Application of Least Squares Approximation to Predict Sales Outcomes at Bandar Rajut CR04 Bandung Knitted Store

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***Abstract*—Over the past 10 years, the popularity of online shopping is experiencing a drastic increase and has become a preferred mode of transaction for many customers. Among various online sales product categories, fashion retail has the largest sales volume. In this regard, entrepreneurs need accurate sales predictions to manage finances and develop business strategies. Therefore, this study aimed to propose a prediction model for the sales outcomes of Bandar Rajut CR04 Bandung, an online fashion store that sells knitted goods. The model was built using least squares polynomial fitting of degree 1-4. The results showed that the degree 3 and 4 fits fail to have a stable linear system, evidenced by the condition numbers. The modified Legendre and Chebyshev polynomials were used to improve the stability of the linear system for solving $La=b$. The results showed that these methods could give a more stable linear system with smaller condition numbers that allow for obtaining a more accurate solution. This system enabled the models to be used for predicting sales outcomes of Bandar Rajut CR04 Bandung with a highly reliable result.**

***Index Terms*—least squares approximation, sales outcome forecasting, online sales, fashion retail**

I. Introduction

OVER the past 10 years, online shopping is experiencing significant growth in popularity and has taken over as the preferred method of many consumers [1]. The significant development in e-commerce has shifted the business landscape by centralizing primary activities around online transactions and services facilitated by information technology [2]. This transformation has displaced traditional business practices and introduced a new model enabling global transactions without physical meetings, altering the strategies and operations of companies [2]. E-commerce provides easy access to product information, low prices, and attractive promotions, making the method an appealing option for shoppers [1].

Fashion retail has the largest sales volume among the various online product categories [1]. This shows a great interest from consumers in fashion products in e-commerce environment. According to a previous study, product quality, such as the type of fabric used, is a major factor affecting consumer behavior [3]. The quality of goods sold by e-commerce has a close relationship with financial arrangements. This includes calculating costs related to manufacturing, production, or other resources needed to develop high-quality products. In some cases, financing higher-quality products requires higher costs, and managing the finances of an e-commerce business can be challenging. Therefore, entrepreneurs need to consider sales outcome predictions when managing the company finances.

Accurately predicting sales outcomes is important for planning and strategy, resource allocation, marketing, logistics, and supply chain management of a company [4]. The result of a previous study showed that accurate predictions could assist companies in making more precise decisions [5]. Similarly, inaccurate predictions can lead to errors in financial planning, inventory management, and decision-making.

Least squares approximation is a numerical method used to forecast sales outcomes. This method enables business owners to adjust regression models based on past sales data. For instance, Kasmi et al. predicted the sales or demand for ornamental fish exports in Rezky Bahari Commandiatire Vennotschaap (CV) using the linear least squares approximation, and the result showed a high level of prediction accuracy [5]. Chen et al. adjusted and predicted flight data using polynomial least squares and found that "fourth-order polynomial least squares" had advantages and were simpler in data correction and adjustment [6]. Walangadi and Kumala developed a motorcycle sales prediction program at Hasjrat Abadi Gorontalo Ltd. using linear least squares, and the results showed that the prediction results were accurate [7]. In addition, Vibha et al. applied this method in comparing the results of video segmentation in Multimedia Mining [8]. Zhang et al. also obtained the inverse operation of dictionary matrix update in the reconstruction of the super-resolution polymorphic sparsity regularized image [9].

The result of these previous studies showed that least squares approximation is an effective tool for predicting various factors. This study applied least squares approximation to predict the sales outcomes of Bandar Rajut CR04 Bandung, an online fashion knitted store in Bandung, Indonesia. The historical sales data were analyzed to develop

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an accurate prediction model. The results of this study are expected to provide insights that can help companies, such as Bandar Rajut CR04 Bandung predict future sales outcomes, as well as carry out financial planning and determine company strategies.

II. Materials and Methods

A. Sales Outcomes Data

The data used was monthly online sales outcomes of knitted fashion products in Bandar Rajut CR04 Bandung that has been processed, as shown in Table I. From the dataset, the sequence of the months was the dependent variables *x*, which are evenly spaced in the interval [0,1], and sales outcomes (in IDR) were independent variables *y*, as shown in Table II.

