Leader-Following Networks under Model Prediction Event-Triggered Control

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***Abstract*—The consistency of leader-following networks is an important research branch in nonlinear science, and the design better and more effective controllers is currently an open problem. A new event-triggered control (ETC) protocol is investigated for the leader-following network, where the followers can communicate with leaders in real-time. A sufficient criterion in matrix form is derived for the synchronization of the leader-following network under this control protocol, and it is proved that the event-triggered mechanism does not exhibit the Zeno phenomenon. Meanwhile, the convergence speed and triggering times of the error systems are compared with the other two control protocols, and the effectiveness and advantages are verified for the proposed control method through two numerical examples. The proposed ETC protocol can reduce the triggering frequency of the controller and the cost of leader-following network. This method avoids the need for continuously monitoring the status of neighboring followers, and it reduces the cost of network communication and the resource consumption in the control process.**

***Index Terms*—leader-following network, event-triggered control, network synchronization, control cost.**

I. Introduction

WITH the development of technology, the network is very important in many fields, such as biological engineering [1], automation technology [2] and information technology [3]. Network synchronization has become one of the hot topics in network research as its potential applications in power systems [4], multi-agent systems [5], and secure communication [6]. The goal of network synchronization is to achieve the expected trajectory of nodes in the entire network through internal coupling and external control. Usually, a leader-following network refers to a network with leaders. The synchronization of the leader-following network is achieved by applying control to the followers, so that the final trajectory of all followers is consistent with that of the leader. After decades of research on network synchronization, many control methods of synchronization have been proposed, such as intermittent control [7], impulsive control [8], pinning control [9], event-triggered control (ETC) [10], etc. The ETC has attracted much attention as the low cost of network synchronization.

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The ETC needs to design a suitable control protocol and triggering condition. If the system meets the triggering condition at a certain moment, then the event is triggered at that moment, and the control protocol updates the state. The early research on ETC is usually to design a discontinuous control protocol and uses static event-triggered conditions to update the control protocol according to the continuous monitoring of neighboring nodes state [11]. In reference [12], the event-triggered input is designed for a switched system by an observer, and the signals of controlled system can be bounded. For uncertain nonlinear systems with unmodeled dynamics, the ETC is designed to keep the convergence of tracking error and bounded signals of the close-loop control system [13]. An adaptive tracking method is proposed for the system with unknown disturbances by the ETC [14]. The synchronization is investigated by disturbance observer-based control and switched-gain ETC for Lur'e systems, and the effectiveness is shown based on the master-slave Chua's circuits [15]. For the switched nonlinear systems, the ETC is designed to save the communication resource and maintain good signal tracking performance [16]. In reference [17], the consensus of multi-agent systems is investigated by an adaptive event-triggered mechanism, and a novel model-free deep reinforcement learning is used to for an approximated linearized control protocol. A sufficient condition is proposed for the synchronization of complex networks by using ETC [18]. The ETC scheme is investigated for the switched nonlinear systems with unmeasurable states, and the communication costs are significantly reduced [19].

In order to improve utilization of resource during network synchronization and effectively reduce the number of updates to the control protocol, some improved static and dynamic event-triggered conditions are proposed [20], [21]. Although the improved triggering conditions have greater advantages in saving resource, these triggering conditions still require continuous monitoring of the state of neighboring nodes, which makes it difficult to effectively reduce the cost of network communication. An ETC by state prediction is proposed to overcome the limitations of aforementioned ETC [22]. This ETC is different from most control protocols, as the follower state is predicted by the system model, and the triggering conditions do not include the state of neighboring nodes. Therefore, the ETC by state prediction does not require continuous monitoring of the state of neighboring nodes. Furthermore, Zhang et al. proposes a control method that the states of some followers are directly connected to the leader in real-time [23]. However, the states among followers are still predicted, so it is difficult to effectively reduce the cost of ETC. Therefore, a good control protocol is necessary to ensure the advantages of ETC by state prediction and reduce the control cost.

In order to reduce the cost of communication and computational resource during network synchronization, the ETC protocol is studied for the leader-following network. In this paper, the main contributions are given as follows: (1) In the proposed control protocol, the states of followers communicate with the leader in real-time, and it is applied to the synchronization research of leader-following networks. (2) A sufficient criterion is proposed for achieving synchronization in the leader-following network, and it is proved that there is no Zeno phenomenon within a finite time in the ETC. (3) Based on the numerical examples of gyroscope systems, the new control method can obtain good results and reduce the cost of ETC.

