Pricing Cancer Reinsurance Premiums Using a Stochastic Treatment Cost Model

Yunita Wulan Sari, Gunardi, Noorma Yulia Megawati, and Susanna Hilda Hutajulu

***Abstract*—Cancer is a costly and life-threatening condition requiring extensive treatment, often compounded by side effects that increase the financial burden. Cancer insurance helps mitigate these costs, but insurers face significant claim risks, necessitating reinsurance. This study presents a model for calculating reinsurance premiums based on a stochastic model of total treatment costs, including side effects, focusing on breast cancer. The findings show that premiums rise with increases in new diagnoses, side effect rates (e.g., cardiotoxicity), and the initial number of patients undergoing chemotherapy, experiencing side effects, or being cancer-free. Conversely, higher recovery rates, mortality rates, and retention limits lower premiums. This model offers a practical tool for reinsurers to set appropriate premiums.**

***Index Terms*—reinsurance premiums, cancer insurance, stochastic model, treatment cost model, retention.**

I. Introduction

CANCER is a non-communicable illness that poses a global challenge, with an annual mortality rate of roughly 13%. In 2020, there were 19,292,789 cancer patients worldwide [1], which is expected to increase yearly. Cancer types are classified based on the specific body tissue where cancer cells originate and proliferate. Breast cancer was the most frequent type in 2020, accounting for 11.7% of all cancer cases [1].

Every cancer patient faces high costs of care and therapy, particularly if additional treatment is needed to manage side effects [2]–[5]. These expenses include direct medical, non-medical, and indirect costs. A stochastic method can estimate the costs associated with cancer treatment, including the expenses related to side effects [6]. The overall high cost of treatment can pose a considerable burden and challenge for patients and their families, leading to a financial strain known as financial toxicity, which can result in the family's financial collapse ([7], [8]).

Insurance is a financial instrument offering guarantees and protection against potential future losses. Specialized cancer insurance is a suitable option to help reduce the financial burden of cancer treatment costs. Insurance companies issue policies and collect premiums with the commitment to disburse the insured amount if specific risks materialize. Cancer insurance companies often offer two payment options. The first option involves a one-time payment of the sum insured upon cancer diagnosis (lump-sum payment). The second option entails paying out the insured amount based on the treatment expenses incurred from the time of diagnosis until the end of the insurance policy (at cost). However, cancer insurance companies risk not being able to fulfill their obligations at the agreed-upon time. This situation could arise from a disaster, the insolvency of the insurance company, and/or insufficient policy value or company assets to cover these commitments. Consequently, reinsurance becomes essential. Reinsurance is insurance purchased by insurance companies (insurers) to protect themselves from adverse risks [9]. Reinsurance helps insurance companies stabilize losses, enhance their capacity, limit liability for specific risks, and protect against catastrophes, thereby reducing the risk of bankruptcy [10].

Insurance companies must pay premiums to reinsurance companies (reinsurers) for risk transfer purposes. The methods for determining and calculating these premiums pose a challenge. Several researchers and actuaries have developed methods for calculating reinsurance premiums. Chambashi et al. [11] proposed a reinsurance premium computation approach based on a composite lognormal model integrated into the risk-adjusted premium principle. This approach calculates premiums by considering the expected losses and the variability of risks the reinsurer faces. Additionally, reinsurance premium determination may also incorporate the risk-adjusted value of liability and the economic reinsurance premium principle, where liability valuation is performed using a cost-of-capital approach, and the capital at risk is quantified using value-at-risk (VaR) or conditional value-at-risk (CVaR) [12]. Moreover, Chicaiza and Cabedo [13] adopted the Black-Scholes method, initially developed for option pricing, to estimate reinsurance premiums specifically for high-cost illness insurance in Colombia.

Previous studies on reinsurance premiums have not explored models that determine premiums while considering the risks associated with treatment costs, including expenses for managing side effects that exceed the retention limit. The retention limit, which represents the cost threshold covered by the reinsurer, is established when the reinsurance contract is agreed upon. While prior research has primarily focused on general methodologies for premium calculation, this study extends existing models by explicitly integrating the financial risks arising from treatment side effects that are not covered within the retention limit. Unexpected treatment side effects can significantly escalate healthcare costs, potentially leading to financial instability for insurers and reinsurers. Therefore, by incorporating this component into the premium calcu-

Manuscript received October 30, 2024; revised March 03, 2025.

This research was supported by Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, under Grant number 2476/UN1/FMIPA.1.3/KP/PT.01.03/2024.

