Anticipating Lag Synchronization Based on BO-CNN-LSTM

Yongqing Wu, Xingxing Bao

Abstract—**Synchronization is crucial for coherent dynamics in coupled chaotic systems. The delayed participation of the response system in lag synchronization provides new insights into complex natural phenomena. This study presents a data-driven prediction framework combining Bayesian Optimization, Convolutional Neural Networks, and Long Short Term Memory networks (BO-CNN-LSTM) to predict lag synchronization, especially when system equations are unknown or mathematical models are difficult to establish. Trained on time series data from asynchronous states, the model predicts the lag synchronization transition by adjusting the control parameters. The BO algorithm optimizes hyperparameters, preventing overfitting and improving model performance. Applied to the Lorenz system with time-varying delayed coupling, our approach accurately captures the effects of coupling coefficients and time delays on lag synchronization, providing a reliable tool for analyzing collective dynamics in coupled systems.**

Index Terms—**Coupled chaotic system, BO-CNN-LSTM neural network, Anticipating synchronization, Lag synchronization.**

I. Introduction

SYNCHRONIZATION is a fundamental phenomenon in dynamic systems, referring to the coherent behavior of coupled units. Chaotic oscillators can exhibit trajectory convergence despite distinct initial states, a phenomenon extensively analyzed in nonlinear dynamics and complex networks[1]. Examples of synchronization include pendulums swinging in unison or the gradual coordination of applause. In nonlinear science and the study of complex systems, synchronization is a key aspect of the collective dynamics of oscillators[2][3]. Beyond its theoretical significance, synchronization has practical applications in real-world systems, such as communication networks, where time delays can significantly influence synchronization dynamics[4][5]. Time delays imply that a system's evolution depends on both its current and previous states, leading to desynchronization and design challenges. Dynamic networks with time-delayed coupling terms are frequently used to model such effects, with delays categorized as constant, time-varying, or distributed, each introducing unique complexities[6][7][8]. Moreover, the inherent unpredictability of chaotic systems further complicates synchronization analysis. For instance, predicting solar storms like the 2012 Carrington-class event that narrowly missed Earth, remains challenging due to the chaotic nature of space weather systems. Traditional synchronization prediction methods, which rely on precise system modeling, may struggle with complex systems. Machine learning, however, offer a data-driven approach by extracting patterns directly from available data without requiring exact equations. The study by Wang et al. introduced a model-free reservoir computing approach for predicting synchronization transitions in coupled oscillators[9]. This method effectively captures both complete and phase synchronization dynamics, although it necessitates substantial data for model training and is sensitive to hyperparameter adjustments. Building on these advancements, this study focuses on predicting lag synchronization transitions in complex networks, where time delays and coupling parameters introduce additional dynamical complexity. Understanding these parameters is critical for elucidating synchronization behaviors. Lodi et al. recently analyzed synchronization in time-delay neural architectures incorporating discontinuous activation functions, leveraging robust adaptive regulation to counteract parameter mismatches[10]. Similarly, Yang et al. employed a machine learning approach to identify nonlinear system instabilities, capturing transient chaos and critical transitions through data-driven analysis. Integrating temporal dynamics into such models could further enhance prediction accuracy[11].

Originally proposed by Hochreiter and Schmidhuber in 1997, the Long Short-Term Memory (LSTM) network is widely recognized for its predictive capabilities in chaotic systems[12]. Similarly, Convolutional Neural Network (CNN) has demonstrated excellent performance across various machine learning problems and computer vision[13]. A notable advantage of CNN lies in its capacity for minimal preprocessing compared to alternative methods. This efficiency is especially important in tasks such as time series classification, where Tang et al. showed that selecting optimal convolutional kernel sizes in Conv1D models is critical for achieving accurate results[14]. While LSTM and CNN models individually yield impressive predictive results, lag synchronization in chaotic systems presents challenges that are difficult to address with a single approach. Hybrid models that combine the strengths of both CNNs for capturing dynamic features and LSTMs for trend prediction are gaining popularity[15]. Another, most machine learning algorithms require parameters, highlighting the appeal of developing parameter-efficient alternatives. More flexible take on this issue is to regard optimizing these parameters as an automated process. We

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can particularly regard this tuning process as optimizing an implicit black-box function, utilizing algorithms tailored for these cases. An effective choice is Bayesian Optimization (BO), demonstrated to surpass various advanced global optimization techniques across multiple difficult benchmark tasks[16]. In this study, BO is applied to optimize hyperparameters by constructing a probabilistic model that identifies the optimal parameter combination, thereby minimizing loss and enhancing predictive performance.

Anticipating lag synchronization using machine learning is crucial for understanding complex coupled chaotic systems. This paper introduces a novel data-driven framework that integrates Bayesian Optimization, Convolutional Neural Networks, and Long Short-Term Memory networks (BO-CNN-LSTM) to predict lag synchronization and overcome challenges posed by unknown or intricate governing equations. The model-free nature of the framework allows only time series data from asynchronous states and no prior knowledge of governing equations. It systematically evaluates the impact of coupling strength and time delays on synchronization dynamics, providing quantitative evidence of its predictive accuracy. By successfully capturing transitions from desynchronization to synchronization, the BO-CNN-LSTM model demonstrates its ability to predict critical thresholds and adapt to varying chaotic system conditions. Its applicability to real-world systems, where governing equations may be unknown, underscores its potential for diverse applications. The paper is structured as follows: Section II introduces the fundamentals of Bayesian Optimization combined with Convolutional Neural Networks and lag synchronization in the Lorenz oscillator. Section III presents a numerical analysis of the key factors influencing lag synchronization. Section IV concludes with a summary of the findings.