By implementing threshold-based triggering conditions, the ETC inherently reduces operational costs associated with continuous control. This approach not only conserves network bandwidth and computational resources but also minimizes energy consumption in sensor nodes and control units. However, the ETC must simultaneously ensure stability and performance of the controlled system, and a suboptimal configuration of the controller may potentially induce system oscillations or trigger excessively frequent activations. To address this challenge, this study presents a comparative analysis of control efficacy and operational costs under different ETC methods. Furthermore, we propose a novel time-decaying exponential triggering mechanism, in which the threshold parameter asymptotically decreases over time. Under the identical triggering thresholds, the proposed ETC method achieves consensus of leader-following network by partially predicting methods, while dramatically curtailing control costs and reducing the frequency of event-triggering instances. Consequently, the proposed ETC strategy not only effectively reduces network bandwidth consumption and computational resource utilization but also enhances system efficiency, as empirically validated in Section IV.

The remaining research is given as follows. In Section II, the mathematical model of leader-following network is given. A sufficient condition is proposed for the leader-following network by the ETC in Section III. In Section IV, two numerical examples of leader-following networks are investigated based on the gyroscope systems, and the simulation results verify the effectiveness and feasibility of the ETC. Finally, there is the conclusion in Section V.

II. Model of Leader-Following Network

The nonlinear dynamic systems of a leader-following network are given as follows:

$$\dot{\boldsymbol{x}}_0(t) = \boldsymbol{f}(t, \boldsymbol{x}_0(t)), \tag{1}$$

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}(t, \boldsymbol{x}_i(t)) + \boldsymbol{u}_i(t), i = 1, 2, \cdots, N, \tag{2}$$

where $\boldsymbol{x}_0(t) \in \boldsymbol{R}^n$ denotes the leader's state, $\boldsymbol{f}(t, \boldsymbol{x}_0(t))$ is a continuous nonlinear function of the leader. $\boldsymbol{x}_i(t) \in \boldsymbol{R}^n$ is the ith follower's state, $\boldsymbol{f}(t, \boldsymbol{x}_i(t))$ is a continuous nonlinear function of the ith follower, and $\boldsymbol{u}_i(t) \in \boldsymbol{R}^n$ is the control protocol of the ith follower.

Assumption 2.1: There exists a region U, and the dynamic equation satisfies $\|\boldsymbol{f}(t, \boldsymbol{x}_0(t)) - \boldsymbol{f}(t, \boldsymbol{x}_i(t))\| \leqslant r_1 \|\boldsymbol{x}_0(t) - \boldsymbol{x}_i(t)\|$ when $\boldsymbol{x}_0(0), \boldsymbol{x}_i(0) \in U$, where r_1 is a positive constant.

Similarly, according to the Lemma 1 in reference [24], one yields the following Lemma.

Lemma 2.1: If the constant $s = \min\{l_0, \lambda_0\}, l_0 > 0, \mu_0 > 0, l_0 \neq \lambda_0$, and the scalar function satisfies

$$\dot{u} \leqslant -l_0 u + \mu_0 e^{-\lambda_0 t}, u(t_0) = u_0 \geqslant 0,$$

then there is $u \leqslant (u_0 + \frac{\mu_0}{|l_0 - \lambda_0|})e^{-st}$, and the $u(t) \to 0$ as $t \to +\infty$.

Definition 2.1: If the initial values $\boldsymbol{x}_0(0)$ and $\boldsymbol{x}_i(0)$ are all within a region U, then the $\lim\limits_{t \to +\infty} \|\boldsymbol{x}_0(t) - \boldsymbol{x}_i(t)\| = 0$, it is said that the leader-following networks (1) and (2) can achieve synchronization in the region U.

Note that

$$\boldsymbol{q}_i(t) = c\sum_{j=1}^{N} a_{ij}(\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)) + b_i(\boldsymbol{x}_0(t) - \boldsymbol{x}_i(t)), \tag{3}$$

where $\boldsymbol{A} = (a_{ij})_{N \times N}$ is the adjacency matrix, and it denotes the topological connection relationship between followers. If the ith follower can directly receive information from the jth follower ($i \neq j$), then the jth follower is the neighbor of the ith follower, i.e., $a_{ij} = 1$; otherwise, $a_{ij} = 0$, and $a_{ii} = 0$. One lets N_i be a set of the neighbors of the ith follower. The Laplace matrix $\boldsymbol{L} = (l_{ij})_{N \times N}$, where $l_{ij} = -a_{ij}$ when $i \neq j$, and $l_{ii} = \sum\limits_{j=1, j \neq i}^{N} a_{ij}$ when $i = j$. One lets $\boldsymbol{B} = diag\{b_1, b_2, \cdots, b_N\}$, where $b_i > 0$ means that the ith follower can communicate with the leader; otherwise, $b_i = 0$.