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lation, the proposed model provides a more accurate and sustainable pricing scheme.

The premium model developed in this study is based on the cancer treatment cost model formulated by [6]. Unlike Sari et al. [6], who employed a numerical approach, this research focuses on deriving an analytical solution before applying it to reinsurance premium calculations. By offering an analytical solution, this study provides a more explicit mathematical framework for understanding cost dynamics, allowing for greater interpretability and applicability in premium determination. Furthermore, the proposed reinsurance premium model offers practical insights for insurers and reinsurers in setting more precise premium rates. This approach is particularly relevant in markets where high-cost illnesses pose significant financial risks.

The article is organized as follows. In Section II, we present the analytical solutions of the stochastic model of cancer treatment costs, including treatment side effects. Furthermore, we introduce a reinsurance premium model based on claims for cancer treatment costs that exceed the retention limit. Numerical applications to explore the influence of model parameters are presented in Section III. Finally, the findings and conclusions of the analysis are discussed in the last section.

II. Cancer Reinsurance Premium Model

Cancer has high treatment costs, and insurance remains a widely recognized means of alleviating this financial burden. However, when the expenses for insured patients become excessively high, this imposes considerable financial strain on insurance companies.

To address this challenge, insurers that provide coverage for high-cost diseases must secure reinsurance. Reinsurance is a mechanism where an insurance company transfers a portion of its risk to a reinsurer or reinsurance company to mitigate potential financial losses from large or frequent claims. Essentially, reinsurance redistributes the burden of excessive expenses that the primary insurer would otherwise have to bear, enabling them to manage substantial claims without experiencing severe financial setbacks.

Cancer insurers must pay reinsurance premiums in exchange for protection against the high claim risks associated with cancer insurance policies. The high estimated cost of cancer treatment directly affects the insurance company's reinsurance premium. Therefore, estimating cancer treatment costs is crucial in modeling cancer reinsurance premiums.

A. Cancer Treatment Cost

Each cancer patient requires specialized medications and treatments that must be provided consistently and comprehensively. Additional care is also required if the patient experiences therapeutic side effects. These additional treatments often result in higher costs and extended treatment periods. As a result, in Section II-A, we will introduce a framework describing the flow of patient status compartments for insured cancer patients during therapy. This framework will be the basis for developing a model to predict total treatment costs. These cost estimates will form the basis for cancer insurance claims.

Some cancer patients will be recommended chemotherapy after being diagnosed. After undergoing chemotherapy, various outcomes can be observed: some patients recover, some patients die, and others experience chemotherapy side effects. Consequently, cancer patients who are covered by insurance can be categorized into three groups: those who receive chemotherapy (A), those who have recovered from cancer (F), and those who experience chemotherapy side effects (E). Figure 1 illustrates the flow of insured statuses within the population.

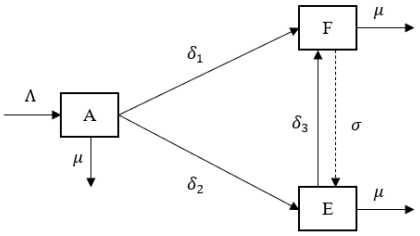

Fig. 1. Patient transfer chart

Referring to the patient status flow compartments developed by [6], a system is defined as follows:

$$\begin{cases} dA_t = (\Lambda - (\delta_1 + \delta_2 + \mu) A_t)\, dt, \\ dE_t = (\delta_2 A_t - (\delta_3 + \mu) E_t)\, dt + \sigma F_t dB_t, \\ dF_t = (\delta_1 A_t + \delta_3 E_t - \mu F_t)\, dt - \sigma F_t dB_t, \end{cases} \tag{1}$$

where B_t is a standard Brownian motion, and information about the model parameters is detailed in Table I. The initial conditions for System (1) are $A_0 > 0, E_0 > 0,$ and$F_0 > 0$.

TABLE I
Description of the Model Parameters

Parameter	Description
Λ	The number of newly diagnosed cancer
δ_1	The rate at which insured will be cured after taking chemotherapy
δ_2	The rate at which insured will experience chemotherapy side effects
δ_3	The rate at which insured who get chemotherapy side effects will be cured of cancer
μ	The mortality rate of cancer patients who are insured
σ	The intensity of the white noise in the rate of survivors who experience chemotherapy side effects

The solution of the System in Equation (1) using the Euler-Maruyama method has been investigated by [6]. This study will first determine the analytic solution of Equation (1) to calculate insurance premiums. The differential equation $dA_t = (\Lambda - (\delta_1 + \delta_2 + \mu) A_t)\, dt$ in Equation (1) has the following analytical solution [14]:

$$A_t = \frac{\Lambda}{a} + \left(A_0 - \frac{\Lambda}{a}\right) e^{-at} \tag{2}$$

where $a = \delta_1 + \delta_2 + \mu$.