TABLE I
SALES OUTCOMES DATA

Month	Sales Outcomes (IDR)
Aug-22	79374470
Sep-22	70151681
Oct-22	65702116
Nov-22	77134699
Dec-22	68238980
Jan-23	57729313
Feb-23	51542381
Mar-23	64676187
Apr-23	37541777
May-23	49605423
Jun-23	55306225
Jul-23	52985409
Aug-23	50154798
Sep-23	42098734

B. Forecasting

Forecasting is a process that combines quantitative elements, such as data and models, with human intervention across different stages, including the development of forecasts and the use in decision-making [10]. A previous study reported that forecasting was intrinsically related to decision-making [11]. Prediction helps agents make informed decisions in uncertain situations [11]. The fundamental principle of forecasting theory is the belief that predictions can be constructed based on data from the past and present [10]. When different projections lead to decisions, forecast inaccuracies will cost decision-makers [11]. Forecasting can also help identify potential market opportunities in the future by anticipating sales or demand [5]. Riabacke explored the decision-making of managers under risk and uncertainty [12].

C. Knitted Fashion Products

Driven by style, the fashion industry is dominantly important as the largest consumer market [13]. Style plays a significant role in determining the launch of new items and is a key factor in revitalizing product portfolios [13]. For entrepreneurs in the fashion sector, keeping up with trends and changes is important, specifically since the product lifecycle is often only a few months. In 2019, knitting products were in high demand [14], but the trend began to decline over time. Meanwhile, Bandar Rajut CRO4 Bandung is an online shop in the fashion sector that was established in 2016. This store sells a variety of knitted products, such as sweaters, bags, cardigans, and others.

D. Least Squares Approximation

Least squares approximation is a commonly used method in numerical methods, statistics, and time series analysis to forecast trends as well as make predictions based on historical data. This method aims to find the line or curve that best fits the existing data pattern by minimizing the root of mean squared errors between the observed and predicted value. Optimal estimates about trends or patterns in historical data can be estimated with this method for future analyses or predictions.

Suppose $\widehat{f}(x)$ are members of a class of function *D* that approximate to the data $\{(x_i, y_i)\}$ with $i = 1, ..., n$. The root of mean squared errors between the observed value and the predicted value is expressed as [15]

$$E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[\widehat{f}(x_i) - y_i]^2}. \tag{1}$$

The function $\widehat{f^*}(x)$ that minimizes *E* relative to all $\widehat{f}(x)$ in *D*, is called the least squares approximation/ fit to the data $\{(x_i, y_i)\}$.

Let $\widehat{f}(x)$ in *D* be polynomial functions of degree (*m*-1) written as follows:

$$\widehat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + \cdots + a_m\varphi_m(x) \tag{2}$$

These functions can be written in standard form as follows:

$$\widehat{f}(x) = a_1 + a_2 x + ... + a_m x^{m-1} \tag{3}$$

with

$$\varphi_i(x) = x^{i-1}, \qquad i = 1, \ldots, m.$$

Minimizing *E* is equivalent to minimizing the sum in (1), even though the values of the minimum will be different, namely to minimize the function:

$$H(a_1, a_2, \cdots, a_m) = \sum_{i=1}^{n}[a_1 + a_2 x_i + ... + a_m x_i^{m-1} - y_i]^2 \tag{4}$$

where $\{a_i\}$ are unknown numbers, and the resulting choice of $\{a_i\}$ will also minimize *E*.

Using multivariable calculus the values of $\{a_i\}$ that minimize (4) will satisfy

$$\frac{\partial H(a_1, a_2, \cdots, a_m)}{\partial a_i} = 0, \qquad i = 1, \cdots, m$$

Taking the partial derivatives of $H(a_1,a_2,\cdots,a_m)$ and equating to zeros gives the following normal equations:

$$\sum_{k=1}^{m} a_k[\sum_{j-1}^{n} x_j^{\,i+k-2}] = \sum_{j=1}^{n} y_j x_j^{\,i-1},\ \ i=1,2,...,m. \tag{5}$$

In the matrix form, the normal equation can be expressed as

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^{m-1} \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^{2} & \cdots & \sum_{i=1}^{n} x_i^{m} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^{m-1} & \sum_{i=1}^{n} x_i^{m} & \cdots & \sum_{i=1}^{n} x_i^{2(m-1)} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^{m-1} y_i \end{bmatrix}$$

$$La = b \tag{6}$$

where

$$L = \begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^{m-1} \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^{2} & \cdots & \sum_{i=1}^{n} x_i^{m} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^{m-1} & \sum_{i=1}^{n} x_i^{m} & \cdots & \sum_{i=1}^{n} x_i^{2(m-1)} \end{bmatrix}, a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}, b = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^{m-1} y_i \end{bmatrix}.$$