III. Main Results

In order to reduce continuous communication between network nodes, a new controller is designed by

$$\boldsymbol{u}_i(t) = c\sum_{j=1}^{N} a_{ij}(\hat{\boldsymbol{x}}_j(t) - \boldsymbol{x}_i(t)) + b_i(\hat{\boldsymbol{x}}_0(t) - \boldsymbol{x}_i(t)), \tag{4}$$

where c is the control gain. This controller is continuously updated, but the communication with neighbors is discontinuous. The definition of continuous function $\hat{\boldsymbol{x}}_j(t)$ is given as follows

$$\begin{cases} \hat{\boldsymbol{x}}_j(t) = \hat{\boldsymbol{x}}_j(t), & t = t_k^j, \\ \dot{\hat{\boldsymbol{x}}}_j(t) = f(t, \hat{\boldsymbol{x}}_j(t)), & t \in (t_k^j, t_{k+1}^j), \end{cases} \tag{5}$$

where $j \in \{0\} \bigcup N_i, k = 0, 1, 2, 3, \cdots$. t_k^j is the kth event-triggering moment of the jth follower. At this moment, the jth follower sends its state to surrounding neighbors through the network. In control protocol (5), all followers will update the state of the jth follower if they can directly receive the information of the jth follower. Due to the leader's trajectory is only depended on its dynamic equations, then $\hat{\boldsymbol{x}}_0(t) = \boldsymbol{x}_0(t)$.

Note 3.1: The control protocol in reference [22] is given by

$$\boldsymbol{u}_{i1}(t) = c\sum_{j=1}^{N} a_{ij}(\hat{\boldsymbol{x}}_j(t) - \hat{\boldsymbol{x}}_i(t)) + b_i(\hat{\boldsymbol{x}}_0(t) - \hat{\boldsymbol{x}}_i(t)). \tag{6}$$

This control protocol can ensure that communication between leader-following network is discontinuous, but the ith follower can obtain its state $\boldsymbol{x}_i(t)$ without the need for external network. As the state of the ith follower is predicted by the control protocol (5), it not only increases the computational complexity of the controller, but also degrades the state estimation accuracy of the ith follower. Therefore, the control protocol is changed by [23]

$$\boldsymbol{u}_{i2}(t) = c\sum_{j=1}^{N} a_{ij}(\hat{\boldsymbol{x}}_j(t) - \hat{\boldsymbol{x}}_i(t)) + b_i(\boldsymbol{x}_0(t) - \boldsymbol{x}_i(t)). \quad (7)$$

The control protocol requires the predicted state of neighbors and the real-time state of the ith follower and leader.

The proposed control protocol (4) only requires the ith follower's state in real-time and the predicted states of neighbors and leader. Compared with other control protocols, the control protocol (4) reduces dependency on predicted states of neighboring followers and the leader while maintaining the estimation accuracy of the ith follower's state.

The event-triggering time series $\{t_k^i\}$ of the ith follower is determined by the triggering conditions, i.e.,

$$\begin{cases} t_0^i = 0 \\ t_{k+1}^i = \inf\left\{t > t_k^i \mid \|\boldsymbol{\varepsilon}_i(t)\|^2 - g_i(t) \geqslant 0\right\}, \end{cases} \quad (8)$$

where $\boldsymbol{\varepsilon}_i(t) = \boldsymbol{x}_i(t) - \hat{\boldsymbol{x}}_i(t)$, $g_i(t) = \beta_i \exp(-\lambda t)$, and $i = 1, 2, \cdots, N, k = 0, 1, 2, 3, \cdots$.

Note 3.2: According to the triggering conditions (8), the event-triggered mechanism for the ith follower is exclusively dependent on its own current state and predicted state, thus obviating the necessity for persistent monitoring of neighboring agents' states.

According to Eqs. (1) and (2), if the error between the ith follower and leader is denoted as $\boldsymbol{e}_i(t) = \boldsymbol{x}_0(t) - \boldsymbol{x}_i(t)$, then the dynamic equation of the error system can be obtained as follows:

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) =& f(t, \boldsymbol{x}_0) - f(t, \boldsymbol{x}_i) - \\ & c\sum_{j=1}^{N} a_{ij}(\hat{\boldsymbol{x}}_j(t) - \boldsymbol{x}_i(t)) - b_i(\hat{\boldsymbol{x}}_0(t) - \boldsymbol{x}_i(t)).\end{aligned}$$

According to Eq. (3), the error equation is rewritten as

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t) = & f(t, \boldsymbol{x}_0) - f(t, \boldsymbol{x}_i) - \boldsymbol{q}_i(t) + \cdots \\ & c\sum_{j=1}^{N} a_{ij}(\boldsymbol{x}_j(t) - \hat{\boldsymbol{x}}_j(t)).\end{aligned} \quad (9)$$

