# The Dynamics and Applications of an Optimal Iterative Method

Mani Sandeep Kumar Mylapalli, *Member, IAENG*, Navya Kakarlapudi, Virath Singh, Chaganti Pragathi, Perumali Sundarayya

***Abstract*— In this paper, we develop a sixteenth-order iterative scheme to compute the zero of the nonlinear equation in four steps, using five functional evaluations, and achieve optimality with an efficiency index of 1.741. We also discuss the theoretical convergence analysis of the approach and compare the proposed technique with recent methods of equal order regarding successive errors, number of iterations, and functional evaluations by taking several algebraical, and transcendental test functions. The developed method is put into practice in applications in the fields of medical sciences, physics, and chemical engineering. Various approaches are analyzed using basins of attraction to show taking polynomials as test functions in a complex plane.**

***Index Terms*— Nonlinear Equation, Optimal order, Efficiency Index, Order of Convergence, Iterative Method, Functional Evaluations, Basins of Attraction.**

## I. Introduction

SOLVING nonlinear equations are among the most critical challenges in scientific and technical applications. Nonlinear equations can be used to solve several optimization challenges in various applications. Computing their roots in a finite number of arithmetic operations is generally tricky. A subfield of computer science and mathematics known as numerical analysis creates evaluates and applies algorithms to resolve numerical issues. This work is about iterative approaches for finding a simple root $x$, i.e., $h(x)=0$. The most well-known and frequently applied methodology for resolving nonlinear equations is Newton Rahson's method (NR) [2] and its efficiency index is $\sqrt{2}=1.414$. It is given by,

$$x_{n+1}=x_n-\frac{h(x_n)}{h'(x_n)}, \qquad n=0,1,2,\ldots \tag{1}$$

This employs a one-step iterative procedure. Newton's method has a quadratic order of convergence to get a simple zero.

Manuscript received July 23, 2024, revised April 4, 2025.

Mani Sandeep Kumar Mylapalli is an Assistant Professor of Mathematics, GITAM School of Science, GITAM (Deemed to be University), Visakhapatnam 530040, INDIA (corresponding author phone: +91 6303106197; e-mail: mmylapal@gitam.edu).

Navya Kakarlapudi is an Assistant Professor of Mathematics, Centurion University of Technology and Management, Vizianagaram 535003, INDIA (e-mail: kakarlapudinavya@gmail.com).

Virath Sewnath Singh is a Senior Lecturer of Mathematics, University of KwaZulu Natal, Durban 4000, South Africa (e-mail: singhv@ukzn.ac.za).

Pragathi Chaganti is an Associate Professor of Mathematics, Gitam Institute of Science, GITAM (Deemed to be University), Visakhapatnam 530040, INDIA (e-mail: pchagant@gitam.edu).

Perumali Sundarayya is an Associate Professor of Mathematics, Gitam Institute of Science, GITAM (Deemed to be University), Visakhapatnam 530040, INDIA (e-mail: sperumal@gitam.edu).

This work aims to build an effective derivative-free technique for solving nonlinear equations. We developed a new optimal iterative approach to bolster the hypothesis. Based on $N$ functional assessments, Kung-Traub hypothesized that multipoint iteration techniques may attain an optimal convergence order $2N-1$.

This study uses the weight function methodology to build a sixteenth-order iterative method. To improve the iterative method presented, we consider the finite difference approximation and use the approximants of the higher derivatives to avoid computing the high-order derivatives of the function. To evaluate the performance of the new strategy against the current equal-order methods considering a few test functions and real-world application problems. We also studied the dynamic performance of our developed method. The basins of attraction are also shown and contrasted with the other methods of the same order at the study's conclusion.

Some of the existing methods developed recently for solving nonlinear equations are given below:

In the year 2020, an optimal sixteenth-order iterative method (DM) is presented by Dejan Cebic [1] with five functional evaluations and is given as

$$\begin{aligned}
y_n &= x_n-\frac{h(x_n)}{h'(x_n)}\\
z_n &= y_n-\left(3-\frac{2h[y_n,x_n]}{h'(x_n)}\right)\frac{h(y_n)}{h'(x_n)}\\
w_n &= z_n-\frac{h(z_n)}{h'(x_n)}\frac{h'(x_n)-h[y_n,x_n]+h[z_n,y_n]}{2h[z_n,y_n]-h[z_n,x_n]}\\
x_{n+1} &= w_n-\frac{h(w_n)\left(2h[z_n,x_n]-2h[w_n,x_n]+h[w_n,z_n]\right)}{h'(x_n)\left(h[w_n,y_n]-h[z_n,y_n]\right)+h^2[z_n,x_n]-h^2[w_n,x_n]+h^2[w_n,z_n]}
\end{aligned} \tag{2}$$

In the year 2017, A new sixteenth-order optimal iterative method (RM) developed by Rafiullah et al. [4] with six functional evaluations is as follows:

$$\begin{aligned}
y_n &= x_n-\frac{h(x_n)}{h'(x_n)}\\
z_n &= y_n-\frac{h(y_n)}{h'(y_n)}-\frac{\left(h(y_n)\right)^2\left(h'(x_n)-h'(y_n)\right)}{2\left(h(x_n)-h(y_n)\right)\left(h'(x_n)\right)^2}\\
w_n &= z_n-\frac{h(z_n)\left((x_n-y_n)(x_n-z_n)(y_n-z_n)\right)}{\left(-h(z_n)(x_n-y_n)(x_n-2z_n+y_n)+h(y_n)(x_n-z_n)^2-h(x_n)(y_n-z_n)^2\right)}\\
x_{n+1} &= w_n-\frac{h(w_n)}{h'(w_n)}
\end{aligned} \tag{3}$$

where

$$h'(w_n)=\frac{h(w_n)}{w_n-x_n}+\frac{h(w_n)}{w_n-y_n}+\frac{h(w_n)}{w_n-z_n}+\frac{h(x_n)(w_n-y_n)(w_n-z_n)}{(x_n-w_n)(x_n-y_n)(x_n-z_n)}+\frac{h(y_n)(w_n-x_n)(w_n-z_n)}{(w_n-y_n)(x_n-y_n)(y_n-z_n)}+\frac{h(z_n)(x_n-w_n)(y_n-w_n)}{(z_n-w_n)(z_n-x_n)(z_n-y_n)}$$

In the year 2019, Young Hee Geum et al. [9] proposed a new method (YM) with optimal sixteenth-order convergence using five functional evolutions as follows:

$$\begin{aligned} y_n &= x_n-\frac{h(x_n)}{h'(x_n)}\\ z_n &= y_n-\frac{h(y_n)}{h'(x_n)}G(s),\ s=\frac{h(y_n)}{h(x_n)}\\ w_n &= z_n-\frac{h(z_n)}{h'(x_n)}H(s,u),\ u=\frac{h(z_n)}{h(y_n)}\\ x_{n+1} &= w_n-\frac{h(w_n)}{h'(x_n)}I(s,u,v),\ v=\frac{h(w_n)}{h(z_n)} \end{aligned} \tag{4}$$

where

$$G(s)=\frac{1}{1-2s},\ H(s,u)=G(s)\frac{(s-1)^2}{1-2s-u+2s^2u}$$

$$I(s,u,v)=H(s,u)\left(\frac{1-s-s^2-2s^3+u(s^2-s-1)+2su^2}{1-s-s^2-2s^3-u(1+s+s^3+s^4)-v(1-s-s^2-2s^3)}\right)$$

In the year 2018, a four-step optimal sixteenth-order iterative method (JM) was presented by Janakraj Sharma et al. [3]

$$\begin{aligned} y_n &= x_n-\frac{h(x_n)}{h'(x_n)}\\ z_n &= y_n-\frac{h'(x_n)h(y_n)}{(h[y_n,x_n])^2}\\ w_n &= z_n-\frac{1}{u_1-u_2+u_3}\frac{h'(x_n)h(z_n)}{(h[z_n,x_n])^2}\\ x_{n+1} &= w_n-\frac{1}{v_1-v_2+v_3-v_4}\frac{h'(x_n)h(w_n)}{(h[w_n,x_n])^2} \end{aligned} \tag{5}$$

where

$$u_1=(y_n-z_n)/(y_n-x_n)$$

$$u_2=h'(x_n)(z_n-x_n)^2/(y_n-z_n)(y_n-x_n)h[y_n,x_n]$$

$$u_3=h'(x_n)(z_n-x_n)/(y_n-z_n)(h[z_n,x_n])$$

$$v_1=(y_n-w_n)(z_n-w_n)/(y_n-x_n)(z_n-x_n)$$

$$v_2=h'(x_n)(w_n-x_n)^2(w_n-z_n)/(y_n-x_n)(y_n-z_n)(y_n-w_n)h[y_n,x_n]$$

$$v_3=h'(x_n)(w_n-x_n)^2(y_n-w_n)/(z_n-x_n)(z_n-y_n)(z_n-w_n)(h[z_n,x_n])$$

$$v_4=h'(x_n)(w_n-x_n)(2w_n-y_n-z_n)/(y_n-w_n)(z_n-w_n)(h[w_n,x_n])$$

The article's remaining section is organized as follows: Section II discusses the approach's development. An analysis of the proposed scheme's convergence is presented in Section III. Section IV assesses the suggested approach on several test functions, and the outcomes are compared with those of other existing same-order approaches in the Numerical Examples section. A few notions from chemical engineering, medical science, and physics have also been tested in this way. Through complex dynamics, Section V examines the stability of the established techniques. The rational function is reviewed using these methodologies on various nonlinear complex polynomial functions, and their basins of attraction are illustrated. The study conclusions are finally covered in Section VI.