II. Related Work

A. LSTM model and structure

Keras is a Python-based deep learning library, with LSTM as a recurrent neural network designed for sequence data processing[17]. Compared to traditional RNNs, it excels in handling prolonged dependencies in sequential data. Its architecture incorporates three gates and cell states, which are learnable parameters that utilize the prior hidden state h_{t-1} and the present input x_t. These equations are given by:

$$i_t = \sigma\left(W^i h_{t-1} + I^i x_t + b_i\right) \tag{1}$$
$$f_t = \sigma\left(W^f h_{t-1} + I^f x_t + b_f\right) \tag{2}$$
$$o_t = \sigma\left(W^o h_{t-1} + I^o x_t + b_o\right) \tag{3}$$
$$\hat{c}_t = \tanh(W^c h_{t-1} + I^c x_t + b_c)\,, \tag{4}$$

where W^i, I^i, b^i correspond to the cyclic weight matrix, projection matrix and bias vector of the input gate i_t, respectively. Likewise, W^f, I^f, b^f pertain to the forget gate f_t. W^o, I^o, b^o to the output gate o_t. W^c, I^c, b^c to the memory cell $\hat{c}_t$.

LSTM, as a variant of RNN, exhibits strong performance in learning time series data by preserving contextual information and capturing temporal dynamics of events. The improvement of it is mainly reflected in the introduction of new internal state and gating mechanisms[18]. The input gate i_t determines the amount of information retained the candidate state at the current time step. The forget gate f_t regulates the extent to which the prior internal state c_{t-1} is discarded. The output gate o_t, using the updated cell state c_t, controls the amount of information transferred to the external state h_t. As illustrated in Fig. 1, applying the sigmoid function σ to the current input x_t and the prior output h_{t-1} generates the three gates. Within the forget gate, f_t adjusts the prior memory c_{t-1}, removing certain information to refresh the cell state c_t. Then network transforms the current input x_t alongside the prior output h_{t-1} through the $\tanh$ function, obtaining the processed input $\hat{c}_t$. This transformed input is then multiplied by the input gate i_t, which filters the input information. The filtered information is added to the updated cell state c_t, which is subsequently transferred to the next iteration. The equation is given as follows:

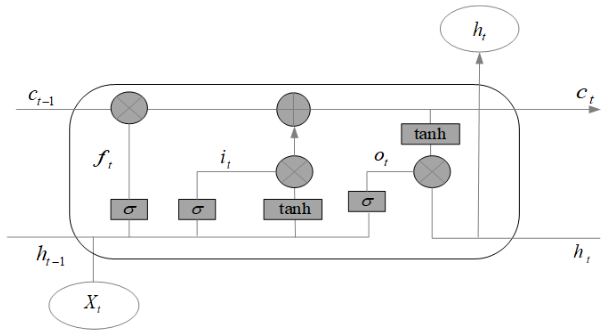

Fig. 1. LSTM network structure.

$$c_t = f_t \odot c_{t-1} + i_t \odot \hat{c}_t, \tag{5}$$
$$h_t = o_t \odot \tanh(c_t), \tag{6}$$

where $\odot$ is the element-by-element product. Multiply the output gate o_t with $\tanh(c_t)$, which is the transformed cell state, used to compute the hidden state h_t. At the next time step, this hidden state is carried over, where it interacts with the next input x_{t+1}.

B. BO-CNN-LSTM model[19]

The CNN-LSTM architecture synergistically combines CNN's superior spatial feature extraction with LSTM's exceptional temporal modeling capabilities, enhancing robust learning of spatiotemporal patterns. And the combination allows for a more comprehensive analysis and prediction of time series changes. Moreover, the model's architecture facilitates the processing of sequential data, making it particularly well-suited for applications involving dynamic systems. However, with growing hyperparameter counts, network sophistication increases, making it more susceptible to overfitting. Typically, hyperparameter combinations are adjusted based on empirical experience, which can lead to suboptimal generalization performance since manual tuning often overlooks relevant factors and fails to effectively utilize past performance data. Additionally, each new hyperparameter combination necessitates retraining the model, entailing prolonged training cycles and intensive manual intervention.

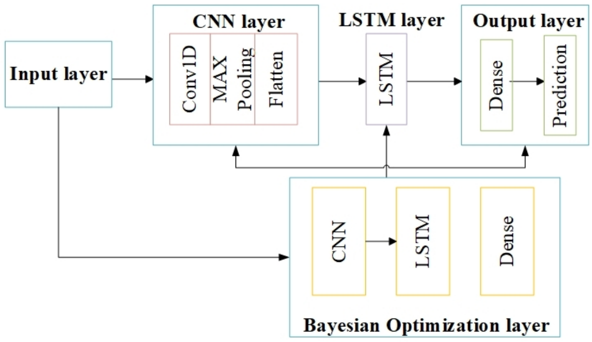

Fig. 2. Architectural diagram of the BO-CNN-LSTM hybrid model.

