Stability Analysis for a Lattice Hydrodynamic Model Considering Driver's Characteristic and On-ramp under V2X Environment

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Abstract—**In this paper, a novel lattice hydrodynamic mode is introduced by considering drivers characteristic and on-ramp under V2X environment. Meanwhile, the self-delayed effect in the process of flux transmission is also a key consideration in the model. By conducting linear and nonlinear analysis on the proposed model, the stability condition and modified Korteweg-de Vries (mKdV) equation are derived. The impact of self-delayed effect in the process of flux transmission, aggressive and conservative driving characteristics, and on-ramp on traffic flow evolution are further analyzed. The results confirm the theoretical analysis and reveal the fact that self-delayed effect in the process of flux transmission can improve traffic stability, while aggressive and conservative driving behavior and traffic flow entering from the on-ramp are not conducive to traffic stability and cause traffic congestion.**

Index Terms—**Lattice hydrodynamic model; On-ramp; Driver's characteristics; Stability analysis.**

I. INTRODUCTION

THE increase in the number of vehicles will inevitably cause some traffic problems. In view of these, the governance of traffic congestion will also become a long-term concern for people and the research of traffic modeling has also become an effective way to solve these problems. Meanwhile, it is particularly important to analyze the dynamic behavior of the system[1]. Lattice hydrodynamic model[2-3] emerge as the times require which combined micro and macro features. Subsequently, lattice model attract widespread attention and some improved models[4-12] have been proposed.

As is known to all, the driver's behavior will inevitably cause changes in the traffic flow state and corresponding research becomes particularly important. For instance, driver's anticipation effect[13-17], individual difference[18-19], driver's desire of driving smoothly[20], driver's memory effect[21-25], and so on. Meanwhile, individual personality traits will also generate different driving characteristics in actual traffic which consist of aggressive, timid and robust driving. Sharma[26] considered timid and aggressive driving behavior in a lattice model. Since then, combining specific traffic models, the changes in traffic flow patterns caused by driver characteristics have also been studied by scholars in [27-29]. However, the above models ignored the self-delayed effect in the perception process of flux information under V2X environment and the changes in road scenes.

In actual traffic, on-ramp effectively handle the traffic volume of urban roads as a common traffic scenario on highways. Tian et al.[30] considered the on-ramp and explore the dynamic congested traffic states. Later, Scholars further analyzed the impact of on-ramp on traffic flow by combining specific lattice models[32-34]. In addition, with the development of artificial intelligence, big data, and vehicle networking technology, information exchange between vehicles and network clouds is possible. Drivers can obtain traffic information via vehicle-to-X(V2X) communication. Jia et al.[35-37] explore the cooperative driving model by considering V2V and V2I communication. Recently, based on V2X environment, Peng et al.[38-39] introduce collaborative information transmission in lattice models. However, the impact of V2X communication on traffic stability lacking of large-scale research. So, a novel lattice hydrodynamic model considering driver's characteristic and on-ramp under V2X environment is put forward in this paper, the stability is the focus of discussion above the proposed model.

II. A NOVEL LATTICE HYDRODYNAMIC MODEL

In actual traffic, on-ramp is commonly found on highway, the schematic of highway with on-ramp is shown in Fig.1.

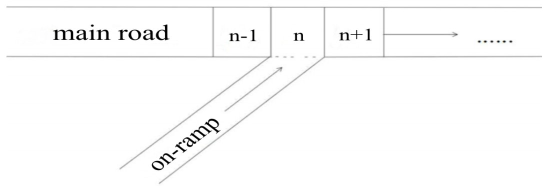

Fig.1. The schematic of highway with on-ramp.

In Fig.1, the density of lattice $n-1$ of the main road is smaller than that of lattice n with the traffic flow of the on-ramp injects into the main road at lattice n. Consequently, the incoming flow at lattice n defined as $\mu|\rho_0^2 V'(\rho_0)|(\rho_n - \rho_{n-1})$. On the basis of the discussion in introduction, a novel lattice hydrodynamic model considering driver's characteristic and on-ramp under V2X environment as follows:

$$\begin{aligned}
&\partial_t(\rho_n(t)v_n(t)) \\
=&\alpha\rho_0(\gamma V(\rho_{n+1}(t+\beta\tau_1)) + (1-\gamma)V(\rho_{n+1}(t-\beta\tau_1))) \\
&-\alpha q_n(t) + \lambda\alpha(\rho_{n+1}(t)v_{n+1}(t) \\
&-\rho_{n+1}(t-\tau_2)v_{n+1}(t-\tau_2))
\end{aligned} \tag{1}$$

