# Event-Triggered Adaptive Fault-Tolerant Control for Linear Multi-Agent Systems with Actuator Faults and Time Delays

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*Abstract*—**This paper solves the cooperative fault-tolerant control (CFTC) issue for linear multi-agent systems (MASs) with actuator faults and state delays. First, in view of the difficulties existing in the system such as actuator failure and compound interference, an adaptive fault-tolerant controller and the corresponding parameter updating laws are designed. Then, state delays in the systems are eliminated by combining Lyapunov-Krasovskii functional (LKF) and the Young inequality. In addition, an event-triggered strategy is proposed according to system characteristics, which greatly reduces the communication resources consumption. It is proved that the designed control scheme ensures that the closed-loop signals are uniformly bounded, and the tracking errors of all subsystems steadily converge to zero. Finally, the simulation results assess the feasibility of the control strategy.**

*Index Terms*—**Multi-agent systems, Event-triggered strategy, State delays, Cooperative fault-tolerant control, Actuator faults**

## I. Introduction

MULTI-AGENT systems with autonomy, distribution and coordination capacities have become a frontier area of control research in the last few years. It has broad application prospects in numerous domains, such as satellite flight [1], wireless sensor networks [2], and traffic control [3], which have been intensively researched by numerous scholars in recent years. However, actuator failures will inevitably occur due to the large scale of MASs, thereby seriously affecting the security and stability of the system. Based on the aforementioned problems, various techniques, including fault compensation, fault estimation and isolation have been presented to realize fault-tolerant control (FTC) of the system. For instance, adaptive fault-tolerant controllers in [4]-[6] were proposed to compensate for actuator faults and parameter uncertainties. The work in [7] considered the convergence fault estimation of the system based on the convergence theorem of the average estimation sequence of differential equations. Additionally, the static compensation decoupling method in [8] was frequently employed to tackle the issue of multiple fault detection in linear systems.

It should be noted that the signals from the controller in the above results were constantly updated, which wasted a lot of communication resources and increased computational effort. Thus, event-triggered mechanisms were widely employed in control systems [9]-[10]. According to the different control inputs, three different updating strategies were given to handle non-deterministic nonlinear systems [11]. Furthermore, event-triggered mechanisms have also been applied to MASs with leaderless [12] and those with leader following [13]. On this basis, an event-driven distributed adaptive control method was introduced for a distributed MAS [14], and the given protocol was completely distributed and scalable, and moreover, it was completely independent of any global information regarding the network topology. In the following study, the authors generalized the event-triggered mechanism to a discrete random system [15]-[16] with input nonlinearity by applying relaxation matrix variables, filter transfer functions and other methods. It is important to note that despite the existence of some academic findings on event-triggered mechanisms, these results did not address the impact of time delays on system stability.

In fact, time delays widely exist in practical control systems. The research results indicate that time-varying delays can seriously affect the safety and stability of control systems. Consequently, the studies on linear time-varying delay systems have yielded considerable results in recent years. It is well known that LKF is a useful means of ensuring the stability of time delays systems [17]. However, for satisfying the system stability condition, the LKF method usually requires a more conservative estimation of the dynamic characteristic of the system. For this reason, many effective methods have been proposed to obtain less conservative stability conditions. Specifically, the approaches of augmented Lyapunov functional and Wirtinger-type inequalities were applied in [18]-[19]. Besides, the estimation of time-varying delays in linear systems was considered in [20] based on two different strategies, including mutual convexity methods and integral inequalities, and there were also results for interval time-varying delays systems [21]-[22], which gave strategies for delay range partitioning (DRP). Although the methods for dealing with time-varying delays are very mature, relatively little work has been done in MASs, especially when actuator failures that are common in the system need to be considered simultaneously. Thus motivating our current investigation.

Inspired by the above discussion, to quickly resolve difficulties such as actuator failures and time-varying delays, while avoiding continuous updating of signals, we propose an effective CFTC scheme. In addition, compared with results already available, the main contributions made are organized as follows:

1) This paper extends the control system [4] and [7]

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to MASs by using relevant state information passed by neighboring nodes. The presented CFTC strategy aims to incorporate the corresponding parameter updating laws to compensate for unknown fault disturbances, thus achieving the boundedness of closed-loop signals and make the error of each subsystem gradually converge to zero.

2) Different from the conventional strategy in [4], [5], [6], the ETC protocol is proposed to minimize the computational requirements of the communication process, and the Zeno behavior can be effectively avoided.

3) Compared to FTC methods in [4], [5] and [17], this article also studies the negative influence of state time-varying delays on MASs. By invoking LKF and designing the corresponding adaptive cooperative controller, the problem of poor dynamic performance caused by time delays is effectively solved.

This paper is organized as follows. Section II presents the graph theory of MASs, some basic assumptions and the devising of fault-tolerant controller. Next, the feasibility analysis of the given control strategy is presented in Section III. An illustrative example is given in Section IV. Finally, Section V draws the conclusion.

## II. Problem statement and preliminaries

### A. Graph Theory

Let $\mathcal{G}(\vartheta, \mathcal{J}, \mathcal{A})$ represent a directed graph with $N$ nodes $\vartheta = \{v_1, v_2, \ldots, v_N\}$. where $\mathcal{J} \subseteq \vartheta \times \vartheta$ refers to a group of edges, and the associated adjacency matrix is denoted by $\mathcal{A} = [r_{ij}] \in R^{N\times N}$. When $\mathcal{G}$ is undirected, the matrix $\mathcal{A}$ is symmetric. If node $i$ and node $j$ can communicate information, then $r_{ij} > 0$, or else, $r_{ij} = 0$. In particular, when $r_{ij} = r_{ji}$ for any $v_i, v_j \in \vartheta$, it indicates that at least one path exists, that is, the graph $\mathcal{G}$ is undirectedly connected. The degree matrix $\mathcal{S}$ is defined as $\mathcal{S} = diag(s_i) \in R^{N\times N}$ with $s_i = \sum_{j=1}^{N} r_{ij}$, so the Laplace matrix is formulated as $\mathcal{L} = \mathcal{S} - \mathcal{A}$. Furthermore, $\mathcal{F} = diag(r_{10}, r_{20}, \ldots, r_{N0})$ is the leader adjacency matrix, where $r_{i0} > 0$ if node $v_i$ is able to acquire leader information, or else, $r_{i0} = 0$.

