Analysis of a Repairable M/G/1 Queue with the Gated Service and Multiple Adaptive Vacation

Tongyu Xu, Zhanyou Ma*, Ying Sun

***Abstract*—This paper considered a repairable M^X/G/1 queueing system with the gated service, setup time and multiple adaptive vacation rules. Customers arrive according to Markov flow. The system has a main queue and an infinite buffer queue separated by a gate. After the customers arrival, they enter the buffer and wait for service in order. Before each service period begins, the system has a random setup period. After the setup period ends, the system opens the gate and customers waiting in the buffer zone enter the main queue according to the original order to receive the service. This paper uses tools such as the intuitive total probability decomposition theorem, regeneration cycle method, PGF and LST transform to analyze the system's queue length and waiting time.The probabilities for the server being in the states of general service, idle, setup, and vacation are presented individually. .**

***Index Terms*—gated service, repairable server, setup time, multiple adaptive vacation, batch arrival, regeneration cycle method**

I. Introduction

IN recent years, extensive research has been conducted on the exhaustive service vacation queueing system, yielding numerous satisfactory results. However, comparatively less research has been dedicated to nonexhaustive service queueing systems.

Leung and Eisenberg [1] built an M/G/1 queueing system subject to gated rule. The system provides a fixed service time to the queue at each visit. Upon completion of this period or after service for all eligible customers, the server stops service and starts vacation. The service mechanism is according to the gated rule, wherein only customers who already entered the system at the initiation of the service are eligible for service during that particular visit, and customers who arrive later are deferred to the next visit. Tian and Zhang [2] summarized the research of queueing systems into two categories: exhausted services and nonexhausted services. In daily life, batch arrivals of customers are more realistic than individual arrivals. Vishnevsky et al. [3] investigated a gated service queueing system incorporating adaptive vacation. The duration of the vacation is contingent upon the count of consecutive vacation finished moments at which the buffer has no customers. Several key performance metrics including the mean sojourn time are computed. Vishnevsky et al. [4] studied a gated service BMAP/G/1 type queueing system subjected to adaptive vacations. This system is effective in solving problems in WiMax networks and broadband wireless Wi-Fi. Dudin et al. [5] studied a gated service BMAP/G/1 type queueing system. The length of vacations is contingent upon the frequency of system vacancies after the previous vacation periods. Banik and Ghosh [6] analyzed a nonexhaustive service BMAP/R/1/N (∞) queues. The system assumes that batch customers arrive according to Markov processes and receive services according to gated service discipline, and the system can provide batch services to customers. Qi et al. [7] established a nonexhaustive queueing system subjected to gated rule, to analyze the indicators of light load traffic blockchain systems. Simulate the operation mechanism of blockchain as a queuing system, and then calculate and analyze its performance. The regeneration cycle method and embedded Markov chain are utilized to analyze the system's indexes.

In the practical application of queueing systems, the server may breakdown and cannot serve customers. Server breakdown is an unavoidable situation. Therefore, queueing systems with unreliable servers have received widespread attention from scholars.

Ke and Huang [8] examined an M^X/G/1 queueing subjected to delayed Repair and randomized vacation. Suppose that once the vacation concludes, the system is devoid of customers. Subsequently, the server has two potential states. It has a $1-p$ probability of starting a new vacation and a p probability of becoming idle. The system can take up to Jth consecutive vacations. If the system still has no customers after Jth consecutive vacations, the system begins an idle period. The repair may be delayed when server failure occurs. The distributions of significant indexes, such as queue size and reliability indices are derived. Ayyappan and Karpagam [9] discussed a repairable queueing with re-service, standby server and batch arrivals. The PGF of queue length along with other system performance indicators are obtained. Furthermore, the specific instances are derived. A retrial MAP/M/1 queueing system subjected to repairable server was examined by Zhou and Zhou [10]. Quasi-birth and death procedures are used to derive the main queueing indices. Additionally, numerical experiments are performed to show how the variables affect a number of performance indexes. Kalita and Choudhury [11] analyzed an unreliable M$^{[X]}$/G/1 batch arrival queue under a randomized vacation strategy. Customers arrival according to a compound Poisson process, and the system offers two distinct types of services. While the server is working it may breakdown and delay

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repair at any instant. Saggou et al. [12] built a repairable $M^{[X]}$/G/1 system subject to delays of verification, geometric loss, special vacation and batch arrivals. The expressions of stationary performance measures are calculated. A repairable $M^{[x]}/G^{K}/1$ queueing was analyzed by Kalyanaraman and Nagarajan [13]. Additionally, the server may experience breakdown and require repair before resuming service. Jain and Kumar [14] considered a service retrial queueing system with vacation interruption, bulk arrival, balking and feedback. The server may break, but it can be repaired. The failures of vacation service time, retrial attempts and repair are represented by employing the supplementary variables method. The expressions of crucial performance metrics and PGF (Probability Generating Function) of the queue size are obtained.

Wan and Lan [15] utilizing the total probability formula and the LST (Laplace-Stieltjes Transform), considered some metrics such as server failure time, unavailability, the failure rate and average number of failures in $(0,t)$. Reliability indicators have been shown to meet the stochastic decomposition property. Li and Li [16] examined a repairable queueing system subjected to working and interruption vacation. During busy periods, negative customer arrival causes server to breakdown. By employing the matrix-analytic and supplementary variable methods, the system indicators are derived. Variable arrival and failure rates were included in the M/M/1 type queueing by Lv et al. [17]. The length of the queue affects the rate at which consumers arrive. The fault rate of the unreliable server varies over time. There are reliable maintenance personnel in the system for repairs. The stationary performance metrics of the system are analyzed.

In vacation queueing systems, multiple adaptive vacation strategy are more commonly applied than classical vacations. Common vacation strategies such as single and multiple vacation are boundary cases of multiple adaptive vacations. With multiple adaptive vacations systems, we can investigate systems under various vacation states based on boundary conditions.