To enhance the model's generalization ability and ensure accurate predictions across various datasets, we propose employing the Bayesian Optimization algorithm for hyperparameter selection. The BO algorithm merges the strengths of manual tuning with the automated selection capabilities of grid and random search by tracking past evaluations to construct a probabilistic model that identifies the hyperparameter combination minimizing loss. This framework is illustrated in Algorithm 1. By integrating this optimization approach, the proposed BO-CNN-LSTM model comprises an input layer, convolutional layers extracting spatial features, recurrent layers capturing temporal dependencies, an output layer, and a Bayesian Optimization layer, as depicted in Fig. 2. This structure not only facilitates efficient hyperparameter tuning but supports robust predictions of lag synchronization in complex coupled chaotic systems.

Algorithm 1: BO Algorithm

for $t = 1, 2, \ldots$ **do**
  Select the next experiment point via acquisition function maximization;
  $x_t = \arg\max_{x \in \mathcal{X}} \alpha(x|D_{1:t-1})$;
  Evaluate the objective function value: $y_t = f(x_t) + \varepsilon_t$;
  Integrate the data: $D_t = D_{t-1} \cup \{(x_t, y_t)\}$;
  Update the probabilistic surrogate model;
end

C. Lag synchronization of coupled Lorenz oscillators

Formally, a dynamical system exhibits chaos when infinitesimally close initial states generate exponentially divergent trajectories under deterministic dynamics. Notably, the synchronization process differs significantly based on coupling configurations, primarily classified into two distinct types: unidirectional and bidirectional coupling. In former scenarios, a drive-response configuration emerges where a master subsystem governs the evolution of a slave subsystem, resulting in external synchronization, as exemplified by chaos-based communication systems. Conversely, bidirectional coupling involves mutual interaction between subsystems, inducing synchronized behavior through rhythm adjustment onto a shared synchronization manifold, as observed in physiological systems (e.g., cardiorespiratory interactions) and nonlinear optics[20].

Additionally, neural networks are often influenced by impulses and time delays, making lag synchronization a valuable approach for achieving rapid system synchronization. Extensive research has been devoted to studying synchronization phenomena in linear dynamical systems with fixed time delays. Nevertheless, many challenges remain in proving whether time-varying delay systems can achieve neural network synchronization. Therefore determining sufficient synchronization conditions for neural networks constitutes a crucial research objective. This study examines a coupled system comprising two identical dynamic Lorenz equations discussed on the basis of time-delay coupled chaotic maps:

$$\begin{cases} \dot{x_1}(t) = \mu(y_1(t) - x_1(t)) + \varepsilon(x_2(t) - x_1(t-\tau)), \\ \dot{y_1}(t) = x_1(t)(\beta - z_1(t)) - y_1(t) + \varepsilon(y_2(t) - y_1(t-\tau)), \\ \dot{z_1}(t) = x_1(t)y_1(t) - \gamma z_1(t) + \varepsilon(z_2(t) - z_1(t-\tau)), \end{cases} \tag{7}$$

$$\begin{cases} \dot{x_2}(t) = \mu(y_2(t) - x_2(t)) + \varepsilon(x_1(t) - x_2(t-\tau)), \\ \dot{y_2}(t) = x_2(t)(\beta - z_2(t)) - y_2(t) + \varepsilon(y_1(t) - y_2(t-\tau)), \\ \dot{z_2}(t) = x_2(t)y_2(t) - \gamma z_2(t) + \varepsilon(z_1(t) - z_2(t-\tau)), \end{cases} \tag{8}$$

where $(x_1(t), y_1(t), z_1(t))$and $(x_2(t), y_2(t), z_2(t))$represent the state vectors of the first and second coupled systems at discrete time t, respectively. The systems exhibit chaotic dynamics when isolated under parameters $(\mu, \beta, \gamma) = (10.0, 28.0, 2.0)$, with τ denoting the time delay and ε denoting the coupling parameter.

Definition 1. *[21] Let $\xi = (x_1, y_1, z_1)^T$ and $\eta = (x_2, y_2, z_2)^T$, and the dynamical system Eq. (7) is said to lag synchronize with the dynamical system Eq. (8) at time τ if*

$$\lim_{t \to +\infty} \|\eta(t) - \xi(t-\tau)\| = 0, \tag{9}$$

where τ is a given positive time delay.

Remark 1. *When the delay τ approaches zero, the discussion of lag synchronization transitions to complete synchronization. In the absence of time delay, the system's dynamics exhibit instantaneous and synchronized updates and responses across all states. In the coupled Lorenz system discussed, complete synchronization emerges when the coupling coefficient exceeds the critical threshold obtained from the master stability framework. Specifically, the system achieves complete synchronization when $\varepsilon > \varepsilon_0 \approx 0.41$[9][22].*

III. Simulation Results

Due to modeling errors and external disturbances, the parameters of the neural network may be subject to fluctuations, resulting in parameter uncertainty. Given the sensitivity of chaotic systems to initial values, small parameter variations may substantially alter system dynamics[23]. This paper will discuss two factors that influence this phenomenon.

