Consistency of Time-varying Multi-agent Systems with Intervening Directed Topological Connectivity Improved Speed of Analysis and Convergence

Tengfei Wei, Shihua Ma, Rui Hao

***Abstract*—Aiming to enhance the consensus convergence speed in multi-agent systems characterized by directed topology while extending the applicability of consensus control conditions. Consensus and group consensus of nonlinear time-varying of the directed topology node algebraic connectivity of the system with intervention is studied in this paper and the convergence sufficient condition is given. The distribution of system eigenvalues under these conditions is analyzed using the Gerschgorin theorem. The correctness of the proposed convergence condition is rigorously proven using Lyapunov's first method. Finally, Simulation results validate the proposed convergence conditions, and comparative experiments demonstrate the effectiveness of intervention control in improving the system's convergence speed.**

***Index Terms*—multi-agent systems, consensus, stability, intervention control, convergence speed**

I. INTRODUCTION

IN recent years, the coherent control of multi-agent systems has emerged as a frontier and prominent topic of research. Research findings on coherent control have been applied to various fields, including formation control of autonomous vehicle swarms, coordinated control of multi-robot systems, and distributed sensor networks [1-7].

Olfati-Saber and Murray initially proposed a description of the coherence problem for multi-agent systems [8-9]. A consistency control algorithm is also provided to demonstrate that when the topology is a directed strongly connected equilibrium graph, the multi-agent system achieves average consistency. It has been found that the rate of consistency convergence in a multi-agent system is related to the non-zero minimum eigenvalue of its Laplacian matrix and that increasing this eigenvalue can improve the convergence speed [8]. Further research has revealed that the non-zero minimum eigenvalue of the Laplacian matrix can be increased by altering the connectivity between nodes without changing the topology [10]. In the literature [12], the topology of a multi-agent system is designed to allow significant algebraic connectivity between nodes, thereby increasing the convergence speed. In the literature [13], topologies with greater connectivity were identified to accelerate convergence by determining the optimal node locations. An important metric for evaluating the convergence performance of a multi-agent system is its convergence speed [10-14].

Due to the instability of communication channels caused by external environmental disturbances, the connectivity between nodes in the directed topology changes over time. To address this issue, it is necessary to investigate consistency and enhance the convergence rate of multi-agent systems with time-varying directed topologies [15-18].

This paper separately discusses the coherence and group coherence of systems with directed, nonlinear, time-varying topological connectivity and first-order integral interventions in multi-agent systems. Additionally, it investigates the convergence conditions for both types of coherence and demonstrates, through experimental results, how intervention actions can enhance the system's convergence speed.

II. PROBLEM DESCRIPTION

A theoretical framework is first presented for a time-varying directed topological multi-agent system under intermediate connectivity. Let $G(t)=\{V,E(t),A(t)\}$ be a topology with time-varying connectivity, where $V=\{v_1,v_2,\cdots,v_n\}$ is the set of topological nodes and $E=\{e_{ij}=(x_i,x_j)\,|\,i,j=1,2,\cdots,n\}$ is the set of topological edges. Let $A(t)=(a_{ij}(t))\in\mathbb{R}^{n\times n}$ denote the time-varying adjacency matrix of the multi-agent system, where the vectors $a_{ii}=0$. If the set $(v_i,v_j)\in E$, then the node i is said to be a neighbour node of node j, and the neighbours of node i at time t are expressed using the set $N_i(t)=\{x_i\,|\,e_{ij}\in E, j=1,2,\cdots,n\}$. The degree matrix $D(t)=diag\{d_1(t),d_2(t),\cdots,d_n(t)\}$ of the topological map at time t then has

$$d_i(t)=\sum_{j=1}^{n}a_{ij}(t) \tag{1}$$

Manuscript received October 2, 2024; revised March 20, 2025.

Tengfei Wei is an Associate Professor of College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou 730050, China (corresponding author to provide phone: 15214027124; e-mail: tfwei@lut.edu.cn).

Shihua Ma is a graduate student of College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou 730050, China. (e-mail: 2806892555@qq.com).

Rui Hao is a graduate student of College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou 730050, China. (e-mail: 2467888393@qq.com).

Where $i = 1, 2, \cdots, n$. The Laplace moment of a multi-agent system is $L(t) = D(t) - A(t)$.

Considering a system of n intelligences then there are

$$\dot{x}(t) = \imath(t) \tag{2}$$

Equation (2) $x(t) \in \mathbb{R}^{n\times 1}$ is the system state vector and the vector $\imath(t) \in \mathbb{R}^{n\times 1}$ is the consistency control protocol.