### B. Problem Statement

Consider the following MASs with a leader, and the dynamics of the leader is represented as

$$\dot{x}_0(t) = Ax_0(t) \tag{1}$$

where $x_0(t) \in R^n$ denote the state of the leader.

Furthermore, the dynamic equation for agent $i$ is given by

$$\begin{aligned}\dot{x}_i(t) =& Ax_i(t) + B_d d_i(t) + B\left(u_i(t) + u_{si}(t)\right)\\ &+ B_\tau x_i(t - \tau_i(t)), \quad i = 1, 2, \cdots, N\end{aligned} \tag{2}$$

where the states of the follower and the actuator input for later design are represented by $x_i(t) \in R^n$ and $u_i(t) \in R^m$, $u_{si}(t) \in R^m$ indicates the unknown actuator stuck fault, $d_i(t) \in R^l$ stands for unknown interference. In particular, both $u_{si}(t)$ and $d_i(t)$ are bounded. $A$, $B$, $B_d$ and $B_\tau$ are defined as coefficient matrices with appropriate dimensions. Then, we set

$$B_\tau = BG, B_d = BF \tag{3}$$

where $G, F$ are denoted as known constant matrices. Furthermore, the time-varying delays $\tau_i(t)$ meet $0 \le \tau_i(t) \le \tau_i^*$, $\dot{\tau}_i(t) \le \bar{\tau}_i \le 1, i = 1, 2, \ldots, N$, and $x_i(t) = \varphi_i(t)$, $t \in [-\tau_0^*, 0]$ represents the initial condition, in which $\tau_0^* = \max_{1\le i\le N}\{\tau_i^*\}$.

**Remark 1.** Notably, the FTC issue presented in [5] failed to take into account the impact of state time-varying delays on MASs. Meanwhile, continuous signal updates will also impose a significant overload on the system.

### C. Communication Topology and Basic Assumptions

Assuming that the controller design can only use the state information of adjacent subsystems, the neighborhood tracking error of the $i$th follower $e_i(t)$ is denoted as

$$e_i(t) = \sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t)) + g_i(x_i(t) - x_0(t)) \tag{4}$$

Next, for the convenience of subsequent discussions, we introduce the following basic Assumptions and Lemmas:

**Assumption 1** [23]: Suppose that the graph $\mathcal{G}$ is undirectedly and connected, then it can be shown that at least one follower (2) gets signals from the leader (1).

**Lemma 1** [23]: Assume that Assumption 1 is valid. Consequently, the matrix $\mathcal{H} = \mathcal{L} + \mathcal{G}$ is positive definite.

Since only certain agents possess access to the leader dynamics, the ultimate control objective of this article is to propose a CFTC strategy with the help of local state information about neighboring nodes. Even if state delays, actuator failures and external interferences occur during system operation, all follower states can asymptotically track the leader, that is to say, $\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0$, for $i = 1, 2, \ldots, N$.

**Assumption 2** [5]: Stuck faults and external disturbances are both bounded continuous functions, which means that $\|u_{si}\| \le \bar{u}_{si}$ and $\|d_i(t)\| \le \bar{d}_i$, $i = 1, 2, \ldots, N$.

**Lemma 2** [5]: There is a positive constant $\mu_i$ such that

$$\left\|e_i^T PBB^T Pe_i\right\| \ge \mu_i \left\|e_i^T PB\right\|^2 \tag{5}$$

**Remark 2**: It is clear at this point that Assumption 1 and Lemma 1 are standard, that is, the leader is the source of information for all nodes. Besides, Assumption 2 indicates that the external disturbances and stuck faults to each agent are bounded. Finally, Lemma 2 provides an inequality scaling condition for system stability.

Furthermore, choosing an appropriate positive definite matrix $P$ makes it satisfy the Riccati inequality

$$A^T P + PA - 2c\lambda_i PBB^T P + \frac{1}{\beta} I_n < 0 \tag{6}$$

where $\lambda_i$ represents the $ith$ eigenvalue of $L + G$, $\beta$ is a positive design constant.

**Proof**: Inspired by Wang et al. [5], there exists a solution $P > 0$ that makes (7) holds

$$A^T P + PA + Q - PBB^T P = 0 \tag{7}$$

where $Q \in R^{n\times n}$ denotes the positive definite matrix.

In particular, we define $\lambda_i$ for $i = 1, 2, \ldots, N$ meeting $0 < \lambda_1 \le \lambda_2 \le \cdots \le \lambda_N$, and one can obtain

$$A^T P + PA - 2c\lambda_i PBB^T P = -Q - (2c\lambda_i - 1)PBB^T P \tag{8}$$

Thus, inequality (5) is guaranteed to be satisfied by selecting a constant $c$ greater than zero such that $c > \frac{1}{2\lambda_i}$ and a sufficiently large constant $\beta$. Then, the proof is completed.

*D. Event-Triggered Adaptive Fault-Tolerant Control Design*

Within this section, the collaborative fault-tolerant controller is described as

$$\alpha_i(t) = (cK_0 + K_{i1}(t) + K_{i2}(t))\, e_i(t) \tag{9}$$

where $c$ indicates the coupling strength and $c > \frac{1}{2\lambda_{\min}(\mathcal{H})}$.

Set the following event-triggered mechanism

$$\begin{aligned} &u_i(t) = \alpha_i(t_k), \forall t \in [t_k, t_{k+1}) \\ &t_{k+1} = \inf\left\{t \in R^+ : |\alpha_{ij}(t) - u_{ij}(t)| \geq E_{ij}\right\} \end{aligned} \tag{10}$$

where $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{im}]^T$, $E_{ij}$, $j = 1, 2, \ldots, m$ indicate the fixed threshold that will be given later. $t_k$ is the controller update time, that is, when (10) is triggered, then the time $t_k$ is updated to $t_{k+1}$. In the meantime, the controller $u(t_{k+1})$ will be used in the system.

Specially, if $|\alpha_{ij}(t) - u_{ij}(t)| \leq E_{ij}$ for $j = 1, 2, \ldots, m$, there exists a positive time-varying parameter $\omega_{ij}$ fulfilling $|\omega_{ij}(t_k)| = 0, |\omega_{ij}(t_{k+1})| = 1$ and $|\omega_{ij}(t)| \leq 1$, it leads to

$$\alpha_i(t) = u_i(t) + \chi_i(t) \tag{11}$$

where $\chi_i = [\omega_{i1}E_{i1}, \omega_{i2}E_{i2}, \ldots, \omega_{im}E_{im}]^T$ and $\|\chi_i\| \leq E_i^*$, with $E_i^*$ being an unknown constant.