Suppose $N_t = E_t + F_t$, then by differentiating both sides, we obtain

$$dN_t = dE_t + dF_t,$$

where dE_t and dF_t are defined in Equation (1). Substituting

these definitions, we get

$$\begin{aligned} dN_t &= \left(\left(\delta_1+\delta_2\right)A_t-\mu\left(E_t+F_t\right)\right)dt \\ &= \left(\left(\delta_1+\delta_2\right)\frac{\Lambda}{a}+\left(\delta_1+\delta_2\right)\right. \\ &\quad \left.\left(A_0-\frac{\Lambda}{a}\right)e^{-at}-\mu N_t\right)dt. \end{aligned} \tag{3}$$

Analogous to the solution of Equation (2), the analytical solution of Equation (3) is given by

$$\begin{aligned} N_t &= N_0e^{-\mu t}+\frac{\Lambda(\delta_1+\delta_2)}{a\mu}\left(1-e^{-\mu t}\right)+ \\ &\quad \frac{\left(\delta_1+\delta_2\right)\left(A_0-\frac{\Lambda}{a}\right)}{\mu-a}\left(e^{-at}-e^{-\mu t}\right), \end{aligned}$$

where $N_0 = E_0 + F_0$. Since $N_t = E_t + F_t$, we obtain

$$\begin{aligned} F_t &= (E_0+F_0)e^{-\mu t}+\frac{\Lambda(\delta_1+\delta_2)}{a\mu}\left(1-e^{-\mu t}\right) \\ &\quad +\frac{\left(\delta_1+\delta_2\right)\left(A_0-\frac{\Lambda}{a}\right)}{\mu-a}\left(e^{-at}-e^{-\mu t}\right)-E_t. \end{aligned} \tag{4}$$

Next, substituting Equation (4) into the stochastic differential equation $dE_t = (\delta_2 A_t - (\delta_3+\mu)E_t)\,dt + \sigma F_t dB(t)$ defined in System (1), we obtain

$$\begin{aligned} dE_t = &\left(\delta_2\left(\frac{\Lambda}{a}+\left(A_0-\frac{\Lambda}{a}\right)e^{-at}\right)-(\delta_3+\mu)E_t\right)dt \\ &+\sigma\left((E_0+F_0)e^{-\mu t}+\frac{\Lambda(\delta_1+\delta_2)}{a\mu}\left(1-e^{-\mu t}\right)\right. \\ &\left.+\frac{\left(\delta_1+\delta_2\right)\left(A_0-\frac{\Lambda}{a}\right)}{\mu-a}\left(e^{-at}-e^{-\mu t}\right)-E_t\right)dB_t. \end{aligned} \tag{5}$$

Define the functions

$$\begin{aligned} f(t,E_t) &= \delta_2\left(\frac{\Lambda}{a}+\left(A_0-\frac{\Lambda}{a}\right)e^{-at}\right)-(\delta_3+\mu)E_t \\ &= b(t)+BE_t \end{aligned}$$

and

$$\begin{aligned} g(t,E_t) =&\sigma\left(N_0e^{-\mu t}+\frac{\Lambda(\delta_1+\delta_2)}{a\mu}\left(1-e^{-\mu t}\right)\right. \\ &\left.+\frac{\left(\delta_1+\delta_2\right)\left(A_0-\frac{\Lambda}{a}\right)}{\mu-a}\left(e^{-at}-e^{-\mu t}\right)-E_t\right), \\ =&c(t)+CE_t. \end{aligned}$$

Thus, Equation (5) can be rewritten as

$$\begin{aligned} dE_t &= f(t,E_t)dt+g(t,E_t)dB_t \\ &= \left(b(t)+BE_t\right)dt+\left(c(t)+CE_t\right)dB_t. \end{aligned} \tag{6}$$