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$$\begin{aligned}&\partial_t\rho_n(t)+\rho_0(\rho_n(t)v_n(t)-\rho_{n-1}(t)v_{n-1}(t))\\&=\mu|\rho_0^2V^{'}(\rho_0)|(\rho_n(t)-\rho_{n-1}(t)),\end{aligned}\tag{2}$$

where, $\lambda\alpha(q_{n+1}(t)-q_{n+1}(t-\tau_2))$ represents the self-delayed effect of flux and λ shows the reaction coefficient, μ denotes the rate of on-ramp at lattice n, $V(\rho)$ is the optimal velocity function:

$$V(\rho)=\frac{v_{max}}{2}(tanh(\frac{2}{\rho_0}-\frac{\rho}{\rho_0^2}-\frac{1}{\rho_c})+tanh(\frac{1}{\rho_c})),\tag{3}$$

where, ρ_0 is the initial density, ρ_c represents the safety density and v_{max} is the maximal velocity. Combinating Eqs.(1) and (2), we obtain the following equation:

$$\begin{aligned}&\partial_t^2\rho_n(t)-\partial_t\mu|\rho_0^2V^{'}(\rho_0)|(\rho_n(t)-\rho_{n-1}(t))\\&+\alpha\rho_0^2[V(\rho_{n+1}(t))-V(\rho_n(t))]+\alpha\rho_0^2(2\gamma-1)(\beta\tau_1)\\&\times[V'(\rho_{n+1}(t))\partial_t\rho_n(t)]+\alpha[\partial_t\rho_n(t)\\&-\lambda\partial_t\rho_{n+1}(t)+\lambda\partial_t\rho_{n+1}(t-\tau_2)]\\&-\alpha\gamma|\rho_0^2V^{'}(\rho_0)|(\rho_n(t)-\rho_{n-1}(t))\\&+\lambda\alpha\mu|\rho_0^2V^{'}(\rho_0)|(\rho_{n+1}(t)-\rho_n(t))\\&-\lambda\alpha\mu|\rho_0^2V^{'}(\rho_0)|(\rho_{n+1}(t-\tau_2)-\rho_n(t-\tau_2))\\&=0.\end{aligned}\tag{4}$$

III. Linear Stability analysis

It is obvious that the solution of the steady state for Eq.(1) and Eq.(2) is:

$$\rho_n(t)=\rho_0,v_n(t)=V(\rho)\tag{5}$$

Let $y_n(t)$ is a small deviation from the steady state.

$$\rho_n(t)=\rho_0+y_n(t).\tag{6}$$

Then, inserting Eq.(6) into Eq.(4), the following equation is given:

$$\begin{aligned}&\partial_t^2y_n(t)-\partial_t\mu|\rho_0^2V^{'}(\rho_0)|(y_n(t)-y_{n-1}(t))\\&+\alpha\rho_0^2V'(\rho_0)(y_{n+1}(t)-y_n(t))\\&+\alpha\rho_0^2(2\gamma-1)(\beta\tau_1)V'(\rho_0)[\partial_ty_{n+1}(t)-\partial_ty_n(t)]\\&+\alpha[\partial_ty_n(t)-\lambda\partial_ty_{n+1}(t)+\lambda\partial_ty_{n+1}(t-\tau_2)]\\&-\alpha\mu|\rho_0^2V^{'}(\rho_0)|(y_n(t)-y_{n-1}(t))\\&+\lambda\alpha\mu|\rho_0^2V^{'}(\rho_0)|(y_{n+1}(t)-y_n(t))\\&-\lambda\alpha\mu|\rho_0^2V^{'}(\rho_0)|(y_{n+1}(t-\tau_2)-y_n(t-\tau_2))=0.\end{aligned}\tag{7}$$