In this paper, the consistency of the system under intervention is studied, and the system's consistency control protocol is expressed as:

$$\imath_i(t) = \sum_{j\in N_i(t)} a_{ij}(t)(x_j(t) - x_i(t)) - u_i(t) \tag{3}$$

Equation (3), $\imath_i(t) \in \mathbb{R}$ is the consistency control protocol of the i node of the system at the time t , $x_i(t) \in \mathbb{R}$ is the i state variable of the system, and $u_i(t) \in \mathbb{R}$ is the rate of control by intervention of the i node of the system.

Then the multi-agent system written in matrix form can be expressed as：

$$\dot{x}(t) = -L(t)x(t) - u(t) \tag{4}$$

Here, $u(t) \in \mathbb{R}^{n\times 1}$ represents the control vector of the multi-agent system under intervention. If the connectivity between the nodes of the topology is time-varying, the elements of the Laplacian matrix also vary over time. The topological connectivity of many real-world systems is often influenced by external environmental factors, justifying the above conditions.

For the intervention control rate $u(t)$ of a multi-agent system to be practically meaningful, it is typically derived as feedback from the state variable, leading to:

$$u(t) = Q(t)x(t) \tag{5}$$

In equation (5), $Q(t)$ is the state feedback matrix for the intervention control rate.

If the intervention control of the system follows the relationship in equation (5), it indicates that the multi-agent system evolves under external influence; if $Q(t)$ is a zero matrix, then $u(t)$ is also a zero matrix, indicating that the system evolves without external influence and evolves towards full autonomy, i.e., a conventional coherent system.

III. Key Finding

The mutual weights between nodes in the directed topological multi-agent system are unequal and time-varying, meaning the exchange of information between nodes is asymmetric. Consequently, the adjacency matrix $A(t)$ and the Laplacian matrix $L(t)$ of the system are no longer symmetric and their eigenvalues are not necessarily real. Therefore, the consistent convergence of the system can only be determined by analyzing the distribution range of its eigenvalues.

A. Coherence of Multi-intelligence Systems Without Grouping

Theorem 1: For orderless multi-agent systems with continuous time-varying strongly connected connectivity topology (4). The intervention control rate is expressed as shown in equation (5), and the state feedback matrix $Q(t)$ is a diagonal array whose main diagonal element $\lambda_{i,Q}(t)$ is also continuously time-varying. For $\forall i = 1, 2 \cdots n$, with elements $\lambda_{i,Q}(t) > 0$, the multi-agent system (4) converges consistently to the following equilibrium state:

$$x(\infty) = 0 \tag{6}$$

Where the 0 vector is a column vector of order n .

Proof: Based on the properties of the Laplacian matrix $L(t)$ in a strongly connected directed topology, it follows that 0 is an eigenvalue of it, given by the formula $rank(L) = n - 1$. Since the matrix $L(t)$ is an asymmetric real matrix whose eigenvalues are not necessarily real, and the eigenvalues $\lambda_{1,L} = 0$, $\lambda_{2,L}, \lambda_{3,L}, \cdots \lambda_{n,L}$ satisfy $\lambda_{i,L}(t) \in \mathbb{C}$ is the eigenvalue of the Laplacian matrix $L(t)$. The range of eigenvalue distribution of the Laplace matrix $L(t)$ is analyzed below utilizing Gerschgorin's disc theorem.

$$|z - \ell_{ii}(t)| \le R_i(t) \tag{7}$$

$$R_i(t) = \sum_{j=1, j\ne i}^{n} |\ell_{ij}(t)| \tag{8}$$

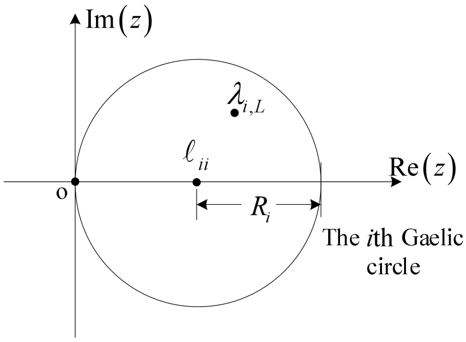

Fig. 1. The ith Gaelic circle of a directed topological Laplacian matrix

Equation (7) describes the complex domain of the ith Gaelic circle, where the radius of the ith Gaelic circle of the matrix $L(t)$ is R_i defined by equation (8). The center of the Gaelic circle ℓ_{ii} corresponds to the element located at the ith row and jth column of the matrix $L(t)$. Fig. 1 illustrates the matrix $L(t)$ of a strongly connected directed topology along with its Gaelic circle. Since the eigenvalues of the matrix $L(t)$ are not necessarily real, their possible locations $\lambda_{i,L}$, as depicted in Fig. 1, lie entirely on the right-hand side of the complex plane. According to the disc theorem, all eigenvalues of the matrix $L(t)$ are distributed within the union of its n Gaelic circles.