A. Effect of coupling parameter on lag synchronization

Different from traditional approaches that utilize fixed coupling parameters[24], we consider a scenario where

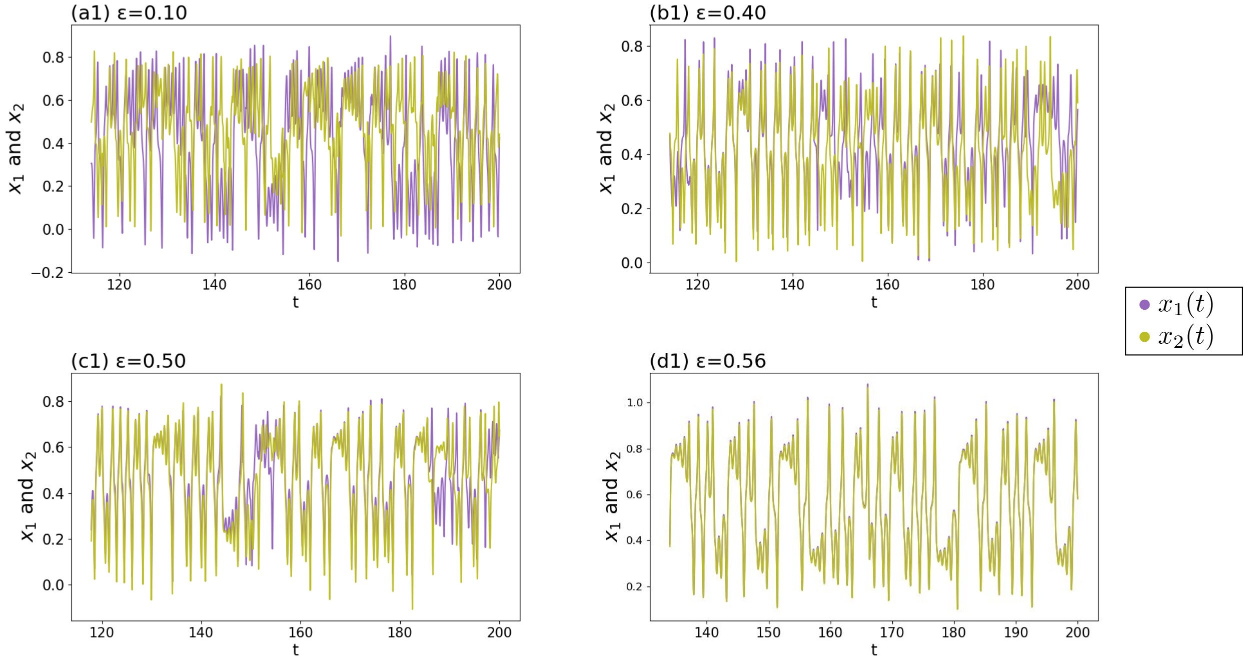

Fig. 3. Predicted the synchronization transition in coupled Lorenz system with different coupling parameters (Purple and chartreuse represent the trained trajectory of x_1 and x_2, respectively).

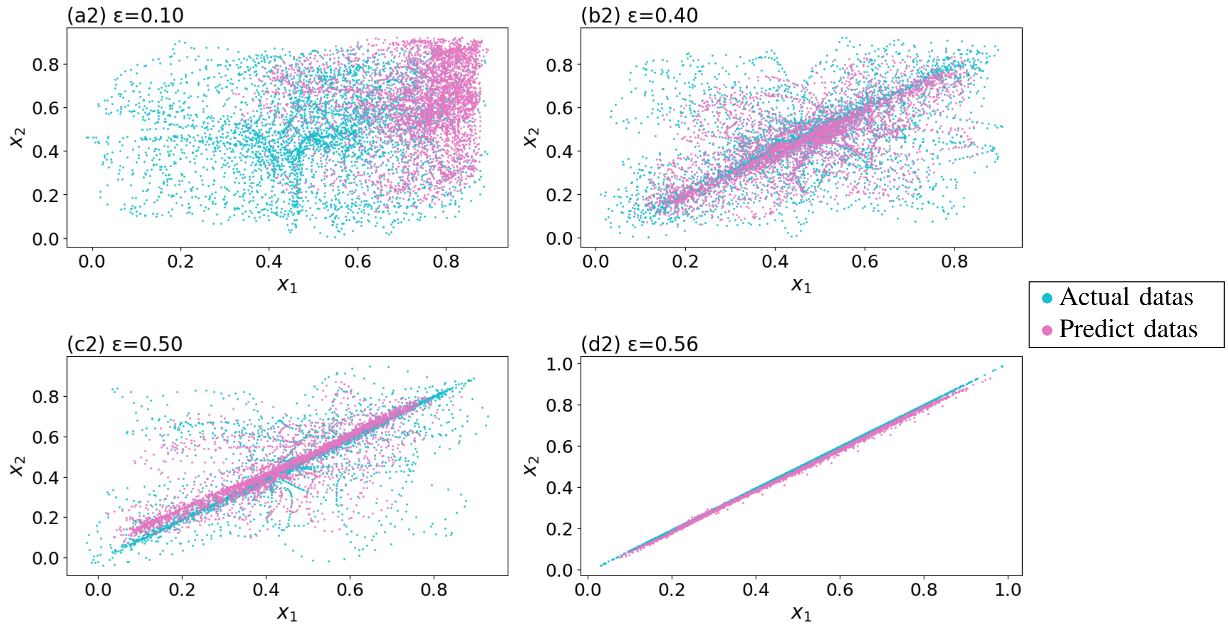

Fig. 4. Dynamic state of coupled Lorenz system variables x_1 and x_2 with different coupling parameters (Blue dots are the variable actual values, pink dots are the variable predicted values).

coupling parameters change over time. In this context, ε is treated as a step function of time, while a delay $\tau = 1$ is set to examine the impact of coupling parameters on system synchronization.