Let $y_n=e^{(ikn+zt)}$, the following equation is obtained:

$$\begin{aligned}&z^2-\mu|\rho_0^2V'(\rho_0)|(z-ze^{-ik})+\alpha\rho_0^2V'(\rho_0)e^{ik-1}\\&+\alpha\rho_0^2(2\gamma-1)(\beta\tau_1)V'(\rho_0)(ze^{ik}-z)\\&+\alpha(z-\lambda ze^{ik}+\lambda ze^{ik-z\tau_2})\\&-\alpha\mu|\rho_0^2V'(\rho_0)|(1-e^{-ik})\\&-\lambda\alpha\mu|\rho_0^2V'(\rho_0)|(e^{ik-z\tau_2}-e^{-z\tau_2})\\&=0.\end{aligned}\tag{8}$$

By substituting $z=z_1(ik)+z_2(ik)^2+...$ into Eq.(8) and neglecting higher order terms of ik. Further, the following equations are obtained:

$$z_1=-(\mu-1)\rho_0^2V'(\rho_0),\tag{9}$$

$$\begin{aligned}z_2=&\lambda\tau_2(\mu-1)-\frac{1+\mu}{2}\\&-[\alpha(2\gamma-1)(\beta\tau_1)(\mu-1)\\&-\lambda\alpha\tau_2\mu(\mu-1)]\rho_0^2V'(\rho_0)\\&-\frac{(\mu-1)^2+(-\mu)(\mu-1)}{\alpha}\rho_0^2V'(\rho_0)\end{aligned}\tag{10}$$

Therefore, if $z_2<0$, traffic flow is unstable. Conversely, traffic flow is stable if $z_2>0$.

Let $z_2=0$, there is the neutral stability condition for proposed model:

$$\alpha=\frac{2\Lambda\rho_0^2V'(\rho_0)}{-2\lambda\tau_2\Lambda-\mu-1+2\Lambda\Gamma\rho_0^2V'(\rho_0)}.\tag{11}$$

where, $\Lambda=\mu-1$, $\Gamma=\lambda\tau_2\mu+\alpha\beta\tau_1(\gamma-1)$.

So, the linear stability condition is obtained as below:

$$a>\frac{2\Lambda\rho_0^2V'(\rho_0)}{-2\lambda\tau_2\Lambda-\mu-1+2\Lambda\Gamma\rho_0^2V'(\rho_0)}.\tag{12}$$

IV. Nonlinear Stability analysis

In this section, the nonlinear stability is analysed and mKdV equation is obtained. Let

$$X=\epsilon(n+bt),T=\epsilon^3t,\tag{13}$$

where $0<\epsilon\leq1$, and b is a constant.
Further, let

$$\rho_n(t)=\rho_c+\epsilon R(X,T).\tag{14}$$

Subsitituting Eqs.(13) and (14) into Eq.(4), there is:

$$\begin{aligned}&\epsilon^2\kappa_1\partial_XR+\epsilon^3\kappa_2\partial_X^2R+\epsilon^4(\kappa_3\partial_X^3R+\kappa_4\partial_XR^3)\\&+\epsilon^5(\kappa_5\partial_T\partial_XR+\kappa_6\partial_X^4R+\kappa_7\partial_X^2R^3)\\&=0,\end{aligned}\tag{15}$$

where

$$\begin{aligned}\kappa_1=&b+(1-\mu)\rho_0^2V'(\rho_c),\\ \kappa_2=&\frac{b^2}{\alpha}-\frac{\mu b}{\alpha}\rho_0^2(\rho_c)-\lambda b^2\tau_1+\frac{(1+\mu)}{2}\rho_0^2V(\rho_c)\\&+[(2\gamma-1)\beta\tau_1b+\lambda\mu b\tau_2]\rho_0^2V(\rho_c),\\ \kappa_3=&\frac{1}{2\alpha}\mu b\rho_0^2V'(\rho_c)+\frac{1}{2}(2\gamma-1)(\beta\tau_1)b\rho_0^2V'(\rho_c)-b^2\tau_2\\&+\frac{1}{2}\lambda b^3{\tau_2}^2+\frac{1}{6}[1+\mu+\mu\lambda+\lambda\mu((1-b\tau_2)^3\\&-b^3{\tau_2}^3)]-(2-b\tau_2)^3)\rho_0^2V'(\rho_c),\\ \kappa_4=&\frac{\rho_0^2V_{op}'''(\rho_c)}{2},\\ \kappa_5=&\frac{2b}{\alpha}-\lambda b\tau_2-\frac{\mu}{\alpha}\rho_0^2V'(\rho_c)+(2\gamma-1)(\beta\tau_1)\rho_0^2V'(\rho_c),\end{aligned}$$

