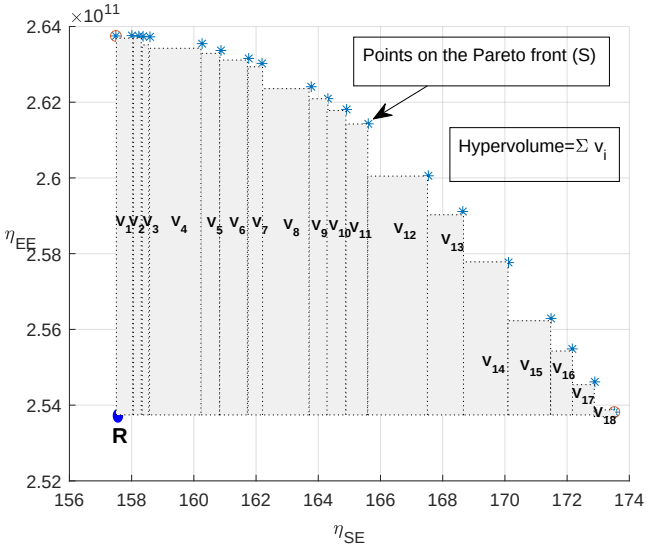

Fig. 3. Hypervolume calculation for the set S of non-dominated solutions determined by the EMRP heuristic with $K = 16$, and a point R dominated by all points of S. The hypervolume is obtained as the sum of the areas v_i. Note that the larger the hypervolume, the better the algorithm performance.

D. Numerical Results Comparison of EORP, EMRP, WS-HC, and WS-PSO Methods

In this section, we present the numerical experiments conducted. For the tests, we considered a network where $K^{\mathrm{CLASS}} = [4, 8, 16, 32, 48]$, and $K \equiv K^{\mathrm{CLASS\ 1}} \cup K^{\mathrm{CLASS\ 2}} \cup K^{\mathrm{CLASS\ 3}}$, with user configurations such as $[2; 1; 1], [4; 2; 2], [8; 4; 4], [16; 8; 8], [24; 12; 12]$. The algorithms were implemented in MATLAB 8.0, and the tests were conducted on a computer running Windows 11 Home Single Language, version $23H2$, equipped with an Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz, 15.9GB of RAM, and a 64-bit operating system. For the WS-HC and WS-PSO methods, we considered $\delta = \frac{1}{19}$, his way λ will assume the following values $\lambda = 0, \frac{1}{19}, \frac{2}{19}, \ldots, 1$, resulting in 20 distinct values. Specifically, we set the parameters for the HC method as $\rho = 10^8$ and Max-It $= 10^4$.

Table I presents the performance of the EORP, EMRP, WS-PSO, and WS-HC algorithms in addressing the EE-SE trade-off problem. The table summarizes key metrics, including execution time, convergence success rate, the number of unique solutions on the Pareto frontier, function evaluations, and the total number of iterations for the EE-SE OCDMA algorithms across varying numbers of optical nodes. Specifically, execution time (Time) is measured in seconds, the number of distinct solutions on the Pareto frontier is denoted as SPF, function evaluations are represented by FunC, and the total number of external iterations for the EE-SE OCDMA algorithms considering different optical node counts (K) is given by $\mathcal{I}_{\mathrm{tot}}$.

In Table I, we observe that both methods converged to the Pareto front in all cases and exhibited many points on the frontier. It is noteworthy that the EMRP method achieved a significantly higher number of points on the Pareto frontier and also showed slightly higher hypervolume values see Figure 5, indicating that the EMRP method provides better spread over the Pareto front, consequently offering a better approximation of it. Regarding convergence time and

the number of function evaluations, the EMRP and EORP methods showed better values in most cases considered, as shown in Figure 4.

Regarding the WS-HC and WS-PSO methods, both obtained a smaller number of distinct points on the Pareto front. Although we used 20 different values for λ, which would allow a maximum of 20 distinct solutions, the reduced number of solutions is due to a characteristic of the method. When λ approaches 1, there is a tendency for solutions to repeat, as discussed in [5]. Upon analyzing the variations of the WS method considered, we observe that WS-HC demonstrated superior performance, exhibiting behavior closer to that of the EORP and EMRP methods concerning execution time, number of function evaluations, and hypervolume. This enhanced performance can be attributed to the characteristics of the method, which employs a local search and benefits from previous solutions as starting points for $\lambda > 0$. In contrast, the WS-PSO requires the consideration of an initial swarm for each value of λ, which may explain the higher FuncC value observed in Table I.