Training datasets were constructed by sampling three characteristic coupling strengths $\varepsilon = [0.10, 0.40, 0.50]$ spanning network spectral radius, all within the asynchronous states, which provides the basis for the model to learn the complex behavior of the system. For each value of ε, we collect time series data of length $T = 4 \times 10^3$ at time steps $\Delta t = 0.02$, after disregarding a transient phase of length $T_0 = 1 \times 10^3$. The input vector $u(t) \equiv [x_1(t), x_2(t), y_1(t), y_2(t), z_1(t), z_2(t)]^T$ and the control parameter signal $\varepsilon(t)$ are then input into the BO-CNN-LSTM model for output training.

This study employs machine learning to predict the synchronization behavior of two variables in a coupled Lorenz system under varying coupling parameters. The state variables of the system are coupled to each other in Eqs. (7)-(9). If x_1 and x_2 achieve synchronization, the remaining vector sets exhibit synchronous characteristics, and vice versa. In this paper, we focus on the relationship between x_1 and x_2 in two identically coupled Lorenz systems, though similar behaviors are observed for other variables.

The RMSE results for various models under different values of ε (0.10, 0.40, 0.50, and 0.56) are shown in Table I. The table highlights the performance of LSTM, KAN, and BO-CNN-LSTM in capturing lag synchronization. BO-CNN-LSTM consistently achieves the lowest RMSE across all tested values of ε, demonstrating its superior ability to handle lag synchronization and outperform other approaches in capturing synchronization transitions.

TABLE I
RMSE-BASED COMPARISON OF LAG SYNCHRONIZATION MODELS.

Datasets	$\varepsilon = 0.10$	$\varepsilon = 0.40$	$\varepsilon = 0.50$	$\varepsilon = 0.56$
LSTM	0.328	0.331	0.337	0.200
KAN	0.305	0.314	0.327	0.194
BO-CNN-LSTM	0.084	0.058	0.076	0.087

Table II shows the loss values for various models evaluated on the test dataset under different values of ε. The loss metric evaluates the synchronization error between the predicted and observed dynamical states, characterizing the attractor reconstruction accuracy in the network's phase space. The proposed BO-CNN-LSTM model consistently achieves the lowest loss values across all tested conditions, demonstrating its superior capability to accurately predict lag synchronization transitions and further affirming its robustness compared to other models.

TABLE II
COMPARISON OF LAG SYNCHRONIZATION MODELS BASED ON LOSS METRICS

Datasets	$\varepsilon = 0.10$	$\varepsilon = 0.40$	$\varepsilon = 0.50$	$\varepsilon = 0.56$
LSTM	0.110	0.123	0.116	0.040
KAN	0.097	0.102	0.109	0.038
BO-CNN-LSTM	0.010	0.006	0.002	0.001

While RMSE and loss effectively quantify prediction accuracy, they do not account for the statistical significance of performance differences across models. To establish whether the improvements achieved by BO-CNN-LSTM are consistent and reliable, we further conduct paired t-tests and calculate 95% confidence intervals (CIs). These statistical analyzes quantify whether the reduction in prediction errors is consistent across different samples and determine the reliability of the model's performance. The dependent samples t-test assesses whether the mean difference in prediction errors between paired model outputs achieves statistical significance. The t-statistic is calculated as follows:

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}, \tag{10}$$

where $\bar{d}$ is the mean of the differences in prediction errors, s_d is the standard deviation of the differences and n is the number of data points. A p-value below 0.05 indicates statistical significance.

In addition, the 95% confidence interval estimates the uncertainty of the model's prediction errors. It is calculated as follows:

$$CI = \bar{x} \pm t_{\alpha/2,n-1} \cdot \left(\frac{s}{\sqrt{n}}\right) \tag{11}$$

here $\bar{x}$ is the mean prediction error, $t_{\alpha/2,n-1}$ is the critical value from the t-distribution ($\bar{\alpha} = 0.05$), s is the standard deviation of the prediction errors and n is the number of data points. The 95% confidence interval provides a range in which the true mean is likely to fall in 95% of repeated experiments.

The statistical analysis shows that BO-CNN-LSTM achieves significantly lower prediction errors compared to LSTM and KAN. The mean RMSE of BO-CNN-LSTM is 0.07625, which reflects a lower average prediction error. The paired-sample analysis reveals a statistically significant difference, with BO-CNN-LSTM outperforming LSTM ($t = -6.01$, $p = 0.0092$) and KAN ($t = -6.00$, $p = 0.0093$). Since both p-values are below the commonly used significance threshold of 0.05, the evidence sufficiently contradicts the null hypothesis, indicating that the observed performance improvements are unlikely to have occurred by chance. Furthermore, the 95% confidence interval for BO-CNN-LSTM is (0.05553, 0.09697), suggesting that if the experiment were repeated multiple times, the true mean prediction error would fall within this range in 95% of cases. The relatively narrow confidence interval suggests that the prediction errors remain stable and do not exhibit large fluctuations, demonstrating greater reliability. These findings confirm that BO-CNN-LSTM not only achieves higher accuracy but also maintains more consistent predictions, ensuring its reliability in forecasting lag synchronization transitions.