Based on ([15], [16]), the solution to Equation (6) is given by

$$\begin{aligned} E_t &= \phi(t)\left[E_0+\int_0^t\phi(s)^{-1}\left(b(s)-c(s)C\right)ds+\right. \\ &\quad \left.\int_0^t\phi(s)^{-1}c(s)dB_s\right] \end{aligned}$$

where $\phi(t) = e^{\int_0^t\left(B-\frac{C^2}{2}\right)ds+\int_0^t CdB_s}$. Thus, the analytical solution for E_t is

$$\begin{aligned} E_t &= e_1+e^{-at}e_2+e^{-\mu t}e_3+(E_0-e_1-e_2-e_3) \\ &\quad e^{-\left(\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t+\sigma B_t\right)} \end{aligned} \tag{7}$$

where

$$\begin{aligned} a &= \delta_1+\delta_2+\mu, \\ e_1 &= \frac{\Lambda}{a\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)}\left(\delta_2+\frac{\sigma^2(\delta_1+\delta_2)}{2\mu}\right), \\ e_2 &= \frac{aA_0-\Lambda}{a\left(\delta_3+\mu+\frac{\sigma^2}{2}-a\right)}\left(\delta_2+\frac{\sigma^2(\delta_1+\delta_2)}{2(\mu-a)}\right), \\ e_3 &= \frac{\sigma^2}{2(\delta_3+\frac{\sigma^2}{2})}\left(E_0+F_0-\frac{\Lambda(\delta_1+\delta_2)}{a\mu}-\right. \\ &\quad \left.\frac{(\delta_1+\delta_2)(aA_0-\Lambda)}{a(\mu-a)}\right). \end{aligned}$$

Sari et al. [6] presented a model for the total cost of cancer therapy at the period t as

$$\begin{aligned} T_{cost_t} &= C_{plan}(A_t+E_t)+C_{SE}E_t \\ &= C_{plan}A_t+(C_{plan}+C_{SE})E_t \end{aligned} \tag{8}$$

where C_{plan} is the planned cost of chemotherapy, and C_{SE} is the cost of treating the side effects of chemotherapy. Using the analytical solutions from Equations (2) and (7), the estimated total cost of cancer treatment for insured patients undergoing chemotherapy at period t (Equation (8))is given by

$$\begin{aligned} T_{cost_t} &= C_{plan}\left(\frac{\Lambda}{a}+\left(A_0-\frac{\Lambda}{a}\right)e^{-at}\right) \\ &\quad +\left(C_{plan}+C_{SE}\right)\left(e_1+e^{-at}e_2+e^{-\mu t}e_3\right) \\ &\quad +\left(C_{plan}+C_{SE}\right)\left(E_0-e_1-e_2-e_3\right) \\ &\quad e^{-\left(\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t+\sigma B_t\right)}. \end{aligned} \tag{9}$$

Next, define

$$\begin{aligned} S_t =&\left(C_{plan}+C_{SE}\right)\left(E_0-e_1-e_2-e_3\right)e^{-\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t} \\ &e^{-\sigma B_t} \end{aligned}$$

where B_t follows a standard Brownian motion, i.e., $B_t \sim N(0,t)$. Since S_t is lognormally distributed, it can be expressed as

$$\begin{aligned} S_t \sim &LN\left(\ln\left(\left(C_{plan}+C_{SE}\right)\left(E_0-e_1-e_2-e_3\right)\right)-\right. \\ &\left.\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t,\sigma^2t\right). \end{aligned}$$

Furthermore, let $p_t = C_{plan}\left(\frac{\Lambda}{a}+\left(A_0-\frac{\Lambda}{a}\right)e^{-at}\right)$ and $q_t = \left(C_{plan}+C_{SE}\right)\left(e_1+e^{-at}e_2+e^{-\mu t}e_3\right)$. Then, the total cost of cancer treatment at period t (Equation (9)) simplifies to

$$T_{cost_t} = p_t+q_t+S_t. \tag{10}$$

A 95% confidence interval for T_{cost_t} is

$$\left[LB_{T_{cost_t}}, LB_{T_{cost_t}}\right]$$

where

$$\begin{aligned} LB_{T_{cost_t}} =&p_t+q_t+\left(C_{plan}+C_{SE}\right)\left(E_0-e_1-e_2-e_3\right) \\ &e^{-\left(\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t+1.96\sigma\sqrt{t}\right)}, \\ UB_{T_{cost_t}} =&p_t+q_t+\left(C_{plan}+C_{SE}\right)\left(E_0-e_1-e_2-e_3\right) \\ &e^{-\left(\left(\delta_3+\mu+\frac{\sigma^2}{2}\right)t-1.96\sigma\sqrt{t}\right)}. \end{aligned}$$