$$\begin{aligned}\kappa_6 =& -\frac{\lambda}{24}\mu[(1-b{\tau_2}^4)-b^4{\tau_2}^4]\rho_0^2V'(\rho_c)\\ &+\frac{1}{24}(1+\mu+8\gamma\beta\tau_1-4\beta\tau_1+\lambda\mu)\rho_0^2V'(\rho_c)\\ &-\frac{\lambda b^4{\tau_2}^3+3\lambda b^2\tau_2-3\lambda b^3{\tau_2}^2}{6}\\ &+\frac{b}{6\alpha}(-\mu)\rho_0^2V'(\rho_c),\\ \kappa_7 =& \frac{\rho_0^2V(\rho_c)}{4}\end{aligned}$$

Near the critical point (ρ_c, α_c), let $\alpha_c = \alpha(1+\epsilon^2)$. Further, let $b = -(1-\mu)\rho_0^2V'(\rho_c)$, eliminating the quadratic and cubic terms of ϵ. There is:

$$\begin{aligned}&\epsilon^4(\partial_T R-\omega_1\partial_X^3R+\omega_2\partial_X R^3)\\ &+\epsilon^5(\omega_3\partial_X^2R+\omega_4\partial_X^4R+\omega_5\partial_X^2R^3)\\ &=0\end{aligned}$$

where

$$\begin{aligned}\omega_1 =& -\frac{1}{6}[1+\mu+\mu\lambda-\lambda\mu(1-b\tau_2)^3\\ &-b^3{\tau_2}^3-(2-b\tau_2)^3]\\ &+\frac{\mu(1-\mu)}{2a_c}\rho_0^4V'(\rho_c)^2\\ &+\frac{1}{2}\lambda(1-\mu)^3{\tau_2}^2\rho_0^6V'(\rho_c)^3\\ &+(1-\mu)\times[(\gamma-\frac{1}{2})(\beta\tau_1)\\ &+(1-\mu)\tau_2]\rho_0^4V'(\rho_c)^2\\ \omega_2 =& \frac{\rho_0^2V'''(\rho_c)}{2},\\ \omega_3 =& [\lambda\mu\tau_2+(2\gamma-1)(\beta\tau_1)](1-\mu)\rho_0^4V'(\rho_c)^2\\ &-\frac{1}{2}(1+\mu)\rho_0^2V'(\rho_c)\\ \omega_4 =& [\frac{-2+\mu}{a_c}\rho_0^2V'(\rho_c)+(2\gamma\beta\tau_2+(\lambda-\beta)\tau_1\\ &-\lambda\tau_2\mu\rho_0^2V'(\rho_c))][\frac{\mu(1-\mu)}{2a_c}\rho_0^4V'(\rho_c)^2\\ &-\frac{1}{6}(1+\mu+\mu\lambda-\lambda\mu((1-b\tau_2)^3-b^3{\tau_2}^3))]\\ &+\frac{1}{2}\lambda(1-\mu)^3{\tau_2}^2\rho_c^6V'(\rho_c)^3\\ &+(1-\mu)(\gamma-\frac{1}{2})\beta\tau_1\\ &+(1-\mu)\tau_2\rho_c^4V'(\rho_c)^2\\ \omega_5 =& \frac{\rho_0^2V'''(\rho_c)}{4}-\frac{-2+\mu}{2a_c}\rho_0^4V'(\rho_c)V'''(\rho_c)\\ &-\frac{2\gamma\beta\tau_1+(\lambda-\beta)\tau_1-\mu\lambda\tau_2}{2}\rho_0^4\\ &\times V'(\rho_c)V'''(\rho_c)\end{aligned}$$

Next, conducting the following transformation:

$$T=\frac{1}{\omega_1}\bar{T}, R=\sqrt{\frac{\omega_1}{\omega_2}}\bar{R} \tag{16}$$

Further, substituting Eq.(16) into Eq.(15), there is the following equation:

$$\begin{aligned}\partial_{\bar{T}}\bar{R}-\partial_{\bar{X}}^3\bar{R}+\partial_{\bar{X}}\bar{R}^3+\frac{\epsilon}{\omega_1}(\omega_3\partial_{\bar{X}}^2\bar{R}+\omega_4\bar{X}^4\bar{R}\\ +\frac{\omega_1\omega_2}{\omega_2}\partial_{\bar{X}}^2\bar{R}^3)=0\end{aligned} \tag{17}$$

In Eq.(17), ignoring the correction term $O(\varepsilon)$ and further have the kind-antikind solution:

$$\bar{R}_0(X,\bar{T})=\sqrt{c}tanh(\frac{c}{2}(X-c\bar{T})), \tag{18}$$

where c is the propagation velocity for kind-antikink density wave.