As showed in Fig. 3, the trained model predicts how synchronization transitions occur for different coupling parameters. Figs. 3(a1)-(c1) show the predicted state changes of the variables x_1 and x_2 in a coupled Lorenz system for three different values when ε is 0.10, 0.40 and 0.50 respectively, where purple denotes the trained motion trajectory of x_1, while chartreuse denotes the trained motion trajectory of x_2. The system exhibits asynchronous dynamics across all three parameter configurations. With coupling strength tuned to 0.56, the trained machine predicts the synchronization behavior of variables x_1 and x_2, as shown in Fig. 3(d1). This figure clearly illustrates the impact of coupling strength on synchronization by comparing the behavior of the system under different parameters, highlighting how adjusting the coupling strength can control synchronization transitions.

To visually illustrate the synchronization process of the variables x_1 and x_2, Figs. 4(a2)-(d2) shows the relationship between the two variables under different coupling parameters (where blue dots denote the actual values of the variables x_1 and x_2, pink dots denote the predicted values of the variables x_1 and x_2), which show the process of mutual coupling changes of the two variables. During the study, it was found that predicting synchronization transition is possible as long as data with at least two different parameter values are used for training. Prediction accuracy exhibits critical enhancement as training parameters approach the bifurcation threshold. As the value of ε increases from 0.10 to 0.56, with a fixed time lag, the

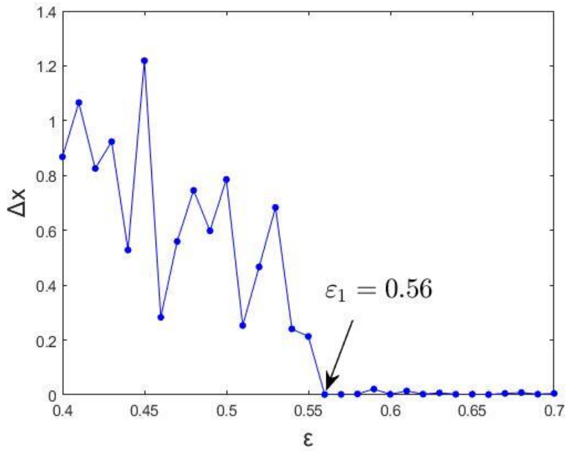

Fig. 5. Effect of coupling parameters on synchronization of coupled Lorenz system.

points (x_1, x_2) gradually converge to the diagonal and begin to exhibit the characteristics of synchronization.

As validate the prediction of the critical transition point ε_1 for lag synchronization, we systematically increased the coupling parameter ε from 0.4 to 0.7. The synchronization error Δx (discarding transients period) is calculated over the time span $T = 4 \times 10^3$, which is as follows:

$$\Delta x = \frac{\sum_{i=T_0}^{T} |x_{1i}(t-\tau) - x_{2i}(t)|}{T - T_0} \tag{12}$$

where x_{1i} and x_{2i} denote the dynamical variables of x_1 and x_2 of the two systems at the ith time node and $T_0 = 1 \times 10^3$ is the transient period.

Fig. 5 illustrates that as the coupling parameter increases, the synchronization error Δx progressively decreases, approaching zero around $\varepsilon_1 = 0.56$. It turns out that a well-trained model can accurately identify the critical transition point to synchronization.

In this study, we developed a BO-CNN-LSTM neural network prediction model. The model was trained using comprehensive dynamic system data generated through numerical integration, encompassing a range of parameters from weak to strong coupling. To enhance prediction accuracy, we meticulously optimized the network architecture and fine-tuned the hyperparameters, aiming to minimize the discrepancies between the model's predictions and actual observations.

B. *Effect of time delay on lag synchronization*

The system achieves complete synchronization when $\varepsilon_1 \geq 0.41$. In this example, the control variable of is set as $\varepsilon = 0.5$ to further consider the influence of time delay on synchronization, indicating that the system achieves complete synchronization without delay. In particular, the time delay τ is treated as a step function of time.

As shown in Table III, the RMSE results for different values of τ (0.50, 1.00, 1.55, and 2.00) offer a comparative assessment of the LSTM, KAN, and BO-CNN-LSTM models. The proposed BO-CNN-LSTM model consistently achieves the lowest RMSE across all tested time-delay conditions. At the critical delay value $\tau = 1.55$, the model demonstrates substantial improvements over other approaches, underscoring its robustness in predicting synchronization transitions under time-delayed dynamics.

TABLE III
RMSE FOR LAG SYNCHRONIZATION MODELS UNDER DIFFERENT TIME DELAYS

Datasets	$\tau = 0.50$	$\tau = 1.00$	$\tau = 1.55$	$\tau = 2.00$
LSTM	0.328	0.343	0.175	0.343
KAN	0.337	0.345	0.190	0.322
BO-CNN-LSTM	0.031	0.084	0.024	0.072

Table IV presents the loss values for the models under various time delays. The BO-CNN-LSTM model consistently outperforms both LSTM and KAN, achieving the lowest loss values in all scenarios. These findings confirm the model's effectiveness and reliability in anticipating lag synchronization transitions, even under complex and diverse time-delay conditions.