Ma [18] considered a multiple adaptive vacation M/G/1 queueing subjected to a gated service policy by utilizing the regeneration cycle method. The service cycle of the system is analyzed, and The probability expression of the system at different stages is solved. Additionally, the stochastic decomposition structure of some reliability performance indicators are calculated. Numerical experiments are used to analyze how various factors affect the system performance indices. Liu et al. [19] introduced the multiple adaptive vacation into the repairable queueing system, and the impacts of a replaceable facility on the system reliability are also considered. Employing the probability decomposition method and renewal process theory, the system availability, the mean failure numbers during $(0,t)$ and the renewal time distribution of the system are obtained. Luo et al. [20] analyzed antypical queueing model subjected to a delayed multiple adaptive vacation strategy. Building upon the workload of servers and the number of customers, a modified Min(N, D) strategy is added. Under this strategy, server interruption of vacations and begin service when the customer count exceeds threshold N or when the cumulative service requirement surpasses threshold D within the vacation period. The distribution of steady state along with transient queue length is analyzed. Qin and Tang [21] discussed a complicated queueing subjected to a random number of vacations, a setup and Min(N, V)-policy control. Employing a total probability decomposition approach, the stationary and transient distributions of the queue length are analyzed under arbitrary initial conditions. The expression of LST of the transient queue size is calculated.

For queuing systems with setup time, Choudhury [22] deals with a batch arrival queueing model subjected to random setup period. The server is shut off whenever the idle period begins or the system is empty. The system begins the setup and starts a busy period when customers arrival. The stationary distribution and some system indexes are calculated. The system's explicit expressions as well as a few performance metrics are obtained. Gao [23] considered a system with single vacation, startup period and batch arrival. The LST of sojourn time and the system's online period are derived, together with the PGF of queue size. An investigation of an repairable M/G/1 queueing model with setup period was conducted by Bu and Liu [24]. The server may breakdown in busy and setup periods. Various performance indexes are studied. Additionally, a cost analysis of the system is conducted. Combined with setup time and N-policy, Kalita and Choudhury [25] stuied a repairable $M^{[X]}$/G/1 system. The server must go through a setup period before the service begins, and it is always shut down until the system is empty. Ayyappan and Deepa [26] analyzed the batch arrival M/G(a, b)/1 system subjected to admittance control, limited service, closedown and setup period. When the service period is finished, the customer requests for re-service, and the system accepts the request both occur with a certain probability. If the queue size is less than a once a service is finished, the system shuts down and begins vacation. When the system ends its vacation status, the server is enabled and service is resumed if the queue size is at least a.

The queueing rule examined in this study is gated service within a nonexhaustive service system. In the queueing system, the system may stop service and enter vacation. At the end of the vacation, there may be a random setup time before resuming service. Compared with ordinary vacation policies, multiple adaptive vacation rules are more general. Furthermore, this article presents two key factors: the arrival of customers in batches and the potential for server failures during operation. These elements are incorporated to make the developed queueing model more related to reality. In this study, we integrate these strategies—batch arrivals, setup times, server failures, multiple adaptive vacations, and the gate service rule—into an M/G/1 queueing. In the current study no prior research has specifically addressed such a queueing system. The analysis and computation in this paper primarily utilize total probability decomposition and the regeneration cycle method.

The contents of this paper are divided into the following sections. Section II established the queueing system, providing the average number of customers at the beginning of service period. Section III analyzed the steady state indicators of the system. The system cycle time is given in section IV. Some special cases of the system are provided in the section V. The section VI is the conclusion of this paper.

II. System Analysis

A. System Description

A repairable M/G/1 queueing system with the gated service, setup time, batch arrival and multiple adaptive vacation rules is considered. The following are the system's descriptions.

1) Customers arriving in bacth according to a Poisson process with rate $\lambda(\lambda > 0)$. The interarrival T obeys the exponential distribution. The size of the batch is a variable X, with the distribution $x_i = P(X = i)$, $i = 1,2,3,\cdots$. It has the mean $E(X)$, the PGF $X(z) = \sum_{i=1}^{\infty} x_i z^i$ and the second-order moment $E(X^2)$.
2) When the system starts serving, the system closes the gate and only receives those customers who are already waiting in the system. Customers who arrive in the current service period are required to wait outside the gate and accept services in the next service period. The mechanism flowchart of gated service is described in Fig. 1. The service time B obeys a general distribution, which distribution function is expressed as $B(t)$, $t \geq 0$. The mean, second-order moment and LST of B are represented by
$$E(B) = \int_0^{\infty} t\mathrm{d}B(t),$$
$$E(B^2) = \int_0^{\infty} t^2\mathrm{d}B(t),$$
$$B^*(s) = \int_0^{\infty} \mathrm{e}^{-st}\mathrm{d}B(t), \quad \Re(s) > 0,$$
where $\Re(s)$ is the real part of s.
3) The server experiences breakdown randomly during the service period, breakdown occurs according to a Poisson process with rate $\alpha(\alpha > 0)$, but remains stable during idle period. It is assumed that the server is fixed right away once the breakdown takes place. In repair time, the server does not provide service, and existing customers are waiting in place. The server completes the remaining service after its recovery. The repair time R obeys a general distribution, which distribution function is expressed as $R(t), (t > 0)$. The mean, second-order moment and LST of R are represented by
$$E(R) = \int_0^{\infty} t\mathrm{d}R(t),$$
$$E(R^2) = \int_0^{\infty} t^2\mathrm{d}R(t),$$
$$R^*(s) = \int_0^{\infty} \mathrm{e}^{-st}\mathrm{d}R(t).$$
4) Following the conclusion of the service period, the server experiences consecutive H vacations, H is a random variable with a positive integer. The system has the following two types of vacation situations. If customers arrives in the kth $(k = 1,2,3,\cdots,H)$ vacation. The system finishes the vacation after the kth vacation. When there are no customer arrivals throughout H vacations, the system finishes vacation after Hth vacation. H is considered a discrete random variable, with the distribution $h_j = P\{H = j\}, j = 1,2,3,\cdots$, the PGF of H is $H(z) = \sum_{j=1}^{\infty} h_j z^j$. The single vacation time V is an independent and identically distributed (i.i.d) random variable, which has the distribution function $V(t)$, the mean $E(V)$, the second-order moment $E(V^2)$ and the LST $v^*(s)$.
5) The system includes a setup period denoted as U, before the start of each service period. U has the distribution function $U(t)$, the LST $u^*(s)$ and the mean $E(U)$.
6) Since the server may malfunction, the actual time spent by a single customer receiving service is the aggregate of the regular service time and potential repair time needed. The general service time $\tilde{B}$ represents the actual duration of service for the customer. The mean, second-order moment and LST of $\tilde{B}$ are given by
$$\begin{aligned}\tilde{B}^*(s) &= \sum_{k=0}^{\infty} \int_0^t R^{(k)}(t-x) \frac{(\alpha x)^k}{k!} \mathrm{e}^{-\alpha x} \mathrm{d}\tilde{B}(x) \\ &= B^*(s + \alpha - \alpha R^*(s)),\end{aligned} \tag{1}$$
$$E(\tilde{B}) = -\left(\tilde{B}^*(s)\right)'\big|_{s=0} = (1 + E(R)\alpha)E(B),$$
$$\begin{aligned}E(\tilde{B}^2) &= \left(\tilde{B}^*(s)\right)''\big|_{s=0} \\ &= E(B)E(R^2)\alpha + (1 + E(R)\alpha)^2 E(B^2).\end{aligned}$$
7) The service time, vacation time, arrival interval, setup time, breakdown occurrence interval and repair time are mutually independent. The system serves based on the FCFS (First Come First Served) rule.