The state feedback matrix $Q(t)$ is given by equation (9), and the matrix $Q(t)$ is diagonal and $\lambda_{i,Q}(t) > 0, \lambda_{i,Q}(t) \in \mathbb{R}$ by the conditions of the Theorem 1 question. Then we have:

$$Q(t) = diag\left\{\lambda_{1,Q}(t), \cdots, \lambda_{n,Q}(t)\right\} \tag{9}$$

From Theorem 1, the intervention control is a feedback of the system state vector, so that equation (5) can be brought into the multi-agent system equation (4) to obtain equation (10).

$$\dot{x}(t) = -(L(t) + Q(t))x(t) \tag{10}$$

The coefficient matrix $L(t) + Q(t)$ is expressed as equation (11).

$$L(t) + Q(t) = \begin{bmatrix} \ell_{11} + \lambda_{1,Q} & \cdots & \ell_{1n} \\ & \ddots & \\ \ell_{n1} & \cdots & \ell_{nn} + \lambda_{n,Q} \end{bmatrix} \tag{11}$$

The ith Gaelic circle equation for the coefficient matrix $L(t) + Q(t)$ is shown in equation (12).

$$\left| z - \left[\ell_{ii}(t) + \lambda_{i,Q}(t) \right] \right| \le R_i(t) \tag{12}$$

Where R_i is the radius of the ith Geier circle of the coefficient matrix, which is the same as the radius of the ith Geier circle of the matrix $L(t)$, as shown in equation (8). The ith Gaelic circle of the coefficient matrix $L(t) + Q(t)$ is shown in Fig. 2. Since the matrix $L(t) + Q(t)$ is also a real asymmetric matrix, its eigenvalues $\lambda_{i,C}$ are not necessarily real, and the possible locations $\lambda_{i,C}$ are given in Fig. 2, all on the right-hand side of the complex plane. Under the conditions of the question, there are $\lambda_{i,Q}(t) > 0$, and all have $\ell_{ii}(t) > 0$ in the strongly connected directed topology. From the above conditions, then inequality (13) holds constantly.

$$R_i(t) < \ell_{ii}(t) + \lambda_{i,Q}(t) \tag{13}$$

Therefore, the ith eigenvalue of the $L(t) + Q(t)$ matrix in equation (10) has a negative real part. Repeating the above steps will eventually show that all the eigenvalues of the $-(L(t) + Q(t))$ matrix in equation (10) have negative real parts. Repeating the above steps in the same way eventually shows that all the eigenvalues of the $-(L(t) + Q(t))$ matrix in equation (10) have negative real parts. From Lyapunov's first method of stability determination, it is clear that the system (10) is asymptotically stable at its equilibrium point.

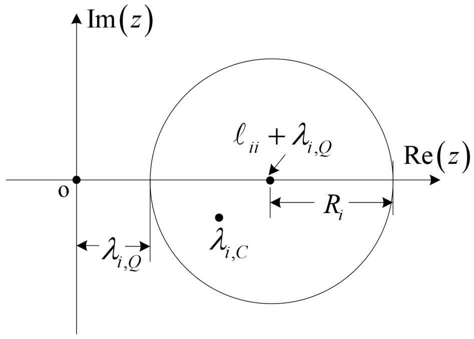

Fig. 2. The ith Gaelic circle of the directed topological coefficient matrix

Since all Gaelic circles of the $L(t) + Q(t)$ matrix are to the right of the complex plane, none of their eigenvalues are zero. It is known that $rank(L + Q) = n$, then the matrix equation of Eq. (14) has and has only zero solutions. Therefore, the equilibrium state of the multi-agent system Eq. (10) is the state origin.

$$(L(t) + Q(t))x(t) = 0 \tag{14}$$

Finally, the multi-agent system described by equation (4) exhibits consistent convergence, and its equilibrium state is the origin. At this point, Theorem 1 is proven.