If one denotes

$$\begin{aligned}
&\boldsymbol{e}(t) = (\boldsymbol{e}_1(t)^{\mathrm{T}}, \boldsymbol{e}_2(t)^{\mathrm{T}}, \cdots, \boldsymbol{e}_N(t)^{\mathrm{T}})^{\mathrm{T}}, \\
&\boldsymbol{q}(t) = (\boldsymbol{q}_1(t)^{\mathrm{T}}, \boldsymbol{q}_2(t)^{\mathrm{T}}, \cdots, \boldsymbol{q}_N(t)^{\mathrm{T}})^{\mathrm{T}}, \\
&\boldsymbol{\varepsilon}(t) = (\boldsymbol{\varepsilon}_1(t)^{\mathrm{T}}, \boldsymbol{\varepsilon}_2(t)^{\mathrm{T}}, \cdots, \boldsymbol{\varepsilon}_N(t)^{\mathrm{T}})^{\mathrm{T}}, \\
&\boldsymbol{F}(t) = ((f(t, \boldsymbol{x}_0) - f(t, \boldsymbol{x}_1))^{\mathrm{T}}, (f(t, \boldsymbol{x}_0) - f(t, \boldsymbol{x}_2))^{T}, \\
&\qquad \cdots, (f(t, \boldsymbol{x}_0) - f(t, \boldsymbol{x}_N))^{T})^{\mathrm{T}},
\end{aligned}$$

then the error system of the leader-following network is given by

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{F}(t) - ((c\boldsymbol{L} + \boldsymbol{B}) \otimes \boldsymbol{I}_n)\boldsymbol{e}(t) - (c\boldsymbol{P} \otimes \boldsymbol{I}_n)\boldsymbol{\varepsilon}(t), \quad (10)$$

where $\boldsymbol{P} = (\boldsymbol{L} - diag\{|N_1|, |N_2|, \cdots, |N_N|\})$, $|N_i|$ represents the number of elements in N_i.

Theorem 3.1: Under the condition that Assumption 2.1 is satisfied, if there is an appropriate control gain c, positive scalar l and matrices $\boldsymbol{L}, \boldsymbol{B}$, the following inequality holds

$$(r_1 + l)\boldsymbol{I}_N - \frac{1}{2}((c\boldsymbol{L} + \boldsymbol{B})^{\mathrm{T}} + (c\boldsymbol{L} + \boldsymbol{B})) \leqslant 0,$$

where $l \neq \frac{\lambda}{2}$, λ is determined by the triggering function (8), then the leader-following networks (1) and (2) can achieve synchronization in the region U by the control protocol (4).

Proof: If Lyapunov function $V(t) = \frac{1}{2}\boldsymbol{e}(t)^{\tau}\boldsymbol{e}(t)$, then the derivative of $V(t)$ is given as follows

$$\begin{aligned}
\dot{V}(t) = & \boldsymbol{e}(t)^{\mathrm{T}}(\boldsymbol{F}(t) - \frac{1}{2}(((c\boldsymbol{L} + \boldsymbol{B})^{\mathrm{T}} + \\
& (c\boldsymbol{L} + \boldsymbol{B})) \otimes \boldsymbol{I}_n)\boldsymbol{e}(t) - (c\boldsymbol{P} \otimes \boldsymbol{I}_n)\boldsymbol{\varepsilon}(t)) \\
\leqslant & \boldsymbol{e}(t)^{\mathrm{T}}(((r_1 + l)\boldsymbol{I}_N - \frac{1}{2}((c\boldsymbol{L} + \boldsymbol{B})^{\mathrm{T}} + \\
& (c\boldsymbol{L} + \boldsymbol{B}))) \otimes \boldsymbol{I}_n)\boldsymbol{e}(t) - \boldsymbol{e}(t)^{\mathrm{T}}(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\boldsymbol{\varepsilon}(t) - \\
& l\boldsymbol{e}(t)^{\mathrm{T}}\boldsymbol{e}(t) \\
\leqslant & -2lV(t) + \|\boldsymbol{e}(t)\| \, \|(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\| \, \|\boldsymbol{\varepsilon}(t)\|.
\end{aligned}$$

If the Lyapunov function $W(t) = \sqrt{V(t)}$, then one has

$$\begin{aligned}
\dot{V}(t) = & 2W(t)\dot{W}(t) \\
\leqslant & -2lW(t)W(t) + \sqrt{2}W(t)\,\|(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\|\,\|\boldsymbol{\varepsilon}(t)\|,
\end{aligned}$$

and

$$\dot{W}(t) \leqslant -lW(t) + \frac{\sqrt{2}}{2}\,\|(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\|\,\|\boldsymbol{\varepsilon}(t)\|. \quad (11)$$

According to the triggering condition (8), one has

$$\|\boldsymbol{\varepsilon}(t)\| \leqslant \sqrt{\sum_{i=1}^{N}\beta_i} \exp(-\frac{\lambda}{2}t),$$

then Eq. (11) is given by

$$\dot{W}(t) \leqslant -lW(t) + \frac{\sqrt{2}}{2}\,\|(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\|\sqrt{\sum_{i=1}^{N}\beta_i} \exp(-\frac{\lambda}{2}t).$$

By Lemma 1, it can be obtained that

$$\begin{aligned}
W(t) \leqslant & (W(0) + \frac{\sqrt{2}\,\|(c\boldsymbol{P} \otimes \boldsymbol{I}_n)\|\sqrt{\sum_{i=1}^{N}\beta_i}}{\left|l - \frac{\lambda}{2}\right|}) \cdot \\
& \exp(-\min\{\frac{\lambda}{2}, l\}t).
\end{aligned}$$

If $\lim_{t\to+\infty} W(t) = 0$, then the error system (10) of the leader-following networks (1) and (2) is stable at the origin, i.e., the leader-following networks (1) and (2) can finally synchronize.