TABLE I
PERFORMANCE OF THE CONSIDERED METHODS

K	Algorithm	Time	SPF	FunC	$\mathcal{I}_{\text{tot}}$	HvEM
4	EMRP	1.0884	21	33195	41	1.2995e+11
4	EORP	1.4275	17	49161	67	1.2760e+11
4	WS-PSO	5.1259	12	248920	20	1.2334e+11
4	WS-HC	1.3442	12	328980	20	1.2275e+11
8	EMRP	1.5913	25	79630	41	2.1282e+11
8	EORP	1.3036	17	46789	47	2.0032e+11
8	WS-PSO	10.8279	11	511720	20	2.0085e+11
8	WS-HC	1.8472	11	58724	20	2.0092e+11
16	EMRP	3.4201	21	196420	64	9.5635e+10
16	EORP	3.3531	18	176419	74	9.4583e+10
16	WS-PSO	21.4873	10	1016120	20	9.1460e+10
16	WS-HC	3.0299	10	154932	20	9.1062e+10
32	EMRP	6.3706	18	363917	61	2.2451e+11
32	EORP	7.7027	16	403648	83	2.1855e+11
32	WS-PSO	48.4683	10	2059320	20	2.1043e+11
32	WS-HC	8.1423	10	399348	20	2.1292e+11
48	EMRP	17.5633	27	1013806	78	2.8989e+12
48	EORP	12.3381	17	677029	91	2.8214e+12
48	WS-PSO	70.9465	15	3124920	20	2.7536e+12
48	WS-HC	23.9461	15	1112148	20	2.8101e+12

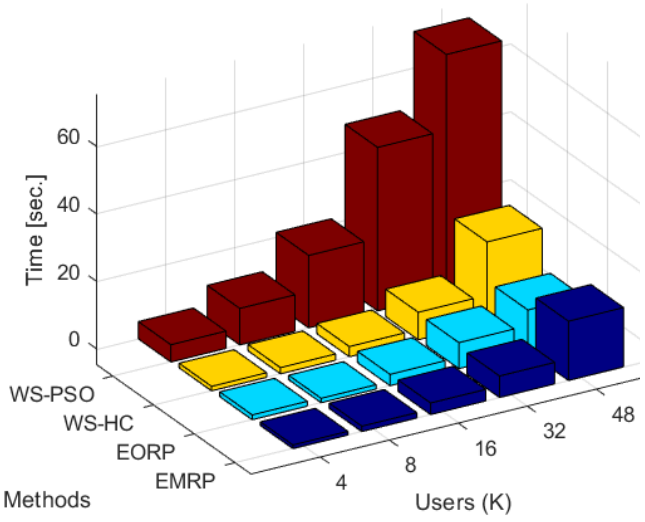

Fig. 4. Elapsed time for the four bi-objective optimization methods evaluated.

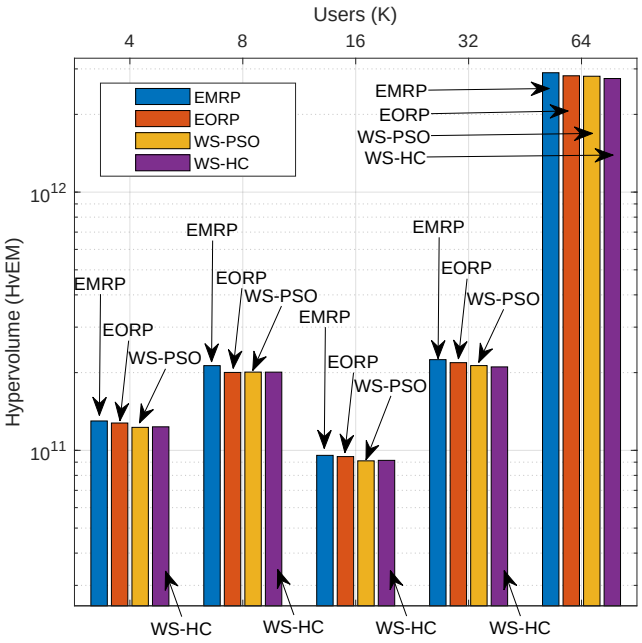

Fig. 5. Hypervolume achieved by the four evaluated bi-objective optimization methods.

Figure 6 presents the approximations of the Pareto front obtained using the EORP, EMRP, WS-HC, and WS-PSO methods for the bi-objective EE-SE optimization problem with $K = 32$. The EMRP method produces a greater number of points and achieves a better distribution, as demonstrated by the hypervolume metric in Table I.