Solving $La = b$ for a, results in the least squares approximation $\widehat{f^*}(x)$ that minimizes E. The solution vector a can be written as:

$$a = L^{-1} b \tag{7}$$

The normal Equations (5) or (6) of least squares fits for degree greater than two are known as ill-conditioned. Therefore, orthogonal polynomials were required to obtain a well-behaved system of linear equations discussed in the following 2 sections.

E. Legendre Polynomials

Legendre polynomials have some nice properties that enable obtaining an accurate solution of Equation (6). These polynomials follow the triple recursion relation defined as follows:

$$\begin{aligned} &P_0(x) = 1,\ P_1(x) = x \\ &P_{n+1}(x) = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} P_{n-1}(x), \quad n \geq 1 \end{aligned} \tag{8}$$

By introducing

$$(f, g) = \int_a^b f(x) g(x) dx$$

for general functions $f(x)$ and $g(x)$, the orthogonality property was observed, namely:

TABLE II
LEAST SQUARES FITS DATA

x_i	y_i
0.07	79374470
0.14	70151681
0.21	65702116
0.29	77134699
0.36	68238980
0.43	57729313
0.50	51542381
0.57	64676187
0.64	37541777
0.71	49605423
0.79	55306225
0.86	52985409
0.93	50154798
1.00	42098734

$$(P_i, P_j) = \begin{cases} 0, & i \neq j \\ \dfrac{2}{2j+1}, & i = j \end{cases}$$

Therefore, the equations can be used as a basis allowing for every polynomial $p(x)$ of degree $\leq n$ to be written in the form

$$p(x) = \sum_{i=0}^{n} \alpha_i P_i(x)$$

with the coefficients $\alpha_0, \alpha_1, \cdots, \alpha_n$ uniquely determined from $p(x)$.

F. Chebyshev Polynomials

Chebyshev polynomials have the same properties as Legendre polynomials. The triple recursion relation owned by Chebyshev polynomials is defined as:

$$\begin{aligned} &T_0(x) = 1,\ T_1(x) = x \\ &T_{n+1}(x) = 2x T_n(x) - T_{n-1}(x), \quad n \geq 1 \end{aligned} \tag{9}$$

where

$$T_n(x) = \cos(n\theta),\ \ 0 \leq \theta \leq \pi.$$

The equations then have the minimum size property as follows:

$$|T_n(x)| \leq 1,\ \ -1 \leq x \leq 1$$

for all $n \geq 0$.

The orthogonality property owned by Chebyshev polynomials enables the use as basis for every polynomials $p(x)$ of degree $\leq n$ with a stable linear system of Equations (6).