B. The Number of Customers at the Initial Moment of the Service Period

The duration of nth general service period is denoted by ${S_p}^{(n)}$. At the initial moment of the ${S_p}^{(n)}$, the number of customers is denoted by ${Q_b}^{(n)}$. The ${Q_b}^{(n+1)}$ is analyzed as follows.

1) If the system has customers arriving in ${S_p}^{(n)}$, then the ${Q_b}^{(n+1)}$ is equivalent to the customers arriving in ${S_p}^{(n)}$, V and U. Fig. 2 describes the state transition of this situation.
2) If the system has no customers arriving in ${S_p}^{(n)}$, but has customers arriving in V_k, then the ${Q_b}^{(n+1)}$ is equivalent to the customers arriving in V and U. Fig. 3 describes the state transition of this situation.
3) If the system has no customers arriving in ${S_p}^{(n)}$ and H vacation periods, then the ${Q_b}^{(n+1)}$ is equivalent to the coutomers arriving in idle period I and U. Fig. 4 describes the state transition of this situation.

The PGF of customers arriving in U, ${S_p}^{(n)}$ and V are $u^*(\lambda - X(z)\lambda)$, ${S_p}^*(\lambda - X(z)\lambda)$ and $v^*(\lambda - X(z)\lambda)$ respectively. The probability that no customers arriving in ${S_p}^{(n)}$, V and U are ${S_p}^{(n)*}(\lambda)$, $v^*(\lambda)$ and $u^*(\lambda)$, respectively.

At the initial moment of U, if no customers arrive in the previous general service period, let P_I denote the probability that U commences following the ending of an idle period, and let P_V denote the probability that U commences following the ending of the vacation period. P_I and P_V are represented by
$$\begin{aligned}P_I &= \sum_{j=1}^{\infty} P\{H = j\} P\left\{T > V^{(j)}\right\} = H(v^*(\lambda)); \\ P_V &= 1 - H(v^*(\lambda)),\end{aligned} \tag{2}$$
where $V^{(j)}$ is the jth convolution of $V(t)$. Using the total probability theorem, the PGF of the count of customers at

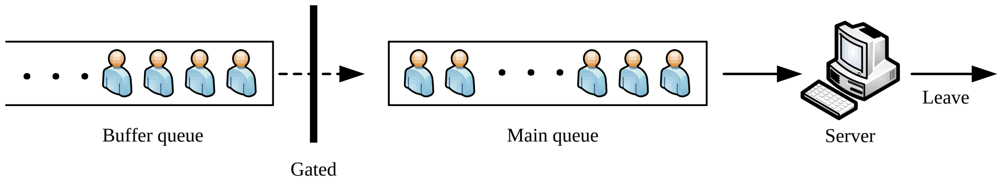

Fig. 1. Mechanism Flowchart of Gated Service

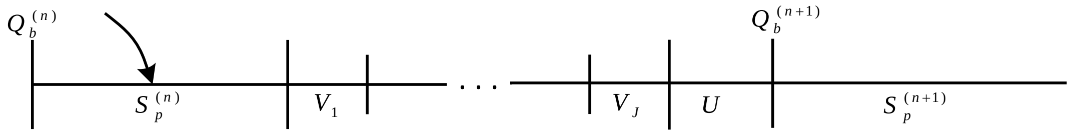

Fig. 2. Customers arriving in $S_p{}^{(n)}$

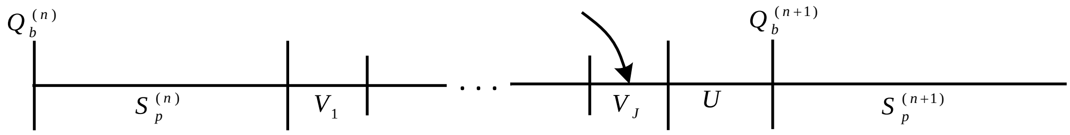

Fig. 3. Customers arriving in V

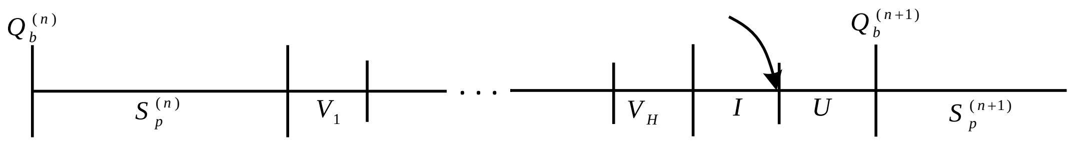

Fig. 4. Customers arriving in I

the initial moment of $S_p{}^{(n+1)}$ can be obtained

$$\begin{aligned} Q_b{}^{(n+1)}(z) = \Bigg[& v^*(\lambda - \lambda X(z))(1 - S_p{}^{(n)*}(\lambda)) \\ & \times \frac{S_p{}^{(n)*}(\lambda - \lambda X(z)) - S_p{}^{(n)*}(\lambda)}{1 - S_p{}^{(n)*}(\lambda)} \\ & + S_p{}^{(n)*}(\lambda) P_V \frac{v^*(\lambda - \lambda X(z)) - v^*(\lambda)}{1 - v^*(\lambda)} \\ & + S_p{}^{(n)*}(\lambda) P_I X(z) \Bigg] u^*(\lambda - \lambda X(z)). \end{aligned} \tag{3}$$