## II. Development of Method

This section covers the study's primary contribution. A novel iterative algorithm of an optimal sixteenth order based on a finite interpolation approach will be provided.

Consider the optimal eighth-order convergent method [6]

$$\begin{aligned} y_n &= x_n-\frac{h(x_n)}{h'(x_n)}\\ z_n &= y_n-H(\tau)\left[\frac{2h(y_n)}{h'(y_n)}.\frac{1}{1+\sqrt{1-2\rho_n}}\right]\\ w_n &= z_n-\frac{h(z_n)}{h'(z_n)} \end{aligned} \tag{6}$$

where

$$\rho_n=\frac{h'(x_n)-h'(y_n)}{h'(x_n)},\ h'(y_n)=2h[y_n,x_n]-h'(x_n),$$

$$H(\tau)=1-\tau\left[\because \tau=\frac{h(y_n)}{h(x_n)}\right]$$

and

$$h'(z_n)=h'(x_n)+(h[x_n,y_n,z_n]+h[x_n,x_n,y_n])(z_n-x_n)+2(h[x_n,y_n,z_n]+h[x_n,x_n,y_n])(z_n-y_n)$$

Consider Newton's method in the fourth step of (6) to get the optimality and better efficiency. Thus, we have

$$x_{n+1}=w_n-\frac{h(w_n)}{h'(w_n)} \tag{7}$$

For reducing functional values, we consider the interpolation approximation of $h'(w_n)$ as follows:

$$h'(w_n)=\begin{cases} h[w_n,z_n]+(w_n-z_n)h[w_n,z_n,y_n]+\\ (w_n-z_n)(w_n-y_n)h[w_n,z_n,y_n,x_n] \end{cases} \tag{8}$$

Thus, we developed the new four-step algorithm, as shown below.

**Algorithm:** The iterative scheme is

1. $y_n = x_n-\frac{h(x_n)}{h'(x_n)}$

2. $z_n = y_n-H(\tau)\left[\frac{2h(y_n)}{h'(y_n)}.\frac{1}{1+\sqrt{1-2\rho_n}}\right]$

where, $\rho_n=\frac{h'(x_n)-h'(y_n)}{h'(x_n)}, h'(y_n)=2h[y_n,x_n]-h'(x_n),$

$H(\tau)=1-\tau$ and $\tau=\frac{h(y_n)}{h(x_n)}$

3. $w_n = z_n - \frac{h(z_n)}{h'(z_n)}$

where, $h'(z_n) = h'(x_n) + (h[x_n, y_n, z_n] + h[x_n, x_n, y_n])(z_n - x_n)$
$+ 2(h[x_n, y_n, z_n] + h[x_n, x_n, y_n])(z_n - y_n)$

4. $x_{n+1} = w_n - \frac{h(w_n)}{h'(w_n)}$ (9)

where, $h'(w_n) = h[w_n, z_n] + (w_n - z_n) h[w_n, z_n, y_n]$
$+ (w_n - z_n)(w_n - y_n) h[w_n, z_n, y_n, x_n]$

Consequently, the sixteenth-order technique (9) with five functional assessments is the best one.

## III. Convergence Criteria

*Theorem [5, 6]:* For an open interval $I$, let $x_0 \in I$ be the simple root of a suitably differentiable function. If $x_0$ is the neighborhood of $x^*$. Then, the algorithm (9) has an optimal sixteenth-order convergence with an error equation,

$$\varepsilon_{n+1} = \left(c_2^7 c_3^4 + c_2^5 c_3^2 c_4^2 - 2c_2^6 c_3^3 c_4\right) e^{16} + O\left(e^{17}\right).$$

*Proof:* Let the simple root of $h(x) = 0$ be $x^*$ and $x^* = x_n + \varepsilon_n$. Thus,

$$h(x^*) = 0$$

Using Taylor's series expansion, expand $h(x_n)$ about $x^*$, we obtain

$$h(x_n) = h'(x^*)\left(\varepsilon_n + c_2 \varepsilon_n^2 + c_3 \varepsilon_n^3 + c_4 \varepsilon_n^4 + ...\right) \quad (10)$$

$$h'(x_n) = h'(x^*)\left(1 + 2c_2 \varepsilon_n + 3c_3 \varepsilon_n^2 + 4c_4 \varepsilon_n^3 + ...\right) \quad (11)$$

Dividing (10) by (11), we get

$$\frac{h(x_n)}{h'(x_n)} = \varepsilon_n - c_2 \varepsilon_n^2 - \left(2c_3 - 2c_2^2\right)\varepsilon_n^3 - \left(3c_4 - 7c_2 c_3 + 4c_2^3\right)\varepsilon_n^4 + \dots \quad (12)$$

Replacing (12) in the first step of (9), we get

$$y_n = x^* + Y \quad (13)$$

where

$$Y = c_2 \varepsilon_n^2 + \left(2c_3 - 2c_2^2\right)\varepsilon_n^3 + \left(3c_4 - 7c_2 c_3 + 4c_2^3\right)\varepsilon_n^4 + ..$$

Again expanding $h(y_n)$ about $x^*$ through the Taylor series, we obtain

$$h(y_n) = h'(x^*)\left(c_2 \varepsilon_n^2 + \left(2c_3 - 2c_2^2\right)\varepsilon_n^3 + \left(3c_4 - 7c_2 c_3 + 5c_2^3\right)\varepsilon_n^4 + \dots\right) \quad (14)$$

$$h'(y_n) = h'(x^*)\left(1 + \left(2c_2^2 - c_3\right)\varepsilon_n^2 + \left(6c_2 c_3 - 4c_2^3 - 2c_4\right)\varepsilon_n^3 + ...\right) \quad (15)$$

Dividing (14) by (15), we get

$$\frac{h(y_n)}{h'(y_n)} = c_2 \varepsilon_n^2 + \left(2c_3 - 2c_2^2\right)\varepsilon_n^3 + \left(3c_2^3 - 6c_2 c_3 + 3c_4\right)\varepsilon_n^4 + \dots \quad (16)$$

From (11) and (15), we obtain

$$\rho_n = 2c_2 \varepsilon_n + \left(4c_3 - 6c_2^2\right)\varepsilon_n^2 + \left(6c_4 + 16c_2^3 - 20c_2 c_3\right)\varepsilon_n^3 + ... \quad (17)$$

and
$$H(\tau) = 1 - \frac{h(y_n)}{h(x_n)} = \begin{pmatrix} 1 - c_2 \varepsilon_n - \left(2c_3 - 3c_2^2\right)\varepsilon_n^2 - \\ \left(3c_4 - 10c_2 c_3 + 8c_2^3\right)\varepsilon_2^3 + ... \end{pmatrix} \quad (18)$$

From the second step of (9), we get

$$z_n = x^* + Z \quad (19)$$

where

$$Z = \left(-c_2 c_3\right)\varepsilon_n^4 + \left(c_2 c_4 - c_3^2 + c_2^4\right)\varepsilon_n^5 +$$
$$\left(c_2 c_5 + 6c_2^2 c_4 + 4c_2 c_3^2 + 5c_2^3 c_3 - c_2^5 - c_3 c_4 - 13c_2 c_3 c_4\right)\varepsilon_n^6 + ...$$

Expanding $h(z_n)$ about $x^*$ through the Taylor expansion as follows:

$$h(z_n) = h'(x^*)\left(Z + c_2 Z^2 + c_3 Z^3 + ...\right) \quad (20)$$

and

$$h'(z_n) = h'(x^*)\begin{pmatrix} 1 + 2c_2 Z + 3c_3 Z^2 - c_4 Z \varepsilon_n^2 + \\ 4c_4 Y^2 Z + c_4 Y \varepsilon_n^2 + 2c_4 Z Y \varepsilon_n + ... \end{pmatrix} \quad (21)$$

Substituting (19), (20), and (21) are in the third step of (9), we get

$$w_n = x^* + W \quad (22)$$

where $W = \left(c_2^3 c_3^2 - c_2^2 c_3 c_4\right)\varepsilon_n^8 + o\left(\varepsilon_n^9\right)$

Expanding $h(w_n)$ about $x^*$ by using the Taylor series, we get

$$h(w_n) = h'(x^*)\left(W + c_2 W^2 + c_3 W^3 + ...\right) \quad (23)$$

On simplification

$$h[w_n, z_n] = \frac{h(w_n) - h(z_n)}{w_n - z_n} = 1 + c_2(W + Z) +$$
$$c_3\left(W^2 + WZ + Z^2\right) + c_4\left(W^3 + W^2 Z + WZ^2 + Z^3\right) + ... \quad (24)$$

$$h[z_n, y_n] = \frac{h(z_n) - h(y_n)}{z_n - y_n} = 1 + c_2(Y + Z)$$
$$+ c_3\left(Y^2 + YZ + Z^2\right) + c_4\left(Y^3 + Y^2 Z + YZ^2 + Z^3\right) + ... \quad (25)$$

$$h[y_n, x_n] = \frac{h(y_n) - h(x_n)}{y_n - x_n} = 1 + c_2(Y + \varepsilon_n)$$
$$+ c_3\left(Y^2 + Y\varepsilon_n + \varepsilon_n^2\right) + c_4\left(Y^3 + Y^2 \varepsilon_n + Y\varepsilon_n^2 + \varepsilon_n^3\right) + ... \quad (26)$$

From (24) and (25)

$$h[w_n, z_n, y_n] = \frac{h[w_n, z_n] - h[z_n, y_n]}{w_n - y_n}$$
$$= c_2 + c_3(W + Z + Y) + c_4\begin{pmatrix} W^2 + Z^2 + Y^2 \\ + WZ + ZY + YW \end{pmatrix} + ... \quad (27)$$

and $(w_n - z_n) h[w_n, z_n, y_n] = \begin{pmatrix} c_2 W - c_2 Z - c_3 YZ \\ -c_3 Z^2 - c_4 Y^2 Z + ... \end{pmatrix}$ (28)