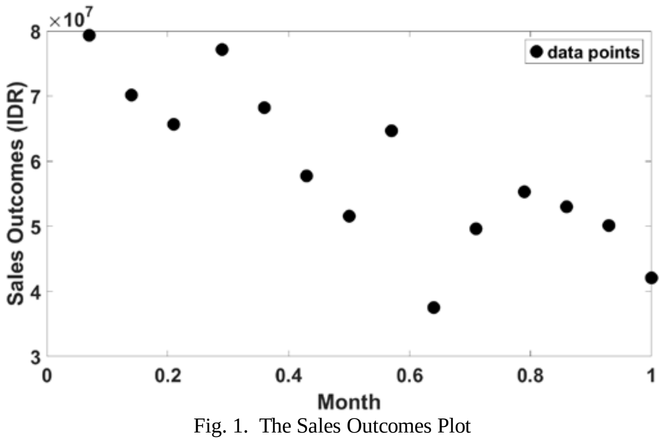

Fig. 1. The Sales Outcomes Plot

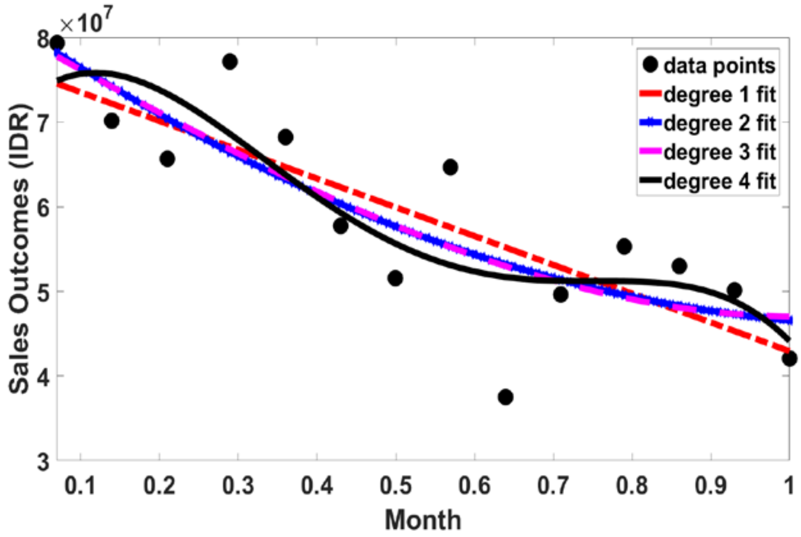

Fig. 2. The least squares fits of degree 1 to 4 approximating the data.

III. Results and Discussion

Based on the dataset in Table I, the sequence of the months becomes the dependent variables x, which are evenly spaced in the interval [0,1]. Meanwhile, sales outcomes (in IDR) become independent variables y (Table II), as shown in Figure 1.

The result in Figure 1 showed that sales outcomes fluctuated every month, but decreased over time. Furthermore, least squares fits of degree 1, 2, 3, and 4 were determined for the sales outcomes data.

For the fit of degree 1, the system of Equations (6) can be written as

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix} \tag{10}$$

The coefficients a_1 and a_2 can be written as follows:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix} \tag{11}$$

Solving (10) for the vector a, requires specifying the values of $\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i^2, \sum_{i=1}^{n} y_i$, and $\sum_{i=1}^{n} x_i y_i$. Calculating these values and substituting into (10) gives least squares fits of degree 1 as follows:

$$\hat{f}(x) = 76948428 - 34004774x.$$

Least square fits of degree 2 to 4 are given respectively as follows:

Degree 2 fit:

$$\hat{f}(x) = 82536825 - 63430575x + 27477810x^2$$

Degree 3 fit:

$$\hat{f}(x) = 81456123 - 52933550x + 3687374x^2 + 14836656x^3$$

Degree 4 fit:

$$\hat{f}(x) = 69341755 + 121738860x - 673897756x^2 + 975560811x^3 - 448592918x^4$$

All the degree 1 to 4 fits of the standard form (3) are shown in Figure 2, while the error values obtained by using Equation (1), as well as the condition numbers are shown in Table III.

TABLE III
RMS Errors and Condition Numbers of Least Squares Fits

Degree	RMS Error of Standard Form	RMS Error of Modified Legendre	RMS Error of Modified Chebyshev	Condition Number of Standard Form	Condition Number of Modified Legendre	Condition Number of Modified Chebyshev
One	3.69099e+6	1.13684e+7	1. 16364e+7	2.04867e+1	3.04624	3.04624
Two	3.65111e+6	1. 14227e+7	1. 08064e+7	5.98605e+2	5.57878	4.87042
Three	6.12931e+6	1.17656e+7	1. 09442e+7	2.0171e+4	9.79645	8.65201
Four	6.353181e+6	1.13954e+7	1. 13706e+7	7.13982e+5	17.59002	16.15024

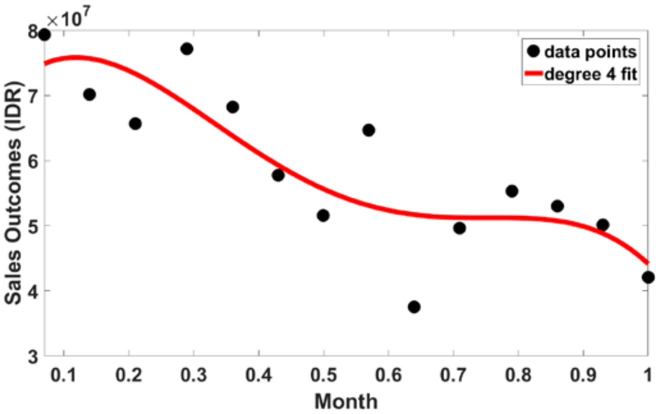

Fig. 3. The least squares fit of degree 4 approximating the data.