When no intervention control is applied, both the system analyzed in Theorem 1 and the directed topologically consistent system have identical state consistency. Under intervention control, system consistency remains unaffected by the time-varying nature of the algebraic connectivity of the topological nodes. As long as the conditions of Theorem 1 are satisfied, the system states will consistently converge to zero.

The Laplacian matrix is determined by the actual topology and cannot be altered, thereby limiting the system's convergence speed. Then the state feedback matrix can be artificially configured according to the actual situation, which can play a role in accelerating the speed of coherent convergence.

The results of Theorem 1 allow for a more generalized application of the consistency conditions. Moreover, when the state feedback matrix is not diagonal, the polar configuration theorem can be employed to determine the state feedback matrix based on system consistency and performance requirements. When the topological connectivity changes, the Laplacian matrix also changes, and the aforementioned steps must be repeated to ensure system consistency. No changes are required in Theorem 1 to ensure system consistency, which enhances the robustness of the system.

B. Multi-intelligence System Group Consistency

Group consistency is a key concept in multi-agent formation control, where each group achieves a distinct consistency convergence value. Therefore, it is necessary to investigate the intervened group consistency of a multi-agent system with a time-varying topology with directed connectivity. From Theorem 1, it is clear that the multi-agent system state consistency converges to zero under its given conditions, then the state variables of the original intervened consistency system (4) and (5) are transformed by a coordinate translation. Then the intervention control for the ith state variable of the multi-agent system with group consistency is expressed as:

$$u_i(t) = -\sum_{j\in N_i(t)} a_{ij}(t)(x_j^* - x_i^*) + \lambda_{i,Q}(t)(x_i(t) - x_i^*) \tag{15}$$

Where x_i^* equation (15) is the reference setting for the ith state variable.

At this point, the matrix form for the control rate of the system grouping consistency intervention is

$$u(t) = Q(t)x(t) - (L(t) + Q(t))\sum_{i=1}^{m} C_i S_i^* \tag{16}$$

$$C_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{in}]^{\mathrm{T}} \tag{17}$$

Where m is the number of consistent groupings of the multi-intelligence system in equation (16). C_i is the ith grouping vector of the grouping consistency system. When the jth system state variable x_j belongs to the ith grouping, there is $\mu_{ij} = 1$, otherwise $\mu_{ij} = 0$. S_i^* is the state reference value of the ith grouping.

Theorem 2: For the nth-order grouped multi-agent system (4) with a continuous time-varying strongly connected topology of connectivity, the intervention control rate has the form expressed in equation (16) and the state feedback matrix $Q(t)$ is a diagonal array whose main diagonal element $\lambda_{i,Q}(t)$ is also continuously time-varying. For $\forall i = 1,2\cdots n$, with $\lambda_{i,Q}(t) > 0$, the grouped multi-agent system (4) converges consistently to the following equilibrium state:

$$x(\infty) = \sum_{i=1}^{m} C_i S_i^* \tag{18}$$

Proof: Substituting the group consistency intervention control equation (16) into the multi-agent system equation (4) results in equation (19).

$$\dot{x}(t) = -(L(t) + Q(t))\left[x(t) - \sum_{i=1}^{m} C_i S_i^*\right] \tag{19}$$

The Laplacian matrix $L(t)$ and the state feedback matrix $Q(t)$ of Theorem 2 have the same properties as $L(t)$ and $Q(t)$ of Theorem 1. Therefore, the results of Theorem 1 for the range of eigenvalue distributions of the matrix $-(L(t)+Q(t))$ still apply here. It follows that all the eigenvalues of the coefficient matrix $-(L(t)+Q(t))$ of equation (19) have negative real parts. Since the $\sum C_i S_i^*$ term in equation (19) is a constant, it follows from Lyapunov's first method of determining stability that the system equation (19) is asymptotically stable at its equilibrium point.

By Theorem 1, $rank(L+Q) = n$, then there are only $\sum C_i S_i^*$ solutions to the matrix equation shown in equation (20). Therefore, the equilibrium state of the multi-agent system equation (19) is $\sum C_i S_i^*$.

$$(L(t) + Q(t))\left[x(t) - \sum_{i=1}^{m} C_i S_i^*\right] = 0 \tag{20}$$

Finally, the multi-agent system shown in equation (4) has group consistent convergence and its equilibrium state is $\sum C_i S_i^*$. At this point, Theorem 2 is proven.