In the system with gated service, the duration of $S_p{}^{(n)}$ is equivalent to the cumulative service time of customers served in $S_p{}^{(n)}$. Thus we have that

$$\begin{aligned} S_p{}^{(n)*}(s) &= \int_0^\infty \mathrm{e}^{-st} \mathrm{d}P\left\{S_p{}^{(n)} < t\right\} \\ &= \sum_{k=1}^{\infty} \int_0^\infty \mathrm{e}^{-st} \mathrm{d}P\{\tilde{B}_1 + \cdots + \tilde{B}_k < t\} P\left\{Q_b{}^{(n)} = k\right\} \\ &= \sum_{k=1}^{\infty} \left(\tilde{B}^*(s)\right)^k P\left\{Q_b{}^{(n)} = k\right\} \\ &= Q_b{}^{(n)}(\tilde{B}^*(s)). \end{aligned} \tag{4}$$

Replacing s with λ and $\lambda - \lambda X(z)$ in (4) respectively, we have that

$$\begin{aligned} & S_p{}^{(n)*}(\lambda) = Q_b{}^{(n)}(\tilde{B}^*(\lambda)), \\ & S_p{}^{(n)*}(\lambda - \lambda X(z)) = Q_b{}^{(n)}(\tilde{B}^*(\lambda - \lambda X(z))). \end{aligned} \tag{5}$$

Substituting (2) and (5) into (3) , it is evident that

$$\begin{aligned} Q_b{}^{(n+1)}(z) = \Bigg[& (1 - Q_b{}^{(n)}(\tilde{B}^*(\lambda))) v^*(\lambda - \lambda X(z)) \\ & \frac{Q_b{}^{(n)}(\tilde{B}^*(\lambda - \lambda X(z))) - Q_b{}^{(n)}(\tilde{B}^*(\lambda))}{1 - Q_b{}^{(n)}(\tilde{B}^*(\lambda))} \\ & + Q_b{}^{(n)}(\tilde{B}^*(\lambda))(1 - H(v^*(\lambda))) \\ & \times \frac{v^*(\lambda - \lambda X(z)) - v^*(\lambda)}{1 - v^*(\lambda)} + Q_b{}^{(n)}(\tilde{B}^*(\lambda)) \\ & \times H(v^*(\lambda)) X(z) \Bigg] u^*(\lambda - \lambda X(z)). \end{aligned} \tag{6}$$

When system reaches a stable state, we have

$$Q_b{}^{(n+1)}(z) \to Q_b(z),$$

(6) simplified as

$$\begin{aligned} Q_b(z) = & u^*(\lambda - \lambda X(z)) \Bigg[Q_b(\tilde{B}^*(\lambda - \lambda X(z))) \\ & \times v^*(\lambda - \lambda X(z)) + Q_b(\tilde{B}^*(\lambda)) \\ & \times \Bigg(\frac{1 - H(v^*(\lambda))}{1 - v^*(\lambda)} (v^*(\lambda - \lambda X(z)) - v^*(\lambda)) \\ & + H(v^*(\lambda)) X(z) - v^*(\lambda - \lambda X(z)) \Bigg) \Bigg]. \end{aligned} \tag{7}$$

Taking the derivative of (7) and letting $z = 1$, the average of

Q_b can be obtained

$$\begin{aligned} E(Q_b) &= (Q_b(z))'\big|_{z=1} \\ &= \frac{Q_b(\tilde{B}^*(\lambda))E(X)}{(1-v^*(\lambda))(1-\tilde{\rho})}[\lambda E(V)(v^*(\lambda) \\ &\quad -H(v^*(\lambda)))+H(v^*(\lambda))(1-v^*(\lambda))] \\ &\quad + \frac{(\lambda E(U)+\lambda E(V))E(X)}{1-\tilde{\rho}}, \end{aligned} \tag{8}$$

where the traffic intensity

$$\tilde{\rho} = \lambda E(\tilde{B})E(X) = \lambda(1+E(R)\alpha)E(B)E(X).$$

III. The Stationary Analysis of System

A. The Stationary Condition of System

1) The Mean of Length of Service Period: The duration of the general service period is the aggregate of service time of all customers during in S_p. By using (8), the $E(S_p)$ can be obtained

$$\begin{aligned} E(S_p) &= -\left(S_p{}^{(n)*}(s)\right)'\Big|_{s=0} \\ &= -\left(Q_b(B^*(s))\right)'(B^*(s))'\big|_{s=0} = E(Q_b)E(\tilde{B}). \end{aligned} \tag{9}$$

2) The Mean of Length of Vacation Period: The count of vacations the server takes is denoted by random J. The continuous vacation series J is a random variable, which is given by

$$J = \min\left\{k : V^{k-1} < T < V^k,\ H\right\}.$$

The definition of J leads us to the conclusion that

$$\begin{aligned} P\{J \geq j\} &= P\left\{V^{(j-1)} \geq T\right\}P\{H \geq j\} \\ &= (v^*(\lambda))^{j-1}\sum_{k=j}^{\infty} h_k, \quad j \geq 2; \\ P\{J \geq 1\} &= 1. \end{aligned}$$

The PGF of continuous vacation series J is calculated by

$$\begin{aligned} J(z) &= 1+(1-\frac{1}{z})\sum_{j=1}^{\infty} z^j P\{J \geq j\} \\ &= 1-\frac{(1-z)(1-H(zv^*(\lambda)))}{1-zv^*(\lambda)}. \end{aligned}$$

Therefore, the LST of the duration of continuous vacation period V_h can be obtained

$$V_h{}^*(s) = J(v^*(s)) = 1-\frac{(1-v^*(s))(1-H(v^*(\lambda)v^*(s)))}{1-v^*(\lambda)v^*(s)}.$$

The mean of V_h is given by

$$E(V_h) = -(V_h{}^*(s))'\big|_{s=0} = \frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}E(V). \tag{10}$$

3) The Mean of Length of Idle Period: If the system has no customer arriving in the general service period and the H consecutive vacation periods. The system starts the idle period after H vacations. The duration of the idle period is the interarrival of one customer. The mean of I is calculated by

$$E(I) = \frac{S_p{}^*(\lambda)H(v^*(\lambda))}{\lambda} = \frac{Q_b(\tilde{B}^*(\lambda))H(v^*(\lambda))}{\lambda}. \tag{11}$$