Similarly, from (25) and (26)

$$h[z_n, y_n, x_n] = \frac{h[z_n, y_n] - h[y_n, x_n]}{z_n - x_n}$$
$$= c_2 + c_3(Z + Y + \varepsilon_n) + c_4\begin{pmatrix} Z^2 + Y^2 + \varepsilon_n^2 + \\ Z\varepsilon_n + Y\varepsilon_n + ZY \end{pmatrix} + ... \quad (29)$$

Finding the dividing difference of (27) and (29), we get

$$h[w_n, z_n, y_n, x_n] = \frac{h[w_n, z_n, y_n] - h[z_n, y_n, x_n]}{w_n - x_n} \quad (30)$$
$$= c_3 + c_4\varepsilon_n + c_4 Y + c_4 Z + c_4 W + ...$$

and

$$(w_n - z_n)(w_n - y_n) h[w_n, z_n, y_n, x_n] = \begin{pmatrix} c_3 YZ + c_4 YZ\varepsilon_n \\ +c_4 Y^2 Z + ... \end{pmatrix} \quad (31)$$

Substituting the above terms in $h'(w_n)$ of (9), we get

$$h'(w_n) = h'(x^*)(1 + 2c_2 W + c_4 YZ\varepsilon_n + ...) \quad (32)$$

Substituting (22), (23), and (32), in the fourth step of (9)

$$\varepsilon_{n+1} = \left(c_2^7 c_3^4 + c_2^5 c_3^2 c_4^2 - 2c_2^6 c_3^3 c_4\right) e^{16} + O\left(e^{17}\right)$$

Therefore, the proposed algorithm's convergence order is sixteen and denoted with (KM). Hence, the efficiency index is $E.I = 16^{1/5} = 1.7411$.

## IV. Numerical Computations

This section is entirely devoted to evaluating the applicability and reliability of the suggested optimal sixteenth-order iterative method. For this reason, we take into four standard test problems and six real-world application-oriented problems from the fields of physics, chemical engineering, and medicine, such as the depth of embedment, vertical stress, the volume of van der Waals, stirred tank reactor, blood rheology, the law of blood flow problem. To compare our suggested approach (KM) to existing iterative methods, namely, DM, RM, YM, and SM, regarding several iterations, associated subsequent errors, the number of functional assessments, and computational time. All computations are conducted using mp math-PYTHON with the halting condition $|f(x_n)| < \varepsilon$, where $\varepsilon = 10^{-199}$ tolerance and 690 decimal place accuracy. Table I shows an analogy of different algorithms; Table II shows the roots of the test functions; and Table III shows all (including test functions and application problems) of the numerical results and it includes starting estimates $x_0$, the number of iterations ($n$), successive error values of each iteration, the number of function evaluations $|h(x_{n+1})|$ and the computational time.

Table I
COMPARISON OF EFFICIENCY INTEX

| Methods | P | N | E. I |
|---|---|---|---|
| DM | 16 | 5 | 1.7411 |
| RM | 16 | 6 | 1.5874 |
| YM | 16 | 5 | 1.7411 |
| SM | 16 | 5 | 1.7411 |
| **KM** | 16 | 5 | 1.7411 |

Where P is the order of convergence, N is the number of functional evaluations per iteration and E.I is the efficiency- index.

Table II
ROOTS OF THE TEST FUNCTIONS

| Test Function | Root |
|---|---|
| $h_1(x) = \sin x - x^2 + 1$ | 1.4096240040 |
| $h_2(x) = sin(2\cos x) - 1 - x^2 + e^{\sin x^3}$ | −0.7848959876 |
| $h_3(x) = \sin x + \cos x + x$ | −0.4566247045 |
| $h_4(x) = e^{\sin x} - x + 1$ | 2.6306641479 |

*Some real-life applications:*

In this section, we give some practical application problems from different fields, such as Physics, Chemical Engineering and Medicine, etc. and the results are discussed in Table III [ $h_5(x) - h_{10}(x)$ ].

*Application 1. ((Depth of Embedment Model, [6,7])*

The embedment depth of a sheet-pile wall is determined using the following nonlinear equation:

$$h_5(x) = \frac{1}{4.62}\left(x^3 + 2.87x^2 - 10.28\right) - x$$

The approximated root is 2.00211877895382.

*Application 2. (The vertical stress, [7])*

One of the primary stresses that finite surface structures experience is vertical stress, which is represented by

$$h_6(x) = \frac{x + CosxSinx}{\pi} - \frac{1}{4}$$

The root of $h_6(x) = 0$ is 0.4160444988100767043.

*Application 3. (Volume from van der Waals equation, [7])*

Van der Waals' equation of a non-ideal gas is given by

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

where $n$ is the number of moles, $V$ is the volume of the gas, $T$ is the temperature in Kelvin, $p$ is the pressure, and $R$ is the gas constant, which is equal to 0.0820578 L-atm/mol-K. It is given by

$$h(V) = pV^3 - n(RT + bp)V^2 + n^2 aV - n^3 ab$$

Put $V = x$, by giving particular values to the parameters, the above equation is the nonlinear polynomial function.

$$h_7(x) = 40x^3 - 95.26535116x^2 + 35.28x - 5.6998368$$

The root of the equation is $x^* \approx 1.9707842194070294$.

*Application 4. (Stirred Tank Reactor, [6])*

Consider a stirred-tank reactor. The reactor receives materials at rates of $\beta$ and $q - \beta$, respectively. The equipment improves mixed reactions, as shown below:

$$H_1 + H_2 \to H_3; H_3 + H_2 \to H_4;$$
$$H_4 + H_2 \to H_5; H_4 + H_2 \to H_6.$$

During their preliminary analysis of this intricate control system, Douglas found the nonlinear polynomial equation:

$$\frac{2.98\times(x+2.25)}{(x+1.45)\times(x+2.85)^2\times(x+4.35)}=\frac{1}{G_C}$$

By taking $G_C=0$, we have

$$h_8(x)=x^4+11.50x^3+47.49x^2+83.06325x-51.23266875=0$$

The real root of the equation is -1.45.

*Application 5. (Blood rheology model, [6, 8])*

In medicine, the study of blood flow and structure is referred to as blood rheology. We take into consideration the following nonlinear equation for analyzing the plug flow of a Caisson fluid flow:

$$h_9(x)=\frac{1}{441}x^8-\frac{8}{63}x^5-0.0571428571x^4+\frac{16}{9}x^2-3.624489796x+0.3$$

Where $x$ is the plug flow of Caisson fluid, and the approximated root is 0.0864335580522467.

*Application 6. (Law of Blood Flow, [8])*

This legislation was proposed in 1840 by French physician Jean Poiseuille. Where $\nu$ is the blood viscosity, $R$ is the radius, $l$ is the length, $P$ is the pressure and $h$ is a function of $x$ with the domain $[0,R]$, blood flows via the vein or artery. This law is stated as the nonlinear model shown below by

$$h_{10}(x)=\frac{P}{\nu l}\left(R^2-x^2\right)$$

Where, $P=4000$, $R=0.008$, $\nu=0.027$, and $l=2$ are taken for the simulations.