Without loss of generality, it can be shown that

$$\left\|\sum_{i=1}^{N} Gx_0(t)\right\| \leq \bar{D}_0 \tag{12}$$

where $\bar{D}_0 > 0$ is a constant.

Besides, for a positive parameter $k_{i1}$, let

$$\frac{r_0 G^2}{4\chi_i(1-\tau_i)} = \mu_i k_{i1} \tag{13}$$

and there also exists a positive parameter $k_{i2}$ such that

$$\begin{aligned} &\left\|u_{si}(t) + Fd_i(t) - \chi_i + \sum_{i=1}^{N} Gx_0(t)\right\| \\ &\leq \bar{u}_{si} + \|F\|\,\bar{d}_i + E_i^* + \bar{D}_0 \leq \mu_i k_{i2} \end{aligned} \tag{14}$$

Then, the controller gains $K_0, K_{i1}(t), K_{i2}(t)$ are chosen as

$$\begin{aligned} K_0 &= -B^T P \\ K_{i1}(t) &= -\hat{k}_{i1} B^T P \\ K_{i2}(t) &= \frac{-\hat{k}_{i2} B^T P}{\left\|e_i^T PB\right\| \hat{k}_{i2} + \sigma_i(t)} \end{aligned} \tag{15}$$

where $\hat{k}_{i1}$ $\hat{k}_{i2}$ are the estimates of $k_{i1}, k_{i2}$, $\sigma_i(t) > 0$ is a uniformly bounded continuous function satisfying

$$\lim_{t\to\infty} \int_{t_0}^{t} \sigma_i(\tau)\, d\tau \leq \bar{\sigma}_i < \infty \tag{16}$$

where $\bar{\sigma}_i > 0$, and the parameter update laws are given by

$$\begin{aligned} \dot{\hat{k}}_{i1} &= -\gamma_{i1}\sigma_i\hat{k}_{i1} + \gamma_{i1}\left\|e_i^T PB\right\|^2 \\ \dot{\hat{k}}_{i2} &= -\gamma_{i2}\sigma_i\hat{k}_{i2} + \gamma_{i2}\left\|e_i^T PB\right\| \end{aligned} \tag{17}$$

where $\gamma_{i,1}, \gamma_{i,2}$ represent positive constants, and $\tilde{k}_{i1}(t) = \hat{k}_{i1}(t) - k_{i1}(t), \tilde{k}_{i2}(t) = \hat{k}_{i2}(t) - k_{i2}(t), i = 1, 2, \ldots, N$.

Therefore, the closed-loop dynamics of agent $i$ is described as

$$\begin{aligned} \dot{x}_i(t) =& Ax_i + B_d d_i + Bu_{si} + BGx_i(t-\tau_i) \\ &+ B\left((cK_0 + K_{i1} + K_{i2})e_i(t) - \chi_i\right) \end{aligned} \tag{18}$$

*E. Error Dynamic Systems*

Define the dynamic error $\delta_i(t) = x_i(t) - x_0(t)$, which is given by (1), (2) and (18) that

$$\begin{aligned} \dot{\delta}_i(t) =& A\delta_i + B_d d_i + Bu_{si} + BGx_i(t-\tau_i) \\ &+ B((cK_0 + K_{i1} + K_{i2})e_i(t) - \chi_i) \end{aligned} \tag{19}$$

Furthermore, to acquire the tracking error for the entire system, let

$$\begin{aligned} e &= [e_1^T, e_2^T, \ldots, e_N^T]^T, \\ \bar{D} &= \left[Fd_1^T, Fd_2^T, \ldots, Fd_N^T\right]^T, \\ \phi_s &= \left[us_1^T, us_2^T, \ldots, us_N^T\right]^T, \\ \chi &= [\chi_1, \chi_2, \ldots, \chi_N]^T, \\ K_1 &= [K_{11}e_1, K_{21}e_2, \ldots, K_{N1}e_N]^T, \\ K_2 &= [K_{12}e_1, K_{22}e_2, \ldots, K_{N2}e_N]^T, \\ \delta(t) &= [\delta_1^T(t), \delta_2^T(t), \cdots, \delta_N^T(t)]^T, \\ \bar{T} &= [x_1(t-\tau_1(t)), x_2(t-\tau_2(t)), \ldots, x_N(t-\tau_N(t))]^T \end{aligned} \tag{20}$$

Afterwards, the global dynamic error can be easily to get that

$$\begin{aligned} \dot{\delta} =& (I_N \otimes A + cH \otimes BK_0)\,\delta \\ &+ (I_N \otimes B)(\bar{D} + \phi_s + G\bar{T} + K_1 + K_2 - \chi) \end{aligned} \tag{21}$$

## III. Stability analysis

This section describes the application of AFTC methods (9)-(11), (15) and (17) to the leader-follower graph under the event-triggered mechanism, and provides an explanation.

**Theorem 1**: For MASs (1) and (2) that meet Assumptions 1-3, based on the designed controllers (9), the solution of the closed-loop system is guaranteed to be bounded and the tracking error of the system tends to zero. Moreover, to minimize the amount of updates to the control signal, a generalized fixed-threshold method is offered, which avoids the Zeno phenomenon and achieves the purpose of saving resources.