Figures 7 and 8 show the evolution of η_{EE} and η_{SE} in problem (7) for $K = 32$. As observed, EE decreases as λ increases, while SE rises with the increase in λ. This is because a higher λ places more emphasis on SE, shifting the balance towards optimizing spectral efficiency. Also, EE and SE stabilize and remain unchanged when the weighting factor reaches $\lambda \geq \frac{10}{19}$. A characteristic of the WS method is the emergence of many repeated solutions as λ approaches 1. This characteristic of the method cannot be controlled, so even using 20 different values for λ, we obtained between 10 and 15 distinct solutions in the Pareto front. However, the solutions are the curve.

Figures 9 and 10 illustrate the iterative evolution of the EORP and EMRP heuristics, respectively, as they converge towards the Pareto frontier. Points marked with $\circ$' represent the path towards maximizing η_{EE} and η_{SE}, while points marked with $+$' denote the evolution of other points toward the Pareto frontier. The EORP method starts from a single point and gradually spreads points towards the Pareto front. Notably, this approach tends to have lower computational costs compared to the EMRP method, especially in the early iterations. The EMRP method incurs a higher computational cost for its initialization; indeed, the EORP method generates random points until obtaining a feasible point to initiate the method. On the other hand, the EMRP method generates K random points. It selects the feasible points for the problem, subsequently classifying them (checking which ones dominate the others) to begin the method. However, this approach for the EMRP method initializes the method in a more favorable condition for convergence, and the method

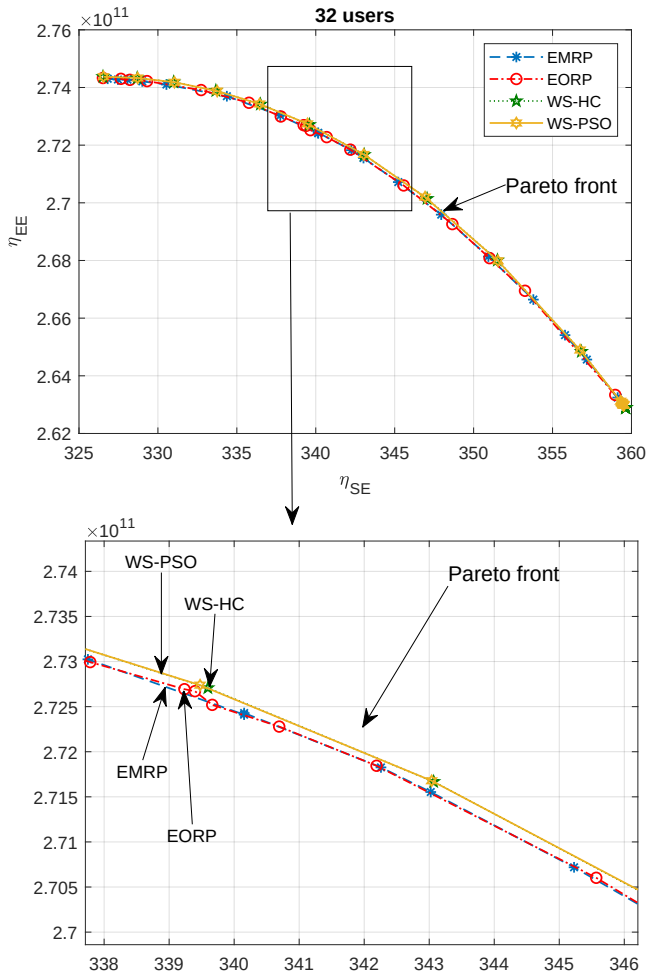

Fig. 6. Graph of the points obtained by the convergence of the EMRP, EORP, WS-HC, and WS-PSO methods.

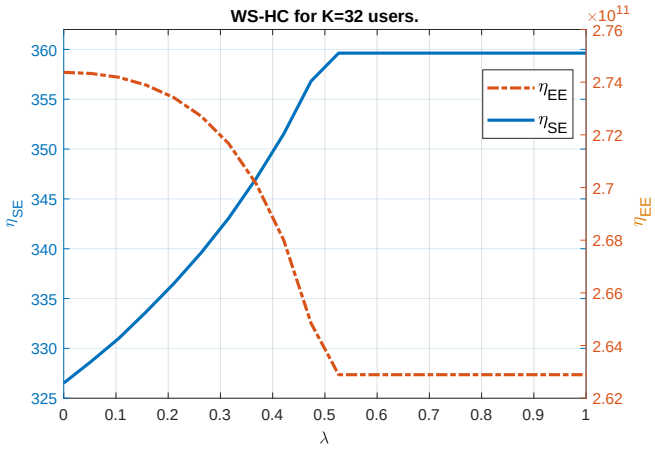

Fig. 7. Impact of the weighting factor λ on the EE-SE trade-off for $K = 32$ using the WS-HC method.