To compute c, the following condition is needed:

$$(\bar{R}_0, M[\bar{R}_0])\equiv\int_{-\infty}^{\infty}dX\bar{R}_0M[\bar{R}_0]=0, \tag{19}$$

where

$$M[\bar{R}_0]=\frac{1}{\omega_1}(\omega_3\partial_{\bar{X}}^2\bar{R}+\omega_4\bar{X}^4\bar{R}+\frac{\omega_1\omega_2}{\omega_2}\partial_{\bar{X}}^2\bar{R}^3) \tag{20}$$

By solving the Eq.(20), the general velocity is:

$$c=\frac{5\omega_2\omega_3}{2\omega_2\omega_4-3\omega_1\omega_5} \tag{21}$$

So, the kind-antikink solution is:

$$\rho_n=\rho_c+\varepsilon\sqrt{\frac{\omega_1 c}{\omega_2}}tanh(\sqrt{\frac{c}{2}}(X-c\omega_1T)) \tag{22}$$

where $\varepsilon^2=(\frac{\alpha_c-1}{\alpha})$.

A is the amplitude:

$$A=\sqrt{\frac{\omega_1\epsilon^2c}{\omega_2}}. \tag{23}$$

Thus, the kink-antikink soulution depicts the coexisting phase which consist of freely moving phase and congested phase.

Based on the above analysis, the density of the freely moving phase is $\rho=\rho_c+A$ and the density of congested phase is $\rho=\rho_c-A$.

V. Numerical simulations

In this section, the influence of self-delayed effect in the process of flux transmission, driver's characteristic and on-ramp on the traffic flow is conducted based on periodic boundary.

In the numerical simulation, the parameters are set as: $N=100$, $\Delta t=0.1$, $\rho_0=\rho_c=0.25$, $\alpha=1.2$, $v_{max}=2$, $\sigma=0.1$, $\lambda=0.35$.

The initial disturbance is:

$$\rho_j(1)=\rho_j(0)=\begin{cases}\rho_0+\sigma, & j=N/2-1,\\ \rho_0-\sigma, & j=N/2+1,\\ \rho_0, & j\neq N/2-1,N/2+1.\end{cases} \tag{24}$$

The results for the evolution of density wave are shown in Fig.2 and Fig.3.