B. Reinsurance Premium Model

This model assumes that all patients in the system are insured, and cancer insurance companies must cover the treatment costs incurred by every insured patient, including the expenses for cancer treatment and its side effects. Equation (10) has been used to model the total costs incurred by cancer patients. Furthermore, this study applies an excess-of-loss non-proportional reinsurance treaty. If the total treatment cost, denoted as T_{cost_t}, exceeds the threshold value or retention limit R, then the risk of claim costs exceeding R will be transferred to the reinsurer. The reinsurer determines the retention limit R based on the risk portfolio analysis of the insurance company and its financial capacity. Consequently, the amount of coverage paid by the reinsurer in period t is given by

$$\begin{cases} 0 & \text{, if } 0 \leq T_{cost_t} \leq R, \\ T_{cost_t} - R & \text{, if } R < T_{cost_t}. \end{cases} \tag{11}$$

Reinsurance policies typically have a specified validity period, generally lasting one year. After one year, the reinsurance contract can be renewed with adjustments based on market conditions, changes in the risk portfolio, and regulatory developments. In this study, it is assumed that treatment is administered once a month. Therefore, based on Equation (11), the single premium for cancer reinsurance (P_{re}) over one year can be calculated as follows

$$P_{re} = \sum_{t=1}^{12} e^{-rt} E\left[\max(T_{cost_t} - R, 0)\right] \tag{12}$$

where e^{-rt} is the discounting factor and r is the continuously compounded interest rate. The expectation of $[\max(T_{cost_t} - R, 0)]$ is given by

$$\begin{aligned} E\left[(T_{cost_t} - R)_+\right] &= E\left[(p_t + q_t + S_t - R)_+\right] \\ &= E\left[(S_t - (R - p_t - q_t))_+\right] \\ &= \int_{R-p_t-q_t}^{\infty} S_t h(S_t) dS_t - \\ &\quad (R - p_t - q_t) \int_{R-p_t-q_t}^{\infty} h(S_t) dS_t, \end{aligned}$$

where $h(S_t$ is the probability density function of S_t.

Based on the lognormal distribution properties mentioned in [9], we obtain

$$\int_{R-p_t-q_t}^{\infty} S_t h(S_t) dS_t = (C_{plan} + C_{SE})(E_0 - e_1 - e_2 - e_3) e^{-(\delta_3+\mu)t} \Phi(d_1) \tag{13}$$

and

$$(R - p_t - q_t) \int_{R-p_t-q_t}^{\infty} h(S_t) dS_t = (R - p_t - q_t)\Phi(d_2) \tag{14}$$

where

$$d_1 = \frac{\ln\left(\frac{(C_{plan}+C_{SE})(E_0-e_1-e_2-e_3)}{R-p_t-q_t}\right) - \left(\delta_3 + \mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}},$$

$$d_2 = \frac{\ln\left(\frac{(C_{plan}+C_{SE})(E_0-e_1-e_2-e_3)}{R-p_t-q_t}\right) - \left(\delta_3 + \mu + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}.$$

Moreover $\Phi(z)$ represents the Cumulative Distribution Function (CDF) of a standard normal random variable, i.e., $\Phi(z) = P(Z \leq z)$ for $Z \sim N(0, 1)$.

Since R remains constant during the policy period, it follows from Equations (13) and (14) that if $E_0 > (e_1+e_2+e_3)$, then $R > \max(p_t + q_t)$. Conversely, if $E_0 < (e_1 + e_2 + e_3)$, then $R < \min(p_t + q_t)$.

III. Numerical application

The cost and side effects of chemotherapy vary depending on the type of cancer. In this section, we utilize numerical applications focusing on breast cancer to explore the results of our proposed premium determination approach. Breast cancer is the most common type of cancer. In this numerical analysis, we consider cardiotoxicity as a potential side effect of chemotherapy, which can disrupt heart function and significantly impact breast cancer treatment.

Cardiotoxicity refers to heart damage resulting from cancer treatment, particularly chemotherapy and targeted therapies. This condition can manifest as impaired cardiac function, including heart failure, arrhythmia, or hypertension, either during or after treatment. Certain breast cancer drugs, such as Anthracyclines and Trastuzumab, are known to carry a high risk of cardiotoxic effects ([17], [18]). As a result, affected patients often require regular cardiac monitoring and additional medical care, significantly increasing overall treatment costs. If breast cancer insurance does not cover these expenses, patients may face a considerable financial burden.