Since the group consistent multi-agent system is derived from the consistent system in Theorem 1 by a coordinate translation transformation, the convergence condition of Theorem 1 still applies, i.e. $\lambda_{i,Q}(t) > 0$. The distinction lies in the group consistency intervention control rate, expressed in equation (16), with the group consistency convergence value determined by S_i^*.

When $S_i^* = 0$ the group consistency of the system studied in Theorem 2 coincides with that of Theorem 1, both converge consistently to zero. Under intervention control, as long as the directed topology is strongly connected and $\lambda_{i,Q}(t) > 0$, then the multi-agent system group consistency is independent of the time-varying nature of the algebraic connectivity of the topological nodes, the system states eventually converge $\sum C_i S_i^*$.

Theorem 2 analyzes the grouping consistency of an intervened multi-agent system, where the intervention control consists of state feedback values and reference feedback values. The rate of grouping consistency convergence is determined by both the Laplacian matrix and the state feedback matrix of the system. Consequently, the state feedback matrix can be artificially configured to improve the grouping consistency convergence rate of the multi-agent system.

Finally, when the state feedback matrix is not diagonal, the state feedback matrix can also be determined by the pole configuration theorem, but when the topological connectivity changes, the Laplacian matrix also changes, and then the above steps need to be repeated to ensure the group consistency of the system. By applying Theorem 2, there are no such problems.

IV. Simulation Examples

The validity of the theorem is further confirmed through simulation experiments on a multi-agent system. Simulations are performed on a non-linear, time-varying multi-agent system with directed topological connectivity to validate system consistency and group consistency convergence.

Additionally, comparative experiments demonstrate the efficacy of intervention control in enhancing the speed of consistent convergence.

A. Multi-intelligence System Topology and Simulation Parameters

The simulation experiment considers a system consisting of five intelligences, the first of which is represented by a node, where $i = 1, 2, \cdots, 5$. The time-varying topology of the directed connectivity is illustrated in Fig. 3, demonstrating that the system has strong connectivity in the directed topology. The simulation time is set to 1 s. The initial state of the multi-agent system $x(t_0) = [-1, 0.3, 5, 1.7, -5]^{\mathrm{T}}$.

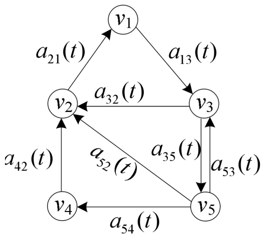

Fig. 3. Time-varying topology diagram for directed connectivity

The grouping consistency system contains 3 groupings with reference values $S_1^* = 2$, $S_2^* = 1$, $S_3^* = -1$. The grouping vectors are

$$C_1 = [1, 0, 1, 0, 0, 0]^{\mathrm{T}} \tag{21}$$

$$C_2 = [0, 1, 0, 0, 0, 0]^{\mathrm{T}} \tag{22}$$

$$C_3 = [0, 0, 0, 0, 1, 1]^{\mathrm{T}} \tag{23}$$

The time-varying adjacency matrix of the multi-agent system for this topology is presented in equation (26).

To evaluate the effect of intervention control on improving the convergence speed of system consistency, two state feedback matrices, $Q_1(t)$ and $Q_2(t)$, were selected and simulated respectively. The diagonal elements of both $Q_1(t)$ and $Q_2(t)$ matrices are greater than zero, thereby satisfying the consistency convergence conditions proposed in Theorem 1 and 2.

$$\begin{aligned} Q_1(t) = diag\{&2t+3, 5t^2+3, \\ &5t^3+1, 2\cos^2 t+3, t\left|\sin(2t)\right|+1\} \end{aligned} \tag{24}$$

$$\begin{aligned} Q_2(t) = diag\{&12t+18, 30t^2+18, \\ &30t^3+6, 12\cos^2 t+18, 6t\left|\sin(2t)\right|+6\} \end{aligned} \tag{25}$$

The corresponding elements $Q_1(t)$ shown in Eq. (24) are all smaller than those $Q_2(t)$ shown in Eq. (25), which indicates that $Q_2(t)$ yields a stronger control of intervention than $Q_1(t)$.

B. Group-free Consistency Simulation Results and Analysis

The results of the coherent state simulation for the ungrouped multi-agent system are presented in Figs. 4 and 6. The strong intervention control for the system depicted in Fig. 4 is obtained from the $Q_1(t)$ feedback system state. The weak intervention control for the system shown in Fig. 6 is obtained by feeding back the system state $Q_2(t)$. Under the above simulation conditions, the system has consistent convergence and the state variables all converge to zero. This further verifies the validity of the convergence condition proposed in Theorem 1.