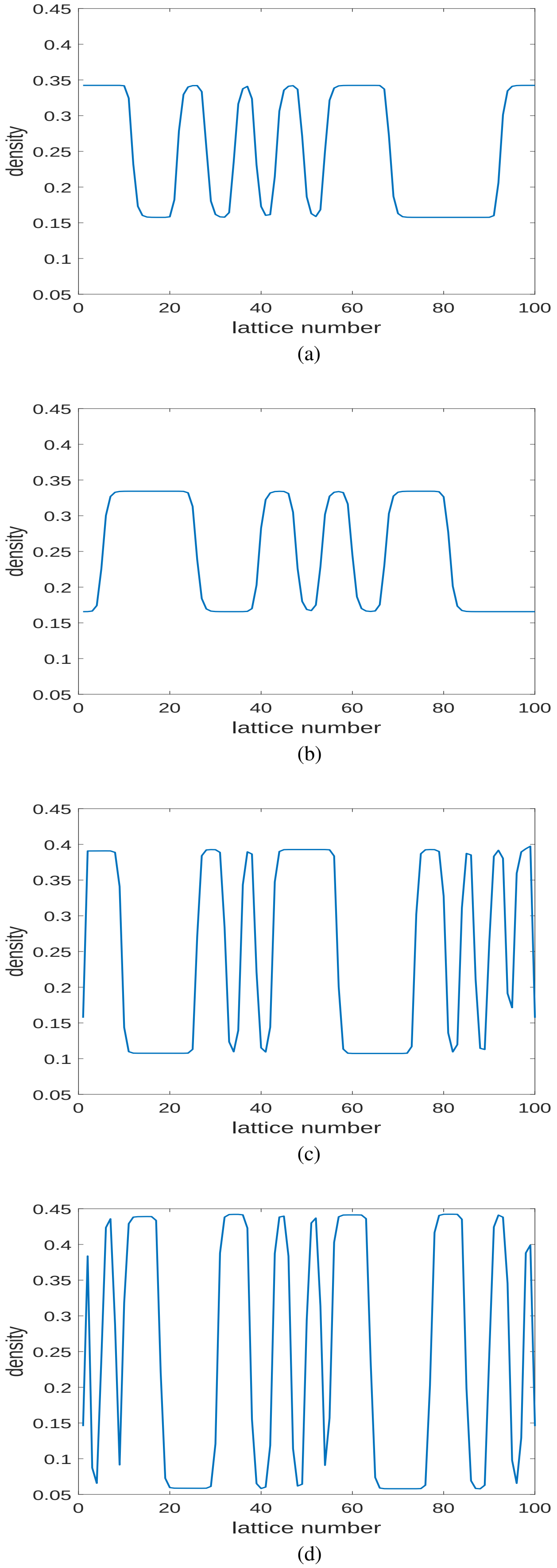

Fig.2. Density wave at $t = 20000$ time step, (a):$\tau_2 = 0, \beta = 0, \tau_1 = 0, \gamma = 0, \mu = 0$, (b):$\tau_2 = 0.2, \beta = 0, \tau_1 = 0, \gamma = 0, \mu = 0$, (c):$\tau_2 = 0.2, \beta = 1, \tau_1 = 2, \gamma = 0.4, \mu = 0$, (d):$\tau_2 = 0.2, \beta = 1, \tau_1 = 2, \gamma = 0.4, \mu = 0.05$.