To ensure the feasibility of the triggering mechanism, it is necessary to prove that there is no Zeno phenomenon (event-triggering happen infinitely within a finite time). The theorem about the Zeno phenomenon is given in Theorem 3.2.

Theorem 3.2: According to the control protocol (4) and event-triggering conditions (8), there is no Zeno phenomenon for all followers within a finite time.

Proof: Assume that the ith follower experiences Zeno phenomenon at time $t = T_0$, i.e. $\lim_{k\to+\infty} t_k^i = T_0$. From the properties of the limit, it can be inferred that there must be $N(\varepsilon_0) > 0$ and $t_k^i \in (T_0 - \varepsilon_0, T_0 + \varepsilon_0)$ for any $\varepsilon_0 > 0$ when $k \geqslant N(\varepsilon_0)$. Hence, one has

$$t_{N(\varepsilon_0)+1}^i - t_{N(\varepsilon_0)}^i < 2\varepsilon_0, \quad (12)$$

and

$$\|\boldsymbol{e}_i(t)\| \leqslant \|\boldsymbol{e}(t)\| \leqslant \sqrt{2}W(t) \leqslant \sqrt{2}W(0) + 4\,\|(c\boldsymbol{P}\otimes \boldsymbol{I}_n)\| \sqrt{\sum_{i=1}^{N}\beta_i \Big/ |2l-\lambda|}.$$

For simplicity, one lets

$$M_0 = \sqrt{2}W(0) + 4\,\|(c\boldsymbol{P}\otimes \boldsymbol{I}_n)\| \sqrt{\sum_{i=1}^{N}\beta_i \Big/ |2l-\lambda|}.$$

As $\|\boldsymbol{\varepsilon}_i(t)\|$ is continuous and differentiable when $t \in [t_k^i, t_{k+1}^i)$, and its Dini derivative satisfies

$$\begin{aligned} &D^{+} \parallel \boldsymbol{\varepsilon}_i(t) \parallel \leqslant \parallel \dot{\hat{\boldsymbol{\varepsilon}}}_i \parallel \\ &\leqslant \parallel \boldsymbol{f}(t,\boldsymbol{x}_i(t)) - \boldsymbol{f}(t,\hat{\boldsymbol{x}}_i(t)) + \\ &c\sum_{j=1}^{N} a_{ij}(\boldsymbol{x}_j(t)-\boldsymbol{x}_i(t)) + b_i(\boldsymbol{x}_0(t)-\boldsymbol{x}_i(t)) - \\ &\quad c\sum_{j=1}^{N} a_{ij}(\boldsymbol{x}_j(t)-\hat{\boldsymbol{x}}_j(t)) \parallel \end{aligned} \tag{13}$$

$$\begin{aligned} &\leqslant r_1\|\boldsymbol{\varepsilon}_i(t)\| + \|\boldsymbol{q}_i(t)\| + c \parallel \sum_{j=1}^{N} a_{ij}\boldsymbol{\varepsilon}_j(t) \parallel \\ &\leqslant r_1\beta_i + (c + c\,|N_i| + b_i)\,\|\boldsymbol{e}(t)\| + c(\sum_{j=1}^{N} a_{ij}\beta_j) \\ &\leqslant r_1\beta_i + (c + c\,|N_i| + b_i)M_0 + c(\sum_{j=1}^{N} a_{ij}\beta_j). \end{aligned}$$

If one lets $M = r_1\beta_i + (c+c\,|N_i|+b_i)M_0 + c(\sum_{j=1}^{N} a_{ij}\beta_j)$, based on the triggering condition (8), then it can be obtained at the moment t_k^{i-},

$$\left\|\boldsymbol{\varepsilon}_i(t_k^{i-})\right\| \geqslant \sqrt{\beta_i}\exp(-\frac{\lambda}{2}t_k^{i-}). \tag{14}$$

According to Eqs. (III) and (14), one has

$$t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} \geqslant \frac{1}{M}\sqrt{\beta_i}\exp(-\frac{\lambda}{2}t^i_{N(\varepsilon_0)+1}).$$

If $\varepsilon_0 > 0$ is a solution to the following equation

$$\frac{1}{M}\sqrt{\beta_i}e^{-\frac{\lambda}{2}T_0} = 2\varepsilon_0\exp(\frac{\lambda}{2}\varepsilon_0),$$

then one has

$$t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} \geqslant \frac{1}{M}\sqrt{\beta_i}\exp(-\frac{\lambda}{2}(T_0+\varepsilon_0)) = 2\varepsilon_0,$$

it contradicts the condition (12), so all followers do not exhibit Zeno phenomenon within a finite time.