TABLE III
COMPARISON OF EFFICIENCY

| Method | $n$ | $\lvert x_1-x_0\rvert$ | $\lvert x_2-x_1\rvert$ | $\lvert x_3-x_2\rvert$ | $\lvert x_4-x_3\rvert$ | $\lvert h(x_{n+1})\rvert$ | Comp. Time |
|---|---|---|---|---|---|---|---|
| h1(x) | $x_0$ | 1.1 | | | | | |
| DM | 4 | 0.309623 | 2.51E-07 | 2.80E-107 | 5.19E-690 | 2.10E-689 | 0.030047 |
| RM | 4 | 0.309624 | 3.06E-10 | 1.01E-157 | 5.68E-315 | 1.51E-314 | 0.030144 |
| YM | 4 | 0.309624 | 7.72E-11 | 2.41E-166 | 1.36E-690 | 5.74E-690 | 0.030021 |
| SM | | Divergent | | | | | |
| **KM** | 3 | 0.309624 | 1.09E-16 | 1.34E-248 | --- | 3.58E-248 | 0.022759 |
| h1(x) | $x_0$ | 1.7 | | | | | |
| DM | 4 | 0.290375 | 5.10E-12 | 2.29E-182 | 4.18E-689 | 5.74E-690 | 0.010013 |
| RM | 3 | 0.290375 | 1.28E-15 | 9.19E-244 | --- | 2.44E-243 | 0.008378 |
| YM | 3 | 0.290375 | 2.30E-14 | 9.58E-223 | --- | 2.54E-222 | 0.008496 |
| SM | | Divergent | | | --- | | |
| **KM** | 3 | 0.290376 | 3.58E-19 | 6.65E-286 | | 1.76E-285 | 0.008129 |
| h2(x) | $x_0$ | -0.2 | | | | | |
| DM | 29 | 30501082 | 1525054 | 7625267 | 3812636 | 1.09E-690 | 0.057922 |
| RM | 27 | 7.166868 | 46.81753 | 2322198 | 1.84E+10 | 8.13E-487 | 0.056854 |
| YM | 5 | 1.301102 | 0.828709 | 0.112502 | 8.93E-16 | 7.51E-242 | 0.028127 |
| SM | | Divergent | | | | | |
| **KM** | 4 | 0.757895 | 0.003354 | 4.32E-36 | 4.54E-559 | 7.13E-558 | 0.027746 |
| h2(x) | $x_0$ | -0.6 | | | | | |
| DM | 4 | 0.184895 | 1.22E-08 | 2.66E-126 | 1.36E-691 | 1.09E-690 | 0.028556 |
| RM | 4 | 0.184895 | 1.61E-11 | 2.19E-174 | 3.35E-348 | 9.44E-348 | 0.028963 |
| YM | 4 | 0.184895 | 1.24E-11 | 5.16E-176 | 1.36E-691 | 1.09E-690 | 0.028543 |
| SM | | Divergent | | | | | |
| **KM** | 3 | 0.184896 | 2.34E-15 | 2.38E-213 | --- | 7.53E-212 | 0.024888 |
| h3(x) | $x_0$ | -2.9 | | | | | |
| DM | 6 | 5.268071 | 1.587129 | 1.662635 | 0.425068 | 4.70E-280 | 0.049482 |
| RM | 9 | 7.209406 | 12.39890 | 1.251174 | 6.842315 | 4.22E-280 | 0.055217 |
| YM | 6 | 13.01157 | 11.52001 | 26.97380 | 0.001152 | 1.09E-690 | 0.052076 |
| SM | 5 | 4.274281 | 1.837365 | 0.006459 | 1.77E-45 | 1.09E-690 | 0.047585 |
| **KM** | 4 | 2.348731 | 0.094644 | 2.61E-27 | 1.42E-410 | 3.32E-410 | 0.041489 |
| h3(x) | $x_0$ | -5 | | | | | |
| DM | 6 | 6.776853 | 7.009183 | 4.609346 | 0.166358 | 6.76E-361 | 0.024375 |
| RM | 5 | 11.67100 | 6.890387 | 0.237241 | 4.87E-22 | 8.34E-353 | 0.020124 |
| YM | 5 | 2.941615 | 1.601759 | 1.03E-06 | 4.73E-107 | 1.09E-690 | 0.020057 |
| SM | | Divergent | | | | | |
| **KM** | 4 | 4.544188 | 0.000812 | 3.51E-58 | 6.84E-691 | 1.09E-690 | 0.017536 |
| h4(x) | $x_0$ | 1.4 | | | | | |
| DM | | Divergent | | | | | |
| RM | 5 | 7.409405 | 8.791936 | 0.151867 | 7.45E-22 | 1.54E-347 | 0.050035 |
| YM | 4 | 0.859028 | 0.371636 | 2.36E-15 | 2.30E-242 | 5.59E-242 | 0.045499 |
| SM | 4 | 1.344396 | 0.113732 | 4.44E-25 | 1.05E-399 | 2.57E-399 | 0.045476 |
| **KM** | 4 | 1.220339 | 0.010325 | 2.22E-38 | 2.06E-573 | 5.95E-573 | 0.045234 |
| h4(x) | $x_0$ | 3.1 | | | | | |
| DM | 4 | 0.469335 | 1.52E-11 | 1.68E-180 | 6.56E-690 | 4.65E-690 | 0.021657 |
| RM | 3 | 0.469335 | 8.93E-13 | 1.14E-202 | --- | 2.77E-202 | 0.019365 |
| YM | 3 | 0.469335 | 4.86E-13 | 2.28E-203 | --- | 5.52E-203 | 0.019356 |
| SM | | Divergent | | | --- | | |
| **KM** | 3 | 0.469336 | 1.34E-16 | 1.31E-273 | --- | 3.32E-273 | 0.019169 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| h5(x) | $x_0$ | 1.9 | | | | | |
| DM | 3 | 0.102118 | 2.75E-18 | 6.53E-284 | --- | 2.67E-283 | 0.011001 |
| RM | 3 | 0.102118 | 4.13E-21 | 1.06E-331 | --- | 4.34E-331 | 0.010568 |
| YM | 3 | 0.102118 | 1.13E-21 | 2.52E-341 | --- | 1.03E-340 | 0.010276 |
| SM | 3 | 0.102118 | 6.60E-20 | 2.87E-311 | --- | 1.17E-310 | 0.010679 |
| **KM** | 3 | 0.102118 | 1.09E-23 | 1.34E-348 | --- | 4.48E-348 | 0.010191 |
| h5(x) | $x_0$ | 2.2 | | | | | |
| DM | 3 | 0.197881 | 4.45E-15 | 1.44E-232 | --- | 5.90E-232 | 0.022114 |
| RM | 3 | 0.197881 | 1.59E-17 | 2.41E-274 | --- | 9.88E-274 | 0.022003 |
| YM | 3 | 0.197881 | 5.01E-18 | 5.75E-283 | --- | 2.35E-282 | 0.021988 |
| SM | 3 | 0.197881 | 3.01E-16 | 1.01E-252 | --- | 4.13E-252 | 0.022048 |
| **KM** | 3 | 0.290376 | 3.58E-19 | 6.65E-286 | --- | 1.76E-285 | 0.021927 |
| h6(x) | $x_0$ | 1.5 | | | | | |
| DM | 5 | 0.751666 | 0.332288 | 8.41E-07 | 6.60E-100 | 1.36E-691 | 0.057561 |
| RM | 7 | 0.500948 | 1.978125 | 13.03318 | 15.423815 | 1.93E-242 | 0.061101 |
| YM | 7 | 20.29210 | 176.6148 | 196.4308 | 0.6070711 | 1.36E-691 | 0.061017 |
| SM | 6 | 22.44969 | 22.21975 | 0.732831 | 0.1211757 | 2.23E-267 | 0.059076 |
| **KM** | 4 | 0.761422 | 0.117886 | 6.18E-16 | 8.92E-271 | 1.75E-271 | 0.055171 |
| h6(x) | $x_0$ | -0.8 | | | | | |
| DM | | Divergent | | | | | |
| RM | 6 | 1.428674 | 7.349666 | 4.674464 | 7.29E-452 | 4.10E-452 | 0.042107 |
| YM | 4 | 1.194142 | 0.021901 | 1.91E-31 | 3.13E-496 | 1.66E-496 | 0.042008 |
| SM | 4 | 1.195349 | 0.020695 | 1.82E-30 | 3.31E-479 | 1.77E-479 | 0.042032 |
| **KM** | 4 | 1.219948 | 0.003903 | 9.16E-42 | 3.15E-621 | 1.67E-621 | 0.041789 |
| h7(x) | $x_0$ | 2.2 | | | | | |
| DM | 4 | 0.229216 | 1.08E-09 | 1.23E-83 | 2.22E-306 | 2.78E-304 | 0.020877 |
| RM | 4 | 0.229216 | 4.32E-12 | 4.80E-122 | 6.91E-262 | 8.70E-260 | 0.020986 |
| YM | 5 | 0.229215 | 3.28E-12 | 3.45E-90 | 1.16E-260 | 1.45E-258 | 0.021154 |
| SM | 5 | 0.229216 | 1.54E-10 | 8.28E-86 | 7.08E-238 | 8.91E-236 | 0.021306 |
| **KM** | 4 | 0.229216 | 1.33E-14 | 1.85E-124 | 3.54E-314 | 4.46E-312 | 0.020310 |
| h7(x) | $x_0$ | 1.8 | | | | | |
| DM | 4 | 0.170784 | 2.72E-07 | 2.24E-71 | 1.33E-269 | 1.67E-267 | 0.017932 |
| RM | 4 | 0.170784 | 1.64E-10 | 4.3E-115 | 4.59E-334 | 5.78E-332 | 0.017854 |
| YM | 5 | 0.170784 | 2.99E-11 | 2.11E-88 | 7.10E-259 | 8.93E-257 | 0.020126 |
| SM | 5 | 0.170784 | 1.12E-09 | 6.49E-83 | 5.55E-235 | 6.98E-233 | 0.020145 |
| **KM** | 4 | 0.170784 | 1.29E-12 | 1.75E-120 | 3.19E-376 | 4.02E-374 | 0.017746 |
| h8(x) | $x_0$ | -1.4 | | | | | |
| DM | 3 | 0.05 | 6.47E-16 | 2.44E-237 | --- | 1.38E-236 | 0.016034 |
| RM | 3 | 0.049999 | 3.82E-18 | 2.09E-275 | --- | 1.18E-274 | 0.015983 |
| YM | 3 | 0.05 | 4.45E-19 | 2.26E-291 | --- | 1.28E-290 | 0.015907 |
| SM | 3 | 0.049999 | 3.28E-17 | 1.31E-259 | --- | 7.47E-259 | 0.016005 |
| **KM** | 3 | 0.05 | 9.85E-21 | 2.20E-300 | --- | 1.26E-299 | 0.015856 |
| h8(x) | $x_0$ | -1.6 | | | | | |
| DM | 4 | 0.14 | 0.000286 | 5.40E-51 | 1.72E-689 | 2.62E-689 | 0.021710 |
| RM | 4 | Divergent | | | | | |
| YM | 4 | 0.15 | 6.56E-09 | 1.11E-128 | 1.91E-690 | 2.62E-689 | 0.021603 |
| SM | 4 | 0.15 | 2.25E-07 | 3.00E-102 | 5.47E-691 | 2.62E-689 | 0.021587 |
| **KM** | 4 | 0.15 | 1.07E-09 | 7.63E-135 | 7.38E-691 | 2.62E-689 | 0.021516 |
| h9(x) | $x_0$ | -0.6 | | | | | |
| DM | 4 | 0.686434 | 2.48E-08 | 1.02E-123 | 3.42E-692 | 1.36E-691 | 0.026226 |
| RM | 4 | 0.686433 | 2.75E-10 | 1.70E-158 | 3.42E-692 | 1.36E-691 | 0.026216 |
| YM | 4 | 0.686433 | 1.56E-09 | 1.24E-145 | 3.55E-692 | 2.05E-691 | 0.026218 |
| SM | | Divergent | | | | | |
| **KM** | 4 | 0.686434 | 1.71E-12 | 2.58E-185 | 4.30E-692 | 1.42E-691 | 0.022628 |
| h9(x) | $x_0$ | 0.2 | | | | | |
| DM | 3 | 0.113566 | 2.65E-20 | 9.30E-319 | --- | 3.08E-318 | 0.020502 |
| RM | 3 | 0.113566 | 2.05E-19 | 9.41E-304 | --- | 3.12E-303 | 0.020526 |
| YM | 3 | 0.113566 | 5.33E-18 | 1.01E-279 | --- | 3.34E-279 | 0.020614 |
| SM | 3 | 0.113566 | 5.33E-18 | 1.01E-279 | --- | 3.34E-279 | 0.020613 |
| **KM** | 3 | 0.113566 | 3.02E-21 | 1.33E-322 | --- | 4.41E-322 | 0.020439 |