The parameter values we use are $\Lambda = 36$, $\delta_1 = 0.36$, $\delta_2 = 0.3$, $\delta_3 = 0.2$, $\mu = 0.56$, and $\sigma = 0.05$. Additionally, we consider initial values of $A_0 = 64$ patients, $E_0 = 10$ patients, and $F_0 = 10$ patients [6]. Mariotto et al. [19] found that uninsured breast cancer patients face a minimum monthly treatment cost of \$290 during the first year after their diagnosis. Furthermore, patients experiencing side effects such as cardiotoxicity had to spend an average of \$289.67 [20]. These costs include primary care, outpatient services, inpatient services, emergency visits before hospitalization, and medication expenses. Based on these findings, we set C_{plan} at \$290 and C_{SE} at \$289.67. In addition, in this case, $E_0 > (e_1+e_2+e_3)$, so R must be greater than $\max(p_t+q_t)$ for $t = 1, 2, \ldots, 12$. Since $\max(p_t + q_t) = 15,331.47$, we set $R = \$16,000$. This study assumes that all patients in the system are covered by breast cancer insurance.

Based on Equations (12)-(14), if it is known at time $t = 0$ that there are 74 breast cancer patients undergoing chemotherapy and ten chemotherapy patients experiencing cardiotoxicity side effects, the net single reinsurance premium for breast cancer insurance is \$6,240.627 at a risk-free annual interest rate of 5.75%. After paying the premium, the breast cancer insurance company can transfer the risk of cancer treatment financing for a year or 12 chemotherapy sessions. This right only applies if the total claim exceeds \$16,000 for each session.

This study also explored the effect of each parameter, initial value, and retention limit on the reinsurance premium. Figure 2 presents the premium values for various Λ, δ_1, δ_2, δ_3, μ, and σ. Figure 3 depicts the reinsurance premium values for different initial numbers of insured individuals

undergoing chemotherapy (A_0), experiencing chemotherapy-induced cardiotoxicity (E_0), and achieving a cancer-free status (F_0). Finally, the reinsurance premiums for various retention limit values are shown in Figure 4.

Figure 2 illustrates that as the number of new breast cancer diagnoses (Λ) increases, the reinsurance premium for breast cancer insurance also increases. Similarly, as the rate of insured individuals experiencing cardiotoxicity after chemotherapy (δ_2) increases, reinsurance premiums become more expensive. In other words, an increase in the number of patients who are newly diagnosed with breast cancer or who experience cardiotoxicity from chemotherapy can lead to total treatment costs that exceed the retention limit. This, in turn, causes the reinsurance premium to increase. Conversely, the reinsurance premium decreases with an increase in the recovery rate of chemotherapy patients (δ_1) and the rate of patients recovering after experiencing chemotherapy-induced cardiotoxicity (δ_3). A higher insured mortality rate (μ) also reduces the reinsurance premium because an increase in the number of patients declared cured of cancer or who have passed away reduces total treatment costs, thereby lowering the risk of treatment costs exceeding the retention limit, which results in a decrease in the reinsurance premium.

Furthermore, in our case study, the reinsurance premium increases as σ rises. A high value of σ indicates a high variability in the uncertainty of patients experiencing cardiotoxicity after recovery and cancer recurrence. Consequently, an increase in σ leads to a higher risk of elevated treatment costs, increasing reinsurance premiums.

The increase in the initial number of patients undergoing chemotherapy (A_0) or experiencing cardiotoxicity during chemotherapy (E_0) leads to a higher risk of rising total treatment costs (Figure 3). Similarly, an increase in the number of patients declared cured of cancer at $t = 0$ (F_0) raises the number of patients developing cardiotoxicity after recovery, thereby increasing the risk of total treatment costs exceeding the retention limit. Consequently, the reinsurance premium rises as the number of insured individuals at $t = 0$ increases, whether they are diagnosed with breast cancer and undergoing chemotherapy, experiencing cardiotoxicity during chemotherapy, or are declared cured of cancer (Figure 3). However, the increase in the reinsurance premium due to the growing number of patients declared cured at $t = 0$ (F_0) is not particularly significant.

The retention limit (R) is a critical factor as it significantly impacts the calculation of reinsurance premiums. In the numerical analysis conducted in this study, higher retention limit values result in lower reinsurance premiums (Figure 4) because larger retention limits decrease the likelihood of total treatment costs exceeding the retention threshold, thereby reducing the probability of claim submissions.