4) Stability Condition of System: For a nonexhaustive queueing system, the equilibrium condition is that the average count of customers arriving in service cycle is lower than served in the general service period. Regarding the queueing system depicted in this paper, a service cycle includes the general service, vacation and setup periods. The amount of customers served in S_p defined as Φ, i.e. $Q_b = \Phi$. From this relation, we obtain that

$$\lambda\left(E(V)+E(S_p)+E(U)\right) < E(\Phi). \tag{12}$$

Substituting (8) and (9) into (12), It can be presented that

$$\begin{aligned} 0 <& \frac{\lambda(E(U)+E(V))(E(X)-1)(v^*(\lambda)-1)}{(v^*(\lambda)-1)(1-\tilde{\rho})} \\ &+ \frac{(1-\lambda E(\tilde{B}))Q_b(\tilde{B}^*(\lambda))E(X)}{(v^*(\lambda)-1)(\tilde{\rho}-1)}[H(v^*(\lambda)) \\ &\times(1-v^*(\lambda))+(v^*(\lambda)-H(v^*(\lambda)))\lambda E(V)]. \end{aligned} \tag{13}$$

Due to

$$1-E(X) < 0;$$

$$0 < v^*(\lambda) < 1;$$

$$H(v^*(\lambda)) = \sum_{j=1}^{\infty}(v^*(\lambda))^j h_j.$$

It can be obtained that

$$H(v^*(\lambda)) < v^*(\lambda). \tag{14}$$

From (14), it can be concluded that inequality (12) holds. For inequality (13) to hold, $1-\tilde{\rho} > 0$ is required. Therefore, the system can stabilize, and the stability condition becomes $1-\tilde{\rho} > 0$.

B. Analysis of Stationary Queue Length

Theorem 1 When $\tilde{\rho} < 1$, the queue length L_v is equal to the aggregate of two parts $L_v = L+L_d$, where L and L_d are independent variables. The one part L is the queue length of the repairable M^X/G/1 queueing system, and its PGF is represented as

$$L(z) = \tilde{B}^*(\lambda-\lambda X(z))\frac{(1-z)(1-\tilde{\rho})}{\tilde{B}^*(\lambda-\lambda X(z))-z}. \tag{15}$$

The other part L_d is the additional queue length, and its PGF represented as

$$\begin{aligned} L_d(z) =& \frac{1}{\eta-z\eta}(1-v^*(\lambda))\Bigg[Q_b(\tilde{B}^*(\lambda-\lambda X(z))) \\ &\times(1-v^*(\lambda-\lambda X(z)))u^*(\lambda-\lambda X(z)) \\ &-u^*(\lambda-\lambda X(z))Q_b(\tilde{B}^*(\lambda)) \\ &\times\Bigg(\frac{(1-H(v^*(\lambda)))}{v^*(\lambda)-1}(v^*(\lambda)-v^*(\lambda-\lambda X(z))) \\ &+X(z)H(v^*(\lambda))-v^*(\lambda-\lambda X(z))\Bigg)\Bigg], \end{aligned} \tag{16}$$

where

$$\begin{aligned} \eta =& Q_b(\tilde{B}^*(\lambda))E(X)[(1-v^*(\lambda))H(v^*(\lambda)) \\ &(v^*(\lambda)-H(v^*(\lambda)))\lambda E(V)] \\ &+\lambda E(X)(E(U)+E(V))(1-v^*(\lambda)). \end{aligned}$$

Proof In the general service period, the amount of customers remaining in the system when the nth customer leaves is denoted as L_n. Hence,

$$L_n = Q_b - n + \sum_{k=1}^{n} A_k, \quad n = 1, 2, \cdots, \Phi,$$

where A_k is the amount of customers arriving in general service period of the kth customer. Note that $A_k (k = 1, 2, \cdots, \Phi)$ are i.i.d, which PGF $A(z) = \tilde{B}^*(\lambda - \lambda X(z))$. Then

$$\begin{aligned}
E\left(\sum_{n=1}^{\Phi} z^{L_n}\right) =& E\left(\sum_{n=1}^{Q_b} z^{Q_b - n} (\tilde{B}^*(\lambda - \lambda X(z)))^n\right) \\
=& \frac{\tilde{B}^*(\lambda - \lambda X(z))}{\tilde{B}^*(\lambda - \lambda X(z)) - z} \\
& \times (Q_b(\tilde{B}^*(\lambda - \lambda X(z)) - Q_b(z)) \\
=& \frac{\tilde{B}^*(\lambda - \lambda X(z))}{\tilde{B}^*(\lambda - \lambda X(z)) - z} \Bigg[Q_b(\tilde{B}^*(\lambda - \lambda X(z))) \\
& \times (1 - u^*(\lambda - \lambda X(z)) v^*(\lambda - \lambda X(z))) \\
& - Q_b(\tilde{B}^*(\lambda)) u^*(\lambda - \lambda X(z)) \\
& \times \Bigg((v^*(\lambda - \lambda X(z)) - v^*(\lambda)) \frac{(H(v^*(\lambda)) - 1)}{v^*(\lambda) - 1} \\
& + H(v^*(\lambda)) X(z) - v^*(\lambda - \lambda X(z)) \Bigg) \Bigg].
\end{aligned}$$

By using regeneration cycle method, the PGF of queue length can be obtained

$$\begin{aligned}
L_v(z) =& \frac{E\left(\sum\limits_{n=1}^{\Phi} z^{L_n}\right)}{E(\Phi)} \\
=& \tilde{B}^*(\lambda - \lambda X(z)) \frac{(1 - z)(1 - \tilde{\rho})}{\tilde{B}^*(\lambda - \lambda X(z)) - z} \\
& \times \frac{1 - v^*(\lambda)}{\eta - z\eta} \Bigg[(1 - u^*(\lambda - \lambda X(z)) \\
& \times v^*(\lambda - \lambda X(z)) Q_b(\tilde{B}^*(\lambda - \lambda X(z))) \\
& - Q_b(\tilde{B}^*(\lambda)) u^*(\lambda - \lambda X(z)) \\
& \times \Bigg((v^*(\lambda) - v^*(\lambda - \lambda X(z))) \frac{(1 - H(v^*(\lambda)))}{v^*(\lambda) - 1} \\
& + H(v^*(\lambda)) X(z) - v^*(\lambda - \lambda X(z)) \Bigg) \Bigg] \\
=& L(z) \times L_d(z).
\end{aligned} \tag{17}$$

Now, substituting (5) into (17), it can be given the final result of the PGF of L_v.