According to Table III, the condition numbers of the degree 3 and 4 fits are very large, namely 20,171 and 713,982 respectively. These results showed possible difficulty in obtaining an accurate solution for $La = b$.

For the degree 4 fit (see Figure 3) the obtained linear system (5), defined here by $La = b$, is given by

$$L = \begin{bmatrix} 14 & 7.5 & 5.1844 & 4.0266 & 3.3349 \\ 7.5 & 5.1844 & 4.0266 & 3.3349 & 2.876 \\ 5.1844 & 4.0266 & 3.3349 & 2.8763 & 2.550 \\ 4.0266 & 3.3349 & 2.8763 & 2.550 & 2.3077 \\ 3.3349 & 2.8763 & 2.550 & 2.3077 & 2.1198 \end{bmatrix}$$

$$a = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{bmatrix}^T$$

$$b = \begin{bmatrix} 822242193 & 400818863 & 264130657 & 199910032 & 162917052 \end{bmatrix}^T$$

The solution is

$$a = \begin{bmatrix} 69341755 & 121738860 & -673897756 & 975560811 & -448592918 \end{bmatrix}^T$$

with

$$\operatorname{cond}(L) = \|L\|\|L^{-1}\| = 713{,}982 \cdot \tag{12}$$

To verify the possible difficulty in obtaining the right solution for $La = b$, b was perturbed by augmenting to the perturbation

$$\begin{bmatrix} 0.01 & -0.01 & 0.01 & -0.01 & 0.01 \end{bmatrix}^T$$

The solution of the new perturbed system is

$$a = \begin{bmatrix} 69341760 & 121738782 & -673897464 & 975560406 & -448592732 \end{bmatrix}^T$$

with errors

$$a - a_{pert} = \begin{bmatrix} -5.607 & 77.696 & -292.082 & 404.412 & -185.422 \end{bmatrix}^T \tag{13}$$

which is different from the earlier result for a. This is due to the use of the standard form (3) leading to a rather ill-conditioned system of linear equations.

The modified Legendre polynomials defined on $[\alpha,\beta]$ was used to obtain the degree 4 least squares fit for the data in Table II, with a better behaved-linear system, as follows:

$$\varphi_k(x) = P_{k-1}\left(\frac{2x-\alpha-\beta}{\beta-\alpha}\right), \quad \alpha \le x \le \beta, \quad k \ge 1 \tag{14}$$

where the nodes $\{x_i\}$ are chosen from an interval $[\alpha,\beta]$, and wrote

$$\hat{f}(x) = a_1\varphi_1(x) + a_2\varphi_2(x) + a_3\varphi_3(x) + a_4\varphi_4(x) + a_5\varphi_5(x) \tag{15}$$

With $[\alpha,\beta] = [0,1]$, then

$$\varphi_1(x) = P_0(2x-1) = 1$$

$$\varphi_2(x) = P_1(2x-1) = 2x-1$$

$$\varphi_3(x) = P_2(2x-1) = \frac{1}{2}\left(3(2x-1)^2 - 1\right)$$

$$\varphi_4(x) = P_3(2x-1) = \frac{1}{2}\left(5(2x-1)^3 - 3(2x-1)\right)$$

$$\varphi_5(x) = P_4(2x-1) = \frac{1}{8}\left(35(2x-1)^4 - 30(2x-1)^2 + 3\right)$$

The linear system minimizing E relative to all $\hat{f}(x)$ has the form

$$\sum_{k=1}^{m} a_k \left[\sum_{j=1}^{n} \varphi_k(x_j)\varphi_i(x_j)\right] = \sum_{j=1}^{n} y_j \varphi_i(x_j), \quad i = 1,\ldots,m \tag{16}$$

which is also denoted by $La = b$, and giving

$$L = \begin{bmatrix} 14 & 1 & 0.1064 & 1 & 0.3164 \\ 1 & 4.7376 & 1 & 0.2264 & 1 \\ 0.1064 & 1 & 2.9931 & 1 & 0.2870 \\ 1 & 0.2264 & 1 & 2.2603 & 1 \\ 0.3164 & 1 & 0.2870 & 1 & 2.0087 \end{bmatrix}$$

$$b = \begin{bmatrix} 822242193 & -20604467 & 2112959 & 61865087 & -5586773 \end{bmatrix}^T$$