IV. Discussion and concluding remarks

Reinsurance is crucial in maintaining financial stability and managing risks for insurance companies covering high-cost, high-risk diseases such as cancer. Cancer insurers can enhance product innovation, expand risk coverage, and offer more affordable premiums by leveraging reinsurance. Additionally, reinsurance strengthens public confidence in cancer insurance products by ensuring that policyholders' claims are honored. Accurately determining reinsurance premiums is essential for sustaining the cancer insurance industry and providing optimal protection for all stakeholders.

This study proposes a method for estimating reinsurance premiums for cancer insurance based on a model that accounts for total cancer treatment costs, including expenses related to treatment side effects. This approach offers reinsurers a practical alternative for premium calculation while maintaining computational simplicity.

This study assumes that all patients in the system are covered by breast cancer insurance. The numerical analysis highlights several key factors influencing breast cancer reinsurance premiums. An increase in newly diagnosed breast cancer cases, as well as a higher incidence of treatment side effects such as cardiotoxicity, leads to higher premiums. Conversely, higher recovery and mortality rates contribute to lower premiums. Initial conditions at $t = 0$, including the number of insured individuals undergoing chemotherapy, experiencing cardiotoxicity, or recovering from cancer, significantly impact premium pricing. A larger initial insured population correlates with higher premium costs.

Another critical factor in premium determination is the retention limit. A higher retention limit reduces the probability of total treatment costs surpassing the threshold, decreasing claim probabilities and leading to lower reinsurance premiums.

The proposed reinsurance premium valuation model aids reinsurers in managing the substantial financial risks associated with cancer treatment claims. Reinsurance companies can utilize these findings for risk management by adjusting the retention level R to optimize the balance between risk and reinsurance premium. Additionally, monitoring the parameters and factors influencing treatment costs can help anticipate surges in medical expenses, enabling a more precise premium-setting strategy. Future research could focus on refining parameter estimation methods and analyzing their impact on premium pricing. Developing a systematic approach for determining retention limits—incorporating risk profiles, claim characteristics, financial capacity, market conditions, medical trends, and regulatory factors—is valuable for further study.

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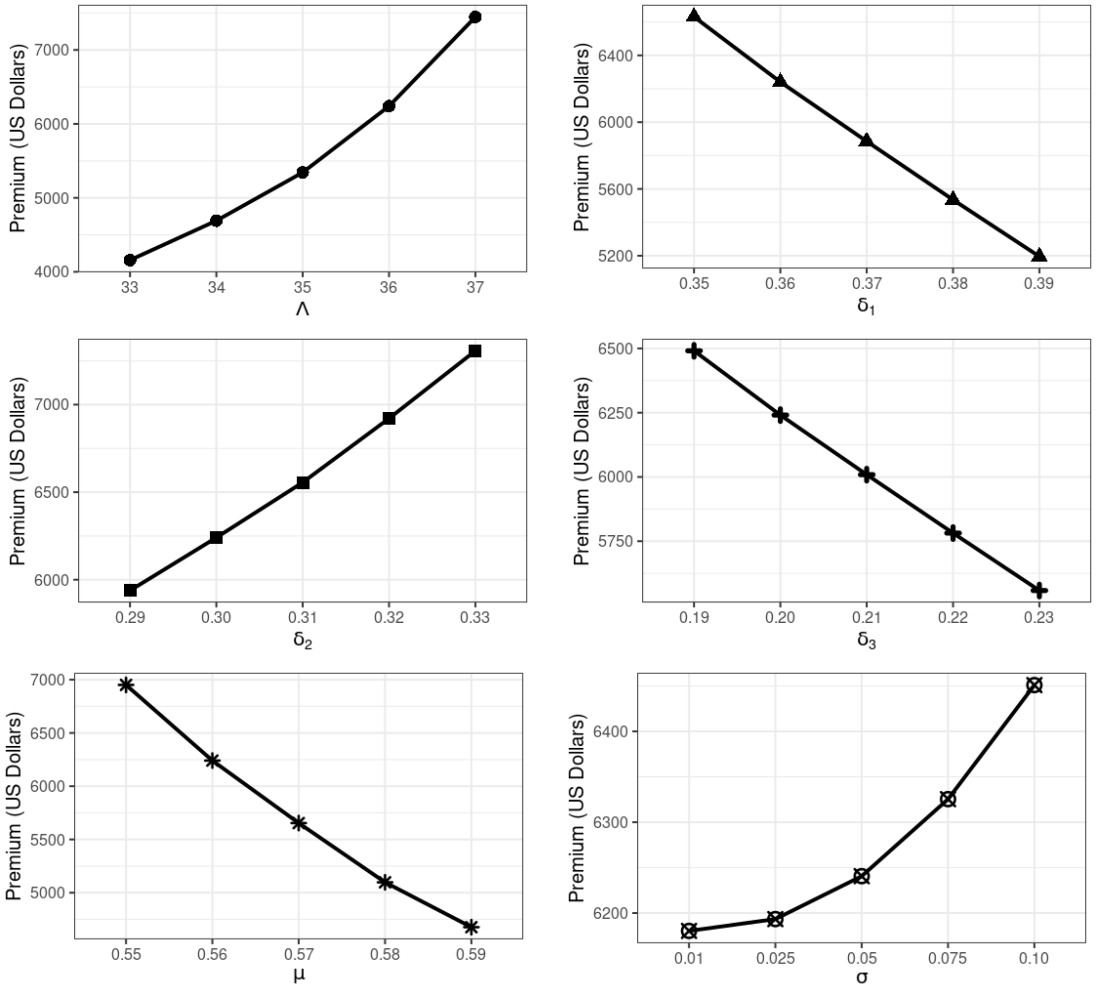