From the results of the stochastic decomposition, the $E(L_v)$ can be obtained

$$E(L_v) = E(L) + E(L_d).$$

Taking the derivative of (15) and letting $z = 1$, the average of L can be obtained

$$\begin{aligned}
E(L) =& (L(z))'|_{z=1} \\
=& \tilde{\rho} + \frac{1}{2 - 2\tilde{\rho}} \Big[\lambda^2 E(X)^2 (E(B) E(R^2) \alpha \\
& + (1 + \alpha E(R))^2 E(B^2)) + \lambda E(B) \\
& \times (1 + \alpha E(R)) E(X^2) \Big].
\end{aligned} \tag{18}$$

Taking the derivative of (16) and letting $z = 1$, the average of L_d can be obtained

$$\begin{aligned}
E(L_d) =& (L_d(z))'|_{z=1} \\
=& \frac{\lambda \tilde{\rho}}{\eta} (E(V) + E(U)) E(X) E(Q_b) (1 - v^*(\lambda)) \\
& + \frac{\lambda}{2\eta} (1 - v^*(\lambda)) [\lambda E(X)^2 (E(U^2) + E(V^2) \\
& + 2E(U)E(V)) + E(X^2)(E(U) + E(V))] \\
& + \frac{\lambda}{2\eta} Q_b(\tilde{B}^*(\lambda)) (v^*(\lambda) - H(v^*(\lambda))) \\
& \times (2\lambda E(X)^2 E(U) E(V) + \lambda E(X)^2 E(V^2) \\
& + E(X^2) E(V)) + \frac{1}{2\eta} Q_b(\tilde{B}^*(\lambda)) H(v^*(\lambda)) \\
& \times (1 - v^*(\lambda)) (2\lambda E(X)^2 E(U) + E(X^2)).
\end{aligned} \tag{19}$$

From (18) and (19), it is easy to get the mean of L_v is

$$\begin{aligned}
E(L_v) =& E(L) + E(L_d) \\
=& \tilde{\rho} + \frac{1}{2 - 2\tilde{\rho}} \Big[\lambda^2 E(X)^2 (\alpha E(B) E(R^2) \\
& + (1 + \alpha E(R))^2 E(B^2)) + \lambda E(B) \\
& \times E(X^2)(1 + \alpha E(R)) \Big] + \frac{\lambda \tilde{\rho}}{\eta} E(X) E(Q_b) \\
& \times (E(U) + E(V))(1 - v^*(\lambda)) + \frac{\lambda}{2\eta} \\
& \times (1 - v^*(\lambda)) [\lambda E(X)^2 (E(U^2) + E(V^2) \\
& + 2E(U)E(V)) + E(X^2)(E(U) + E(V))] \\
& + \frac{\lambda}{2\eta} Q_b(\tilde{B}^*(\lambda)) (v^*(\lambda) - H(v^*(\lambda))) \\
& \times (2\lambda E(X)^2 E(U) E(V) + \lambda E(X)^2 E(V^2) \\
& + E(X^2) E(V)) + \frac{1}{2\eta} Q_b(\tilde{B}^*(\lambda)) H(v^*(\lambda)) \\
& \times (1 - v^*(\lambda)) (2\lambda E(U) E(X)^2 + E(X^2)).
\end{aligned} \tag{20}$$

C. Analysis of Stationary Waiting Time

Theorem 2 When $\tilde{\rho} < 1$, the waiting time W is equal to the aggregate of two parts $W_v = W + W_d$, where W and W_d are independent variables. The part W is the waiting time of the repairable M^X/G/1 queueing system, and its LST is represented as

$$W^*(s) = \frac{s - \tilde{\rho} s}{\lambda X(\tilde{B}^*(s))) - \lambda + s}.$$

The other part W_d is the additional delay, and its LST is represented as

$$\begin{aligned}
W_d^*(s) =& \frac{1 - X(\tilde{B}^*(s)))}{x(1 - \tilde{B}^*(s)))} \times \frac{\lambda - \lambda v^*(\lambda)}{s\eta} \\
& \times \Bigg[Q_b(X(\tilde{B}^*(s)))(1 - u^*(s) v^*(s)) \\
& - u^*(s) Q_b(X(\tilde{B}^*(\lambda))) \\
& \times \Bigg(\frac{(v^*(s) - v^*(\lambda))(1 - H(v^*(\lambda)))}{1 - v^*(\lambda)} \\
& + \frac{(\lambda - s) H(v^*(\lambda))}{\lambda} - v^*(s) \Bigg) \Bigg].
\end{aligned}$$

Proof Assuming a batch of customers arriving simultaneously constitutes a super customer, the performance metrics of a super customer in an M/G/1 system are differentiated using subscript a, contrasting with an M^X/G/1 system.

In this system, the waiting customers are selected randomly, thus the waiting time of one customer comprises two independently composed periods. One part is the waiting time W_X while the customer belongs to a super customer, and the super customer has not received service. The LST of the W_X is denoted by $W_X^*(s)$. The other part is the waiting time W_x within the super customer, and its LST is denoted by $W_x^*(s)$.

The Poisson arrival process has independent incremental characteristics. Within the service period, the waiting time is not associated with the arrival process. The PGF of the service time of single super customer is

$$\tilde{B}_a^*(z) = \sum_{k=1}^{\infty} [\tilde{B}^*(s)]^k x_k = X(\tilde{B}^*(s)). \tag{21}$$

After providing service to a super customer, the PGF of the amount of super consumers is supplied by

$$\begin{aligned} L_X(z) =& \frac{(1-\tilde{\rho})\tilde{B}_a^*(\lambda-\lambda X_a(z))(1-z)}{\tilde{B}_a^*(\lambda-\lambda X_a(z))-z} \\ &\times \frac{1-v^*(\lambda)}{\eta - z\eta}\Bigg[Q_b(\tilde{B}_a^*(\lambda-\lambda X_a(z))) \\ &\times (1-u_a^*(\lambda-\lambda X_a(z))v_a^*(\lambda-\lambda X_a(z))) \\ &-u_a^*(\lambda-\lambda X_a(z))Q_b(\tilde{B}_a^*(\lambda))\Bigg(\Big(1- \\ &H(v^*(\lambda))\Big)\frac{v_a^*(\lambda-\lambda X_a(z))-v^*(\lambda)}{1-v^*(\lambda)} \\ &+H(v^*(\lambda))X_a(z)-v_a^*(\lambda-\lambda X_a(z))\Bigg)\Bigg]. \end{aligned} \tag{22}$$