In addition, as can be seen from Fig. 4, the weakly intervening multi-agent system has all the system state variables converge to zero consistently at around 0.7s, while as can be seen from Fig. 6 the strong intervening system has the states converge to zero at around 0.4s. Therefore, the strongly intervened system exhibits a faster consistent convergence rate compared to the weakly intervened system.

The magnitude of the consistent convergence speed of the multi-agent system can also be known from a comparison of the system state speed plots in Figs. 5 and 7. The velocity of each state of the strong intervention system shown in Fig. 7 is greater than the corresponding state velocity of the weak intervention system shown in Fig. 5.

$$A(t) = \begin{bmatrix} 0 & 0 & 3t^3+t^2+5 & 0 & 0 \\ 2t^2+t+5 & 0 & 0 & 0 & 0 \\ 0 & 2t^2+3 & 0 & 0 & 5t^3+1 \\ 0 & 3t^4+t^2+4 & 0 & 0 & 0 \\ 0 & t\sin^2 t+3 & t\cos^2 t+1 & 3t^3+5t^2+7 & 0 \end{bmatrix} \tag{26}$$

The Laplacian matrix of a multi-agent system is expressed as follows:

$$L(t) = \begin{bmatrix} 3t^3+t^2+5 & 0 & -3t^3-t^2-5 & 0 & 0 \\ -2t^2-t-5 & 2t^2+t+5 & 0 & 0 & 0 \\ 0 & -2t^2-3 & 5t^3+2t^2+4 & 0 & -5t^3-1 \\ 0 & -3t^4-t^2-4 & 0 & 3t^4+t^2+4 & 0 \\ 0 & -t\sin^2 t-3 & -t\cos^2 t-1 & -3t^3-5t^2-7 & 3t^3+5t^2+t+11 \end{bmatrix} \tag{27}$$

The multi-agent coherent convergence speed can then be improved by intervention control, and the stronger the intervention control rate, the faster the coherent convergence speed of the system. Therefore, while the multi-agent system topology remains fixed, the system's consistency convergence speed can be enhanced by configuring the state feedback matrix.

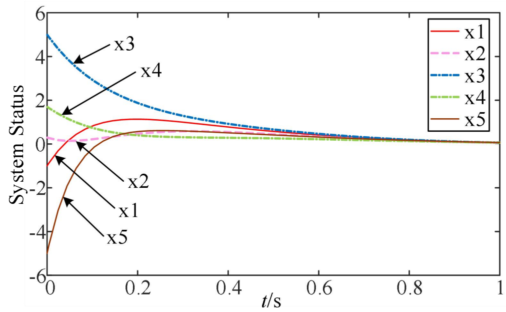

Fig. 4. Weak intervention consistency state diagram

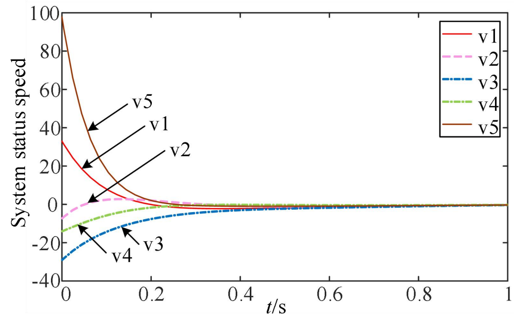

Fig. 5. Weak intervention consistency state velocity diagram

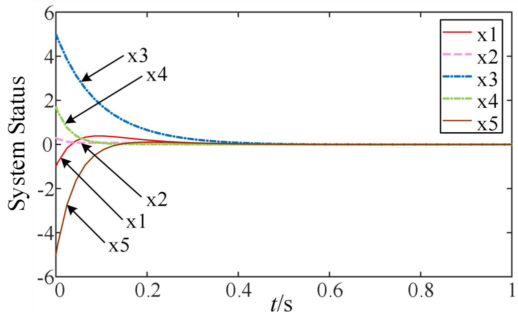

Fig. 6. Strong intervention consistency state diagram

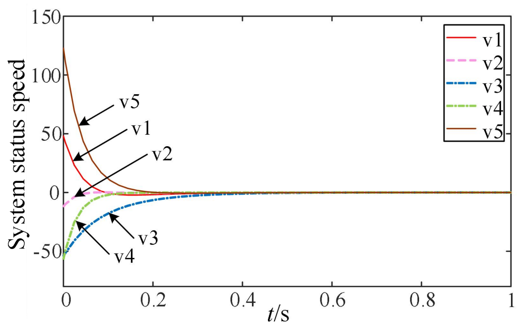

Fig. 7. Velocity diagram of the strong intervention consistency state

C. Group Consistency Simulation Results and Analysis

The results of the grouped coherent state simulation of the multi-agent system are presented in Figs. 8 and 10. In Fig. 8, the strong intervention control is derived from the $Q_1(t)$ feedback system state.