IV. Numerical Examples

Two examples of gyroscope systems are simulated to verify the effectiveness of the new ETC. Based on Matlab R2020b, the numerical simulation is given by the fourth-order Runge-Kutta method, and the step size is $h = 0.001$.

Example 4.1: The dynamic equation of gyroscope system is given by [25]

$$\begin{cases} \dot{x}_1 = -x_2x_3 - 0.5(1+6.5\cos t)x_2 + 0.4x_3 - \\ \qquad 0.002(x_1 + x_1^3), \\ \dot{x}_2 = x_1x_3 - 0.4x_3 + 0.5(1+6.5\cos t)x_1 - \\ \qquad 0.002(x_2 + x_2^3), \\ \dot{x}_3 = -0.2x_1 + 0.2x_2 - 0.2\,x_3 - \\ \qquad 0.001(x_3 + x_3^2) + 1.625\sin t. \end{cases} \tag{15}$$

A leader-following consensus protocol is designed for a network topology consisting of one leader and five followers, and the communication diagram is given in Fig. 1.

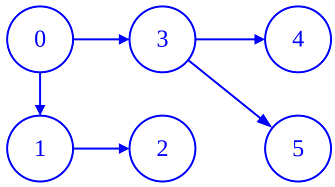

Fig. 1. The communication diagram of leader-following network.

The matrices $\boldsymbol{L}$ and $\boldsymbol{B}$ are given by

$$\boldsymbol{L} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}, \boldsymbol{B} = \mathrm{diag}\{5,0,5,0,0\},$$

the coupling strength $c = 5$, positive scalar $l = 1$. The initial conditions are $x_0(0) = (-0.5, 1, -0.01)^{\mathrm{T}}$ for the leader and $(x_1(0), x_2(0), \cdots, x_5(0))^{\mathrm{T}} = (-0.2, -1,\ 1.5,\ 3, 2.1, -2, 0.5, -0.3, -0.01, -0.6, -0.4, -0.25, -0.2, 0.25, 1)^{\mathrm{T}}$ for the followers.

According to reference [26], $r = 0.2$ can be taken within the attractor range of (15), it is easy to verify that it satisfies Assumption 2.1, and Theorem 3.1 holds if $g_i(t) = \exp(-0.5t)$. According to the control protocol (4), the event-triggering time of the followers is shown in Fig. 2. Based on the control protocol (4), (6) and (7) [22], [23], the comparison of errors $\|e(t)\|$ and the number of event-triggering are shown in Fig. 3 and Table I, respectively.

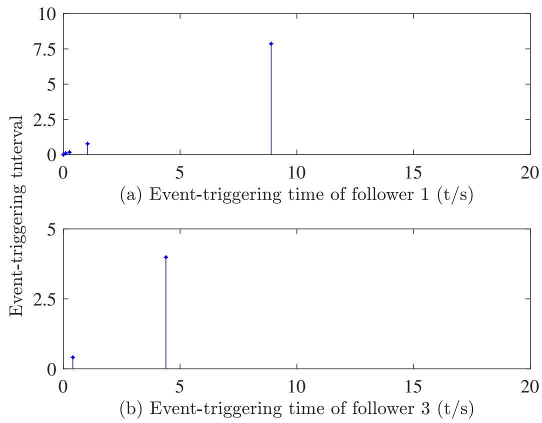

Fig. 2. The event-triggering of followers 1 and 3 by control protocol (4).

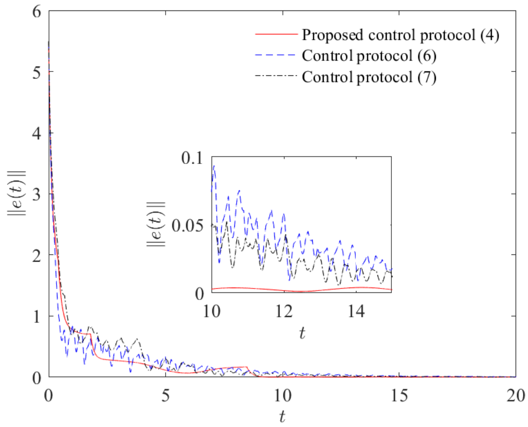

Fig. 3. The errors $\|e(t)\|$ under different control protocols.