**proof**: Design the following Lyapunov-Krasovskii function $V(t)$ as

$$\begin{aligned} V(t) =& \frac{1}{2}\delta^T(H \otimes P)\,\delta + \frac{1}{2}\sum_{i=1}^{N}\mu_i\left(\gamma_{i1}^{-1}\tilde{k}_{i1}^2 + \gamma_{i2}^{-1}\tilde{k}_{i2}^2\right) \\ &+ \sum_{i=1}^{N}\chi_i r_0^{-1}\int_{t-\tau_i}^{t}\delta^T(s)\delta(s)ds \end{aligned} \tag{22}$$

From (15) and (21), one can obtain

$$\begin{aligned} \dot{V} =& \frac{1}{2}\delta^T\left(H \otimes (A^T P + PA) - 2cH^2 \otimes PBB^T P\right)\delta \\ &+ \delta^T(H \otimes PB)(\bar{D} + \phi_s + G\bar{T} + K_1 + K_2 - \chi) \\ &+ \sum_{i=1}^{N}\mu_i\left(\gamma_{i1}^{-1}\tilde{k}_{i1}\dot{\tilde{k}}_{i1} + \gamma_{i2}^{-1}\tilde{k}_{i2}\dot{\tilde{k}}_{i2}\right) + \sum_{i=1}^{N}\chi_i r_0^{-1}\|\delta_i\|^2 \\ &- \sum_{i=1}^{N}\chi_i r_0^{-1}(1 - \dot{\tau}_i(t))\,\|\delta(t-\tau_i)\|^2 \end{aligned} \tag{23}$$

Besides, with the help of the neighborhood tracking error (4) and the definition of $\delta_i$, we infer that $e_i = \sum_{j\in N_i} a_{ij}(\delta_i - \delta_j) + g_i\delta_i$, $e = (H \otimes I_n)\,\delta$. Meanwhile, it can be observed that $\delta^T \left(H \otimes I_n\right)\left(I_N \otimes PB\right) = e^T(I_N \otimes PB)$, then it gives

$$\begin{aligned}\dot{V} =&\frac{1}{2}\delta^T \left(H \otimes (A^T P + PA) - 2cH^2 \otimes PBB^T P\right)\delta \\ &+ \sum_{i=1}^{N} e_i^T PBK_{i1}e_i + \sum_{i=1}^{N} e_i^T PBK_{i2}e_i \\ &+ \sum_{i=1}^{N} e_i^T PB\left(\bar{D} + \phi_s - \chi\right) + \sum_{i=1}^{N} e_i^T PBGx_i(t-\tau_i) \\ &+ \sum_{i=1}^{N} \mu_i\left(\gamma_{i1}^{-1}\tilde{k}_{i1}\dot{\hat{k}}_{i1} + \gamma_{i2}^{-1}\tilde{k}_{i2}\dot{\hat{k}}_{i2}\right) + \sum_{i=1}^{N}\lambda_i r_0^{-1}\left\|\delta_i\right\|^2 \\ &- \sum_{i=1}^{N}\lambda_i r_0^{-1}(1-\dot{\tau}_i(t))\left\|\delta(t-\tau_i)\right\|^2\end{aligned} \tag{24}$$

Combining with (15) and Assumption 3, one can get

$$\begin{aligned}\dot{V} \le&\frac{1}{2}\delta^T \left(H \otimes (A^T P + PA) - 2cH^2 \otimes PBB^T P\right)\delta \\ &- \sum_{i=1}^{N}\frac{\hat{k}_{i2}^2 e_i^T PBB^T Pe_i}{\left\|e_i^T PB\right\|\hat{k}_{i2} + \sigma_i} - \sum_{i=1}^{N}\hat{k}_{i1}e_i^T PBB^T Pe_i \\ &+ \sum_{i=1}^{N}\left|e_i^T PB\right|\left|\bar{D} + \phi_s - \chi + Gx_0(t)\right| \\ &+ \sum_{i=1}^{N}\mu_i\left(\gamma_{i1}^{-1}\tilde{k}_{i1}\dot{\hat{k}}_{i1} + \gamma_{i2}^{-1}\tilde{k}_{i2}\dot{\hat{k}}_{i2}\right) \\ &+ \sum_{i=1}^{N}\lambda_i r_0^{-1}\left\|\delta_i\right\|^2 + \sum_{i=1}^{N} e_i^T PBG\delta(t-\tau_i) \\ &- \sum_{i=1}^{N}\lambda_i r_0^{-1}(1-\dot{\tau}_i(t))\left\|\delta(t-\tau_i)\right\|^2\end{aligned} \tag{25}$$

Considering the positive definite condition for the matrix $\mathcal{H}$ in Lemma 1, we can clearly get that $U^T\mathcal{H}U = diag\left\{\lambda_1, \lambda_2, \cdots, \lambda_N\right\}$ holds. Therefore, a state transformation is defined as $\epsilon = (U^T \otimes I_n)\delta$ with $\epsilon = \left[\epsilon_1^T, \epsilon_2^T, \ldots, \epsilon_N^T\right]^T$. Next, there has

$$\begin{aligned}\left\|e_i^T PB\right\| G\delta(t-\tau_i) \le&\frac{r_0 G^2}{4\lambda_i(1-\tau_i)}\left\|e_i^T PB\right\|^2 \\ &+ \frac{(1-\bar{\tau}_i)\lambda_i}{r_0}\left\|\delta(t-\tau_i)\right\|^2\end{aligned} \tag{26}$$

Accordingly, substituting (13), (14) and (26) into (25), it is true that

$$\begin{aligned}\dot{V} \le&\frac{1}{2}\sum_{i=1}^{N}\lambda_i\epsilon_i^T\left(A^T P + PA - 2c\lambda_i PBB^T P + \frac{I}{r_0}\right)\epsilon_i \\ &- \sum_{i=1}^{N}\frac{\mu_i\hat{k}_{i2}^2\left\|e_i^T PB\right\|^2}{\left\|e_i^T PB\right\|\hat{k}_{i2} + \sigma_i} - \sum_{i=1}^{N}\mu_i\hat{k}_{i1}\left\|e_i^T PB\right\|^2 \\ &+ \sum_{i=1}^{N}\mu_i k_{i1}\left\|e_i^T PB\right\|^2 + \sum_{i=1}^{N}\mu_i k_{i2}\left\|e_i^T PB\right\| \\ &+ \sum_{i=1}^{N}\mu_i\left(\gamma_{i1}^{-1}\tilde{k}_{i1}\dot{\hat{k}}_{i1} + \gamma_{i2}^{-1}\tilde{k}_{i2}\dot{\hat{k}}_{i2}\right)\end{aligned} \tag{27}$$