Fig. 2. Reinsurance premiums for breast cancer insurance at various parameter values over a one-year insurance period

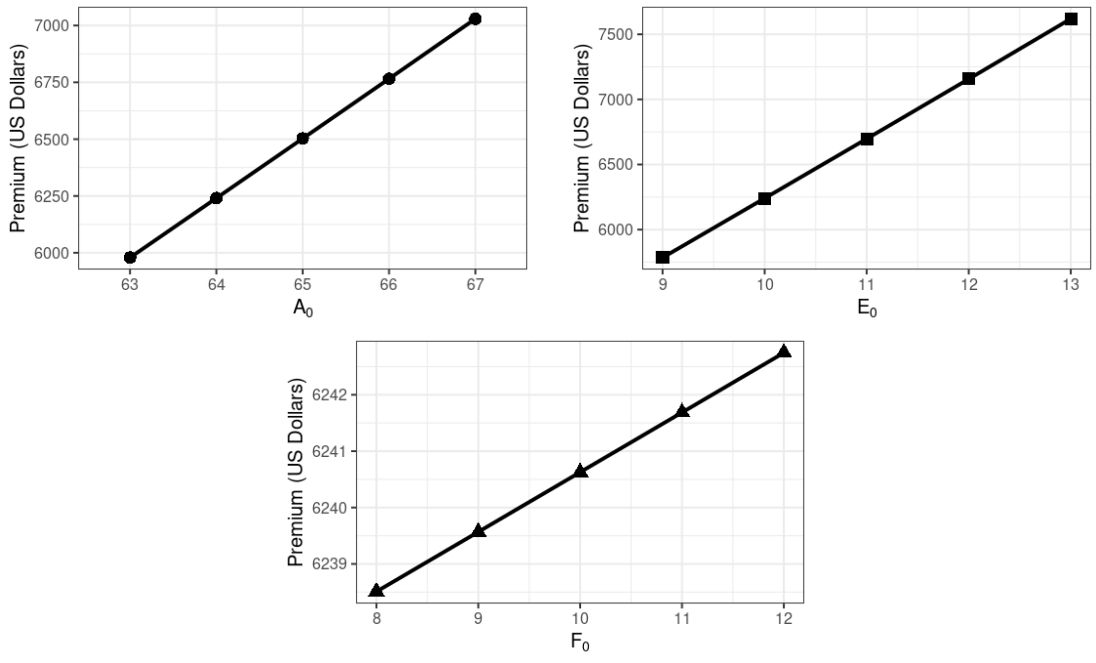

Fig. 3. Reinsurance premiums for breast cancer insurance under various initial values over a one-year insurance period

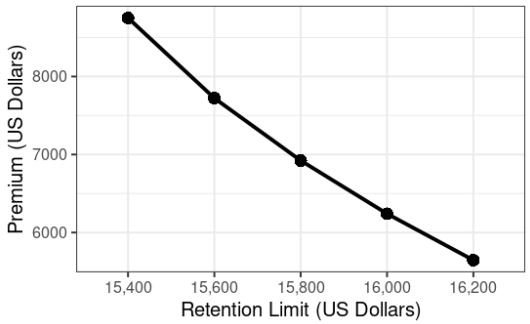

Fig. 4. Reinsurance premiums for breast cancer insurance under various Retention Limit over a one-year insurance period

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