| Method | $n$ | $\lvert x_1 - x_0\rvert$ | $\lvert x_2 - x_1\rvert$ | $\lvert x_3 - x_2\rvert$ | $\lvert x_4 - x_3\rvert$ | $\lvert x_5 - x_4\rvert$ | $\lvert x_6 - x_5\rvert$ | $\lvert h(x_{n+1})\rvert$ | Comp. Time |
|---|---|---|---|---|---|---|---|---|---|
| h10(x) | $x_0$ | 1 | | | | | | | |
| DM | 7 | 0.929072 | 0.065856 | 0.004256 | 1.49E-5 | 2.54E-33 | 2.32E-65 | 2.75E-499 | 0.020004 |
| RM | 6 | 0.940101 | 0.056252 | 0.002845 | 9.11E-7 | 3.18E-66 | 7.05E-102 | 8.35E-329 | 0.016987 |
| YM | 7 | 0.944482 | 0.052058 | 0.002312 | 6.46E-8 | 4.52E-62 | 1.05E-101 | 1.24E-231 | 0.020026 |
| SM | 8 | 0.923933 | 0.070243 | 0.004979 | 4.25E-5 | 2.78E-27 | 6.27E-52 | 7.43E-255 | 0.024250 |
| **KM** | 6 | 0.945671 | 0.030842 | 0.006663 | 7.91E-16 | 1.45E-130 | 2.43E-524 | 4.76E-522 | 0.016902 |
| h10(x) | $x_0$ | 10 | | | | | | | |
| DM | 8 | 9.290750 | 0.658942 | 0.046682 | 0.002824 | 1.50E-06 | 1.61E-24 | 1.61E-623 | 0.025088 |
| RM | 7 | 9.401041 | 0.563078 | 0.033633 | 0.001447 | 5.71E-09 | 1.68E-32 | 1.99E-405 | 0.023592 |
| YM | 8 | 9.448323 | 0.521234 | 0.028638 | 0.001003 | 7.62E-11 | 4.94E-52 | 5.86E-248 | 0.025123 |
| SM | 9 | 9.239366 | 0.702774 | 0.053411 | 0.003636 | 1.30E-05 | 5.42E-15 | 6.42E-263 | 0.027650 |
| **KM** | 6 | 9.442156 | 0.572255 | 0.005868 | 3.01E-05 | 1.28E-40 | 1.72E-218 | 2.12E-218 | 0.021044 |

Where $x_0$ represents the starting approximation, $n$ represents the number of iterations, $\lvert x_{n+1} - x_n\rvert, n = 0,1,2,...$ represents error and $\lvert h(x_{n+1})\rvert$ represents a number of functional evaluations.

The residual error graphs presented below illustrate a comparative analysis between the proposed algorithm and several well-established methods, namely, the DM, RM, YM, and SM methods for solving nonlinear equations. Each graph visually captures the convergence behavior of the iterative methods by plotting the residual error (i.e., the absolute difference between the computed and actual root) against the number of iterations. A steeper decline in the residual error curve indicates faster convergence and higher accuracy of the method.

The proposed algorithm consistently demonstrates superior performance, with more rapid error reduction and fewer iterations required to reach a specified tolerance level. This enhanced convergence behavior confirms the algorithm's efficiency, robustness, and reliability in both controlled test conditions and complex practical problems. By providing a side-by-side comparison through these residual error graphs, the analysis not only highlights the improvements introduced by the new method but also validates its effectiveness over traditional iterative techniques.