Moreover, on account of (17) and (27) yields

$$\begin{aligned}\dot{V} \le&\frac{1}{2}\sum_{i=1}^{N}\lambda_i\epsilon_i^T\left(A^T P + PA - 2c\lambda_i PBB^T P + \frac{I}{r_0}\right)\epsilon_i \\ &- \sum_{i=1}^{N}\frac{\mu_i\hat{k}_{i2}^2\left\|e_i^T PB\right\|^2}{\left\|e_i^T PB\right\|\hat{k}_{i2} + \sigma_i} - \sum_{i=1}^{N}\mu_i\hat{k}_{i1}\left\|e_i^T PB\right\|^2 \\ &+ \sum_{i=1}^{N}\mu_i k_{i1}\left\|e_i^T PB\right\|^2 + \sum_{i=1}^{N}\mu_i k_{i2}\left\|e_i^T PB\right\| \\ &+ \sum_{i=1}^{N}\mu_i\tilde{k}_{i2}\left\|e_i^T PB\right\| + \sum_{i=1}^{N}\mu_i\tilde{k}_{i1}\left\|e_i^T PB\right\|^2 \\ &+ \sum_{i=1}^{N}\mu_i\sigma_i\left(-\tilde{k}_{i1}\hat{k}_{i1} - \tilde{k}_{i2}\hat{k}_{i2}\right)\end{aligned} \tag{28}$$

Obviously, we arrive at

$$\begin{aligned}&\sum_{i=1}^{N}\mu_i\left\|e_i^T PB\right\|(k_{i2} + \tilde{k}_{i2}) - \sum_{i=1}^{N}\frac{\mu_i\hat{k}_{i2}^2\left\|e_i^T PB\right\|}{\left\|e_i^T PB\right\|\hat{k}_{i2} + \sigma_i} \\ &= \sum_{i=1}^{N}\frac{\mu_i\sigma_i k_{i2}\left\|e_i^T PB\right\|}{\left\|e_i^T PB\right\|\hat{k}_{i2} + \sigma_i} \le \sum_{i=1}^{N}\mu_i\sigma_i\end{aligned} \tag{29}$$

and

$$\sum_{i=1}^{N}\mu_i\left\|e_i^T PB\right\|^2\left(k_{i1} - \hat{k}_{i1} + \tilde{k}_{i1}\right) = 0 \tag{30}$$

Then, it also implies

$$\begin{aligned}\dot{V} \le&\frac{1}{2}\sum_{i=1}^{N}\lambda_i\epsilon_i^T\left(A^T P + PA - 2c\lambda_i PBB^T P + \frac{I}{r_0}\right)\epsilon_i \\ &+ \sum_{i=1}^{N}\mu_i\sigma_i\left(-\tilde{k}_{i1}\hat{k}_{i1} - \tilde{k}_{i2}\hat{k}_{i2} + 1\right)\end{aligned} \tag{31}$$

Invoking the inequality constraint equation yields

$$\begin{aligned}&0 \le \frac{pq}{p+q} \le a, \quad \forall p, q > 0 \\ &-\tilde{k}_{i1}\hat{k}_{i1} \le -\tilde{k}_{i1}\left(\tilde{k}_{i1} + k_{i1}\right) \le \frac{1}{4}k_{i1}^2 \\ &-\tilde{k}_{i2}\hat{k}_{i2} \le -\tilde{k}_{i2}\left(\tilde{k}_{i2} + k_{i2}\right) \le \frac{1}{4}k_{i2}^2\end{aligned} \tag{32}$$

and denote $Q$ as a positive definite matrix satisfying $-Q = A^T P + PA - PBB^T P + \frac{1}{r_0}I$. Consequently, it readily follows that

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(Q)\min_{1\le j\le N}\left\{\lambda_j\right\}\left\|\delta\right\|^2 + \sum_{i=1}^{N}\sigma_i\kappa_i \tag{33}$$

where the minimum eigenvalue of matrix $Q$ is expressed as $\lambda_{\min}(Q)$, and $\kappa_i = \mu_i(\frac{k_{i1}^2}{4} + \frac{k_{i2}^2}{4} + 1)$.

Set $\tilde{\delta} = [\delta^T, \tilde{k}_{i1}, \tilde{k}_{i2}]^T$, according to (22), we find that

$$\begin{aligned}V(\tilde{\delta}) \ge&\frac{1}{2}\lambda_{\min}(P)\left\|\delta\right\|^2 + \frac{1}{2}\sum_{i=1}^{N}\mu_i\gamma_{i1}^{-1}\tilde{k}_{i1}^2 \\ &+ \frac{1}{2}\sum_{i=1}^{N}\mu_i\gamma_{i2}^{-1}\tilde{k}_{i2}^2 \ge \tilde{\zeta}_0\left\|\tilde{\delta}\right\|^2\end{aligned} \tag{34}$$

where $\tilde{\zeta}_0 = \min\left\{\frac{1}{2}\lambda_{\min}(P), \frac{1}{2}\mu_i\gamma_{i1}^{-1}, \frac{1}{2}\mu_i\gamma_{i2}^{-1}\right\}$ for $i = 1, 2, \ldots, N$. When $t > t_0$, it becomes

$$\begin{aligned} 0 \le \tilde{\zeta}_0 \left\|\tilde{\delta}\right\|^2 &\le V(\tilde{\delta}) = V(\tilde{\delta}(t_0)) + \int_{t_0}^{t} \dot{V}(\tilde{\delta}(s))ds \\ &\le V(\tilde{\delta}(t_0)) - \frac{1}{2}\int_{t_0}^{t} \lambda_{\min}(Q)\min\nolimits_{1\le j\le N}\lambda_j \left\|\delta(s)\right\|^2 ds \\ &+ \sum_{i=1}^{N} \bar{\sigma}_i\kappa_i \le V(\tilde{\delta}(t_0)) + \sum_{i=1}^{N} \bar{\sigma}_i\kappa_i \end{aligned} \tag{35}$$

This indicates that the solution of the system (18) is globally uniformly and ultimately bounded, and we can summarise that $\frac{1}{2}\int_{t_0}^{t} \lambda_{\min}(Q)\min_{1\le j\le N}\lambda_j \left\|\delta(s)\right\|^2 ds \le V(\tilde{\delta}(t_0)) + \sum_{i=1}^{N} \bar{\sigma}_i\kappa_i$. Because $\tilde{\delta}$ is uniformly bounded, thus the $\delta$ and $\dot{\delta}$ are uniformly bounded. Besides, $\delta$ is uniformly continuous. Then, according to (35), it follows that $\frac{1}{2}\lambda_{\min}(Q)\min_{1\le j\le N}\lambda_j \left\|\delta(s)\right\|^2$ is also uniformly continuous. Finally, we can obtain $\lim_{t\to\infty} \lambda_{\min}(Q)\min_{1\le j\le N}\{\lambda_j\} \left\|\delta(s)\right\|^2 ds = 0$, that is, $\lim_{t\to\infty}\|\delta_i\| = \lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, i = 1, 2, \ldots, N$. That completes the proof.