The solution is

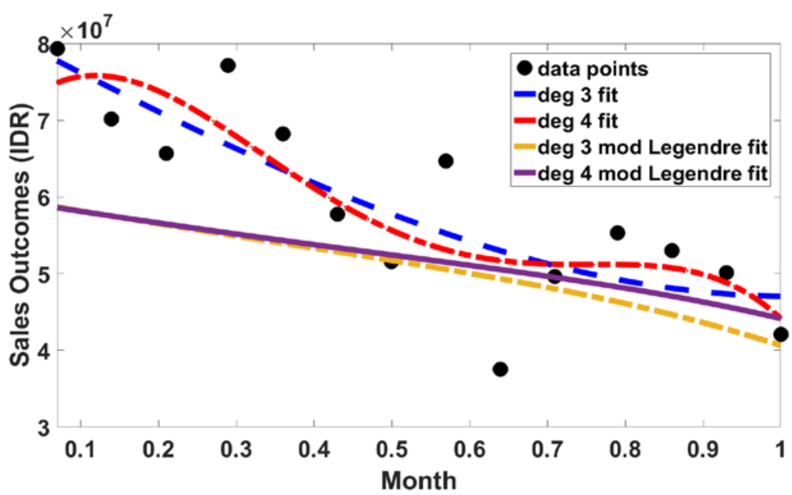

Fig. 4. Plots of actual data and the least squares fits of degree 3 and 4 for the standard form and the modified Legendre.

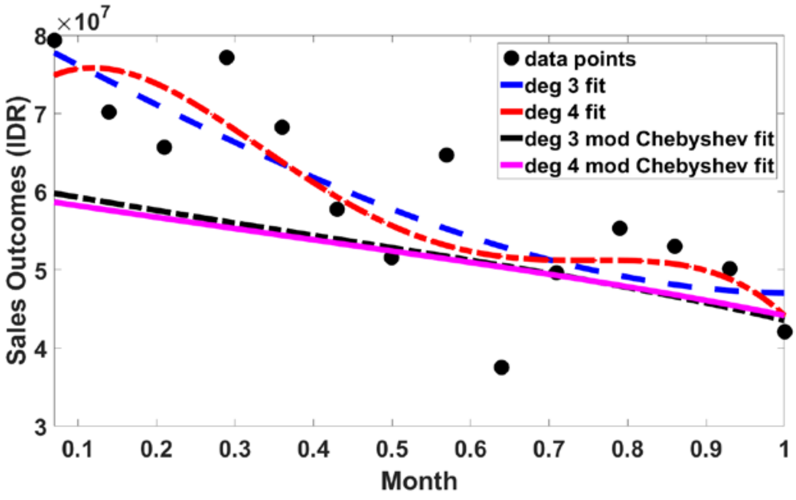

Fig. 5. Plots of actual data and the least squares fits of degree 3 and 4 for the standard form and the modified Chebyshev.

$$a = [59750219 \quad -16514250 \quad 3404505 \quad 3918749 \quad -6408470]^T$$

with

$$\text{cond}(L) = \|L\|\|L^{-1}\| = 17.59002$$

This value was significantly smaller than the previous results in (12). Similarly, giving the same perturbation to b results in the solution of the new perturbed system with errors

$$a - a_{pert} = [-0.00187 \quad 0.0067 \quad -0.00912 \quad 0.01486 \quad 0.01411]^T$$

This value was significantly different from the previous results (13). By taking the norms of the errors, the standard form and the modified Legendre give the following values:

$\|a - a_{pert}\| = 404.412$ and $\|a - a_{pert}\| = 0.01486$.

For the other orthogonal polynomial, the modified Chebyshev polynomials were defined on [0,1] as follows:

$$\varphi_k(x) = T_{k-1}(2x-1), \quad 0 \le x \le 1, \quad k \ge 1$$

where the nodes $\{x_i\}$ are chosen as in Table II, and denote

$$\varphi_1(x) = T_0(2x-1) = 1$$
$$\varphi_2(x) = T_1(2x-1) = 2x-1$$
$$\varphi_3(x) = T_2(2x-1) = 2(2x-1)^2 - 1$$
$$\varphi_4(x) = T_3(2x-1) = 4(2x-1)^3 - 3(2x-1)$$
$$\varphi_5(x) = T_4(2x-1) = 8(2x-1)^4 - 8(2x-1)^2 + 1$$