Because the super customer in (22) is regarded as one customer, $X_a(z)=z$ is established. For a single customer, the waiting time does not affect the arrival process after its arrival. we have that

$$L_X(z) = W_a{}^*(\lambda-\lambda X_a(z))\tilde{B}_a^*(\lambda-\lambda X_a(z)). \tag{23}$$

Substituting (22) into (23) and letting $\lambda - \lambda X_a(z) = s$, we have that

$$\begin{aligned} W_X^*(s) =& \frac{(1-\tilde{\rho})s}{\lambda X(\tilde{B}^*(s))-\lambda+s} \times \frac{\lambda}{s\eta}(1-v^*(\lambda)) \\ &\times \Bigg[Q_b(X(\tilde{B}^*(s)))(1-u^*(s)v^*(s))-u^*(s) \\ &\times Q_b(X(\tilde{B}^*(\lambda)))\Bigg(-v^*(s)+\frac{v^*(s)-v^*(\lambda)}{1-v^*(\lambda)} \\ &\times(1-H(v^*(\lambda)))+\frac{(\lambda-s)H(v^*(\lambda))}{\lambda}\Bigg)\Bigg] \end{aligned} \tag{24}$$

Among single super customer, assuming the number of customers preceding a waiting customer is denoted as X_b. By the renewal processes, The gap between two successive update points equals the batch size X. The probability distribution and PGF of X_b are respectively

$$P(X_b = k) = \frac{1}{x}P(X>k),$$

$$X_b(z) = \sum_{k=0}^{\infty} P(X_b=k)z^k = \frac{1}{x}\sum_{k=0}^{\infty} z^k \sum_{i=k+1}^{\infty} x_i = \frac{x - xX(z)}{1-z}.$$

The LST of W_x can be obtained

$$W_x^*(s) = X_b(\tilde{B}^*(s)) = \frac{1-X(\tilde{B}^*(s))}{x(1-\tilde{B}^*(s))}. \tag{25}$$

Because parts W_X and W_x are independent, by using (24) and (25) the LST of the waiting time is

$$\begin{aligned} W_v^*(s) =& W_X^*(s)\times W_x^*(s) \\ =& \frac{s-\tilde{\rho}s}{\lambda X(\tilde{B}^*(s))-\lambda+s}\times\frac{\lambda-\lambda X(\tilde{B}^*(s))}{s\eta x - s\eta x\tilde{B}^*(s)} \\ &\times(1-v^*(\lambda))\Bigg[Q_b(X(\tilde{B}^*(s)))(1-u^*(s)v^*(s)) \\ &-u^*(s)Q_b(X(\tilde{B}^*(\lambda)))\Bigg(\frac{H(v^*(\lambda))(\lambda-s)}{\lambda} \\ &+\frac{(v^*(s)-v^*(\lambda))(1-H(v^*(\lambda)))}{1-v^*(\lambda)}-v^*(s)\Bigg)\Bigg] \\ =& W^*(s)\times W_d^*(s). \end{aligned} \tag{26}$$

Taking the derivative of (26) and letting $s=0$, the mean of W_v can be obtained

$$\begin{aligned} E(W_v) =& -\left.(W_v^*(s))'\right|_{s=0} \\ =& \frac{1}{2-2\tilde{\rho}}[\lambda E(X)^2(\alpha E(R^2)E(B) \\ &+(1+\alpha E(R))^2E(B^2))+E(\tilde{B})E(X^2)]+\frac{\tilde{\rho}}{\eta} \\ &\times(1-v^*(\lambda))E(X)E(Q_b)(E(U)+E(V)) \\ &+\frac{1}{2\eta}(1-v^*(\lambda))[\lambda E(X)^2(E(U^2)+E(V^2) \\ &+2E(U)E(V))+E(X^2)(E(U)+E(V))] \\ &+\frac{1}{2\eta}Q_b(\tilde{B}^*(\lambda))(v^*(\lambda)-H(v^*(\lambda))) \\ &\times(2\lambda E(X)^2E(U)E(V)+\lambda E(X)^2E(V^2) \\ &+E(X^2)E(V))+\frac{1}{2\lambda\eta}Q_b(\tilde{B}^*(\lambda))H(v^*(\lambda)) \\ &\times(1-v^*(\lambda))(2\lambda E(X)^2E(U)+E(X^2)) \\ =& E(W)+E(W_d). \end{aligned} \tag{27}$$

IV. System Cycle Analysis

The cycle C of the system is represented by the during interval between the moment when one general service period begins and the moment when the next general service period begins, including S_p, I, U and V. From (9), (10), and (11), it is easy to get the mean of C is

$$\begin{aligned} E(C) =& E(V_h)+E(S_p)+E(I)+E(U) \\ =& \frac{Q_b(\tilde{B}^*(\lambda))\tilde{\rho}[(v^*(\lambda)-H(v^*(\lambda)))\lambda E(V)]}{(\lambda\tilde{\rho}-\lambda)(v^*(\lambda)-1)} \\ &+\frac{Q_b(\tilde{B}^*(\lambda))\tilde{\rho}H(v^*(\lambda))(1-v^*(\lambda))}{(1-v^*(\lambda))(\lambda-\lambda\tilde{\rho})} \\ &+\frac{\tilde{\rho}E(U)+\tilde{\rho}E(V)}{(1-\tilde{\rho})}+\frac{E(V)(1-H(v^*(\lambda)))}{1-v^*(\lambda)} \\ &+\frac{Q_b{}^{(n)}(\tilde{B}^*(\lambda))H(v^*(\lambda))}{\lambda}+E(U) \\ =& \frac{\beta}{\lambda(1-v^*(\lambda))(1-\tilde{\rho})}, \end{aligned} \tag{28}$$

where

$$\begin{aligned}\beta =&\tilde{\rho}Q_b(\tilde{B}^*(\lambda))[(v^*(\lambda)-H(v^*(\lambda)))\lambda E(V)\\&+H(v^*(\lambda))(1-v^*(\lambda))]+\lambda\tilde{\rho}(E(U)+E(V))\\&\times(1-v^*(\lambda))+(1-\tilde{\rho})[Q_b(\tilde{B}^*(\lambda))H(v^*(\lambda))\\&\times(1-v^*(\lambda))+\lambda(1-H(v^*(\lambda)))E(V)(1-v^*(\lambda))\\&+\lambda E(U)(1-v^*(\lambda))].\end{aligned}$$