In addition, we can find an upper bound $\bar{t}$ that satisfies $t_{k+1} - t_k > \bar{t}, \forall k \in z^+$.

**Proof**: From (9), we can obtain

$$\dot{\alpha} = \frac{\partial\alpha}{\partial\delta}\dot{\delta} + \frac{\partial\alpha}{\partial\hat{k}_{i1}}\dot{\hat{k}}_{i1} + \frac{\partial\alpha}{\partial\hat{k}_{i2}}\dot{\hat{k}}_{i2} + \frac{\partial\alpha}{\partial\sigma}\dot{\sigma} \tag{36}$$

According to the above proof, it is obtained that $\delta$ , $\hat{k}_{i1}$ and $\hat{k}_{i2}$ are uniformly bounded and continuous. Because $\sigma_i(t)$ is a uniformly bounded continuous function, therefore $\alpha$ is also uniformly bounded and continuous, that is $|\dot{\alpha}| \le \xi$, where $\xi$ is a constant independent of time. In accordance with the event-triggered mechanism (10), the subsequent result (37) holds

$$|\dot{\alpha}_{ij}(t) - \dot{u}_{ij}(t)| \le |\dot{\alpha}_{ij}| \le \xi \tag{37}$$

Then, let $R_{ij}(t) = \alpha_{ij}(t) - u_{ij}(t)$. By observing that $R_{ij}(t_k) = 0$ and $\lim_{t\to t_{k+1}} R_{ij}(t) = E_{ij}$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, m$. Therefore, it is straightforward to derive the time interval $\bar{t}_i$ is satisfied $\bar{t}_i \ge E_{ij}/\xi$ and $\bar{t} = \min\{\bar{t}_1, \bar{t}_2, \ldots, \bar{t}_m\}$. At last, determining that $t_{k+1} - t_k > \bar{t}$, $\forall k \in z^+$, it shows that the Zeno behavior is avoidable. Hereby, the proof is complete.

## IV. Simulation Studies

Ultimately, an example is presented to demonstrate the applicability of the devised control strategy. The network topology depicted in Fig. 1 is comprised of a leader and four followers. Specifically, node 0 denotes the leader, and nodes 1-4 act as the followers.

First, we consider selecting the following system

$$A = \begin{bmatrix} -0.6803 & 0.0002 & -0.1049 & 0 \\ -0.1463 & -0.0062 & -4.6726 & -9.7942 \\ 1.0050 & -0.0006 & -0.5717 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1.5539 & 0.0154 & -0.1556 \\ 0 & 1.3287 & 0.2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

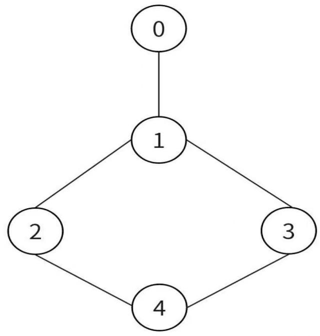


Fig. 1: Undirect communication topology

Based on (5), the positive definite matrix $P$ can be solved as

$$P = \begin{bmatrix} 1.7961 & -0.4256 & 1.2756 & 3.4834 \\ -0.4256 & 0.5505 & -0.7594 & -1.7481 \\ 1.2756 & -0.7594 & 2.4475 & 3.4178 \\ 3.4834 & -1.7481 & 3.4178 & 11.1629 \end{bmatrix}$$

Accordingly, the other parameters of the MASs are taken as $\sigma_1(t) = 5e^{-0.001t}$, $\sigma_2(t) = \sigma_3(t) = \sigma_4(t) = 2e^{-0.005t}$, $r_{11} = r_{32} = 15$, $r_{12} = r_{21} = r_{31} = r_{41} = r_{42} = 10$, $r_{22} = 5$. $F = [1; 0; 0]$, $G = [0.1, 0, 0, 0; 0, 0, 0, 0; 0, 0, 0, 0]$, $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.2 + 0.5\sin(0.5t)$, $c = 5.4$. The initial values are given by $x_0 = [10; -9; 1; 0]^T$, $x_1(0) = [5; -5; 2; 1; 1; 0.8]^T$, $x_2(0) = [5; -5; 2; 1; 1; 0]^T$, $x_3(0) = [5; -5; 2; 1; 1; 0.6]^T$, $x_4(0) = [5; 0.5; 2; 1; 1; 0.8]^T$. The external interferences are determined to be $d_1 = d_2 = d_3 = d_4 = 0.05\sin(t)$. The stuck faults are set as $u_{s1} = [0; 0; 2 + 0.01\sin(t)]$, $u_{s2} = [0; 0; 5 + 0.05\sin(t)]$, $u_{s3} = [0; 0; 3 + 0.01\sin(t)]$, $u_{s4} = [0; 0; 3 + 0.02\sin(t)]$, and the fixed thresholds are established as $E1 = E3 = E4 = 0.5, E2 = 0.8$.

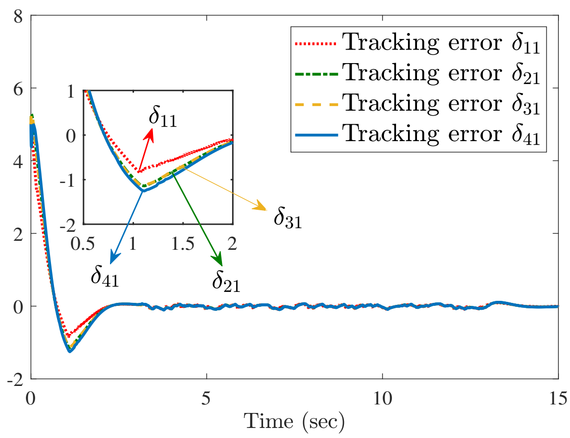


Fig. 2: The state tracking errors of $\delta_{i1}(t) = x_{01} - x_{i1}, i = 1, 2, 3, 4$

The proposed CFTC method relying on event-triggered strategy is applied to the system (2). Figs. 2-8 present the simulation findings. Figs. 2-5 plot the state tracking error performance curves of the following agents and leader, respectively. From the above four simulation images, we can clearly notice that the state tracking errors of all following agents converge asymptotically to a neighbourhood of the origin. That is to say, the designed control scheme has good