In Fig. 10, the weak intervention control is derived from the system $Q_2(t)$ feedback. Under the proposed simulation conditions, the multi-agent system exhibits group consistency convergence, with the state variables converging to their respective group reference values. This further validates the correctness of the convergence condition stated in Theorem 2.

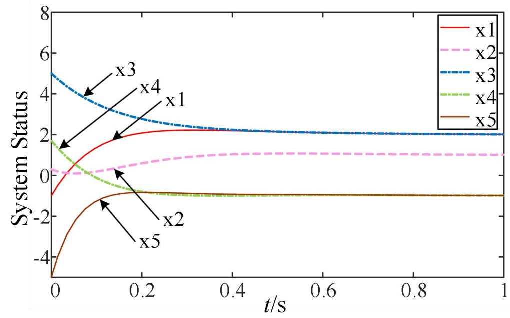

Fig. 8. Weak intervention group consistency state diagram

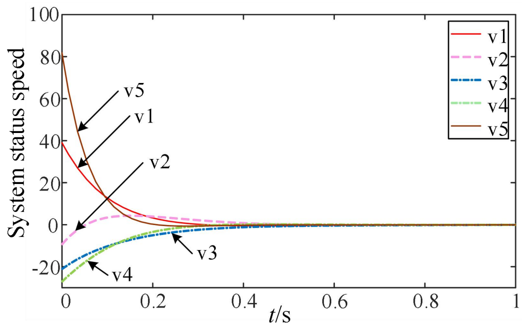

Fig. 9. Weak intervention group consistency state velocity diagram

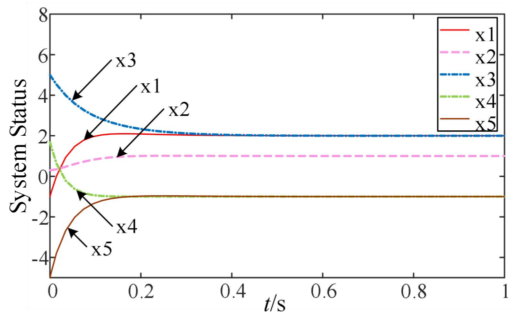

Fig. 10. Consistency state diagram for strong intervention groups

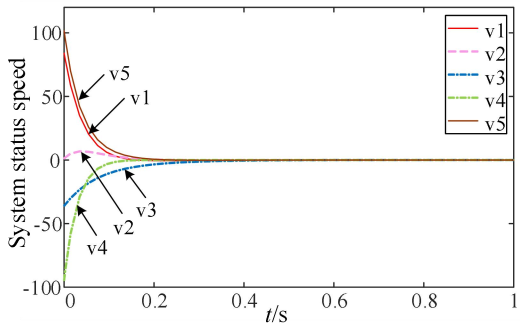

Fig. 11. Consistent state velocity diagram for strong intervention groups

In addition, it can be seen from Fig. 8 that the weak intervention multi-agent system achieves group consistency for the multi-agent system at around 0.5s, while it can be seen from Fig. 10 that the strong intervention system achieves group consistency at around 0.3s. Therefore, the convergence rate of group consistency is faster for the strong intervention system.

The convergence speed of the multi-agent system's grouping consistency can also be assessed by comparing the system state speed graphs in Figs. 9 and 11. In the process of system grouping consistency convergence, each state speed of the strong intervention system shown in Fig. 11 is greater than the corresponding state speed of the weak intervention system shown in Fig. 9. Then the multi-agent grouping consistency convergence speed can be improved by intervention control, and the stronger the intervention control rate, the faster the grouping consistency convergence speed. Therefore, since the directed topology remains fixed, the system's group consistency convergence speed can be improved by adjusting the state feedback matrix.

V. Conclusion

To improve the consensus convergence speed of directed topology of multi-agent systems and make the use conditions of consensus control more general. Consensus and group consensus of nonlinear time-varying of the directed topology node algebraic connectivity of the system with intervention are studied in this paper and the convergence sufficient condition is given. Finally, simulation experiments verify the correctness of the proposed convergence conditions, and the effectiveness of the intervention control in enhancing the convergence speed of system consistency and group consistency is demonstrated through comparative experimental results.