TABLE IV
LOSS METRICS COMPARISON OF LAG SYNCHRONIZATION MODELS UNDER VARYING TIME DELAYS

Datasets	$\tau = 0.50$	$\tau = 1.00$	$\tau = 1.55$	$\tau = 2.00$
LSTM	0.111	0.122	0.031	0.120
KAN	0.116	0.123	0.036	0.107
BO-CNN-LSTM	0.001	0.011	0.001	0.008

Besides, statistical analyzes show that the BO-CNN-LSTM model consistently achieves significantly lower prediction errors under different conditions time delay τ. The average RMSE is 0.05275, indicating a lower mean prediction error. Further validation by paired t-tests shows that the t-value for prediction error is -7.60 with a p-value of 0.0047 compared to LSTM and -8.41 with a p-value of 0.0035 compared to KAN, both values being statistically significant. Additionally, the 95% confidence interval is (0.00549, 0.10001), suggesting that prediction errors fall within a stable range. Consequently, the BO-CNN-LSTM model exhibits superior prediction accuracy and minimal error deviation in various time-delay scenarios, underscoring its stability and reliability in predicting delay synchronization.

Given that the network has been trained to recognize parameter variations in the coupled chaotic system's global dynamics, it should be adept at predicting synchronization transitions. Figs. 6(a1), (b1) and (d1) illustrate that the time-delay coupled chaotic system fails to achieve synchronization when τ is set to 0.50, 1.0, and 2.0, where coral dots denote the trained trajectory of x_1 and olive dots denote the trained trajectory of x_2. In contrast, in Fig. 6(c1) with $\tau = 1.55$, the model predicts synchronization behavior, suggesting the existence of a specific τ value within the interval [1,2] that facilitates synchronization between variables x_1 and x_2 in the system.

Fig. 7 visually demonstrates the synchronization process, illustrating the delay-dependent regime transitions in

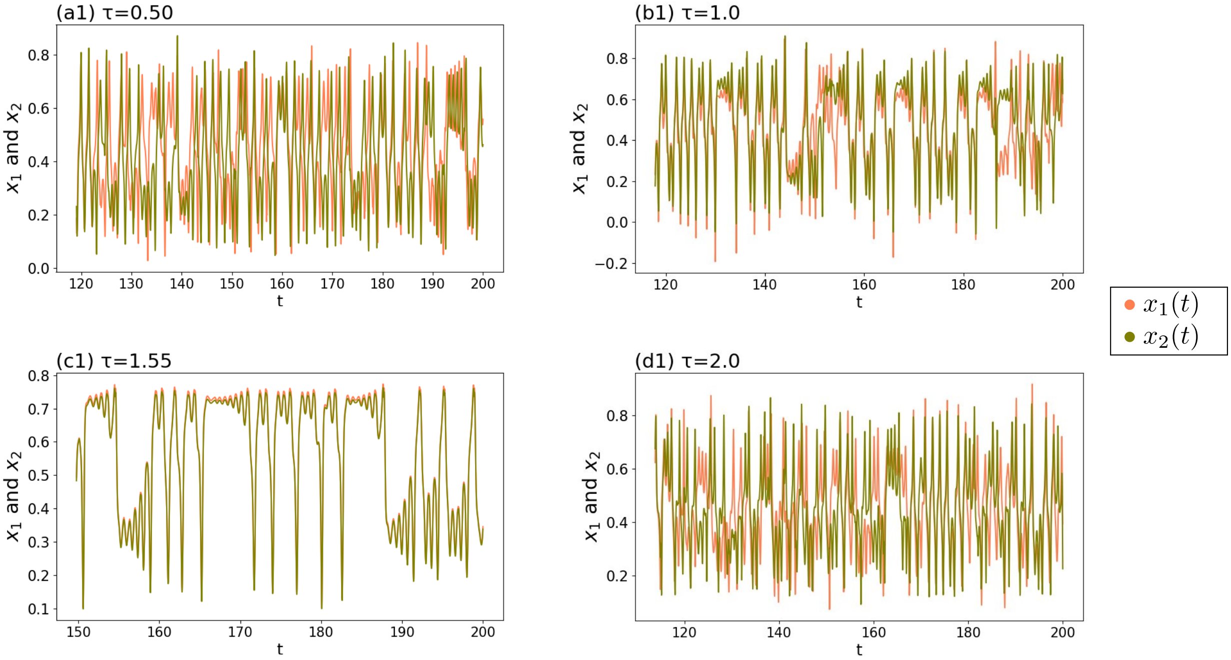

Fig. 6. Coupled Lorenz systems predict synchronization transition with different time delay (Coral and olive denote the trained trajectory of x_1 and x_2, respectively).