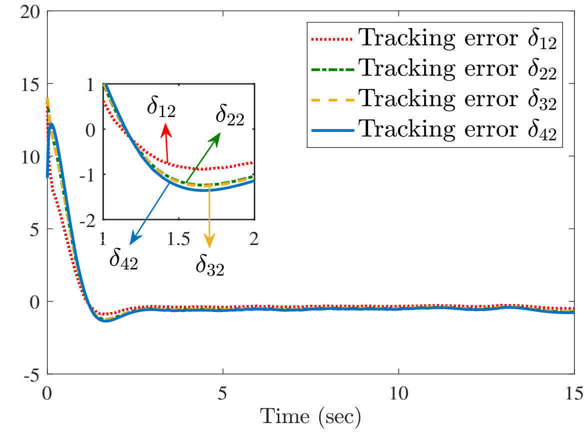


Fig. 3: The state tracking errors of $\delta_{i2}(t) = x_{02} - x_{i2}, i = 1, 2, 3, 4$

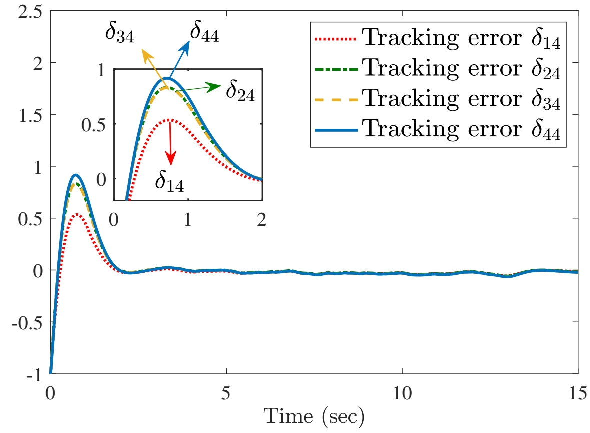


Fig. 5: The state tracking errors of $\delta_{i4}(t) = x_{04} - x_{i4}, i = 1, 2, 3, 4$

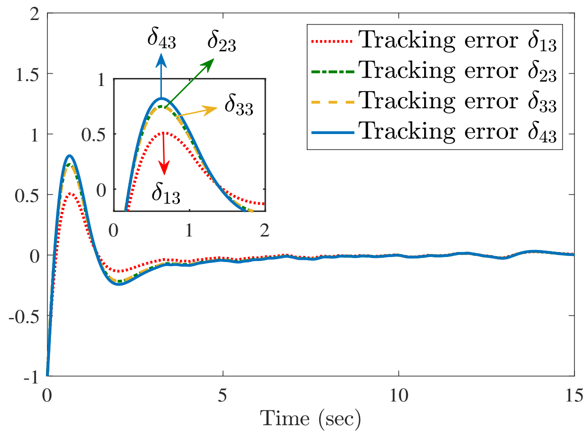


Fig. 4: The state tracking errors of $\delta_{i3}(t) = x_{03} - x_{i3}, i = 1, 2, 3, 4$

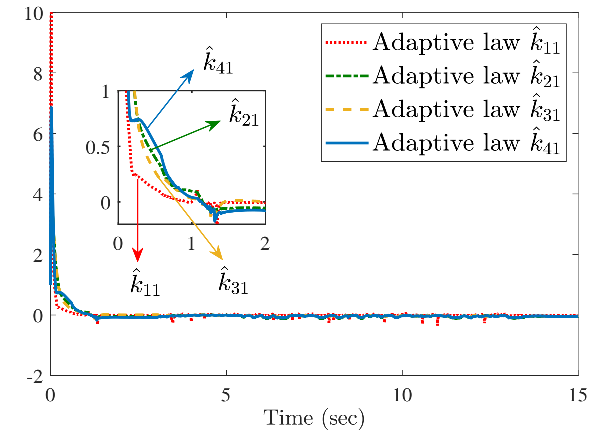


Fig. 6: Adaptive laws $\hat{k}_{i1}$, $i = 1, 2, 3, 4$

state tracking performance for the system. The time interval of the event-triggered is described in Fig. 6. By observing that the number of triggers is controlled within a narrow range and the system state is not updated continuously, which lessens the system of workload and prevents resource waste. Figs. 7 and 8 are the response plots of the parameter update laws $\hat{k}_{i1}$ and $\hat{k}_{i2}$, it is not difficult to see that they have good response curves.

## V. Conclusion

This paper addressed the cooperative control issue of linear MASs affected by time-varying delays and actuator faults. Utilizing the state information of adjacent agents, a cooperative fault-tolerant controller with adaptive update laws is proposed, which eliminates the negative impacts of actuator faults on the system. Then, the Young's inequality and the Lyapunov-Krasovskii stability analysis method are invoked to deal with the state delays. In particular, an event-triggered mechanism relying on the fixed threshold is considered to decrease resource consumption. The stability analysis shows that all closed-loop signals are uniformly bounded, and the tracking errors of all subsystems converge to zero. Even so, further optimizing the event-triggered mechanism and considering more complex heterogeneous MASs will be part of our future research objectives.

## References


[1] F. Yang, W. Wu, Y. Gao, et al, "Multi-agent fingerprints-enhanced distributed intelligent handover algorithm in LEO satellite networks," *IEEE Transactions on Vehicular Technology*, vol. 73, no. 10, pp. 15255-15269, 2024.

[2] L. Wang, G. W. Chen, T. Li, and R. T. Yang, "Fully distributed, event-triggered containment control of multi-agent systems based on wireless sensor networks and time base generators," *Applied Sciences*, vol. 13, no. 19, 11039, 2023.

[3] M. A. S. Kamal, C. P. Tan, T. Hayakawa, S. -I. Azuma, and J. -I. Imura, "Control of vehicular traffic at an intersection using a cyber-physical multiagent framework," *IEEE Transactions on Industrial Informatics*, vol. 17, no. 9, pp. 6230-6240, 2021.

[4] S. Xiao and J. Dong, "Robust adaptive fault-tolerant tracking control for uncertain linear systems with actuator failures based on the closed-loop reference model," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 50, no. 9, pp. 3448-3455, 2020.