The linear system $La = b$ minimizing E relative to all $\hat{f}(x)$ (15) gives

$$L = \begin{bmatrix} 14 & 1 & -4.5248 & 1 & -0.4359 \\ 1 & 4.7376 & 1 & -2.4803 & 1 \\ -4.5248 & 1 & 6.7821 & 1 & -2.2051 \\ 1 & -2.4803 & 1 & 7.0573 & 1 \\ -0.4359 & 1 & -2.2051 & 1 & 7.6503 \end{bmatrix}$$

$$b = [822242193 \quad -20604467 \quad -271263452 \quad 111346820 \quad -66641834]^T$$

The solution is

$$a = [59700154 \quad -15044720 \quad 550732 \quad 2449218 \quad -3504632]^T$$

The modified Chebyshev has the smallest condition numbers among all the degree 2-4 least squares fits discussed in this study, as shown in Table III. For example, the degree 4 modified Chebyshev has the smallest condition number of 16.15024. This degree 4 fit also has the smallest norm of perturbation error, which is of the value

$$\|a - a_{pert}\| = 0.01227$$

Figures 4 and 5 show the plots of the degree 3 and 4 least squares fits for the modified Legendre and Chebyshev respectively, as well as the standard form. The modified Legendre and Chebyshev have larger Root Mean Squared (RMS) errors than the standard form due to the existence of an outlier at $x_9 = 0.64$. However, the smaller condition numbers reflect the stable linear system for the solution of $La = b$. Therefore, a small change to the b values does not significantly affect the solution value a.

The study found that the high-degree least squares fits (degree > 2) diverge rapidly outside the given interval, in contrast to the modified Chebyshev and Legendre fits, as shown in Figures 6 and 7. This results shows that many studies used only a linear fit in terms of least squares approximation to avoid the diverging property for extrapolation outside the given interval. The orthogonal polynomials, such as the modified Legendre and Chebyshev were able to avoid this problem. Therefore, orthogonal polynomials can solve the linear system in terms of least squares approximation with a reliable solution.

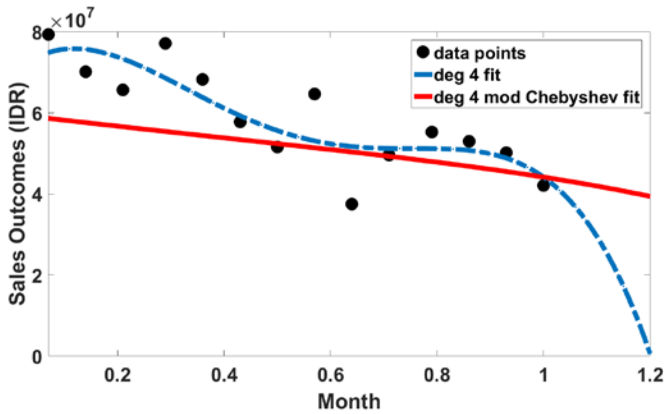

Fig. 6. Comparison of extrapolation between the degree 4 least squares fit and the degree 4 modified Chebyshev fit.

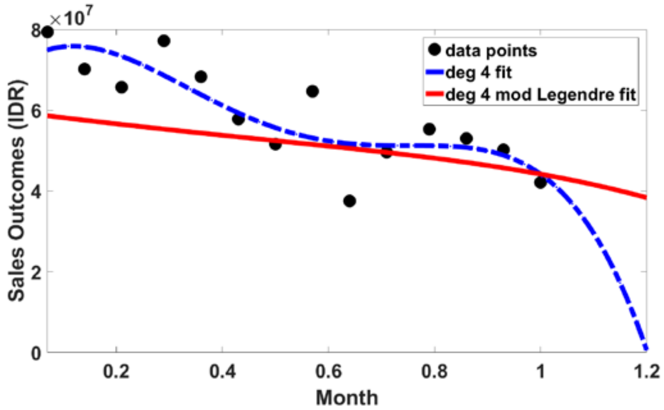

Fig. 7. Comparison of extrapolation between the degree 4 least squares fit and the degree 4 modified Legendre fit.

IV. Conclusions

In conclusion, least squares fits of degree more than 2 were unable to obtain a reliable solution due to the ill-behaved linear system. Therefore, modified orthogonal polynomials, such as modified Legendre and Chebyshev polynomials, were required. The improved linear system could be used to predict sales outcomes in Bandar Rajut CR04 Bandung with a reliable solution.

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