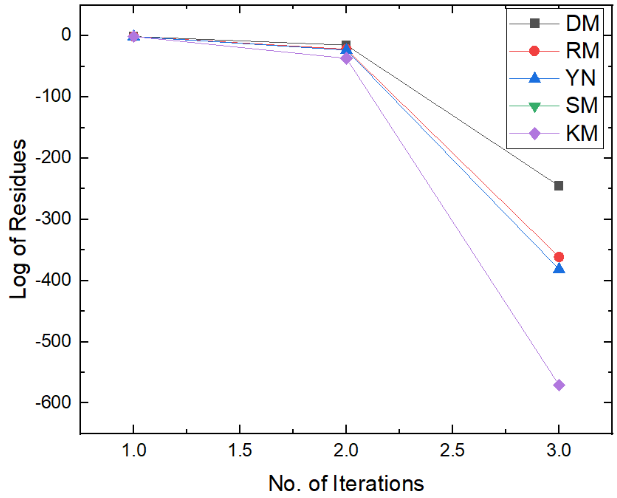


Fig. 1. $h_1(x)$ at $x_0$=1.1

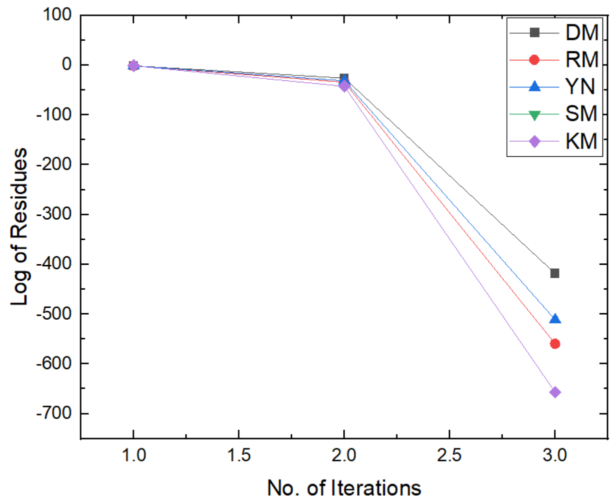


Fig. 2. $h_1(x)$ at $x_0$=1.7

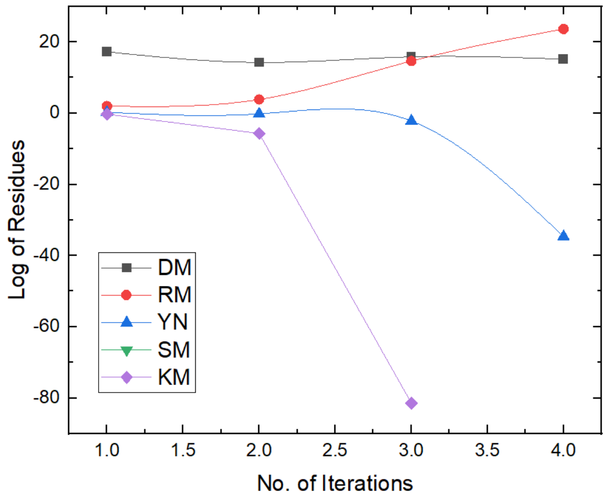


Fig. 3. $h_2(x)$ at $x_0$=1.1

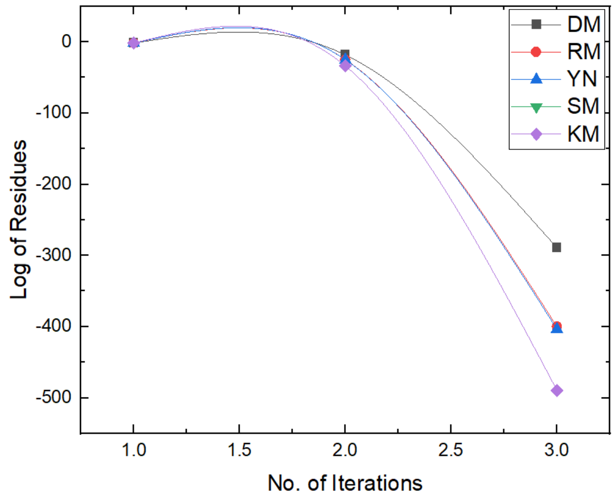


Fig. 4. $h_2(x)$ at $x_0$=-0.6

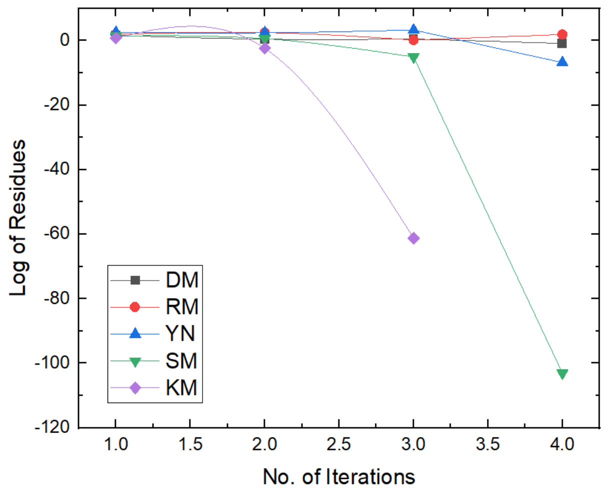


Fig. 5. $h_3(x)$ at $x_0$=-2.9

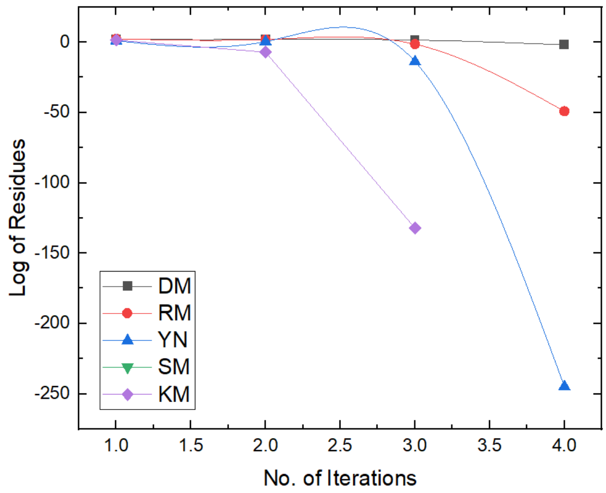



Fig. 6. $h_3(x)$ at $x_0$=-5

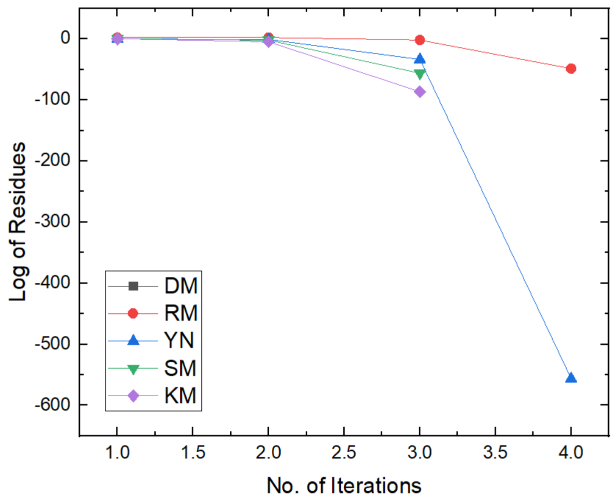



Fig. 7. $h_4(x)$ at $x_0$=1.4

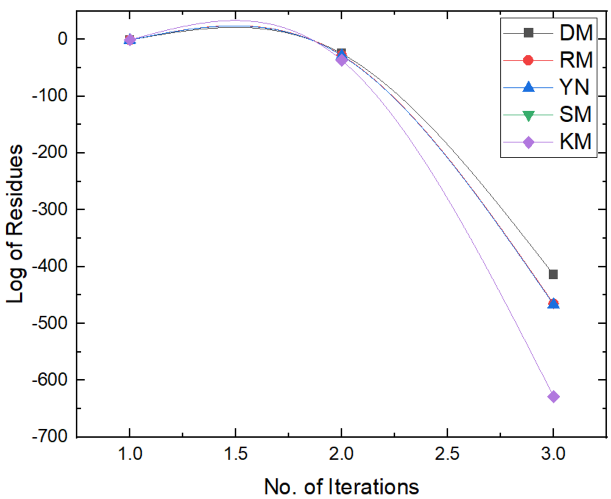



Fig. 8. $h_4(x)$ at $x_0$=3.1

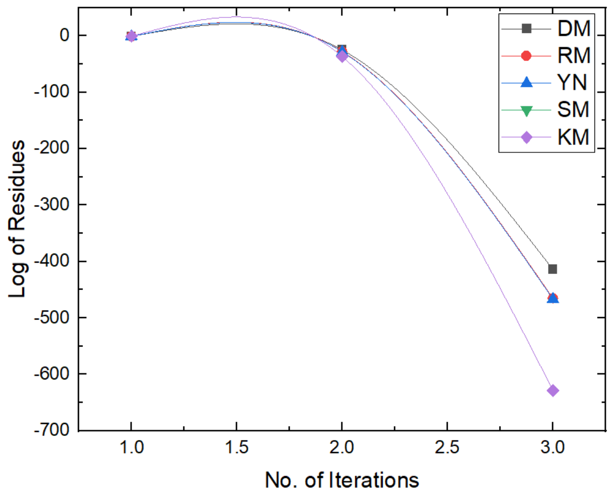



Fig. 9. $h_5(x)$ at $x_0$=1.9

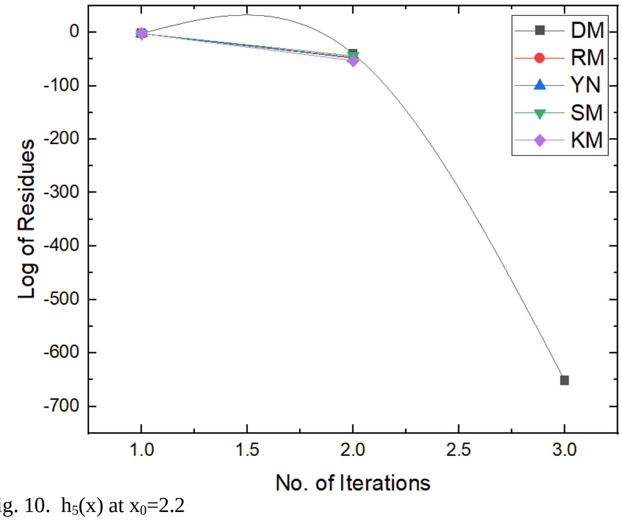



Fig. 10. $h_5(x)$ at $x_0$=2.2

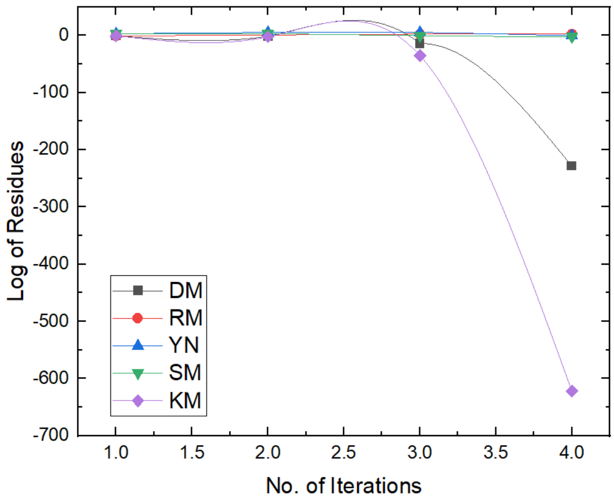



Fig. 11. $h_6(x)$ at $x_0$=1.5

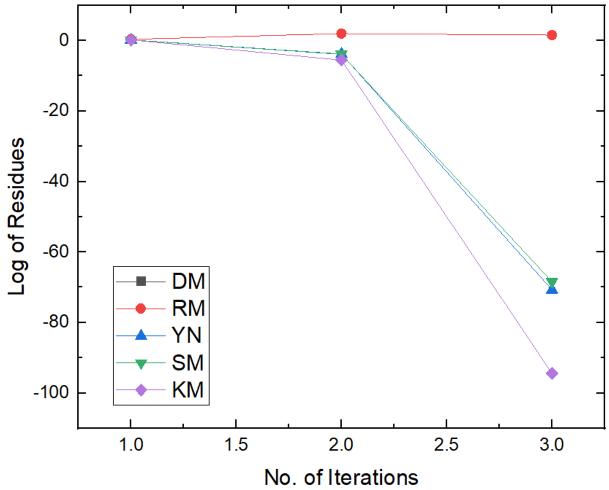



Fig. 12. $h_6(x)$ at $x_0$=-0.8

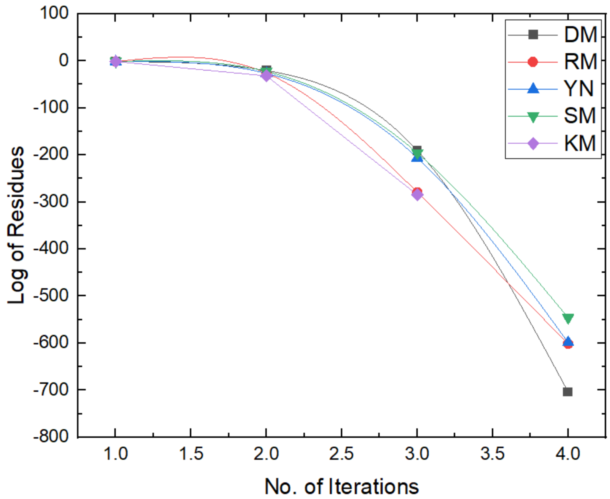



Fig. 13. $h_7(x)$ at $x_0$=2.2

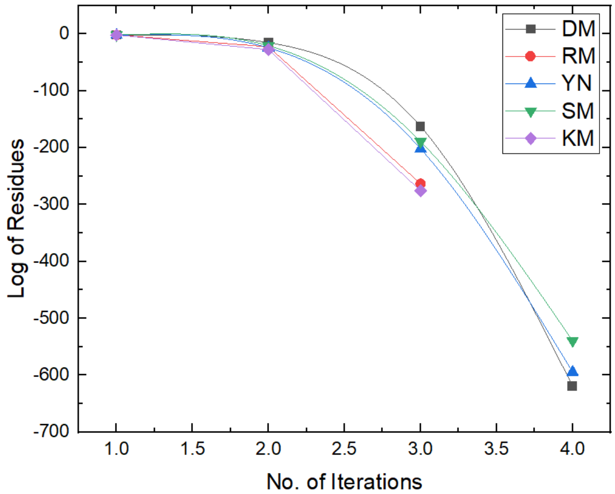



Fig. 14. $h_7(x)$ at $x_0$=1.8

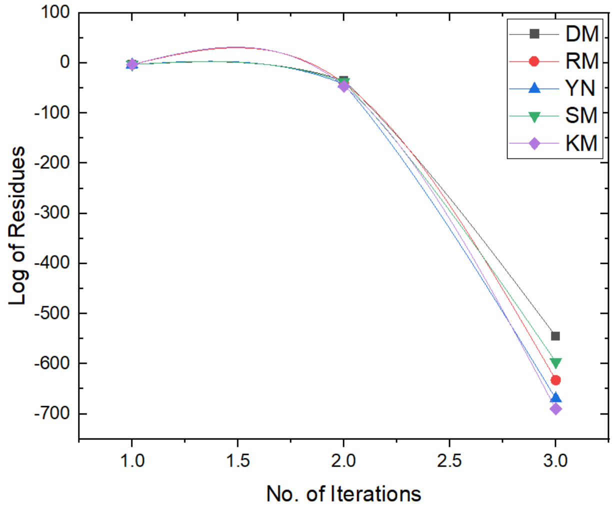



Fig. 15. $h_8(x)$ at $x_0$=-1.6

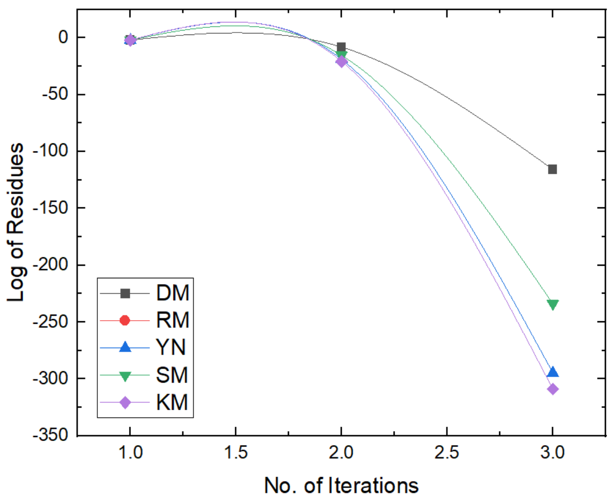



Fig. 16. $h_8(x)$ at $x_0$=-1.4

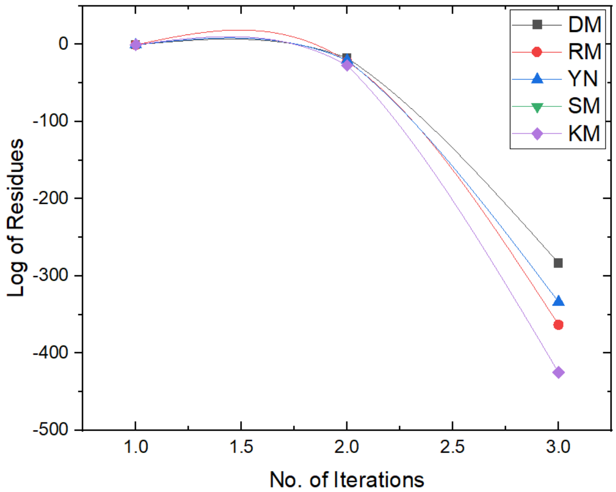



Fig. 17. $h_9(x)$ at $x_0$=-0.6

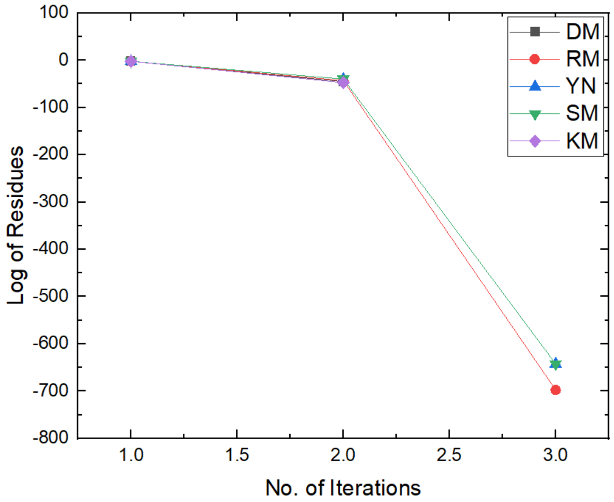



Fig. 18. $h_9(x)$ at $x_0$=0.2

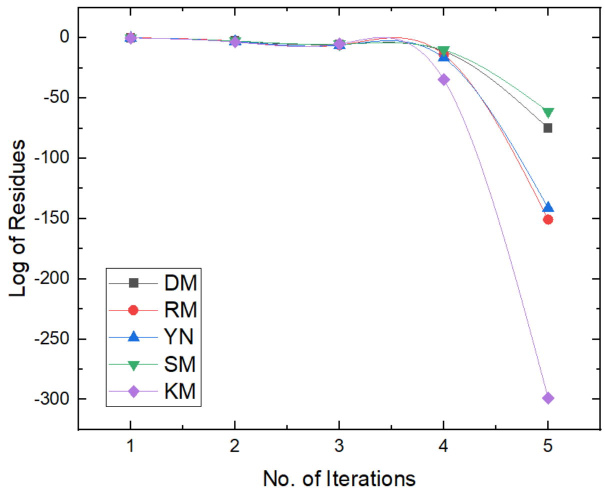



Fig. 19. $h_{10}(x)$ at $x_0$=1

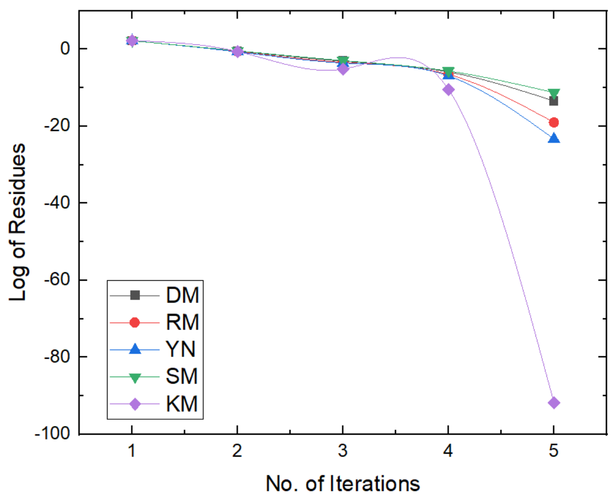



Fig. 20. $h_{10}(x)$ at $x_0$=10

Using Origin Pro software for graphical comparisons, "Fig. 1" through "Fig. 20" display the graphical behaviour of the compared iterative methods DM, RM, YM, SM and KM. We coloured these methods black, red, blue, green, and violet, respectively.

The residual fall graph clearly illustrates the superior performance of the suggested KM in terms of convergence speed and efficiency when compared to other well-established methods, including those by DM, RM, YM, and SM. This enhanced performance is highlighted by the rapid reduction in the residual values, indicating the method's ability to approximate the root with minimal computational effort.