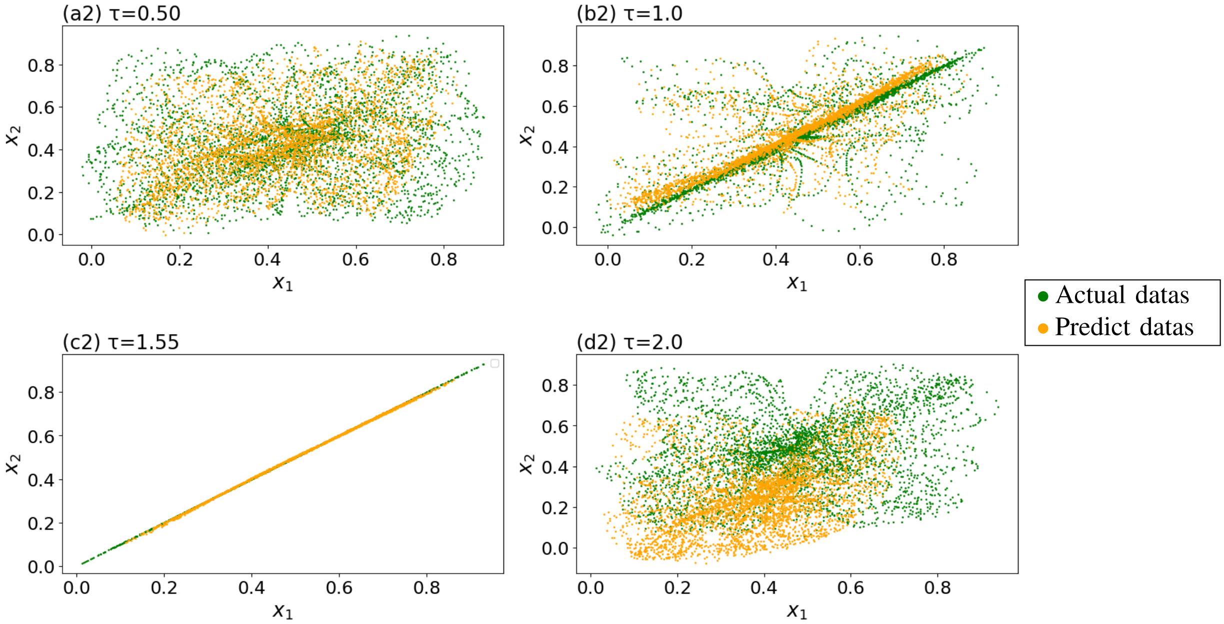

Fig. 7. Dynamic state of coupled Lorenz system variables x_1 and x_2 with different time delay (Green dots are the variable actual values, orange dots are the variable predicted values).

collective dynamics. The experimental results show that lag synchronization occurs within the interval $1.24 \leq \tau \leq 1.57$. Figs. 7(a2)-(d2) further depict the gradual convergence towards the diagonal followed by divergence as the coupling parameter varies from 0.5 to 2.0 for the points (x_1, x_2). The green dots represent the actual values of the variables x_1 and x_2, while the orange dots indicate the predicted values.

To further illustrate the successful prediction of the lag synchronization bifurcation threshold τ_0, we adiabatically vary the control parameters from 1.16 to 1.66, calculating the corresponding synchronization error Δx using Eq. (10). As shown in Fig. 8, the synchronization error Δx gradually decreases with increasing time delay, approaching zero around $\tau_1 = 1.24$. However, the state is not consistently maintained, the synchronization error begins to rise again at $\tau_2 = 1.57$, moving away from zero. These results suggest that a linear approximation of the system may compromise synchronization quality as the time delay changes significantly, even if the response system remains stable. Variations in parameters can lead to desynchronization between the drive and response systems, ultimately affecting synchronization accuracy. Nevertheless, as long as complete synchronization remains stable, the system can sustain stability within a finite range of delays.

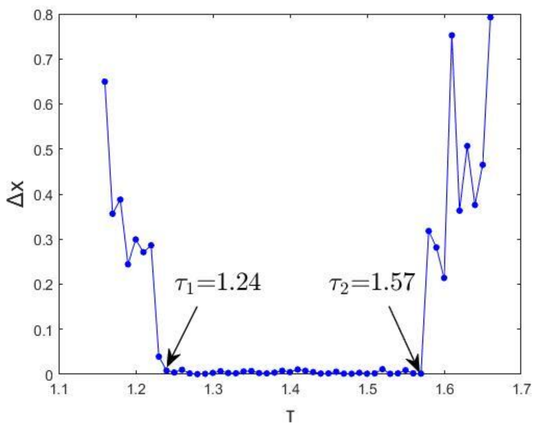

Fig. 8. Effect of time delay on synchronization of systems.

IV. Conclusion

In coupled chaotic systems, synchronization emerges through interactions and mutual adjustments among interconnected nodes. Given the nonlinear complexity of coupled systems, many governing equations remain unknown or too intricate for direct computation. This paper introduces a fully data-driven BO-CNN-LSTM model, which is trained on time series data from asynchronous states, to predict lag synchronization transitions in time-delay coupled chaotic systems. By adjusting coupling parameters and time delays, the model successfully replicates collective dynamics and predicts changes in control parameters, demonstrating its ability to capture lag synchronization phenomena without relying on explicit mathematical models. The reliability of this machine learning approach is confirmed through numerical accuracy, despite the lack of a comprehensive theoretical understanding of neural network layers. Our study focuses on the critical factors of coupling parameters and time delays, examining how their variations influence the synchronization process. This highlights the model's robustness in capturing dynamic behavior under changing conditions. By integrating machine learning with Bayesian Optimization, the BO-CNN-LSTM model provides a innovative method for predicting synchronization transitions using only time series data. This approach represents a significant advancement in analyzing nonlinear and complex systems. Its ability to predict synchronization without prior knowledge of system equations makes it a valuable tool for understanding synchronization dynamics in diverse fields, including biology and engineering.

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