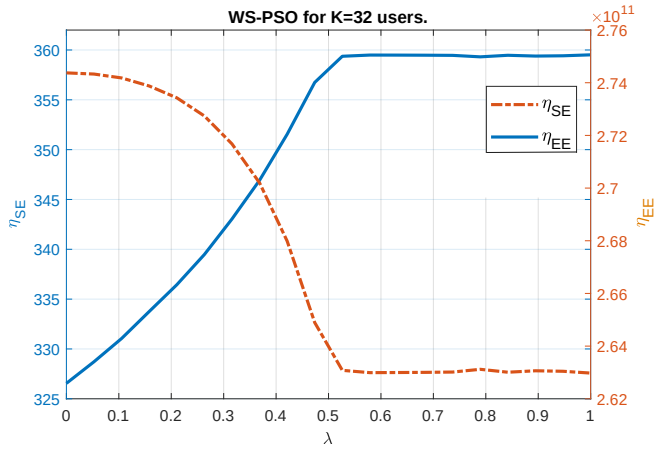

Fig. 8. Impact of the weighting factor λ on the EE-SE trade-off for $K = 32$ using the WS-PSO method.

tends to achieve better spread on the Pareto front, as observed in Figure 10.

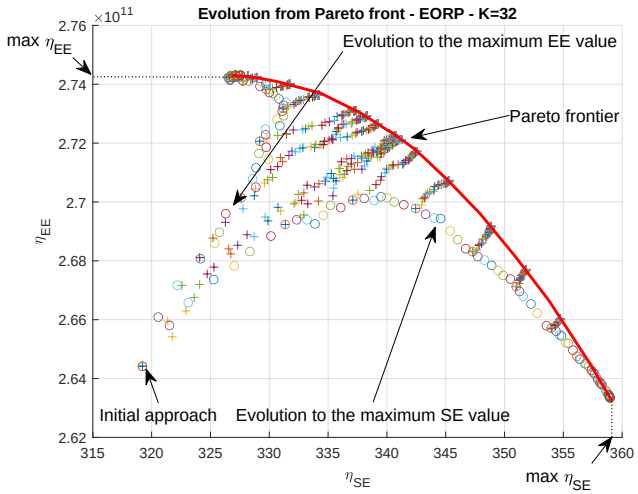

Fig. 9. Evolution achieved by the EORP method from initialization to the Pareto front considering $K = 32$.

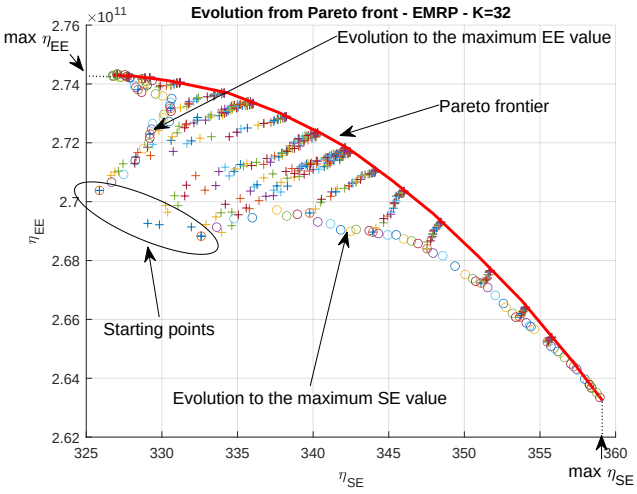

Fig. 10. Evolution achieved by the EMRP method from initialization to the Pareto front considering $K = 32$.

V. Conclusion

This work presents a study and discussion of the method proposed by [6], and we propose an enhanced version called EMRP, which has proven to be as promising as the EORP heuristic. The heuristic method described by EORP can, in principle, be adapted and applied to various bi-objective or multi-objective optimization problems. Motivated by these results, we propose and introduce the EMRP method, which is based on introducing additional initial points in the EORP method.

Through the analysis of the results, the EMRP heuristic has demonstrated the ability to achieve better solutions than the EORP, WS-PSO, and WS-HC heuristics, determining a greater number of points and better spreading along the Pareto front. Since the computational cost of the EMRP method tends to be higher due to its initialization, EMRP may be particularly suitable for problems where determining feasible points incurs a low average cost, meaning that generating feasible points would not significantly impact the method's processing time. However, if determining feasible points is highly costly, the EORP method should be considered a better option. In conclusion, using the EMRP method implies improving the distribution and quality of the Pareto frontier approximation.

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