In Fig. 3, the errors under the control protocol (4) is basically equivalent to the errors in references [22] and [23], so the control protocol (4) ensures the system control performance. According to Fig. 2 and Table I, only the 1th and 3th followers exist event-triggering control, while the other followers do not use event-triggering under the control protocol (4). Because the 2th, 4th and 5th followers do not need to predict their own states in the control protocol (4), so it can save computational resource to some extent. In addition, the control protocol (4) has a greater advantage in reducing the number of event-triggering compared to other control protocols, and its ability to reduce communication pressure is very significant, which has important application in large-scale networks.

In Table II, the costs of consistency control for leader-following network by different control protocols are given based on Example 4.1. Obviously, the costs of five followers are relatively low compared to other control methods, and the total cost is only 20540.

Note 4.1: In order to show the effects of different ETC methods, the triggering conditions of references [22] and [23] are the same, and the initial conditions are also the same.

Note 4.2: Due to the absence of state information transmission to other followers, the 4th and 5th followers do not use ETC. However, they receive state information updates from the 3th follower through continuous communication. So, the continuous state feedback control may lead to increased control costs, which are influenced by the initial relative positions between agents and the number of active communication links among intelligent agents in the network.

TABLE I
THE NUMBER OF EVENT-TRIGGERING FOR FIVE FOLLOWERS UNDER DIFFERENT CONTROL PROTOCOLS

Follower i	1	2	3	4	5	Total
Proposed control protocol (4)	5	0	2	0	0	7
Control protocol (6)	36	37	51	51	52	227
Control protocol (7)	5	57	5	46	49	163

TABLE II
THE COST OF CONSISTENCY CONTROL FOR LEADER-FOLLOWING NETWORK BY DIFFERENT CONTROL PROTOCOLS

Follower i	1	2	3	4	5	Total
Proposed control protocol (4)	3914	9077	2193	2644	2712	20540
Control protocol (6)	14587	20770	14924	16182	16182	72645
Control protocol (7)	5492	21329	2212	12646	14549	56228

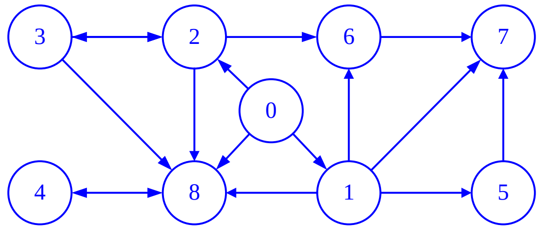

Fig. 4. The communication diagram of leader-following network.

Example 4.2: The gyroscope system in Eq. (15) is still taken as the dynamical system of an isolated node, a leader-following network with one leader and eight followers is given in Fig. 2.

The matrices $\boldsymbol{L}$ and $\boldsymbol{B}$ are given by

$$\boldsymbol{L} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & -1 & 3 & 0 \\ -1 & -1 & -1 & -1 & 0 & 0 & 0 & 4 \end{pmatrix},$$

$$\boldsymbol{B} = \mathrm{diag}\{10, 10, 0, 0, 0, 0, 0, 10\},$$

the coupling strength $c = 5$, and positive scalar $l = 1$. In the leader-following network model, the initial conditions are defined as $x_0(0) = (-0.5, 1, -0.01)^{\mathrm{T}}$, $x(0) = (-0.2, -1, 1.5, 3, 2.1, -2, 0.5, -0.3, -0.01, -0.6, -0.4, -0.25, -0.2, 0.25, 1, -0.5, -1, 2, 1.5, -1.5, 1.1, -2, 1.3, 1)^{\mathrm{T}}$.

Similarly, assuming that $r = 0.2$ and Assumption 2.1 is satisfied, then Theorem 3.1 holds when the triggering condition $g_i(t) = \exp(-0.5t)$. The event-triggering of the followers of the leader-following network under the control protocol (4) is shown in Figs. 5-7. The comparisons of errors $\|e(t)\|$ and the number of event-triggering for three different ETC protocols are shown in Fig. 8 and Table III, respectively.

In Fig. 8, the error $\|e(t)\|$ under the control protocol (4) is not significantly different from the errors in references [22] and [23], so the control protocol (4) has a good control effectiveness. In Fig. 7 and Table III, there is no event-triggering for the 7th follower under the control protocol (4), because the 7th follower does not need to predict its own state, so it reduces the cost of network communication.

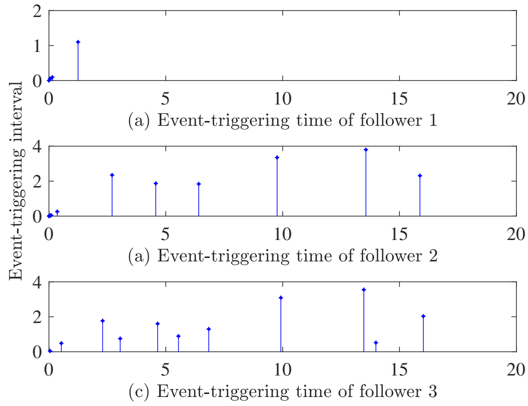

Fig. 5. Event-triggering of followers 1, 2 and 3 by control protocol (4).