References

[1] Rui Wang, Xiwang Dong, Qingdong Li, et al, “Distributed Time-Varying Formation Control for Multiagent Systems With Directed Topology Using an Adaptive Output-Feedback Approach,” *IEEE Transactions on Industrial Informatics*, Vol. 15, no. 8, pp. 4676-4685, 2019.

[2] A. J. Smola and B. Scholkopf, “A tutorial on support vector regression,” *Statistics and Computing*, vol. 14, no. 3, pp. 199-222, 2004.

[3] Jiangping Hu, Zhixin Liu, Jinhuan Wang, et al, “Estimation, Intervention and Interaction of multi-agent systems,” *Journal of Automation*, vol. 39, no. 11, pp. 1796-1804, 2013.

[4] J. Cortes, M. Egerstedt, “Coordinated control of multi-robot systems: A Survey,” *SICE Journal of Control, Measurement, and System Integration*, vol. 10, no. 6, pp. 495-503, 2017.

[5] Qinghua Luo, Chao Liu, Xiaozhen Yan, et al, “A distributed localization method for wireless sensor networks based on anchor node optimal selection and particle filter”, *Sensors*, vol. 22, no. 3, pp. 1003-1003, 2022.

[6] Chenglin Liu and Yuping Tian, “A review of consistency problems for multi-individual systems with time delays,” *Control and Decision Making*, vol. 24, no. 11, pp. 1601-1608, 2009.

[7] Lun Zhang, Yan Lu, Lan Chen, et al, “Evaluating Strategy with Grey Theory Utilized in Wireless Sensor Networks,” *Lecture Notes in Engineering and Computer Science*, vol. 2170, no. 1, pp. 805, 2008.

[8] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520-1533, 2004.

[9] R. Olfati-saber, “Distributed Kalman filtering for sensor networks,” presented at the 46th IEEE Conference on Decision and Control, New Orleans, LA, Dec. 12-14, 2007.

[10] R. Olfati-Saber, “Ultrafast consensus in small-world networks,” *Proceeding of the American Control Conference*, vol. 4, pp. 2374-2378, 2005

[11] Nosrati S, Shafiee M. “Time-Delay dependent stability robustness of small-world protocols for fast distributed consensus seeking,” presented at the 5th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks and Workshops, Limassol, CY, Apr. 16-20, 2007.

[12] Lin Xiao and Stephen Boyd, “Fast linear iterations for distributed averaging,” *Systems and Control Letters*, vol. 53, no. 1, pp. 65-78, 2004.

[13] Y. Kim and M. Mesbahi, “On maximizing the second smallest eigenvalue of a state-dependent graph laplacian,” *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 116-120, 2006.

[14] Qingwen Ma, Xin Xu, Ronghua Zhang, et al, “Robust consensus control of nonlinear multi-agent systems based on convergence rate estimation,” *International Journal of Robust and Nonlinear Control*, vol. 33, no. 3, pp. 2003-2021, 2022.

[15] Tao Li, Minyue Fu, Lihua Xie, et al, “Distributed consensus with limited communication data rate,” *IEEE Transactions on Automatic Control*, vol. 56, no. 2, pp. 279-292, 2011.

[16] Tao Li and Jifeng Zhang. “Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises,” *IEEE Transactions on Automatic Control*, vol. 55, no. 9, pp. 2043-2057, 2010.

[17] Amirkhani, Abdollah, Barshooi, et al, “Consensus in multi-agent systems: a review,” *Artificial Intelligence Review*, vol. 55, no. 5, pp. 1-39, 2021.

[18] Jiahao Dai, Jing Wen Yi and Li Chai, “Accelerating the convergence rate of consensus for second-order multi-agent systems by memory information,” *Automatica*, vol. 166, pp. 111727-, 2024.

Tengfei Wei The corresponding author. He is currently an associate professor in the School of Electrical and Information Engineering, Lanzhou University of Technology. His research interests include intelligent control and information processing, instrumentation and automation equipment, power electronics, smart microgrid system, distributed power systems, photovoltaic power generation control technology, ARM and FPGA.

Shihua Ma He is a graduate student of College of Electrical and Information Engineering, Lanzhou University of Technology. His main research direction is the optimal control of hybrid energy storage.

Rui Hao He is a graduate student of College of Electrical and Information Engineering, Lanzhou University of Technology. His main research direction is the study of the coherence of time-varying multi-agent systems.