In particular, KM demonstrates a remarkable ability to achieve stability in fewer iterations, showcasing its convergence precision. While the other methods exhibit a gradual or slower decline in residuals over multiple iterations, KM outperforms them by exhibiting a steep descent, signaling a more effective approach to solving the nonlinear system. This behavior reflects the method's robust algorithmic design, which minimizes computational cost without compromising accuracy.

Graphical comparisons further substantiate the dominance of KM. The curves for DM, RM, YM, and SM reflect their slower progression toward the root, often requiring additional iterations to achieve comparable levels of precision. In contrast, KM's curve stabilizes significantly earlier, affirming its efficiency and suitability for practical engineering and computational applications.

## V. Basins Of Attraction

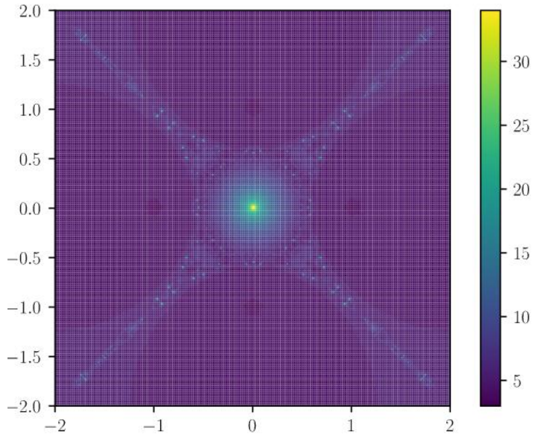
According to the study on basins of attraction covered below, the new method is better than the comparable methods in some crucial areas. Combined with an iterative method acting on a polynomial, this rational function trait provides critical information about the technique's numerical aspects, ensuring its stability and reliability. This is another approach to compare iterative processes without taking initial approximations. To derive the basins of attraction of the root in fractal graphs, assume a square $R\times R=[-2,2]\times[-2,2]$ in which we take $250\times 250$ initial points containing all the roots $\left(z_i^*=1,2,3,...\right)$ of the relevant complex polynomial and use the KM technique starting at each initial point $Z_0$ in the square. We determine that $Z_0$ is in the basins of attraction of the root $Z*j$ of the polynomial if the sequence produced by the iterative technique converges to it after a maximum of 100 iterations and a tolerance of $\left|f\left(z^{(j)}\right)\right|<10^{-16}$. Consider $Z_0$ is given a dark violet colour if $\left|z^{(N)}-z_i^*\right|<10^{-16}$ and the iterative process begins there and reaches a root after $N$ iterations ($N\le 100$). Should $N$ exceed 100, we deduce that the origin has diverged and designate it with the colour yellow. The basins of attraction for the KM and sixteenth-order methods—DM, RM, YM, and SM—are as follows.