[5] X. Wang and G. H. Yang, "Cooperative adaptive fault-tolerant tracking control for a class of multi-agent systems with actuator failures and mismatched parameter uncertainties," *IET Control Theory and Applications*, vol. 9, no. 8, pp. 1274-1284, 2015.

[6] J. Zhang, D. W. Ding, X. M. Sun, et al, "Fault-tolerant bipartite output regulation of linear multi-agent systems with loss-of-effectiveness actuator faults," *International Journal of Control, Automation and Systems*, vol. 20, no. 5, pp. 1473-1483, 2022.

[7] S. J. Huang, D. Q. Zhang, L. D. Guo, et al, "Convergent fault estimation for linear systems with faults and disturbances," *IEEE Transactions on Automatic Control*, vol. 63, no. 3, pp. 888-893, 2018.

[8] Y. J. Fang, M. R. Fei and D. J. Du, "Linear time invariant system fault isolation: a static compensation decoupling method," *International Journal of Critical Infrastructure Protection*, vol. 31, 2020.

[9] Y. B. Zang, X. Y. Ouyang, N. N. Zhao, and J. N. Zhao, "Event-triggered finite-time prescribed performance output-feedback control

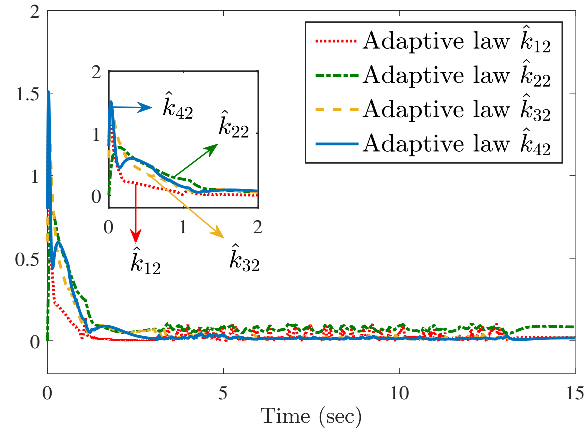


Fig. 7: Adaptive laws $\hat{k}_{i2}$, $i = 1, 2, 3, 4$

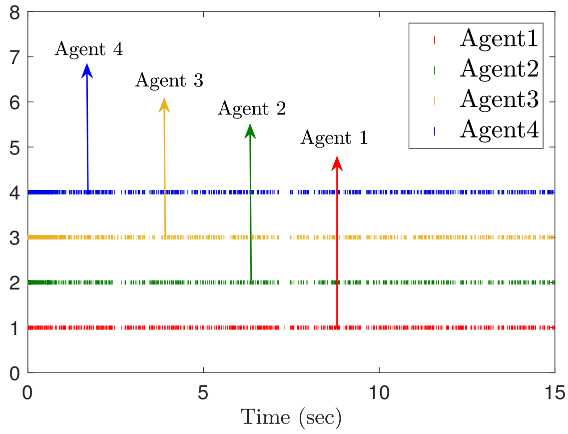


Fig. 8: Event-triggered intervals of agent $i, i = 1, 2, 3, 4$


for nonlinear systems with unmodeled dynamics," *Engineering Letters*, vol. 32, no. 4, pp. 743-752, 2024.

[10] K. Q. Yu, J. X. Ding, L. B. Wu, et al, "Event-triggered prescribed time adaptive fuzzy fault-tolerant control for nonlinear systems with full state constraints," *Engineering Letters*, vol. 32, no. 8, pp. 1577-1584, 2024.

[11] L. T. Xing, C. Y. Wen, Z. T. Liu, et al, "Event-triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2071-2076, 2017.

[12] C. Deng and G. H. Yang, "Leaderless and leader-following consensus of linear multi-agent systems with distributed event-triggered estimators," *Journal of the Franklin Institute*, vol. 356, pp. 309-333, 2019.

[13] Y. N. Shi, H. C. Yan, S. M. Chen, et al, "Distributed event-triggered consensus control for leaderless heterogeneous multiagent systems," *Journal of the Franklin Institute*, vol. 357, no. 6, pp. 3219-3234, 2020.

[14] B. Cheng and Z. K. Li, "Fully distributed event-triggered protocols for linear multi-agent networks," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1655-1662, 2019.

[15] J. R. Zhao, J. H. Park and S. Y. Xu, "Quasi-time-dependent asynchronous filtering for discrete time switched systems via the event-triggered mechanism," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 12, pp. 4633-4651, 2020.

[16] Q. Y. Xu, Y. J. Zhang and S. Y. Xiao, "Event-triggered guaranteed cost consensus of networked singular multi-agent systems," *Asian Journal of Control*, vol. 21, no. 5, pp. 2425-2440, 2019.

[17] H. Zhao, W. D. Li, Z. C. Li, et al, "Dynamic event-triggered control for Markovian jump linear systems with time-varying delay," *International Journal of Robust and Nonlinear Control*, vol. 32, no. 4, pp. 2359-2379, 2022.

[18] M. H. Chen and J. Sun, "Finite time dissipativity-based reliable control for time-varying system with delay and linear fractional uncertainties," *International Journal of Systems Science*, vol. 50, no. 3, pp. 463-478, 2019.

[19] M. H. Chen and J. Sun, "Delay-dependent conditions for finite time stability of linear time-varying systems with delay," *Asian Journal of Control*, vol. 22, no. 2, pp. 924-933, 2020.

[20] B. Yang, Z. F. Yan, X. J. Pan, et al, "Improved stability criteria for linear systems with time-varying delays," *Journal of the Franklin Institute*, vol. 358, no. 15, pp. 7804-7824, 2021.

[21] Z. L. Zhai, H. C. Yan, S. M. Chen, et al, "Hierarchical stability conditions for linear systems with interval time-varying delay," *Journal of the Franklin Institute*, vol. 361, vo. 15, pp. 1403-1415, 2024.

[22] Z. L. Zhai, H. C. Yan, S. M. Chen, et al, "Two novel stability criteria for linear systems with interval time-varying delays," *International Journal of Systems Science*, vol. 361, no. 3, pp. 87-98, 2022.

[23] L. B. Wu, X. Q. He, and D. Q. Zhang, "Cooperative adaptive fuzzy control for a class of uncertain non-linear multi-agent systems with time delays," *Journal of Control and Decision*, vol. 4, no. 3, pp. 131–152, 2016.