In steady state, the probabilities that the system is in S_p, I, V and U are denoted by $P_{\tilde{B}}$, P_I, P_V and P_U, respectively. We obtain that

$$\begin{aligned}P_{\tilde{B}} =&\frac{E(S_p)}{E(C)}=\frac{\tilde{\rho}}{\beta}Q_b(\tilde{B}^*(\lambda))[(v^*(\lambda)-H(v^*(\lambda)))\\&\times\lambda E(V)+H(v^*(\lambda))(1-v^*(\lambda))]\\&+\frac{\lambda\tilde{\rho}}{\beta}(E(U)+E(V))(1-v^*(\lambda));\end{aligned}$$

$$P_I=\frac{E(I)}{E(C)}=\frac{Q_b(\tilde{B}^*(\lambda))H(v^*(\lambda))(1-v^*(\lambda))(1-\tilde{\rho})}{\beta};$$

$$P_V=\frac{E(V_h)}{E(C)}=\frac{\lambda}{\beta}(1-H(v^*(\lambda)))(1-v^*(\lambda))(1-\tilde{\rho})E(V);$$

$$P_U=\frac{E(U)}{E(C)}=\frac{\lambda}{\beta}(E(U)-\tilde{\rho}E(U))(1-v^*(\lambda)).$$

V. Special Cases

For the queueing system, some special queueing systems can be obtained by determining certain variables. If the random variable $H\to\infty$ or $H=1$, a multiple or single vacation queueing system with gated service are considered, respectively.

A. Special Case 1

For the queueing system, if the count of vacations tends to infinity (i.e. $H\to\infty$), a repairable M^X/G/1 queueing with gated service, setup period and multiple vacations is acquired. Under this condition, $H(z)=0$. For this queueing system, the $L_1(z)$ and $W_1^*(s)$ are respectively as

$$\begin{aligned}L_1(z)=&\frac{\tilde{B}^*(\lambda-\lambda X(z))(1-\tilde{\rho})}{\eta_1\tilde{B}^*(\lambda-\lambda X(z))-\eta_1 z}\times[Q_b(\tilde{B}^*(\lambda-\lambda X(z))\\&\times(1-v^*(\lambda))(1-u^*(\lambda-\lambda X(z))v^*(\lambda-\lambda X(z)))\\&+u^*(\lambda-\lambda X(z))Q_b(\tilde{B}^*(\lambda))v^*(\lambda)\\&\times(1-v^*(\lambda-\lambda X(z)))];\end{aligned}$$

$$\begin{aligned}W_1^*(s)=&\frac{1-\tilde{\rho}}{\lambda X(\tilde{B}^*(s))-\lambda+s}\times\frac{\lambda-\lambda X(\tilde{B}^*(s))}{\eta_1 x-\eta_1 x\tilde{B}^*(s))}\\&\times\left[Q_b(X(\tilde{B}^*(s)))(1-v^*(\lambda))(1-u^*(s)v^*(s))\right.\\&\left.+Q_b(X(\tilde{B}^*(\lambda)))v^*(\lambda)u^*(s)(1-v^*(s))\right],\end{aligned}$$

where

$$\begin{aligned}\eta_1=&\lambda E(X)\left[Q_b(\tilde{B}^*(\lambda))v^*(\lambda)E(V)+(1-v^*(\lambda))\right.\\&\left.\times(E(U)+E(V))\right].\end{aligned}$$

B. Special Case 2

For the queueing system, let the count of vacations be 1 (i.e. $H=1$), and a repairable M^X/G/1 queueing with gated service, setup period and single vacation is acquired. Under this condition, $H(z)=z$. For this queueing system, the $L_2(z)$ and $W_2^*(s)$ are respectively as

$$\begin{aligned}L_2(z)=&\tilde{B}^*(\lambda-\lambda X(z))\frac{(1-\tilde{\rho})}{\eta_2\tilde{B}^*(\lambda-\lambda X(z))-\eta_2 z}\\&\times(1-v^*(\lambda))\left[(1-u^*(\lambda-\lambda X(z))\right.\\&\times v^*(\lambda-\lambda X(z)))Q_b(\tilde{B}^*(\lambda-\lambda X(z)))\\&\left.+Q_b(\tilde{B}^*(\lambda))v^*(\lambda)u^*(\lambda-\lambda X(z))(1-X(z))\right];\end{aligned}$$

$$\begin{aligned}W_2^*(s)=&\frac{(1-\tilde{\rho})}{\lambda X(\tilde{B}^*(s))-\lambda+s}\times\frac{\lambda-\lambda X(\tilde{B}^*(s))}{\eta_2 x-\eta_2 x\tilde{B}^*(s))}\\&\times(1-v^*(\lambda))\left[Q_b(X(\tilde{B}^*(s)))(1-u^*(s)v^*(s))\right.\\&\left.+Q_b(X(\tilde{B}^*(\lambda)))v^*(\lambda)u^*(s)\frac{s}{\lambda}\right],\end{aligned}$$

where

$$\begin{aligned}\eta_2=&(E(X)-v^*(\lambda)E(X))\left[\lambda(E(U)+E(V))\right.\\&\left.+Q_b(\tilde{B}^*(\lambda))v^*(\lambda)\right].\end{aligned}$$

From the derivation of the specific indicators of the two special cases mentioned above, we can see that all results for the queueing system and its special cases 1 and 2 in this paper are consistent with reference [2] and [18], which further confirms the system's accuracy and viability in this paper.

VI. Conclusion

The gated service, repairable servers, batch arrival, setup time and multiple adaptive vacation rules are introduced into the M/G/1 queueing system. The average number of customers is calculated at the initial moment of the general service period. The PGF of the queue length is analyzed by employing the regeneration cycle method, and the LST of the waiting time is computed by the independence of the waiting time from the arrival time interval. The average queue length and waiting time of the customers are obtained, and the random decomposition results are provided. Furthermore, the stability condition and average cycle time are given.The likelihoods of the system being in diverse states are calculated. Finally, by controlling the random variable H, the queueing system is transformed into several special systems, and the obtained results are consistent with those in the reference literature. Hence, verified the reliability of the results in this paper.

The exponential distribution in this paper can be extended to more general distributions such as geometric distribution, PH distribution, etc, to establish a more comprehensive theoretical framework for multiple adaptive vacation queueing systems. Moreover, using iterative programs to obtain approximate solutions for $Q_b(\tilde{B}^*(\lambda))$, depicting the trend of steady state indicators as parameters change, and then using this system to analyze and optimize practical problems.

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