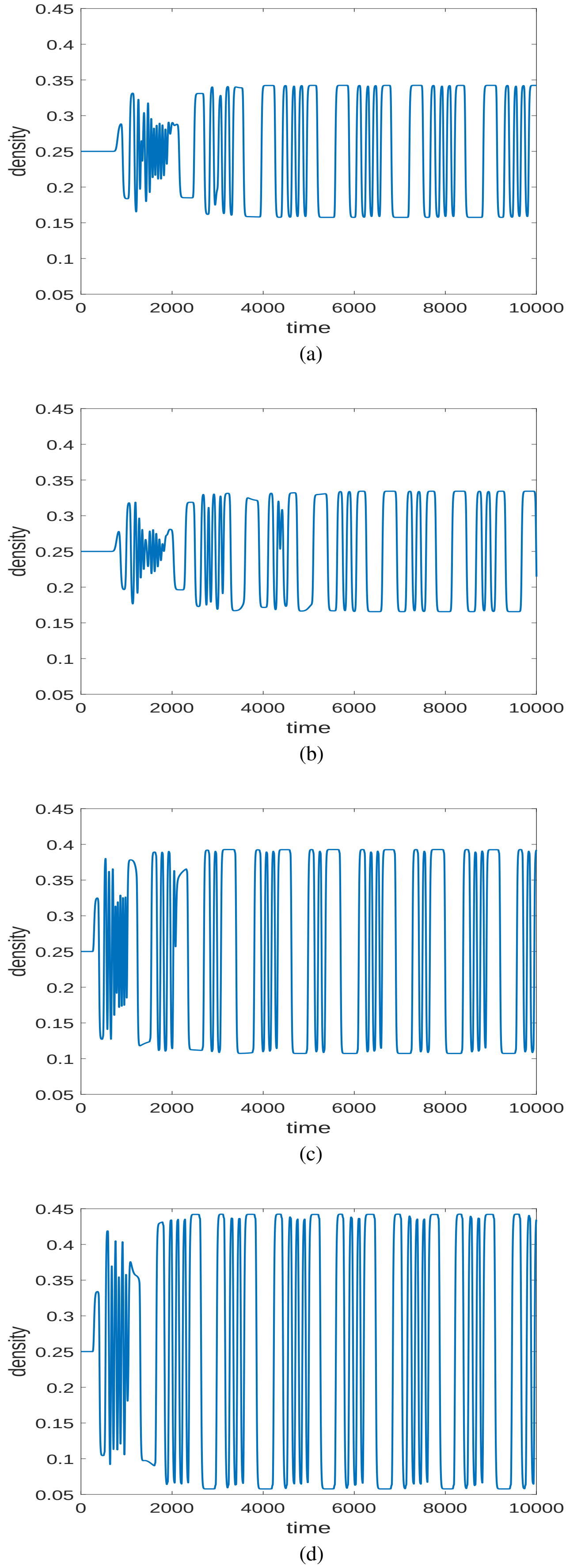

Fig.3. Density wave at lattice $n = 80$,(a):$\tau_2 = 0, \beta = 0, \tau_1 = 0, \gamma = 0, \mu = 0$, (b):$\tau_2 = 0.2, \beta = 0, \tau_1 = 0, \gamma = 0, \mu = 0$, (c):$\tau_2 = 0.2, \beta = 1, \tau_1 = 2, \gamma = 0.4, \mu = 0$, (d):$\tau_2 = 0.2, \beta = 1, \tau_1 = 2, \gamma = 0.4, \mu = 0.05$.

First, let $\beta = 0, \tau_2 = 0, \gamma = 0, \mu = 0$. Namely, exploring the impact of self-delayed effect in the process of flux transmission on traffic flow. In Fig.2(a), self-delayed effect of flux is not considered and let $\tau_2 = 0$. In Fig.2(b), let $\tau_2 = 0.2$, namely, the duration of self-delayed is $0.2s$. It is obvious that the oscillation of density waves in Fig.2(b) is weakened when comparing with Fig.2(a). That is to say, when considering additional self-delay effect of flux, drivers can obtain more accurate traffic information during driving process. So, obtaining the self-delayed flux information can helps to improve stability of traffic system.

Next, in order to explore the impact of driver's characteristic on traffic flow, letting $\tau_2 = 0, \beta = 1, \tau_1 = 0.2, \gamma = 0.4, \mu = 0$ and the result is shown in Fig.2(c). In Fig.2(c), considering the proportion of aggressive drivers is 0.4 and the proportion of timid drivers is 0.6. Obviously, compared with Fig.2(b), the participation of aggressive and timid drivers has caused strong disturbances to the transportation system, and it's corresponding density wave has also produced strong oscillations which is shown in Fig.2(c). Thus, compared with robust drivers, aggressive and timid drivers are not conducive to the smooth operation of the transportation system and are prone to traffic congestion and accidents.

Finally, the impact of on-ramp on the transportation system is explored. Let $\tau_2 = 0.2, \beta = 1, \tau_1 = 0.2, \gamma = 0.4, \mu = 0.05$. That is to say, considering the rate of on-ramp is 0.05. Obviously, compared with Fig.2(c), the amplitude of the density wave has increased and almost oscillating intermittently between $0 - 0.5$. Therefore, vehicles entering the main road from the on-ramp not only create disturbance from on-ramp to main road, but also increase the traffic volume on the main road, which exacerbates traffic congestion.

By simulating the profiles of density-time step profiles at $n = 80th$ lattice within 10000 time steps, it is helpful to clearly analyze the evolution of density waves. By comparison, it can be clearly observed that self-delayed flux information can reduce the oscillation of density waves. However, the participation of aggressive and timid drivers as well as the inflow of traffic flow at the on-ramp exacerbate density wave oscillations. Thus, the control of flow at the on-ramp and the training of drivers are very important in actual traffic.

VI. Conclusion

In the modeling research of traffic flow, driver's behavior and road characteristics are often the focus of consideration. Moreover, on-ramp control on highways has always been a challenging issue in research. Based on these, this paper establishes a novel lattice hydrodynamic model with self-delayed effect in the process of flux transmission, driver's characteristic and on-ramp into consideration. By linear and nonlinear analysis, the stability condition and modified Korteweg-de Vries (mKdV) equation are obtained. Numerical simulation not only validates theoretical analysis, but also reveals the impact of self-delayed flux information, drivers characteristic and on-ramp on traffic flow. The results show that self-delayed flux information can relieve traffic congestion, the participation of aggressive and timid drivers as well as the inflow of traffic flow at the on-ramp can cause serious traffic congestion.

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