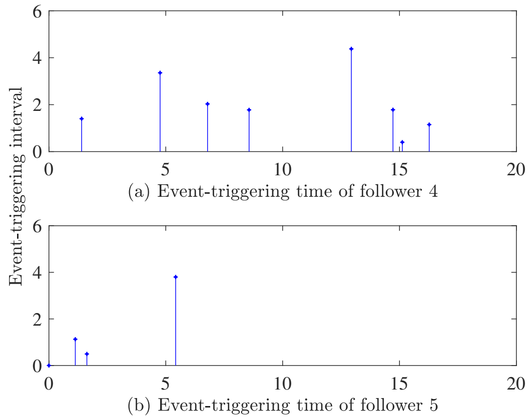

Fig. 6. Event-triggering of followers 4 and 5 by control protocol (4).

TABLE III
THE NUMBER OF EVENT-TRIGGERING FOR EIGHT FOLLOWERS UNDER DIFFERENT CONTROL PROTOCOLS

Follower i	1	2	3	4	5	6	7	8	Total
Proposed control protocol (4)	4	11	11	8	4	8	0	8	54
Control protocol (6)	99	135	42	43	39	105	127	260	850
Control protocol (7)	4	9	43	41	23	80	119	74	393

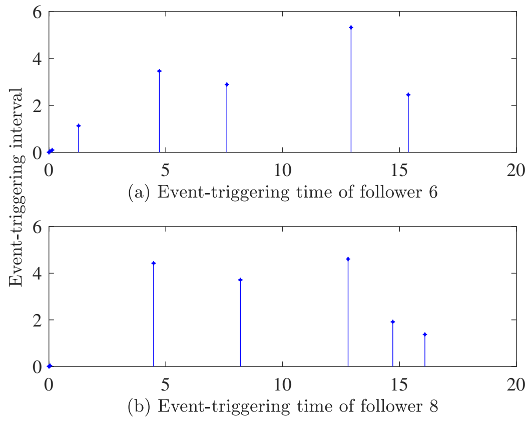

Fig. 7. Event-triggering of followers 6 and 8 by control protocol (4).

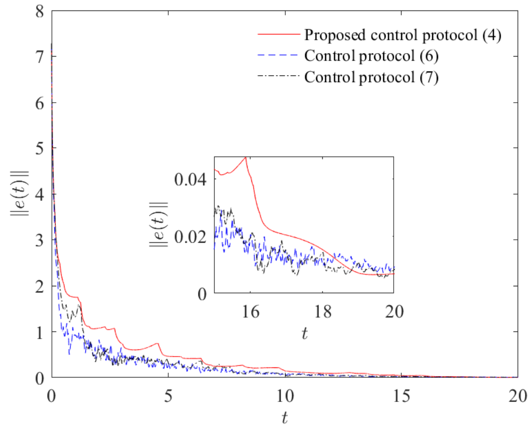

Fig. 8. The errors $\|e(t)\|$ under different control protocols.

TABLE IV
THE COST OF CONSISTENCY CONTROL FOR LEADER-FOLLOWING NETWORK BY DIFFERENT CONTROL PROTOCOLS

Follower i	1	2	3	4	5	6	7	8	Total
Proposed control protocol (4)	4231	8184	5643	3052	3499	6967	6333	6358	44267
Control protocol (6)	31668	42344	17860	18363	18558	37390	39611	70784	276578
Control protocol (7)	4231	14231	17411	14004	11283	28356	37803	35131	152295

In Table IV, the control cost of each follower under the control protocol (4) is lower than the control protocols (6) and (7). Due to the complexity of the topology network in Example 4.2, most followers not only communicate with the leader but also with other followers, which increases the cost of the controllers to some extent.

Note 4.3: The 7th follower operates without ETC, as it does not broadcast its state to other agents in the network. Instead, it employs a simplified feedback controller to maintain the consistency by the state information received from other followers, and the corresponding control cost is given in Table IV.

V. CONCLUSIONS

A new ETC protocol is proposed for leader-following network, and it significantly reduces the cost of communication among multi-agent systems and the resource consumption of network synchronization. A sufficient matrix-form criterion is established to ensure leader-following network synchronization, and there is no Zeno behavior under the proposed ETC protocol. The synchronization of the leader-following networks with 5 and 8 followers is shown by the examples of gyroscope systems. Compared to existing protocols that prioritize tracking error minimization and triggering frequency reduction, the proposed control protocol eliminates continuous state monitoring of neighboring agents while simultaneously reducing event-triggering frequencies and network communication costs. Moreover, the proposed ETC protocol lowers computational costs without compromising consensus in leader-following networks and extends applicability to consensus control problems.

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