Consider the following complex polynomial functions

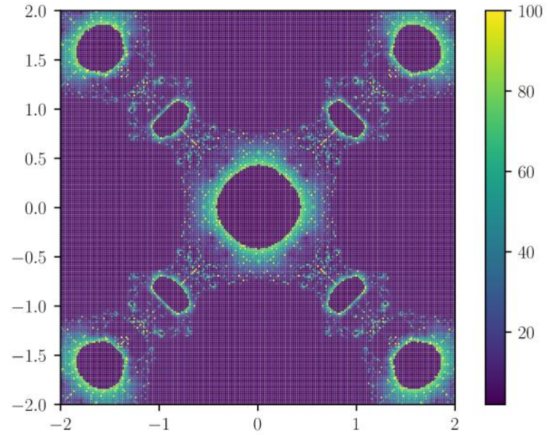
1. $f_1(z)=1-z^4$
2. $f_2(z)=z^{11}-1$

The developed algorithm KM and the comparison methods have the following basins.

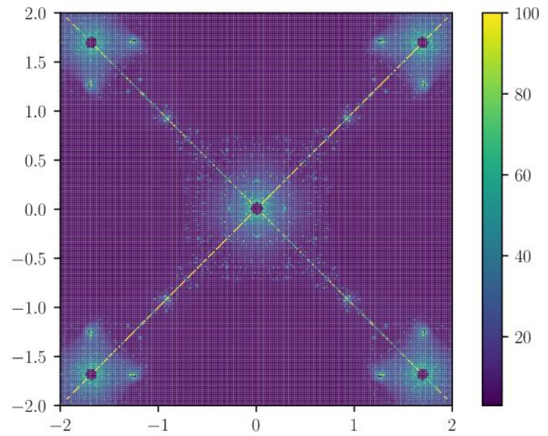
*Example 1.* $f_1(z) = 1 - z^4$



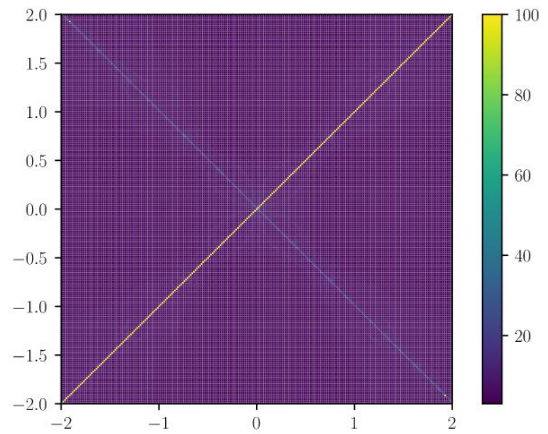

(a) KM



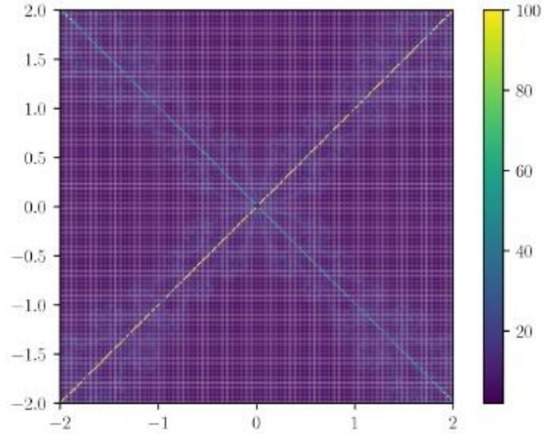

(b) DM




(c) RM



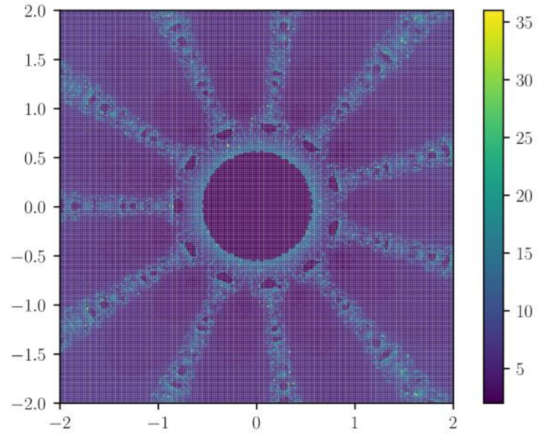

(d) YM



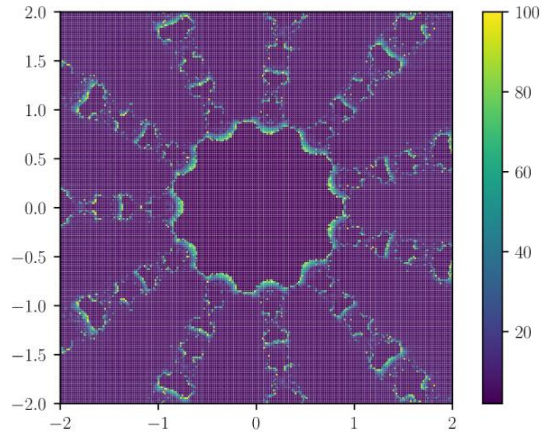

(e) SM

Fig. 1. The polynomiographs for the suggested methods for $f_1(z)$.

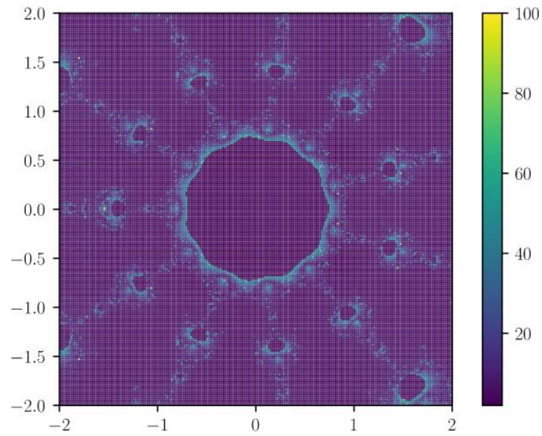
*Example 2.* $f_2(z) = z^{11} - 1$



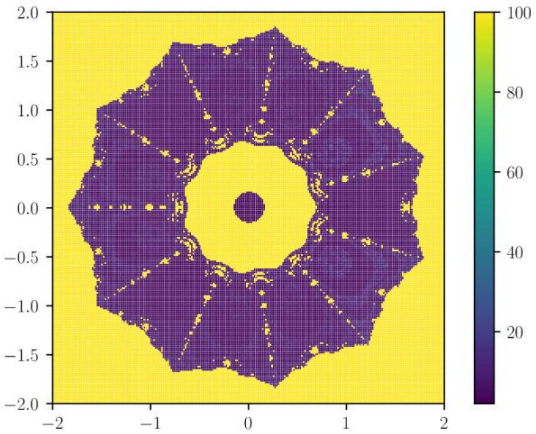

(a) KM



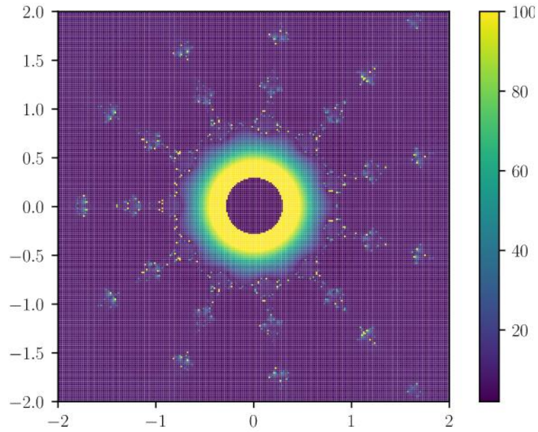

(b) DM




(c) RM




(d) YM




(e) SM

Fig. 2. The polynomiographs for the suggested methods KM, DM, RM, YM and SM for $f_2(z)$.

In Fig. 1, the proposed method KM takes five to twenty iterations for strong convergence, twenty to twenty-five for moderate convergence, and more than twenty-five iterations for weak convergent or divergent to another root. The other methods, DM, RM, YM, and SM, show chaotic behavior.

In Fig. 2, the proposed method KM takes five to twenty iterations for strong convergence, twenty to thirty for moderate convergence, and thirty to thirty-five for weak convergence or divergence to another root. The other methods, DM, RM, YM, and SM, show chaotic behavior.

## VI. Conclusions

In this research, we introduced the sixteenth-order iterative method and created a novel optimal four-step. Based on the convergence study, the suggested strategy has a convergence order of sixteen. There is a computational efficiency index of $16^{1/5} = 1.7411$. The unique approaches outperform the comparative methods regarding results in a few test and application challenges across several areas. Based on the facts gathered, our proposed solutions are superior to the existing techniques and significantly more effective. To explore their areas of interest, we have also looked at the complex field of cyclical techniques. The numerical results of the proposed techniques and related fractal graphs demonstrate that the unique approaches are a valuable alternative to solving the scalar nonlinear equation. The proposed method is compatible with other existing approaches of the same order. The suggested scheme is the most effective approach for each example. Table 3 makes it abundantly evident that, when considering the number of iterations, successive errors, and computational time, the created KM scheme outperforms the other four methods: DM, RM, YM, and SM.

The rapid convergence and reduced iteration count of KM not only highlights its theoretical advantages but also its practical utility in real-world scenarios where time and resource efficiency are critical. This analysis underscores the effectiveness of KM in providing quicker, more reliable solutions compared to the existing methodologies, establishing it as a highly competitive and superior option for solving nonlinear equations.

## Acknowledgment

The authors are highly thankful to GITAM (Deemed to be University) for providing the resources and would like to thank Centurion University (AP Campus) for their continuous support.

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