A One-Dimensional Numerical Simulation of Oil Spill Control in a Coastal Bay Using a Fourth-Order Explicit Finite Difference Method

Teerat Kasamwan, Nopparat Pochai

Abstract—**Oil spills in the sea have both short-term and long-term consequences that need proper management and restoration. The damage can take years or even decades to recover fully. Methods like absorbents, dispersants, bioremediation, mechanical recovery, and in-situ burning are used to mitigate the impacts of oil spills. Each method has its limitations and should be chosen carefully based on the severity of the spill to minimize environmental damage and restore marine ecosystems effectively. This research considers a one-dimensional mathematical model for an oil spill in a coastal bay, incorporating delayed removal mechanisms. The governing equation for an oil spill in this coastal bay context with delayed removal is introduced, alongside the initial condition and boundary conditions associated with oil spill scenarios. A mathematical model is proposed to simulate delayed removal mechanisms. The model solutions are approximated using a fourth-order forward time-centered space finite difference method. The simulations explore two scenarios: instant and delayed removal mechanisms. In the instant removal scenarios, simple average rates of oil removal and basic water flow behaviors are modeled, while the delayed removal scenarios simulate more realistic oil spill conditions. Consequently, the concentration of oil relative to source rate over time is analyzed. The simulations reveal that as the efficiency of the removal mechanism improves, the oil concentration decreases over time. Physically, this reflects that effective management of oil removal leads to a progressive reduction in oil concentration as time advances. According to the research, oil spill concentration is reduced when oil removal mechanisms are more effective. By contrasting a second forward time center space technique and a fourth-order forward time center space technique, it shows the significance of selecting the most effective method for a given simulation circumstance. The simulation results indicate that the concentration associated with the delayed removal mechanism yields less favorable recovery outcomes compared to the prompt removal mechanism across all scenarios. This observation is consistent with the fundamental principle that effective oil spill management should result in a reduction in oil concentration within marine environments. The findings of this study underscore that, in all cases, postponed oil removal exacerbates the detrimental impact on seawater recovery relative to expeditious removal. Consequently, the prompt and efficient removal of oil spills is imperative in mitigating the extent of oil contamination in marine waters.**

Index Terms—**Oil Spill, Coastal bay, Finite difference method, FTCS, High-Order Accuracy**

Manuscript received October 31, 2024; revised March 8, 2025.

This work was supported by King Mongkut's Institute of Technology Ladkrabang.

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I. Introduction

OIL spills in the ocean pose a severe environmental threat and have widespread impacts on marine ecosystems, including marine life, coastal areas, and economies that rely on marine resources. The effects of oil spills are not limited to the immediate aftermath but also result in long-term consequences, which require extensive time and resources for full recovery.

Starting with the impact on marine life, oil floating on the water's surface hinders the ability of sea creatures to breathe and move normally. Marine animals living near the surface may inhale or come into contact with oil and chemicals, leading to respiratory and digestive issues. Seabirds that fly and feed near the surface are at risk of having their feathers coated with oil, which strips their feathers of their waterproofing and temperature-regulating abilities, making it impossible for them to fly and potentially leading to death from exposure to cold.

Moreover, coral reefs, which are rich ecosystems with high biodiversity, often suffer severe damage from oil spills. Oil coating coral reefs disrupts photosynthesis and gas exchange, halting their growth and increasing the risk of coral death. Seagrass beds, another critical component of the marine ecosystem, are similarly affected. Oil covering the surface of seagrass limits photosynthesis, slowing growth, and degrading areas that serve as food sources for some marine species.

The impact on water quality and marine sediment is another significant concern. Oil spills drastically reduce water quality as oil forms a slick on the surface and contaminates the water with harmful chemicals that are toxic to marine life. Additionally, oil that settles into the seabed accumulates in the sediment, becoming a long-term source of toxins. Organisms living in the seabed and coastal areas are directly and indirectly affected by the oil contamination in these sediments.

The economic effects of oil spills are equally devastating, particularly for the fishing and tourism industries. Oil-contaminated marine resources lead to declining fish populations, reducing catch volumes, and potentially contaminating seafood, posing health risks to consumers. Beaches covered in oil become unsuitable for tourism, severely damaging the local tourism industry.

Ultimately, the environmental impacts of oil spills can persist for years. Oil residues left in the sea or along the coastline may take years to be fully removed, and in some cases, the damage may be irreversible, slowing or even preventing the recovery of ecosystems. Affected ecosystems may lose food sources and habitats for marine animals, causing a significant imbalance in the natural environment.

In 1994, W.D. Henshaw [1] A method is presented for solving the time-dependent incompressible Navier-Stokes equations using finite differences on curvilinear overlapping grids in two or three space dimensions. In 2001, W.F. Spotz and G.F. Carey [2] introduced an extension of our prior approaches for steady-state higher-order compact (HOC) difference methods, adapting them to tackle time-dependent problems. Numerical experiments are provided to demonstrate the stability, accuracy, and the oscillatory and dissipative behavior of the methods. In 2005, Y. Zlochower et.al [3] introduced techniques for conducting numerical relativity simulations of binary black holes with fourth-order accuracy. These simulations are implemented within a new coding framework that supports higher-order finite differencing for the Baumgarte-Shapiro-Shibata-Nakamura formulation of Einstein's equations. They calculates gravitational waveforms and demonstrates substantial improvements in waveform accuracy over second-order methods at commonly achievable numerical resolutions.

In 2006, S. Krenk [4] presented a fourth-order accurate time integration algorithm with exact energy conservation for linear structural dynamics. This algorithm is derived by integrating the phase-space representation and evaluating the displacement and velocity integrals through integration by parts, substituting time derivatives from the original differential equations. The method demonstrates unconditional stability, with a fourth-order relative phase error. In 2010, L. Rusu [5] presented the results of a straightforward yet effective model system, ISSM. Additionally, simulations were conducted using the SHORECIRC model as an alternative approach. Lastly, as a case study, the potential spread of pollution toward the Romanian coast, stemming from a hypothetical accident at the Gloria drilling platform, was evaluated. In 2011, J.B.Chen [6] derived a stability formula for Lax-Wendroff methods with fourth-order accuracy in time and arbitrary-order accuracy in space. Additionally, He proved the instability of methods that apply high-order finite-difference approximations directly to the second temporal derivative, thereby addressing and resolving Bording's conjecture.

In 2012, B. Sjögreen and N.A. Petersson [7] presented a fourth-order accurate finite difference method for the two-dimensional elastic wave equation in its second-order formulation. This fourth-order discretization of second derivatives can also be applied to achieve stable, fourth-order accurate discretizations of other partial differential equations beyond the elastic wave equation. In 2015, S. Krenk [8] developed by integrating the differential state-space equations of motion over the time increment and evaluating the resulting time integrals for the inertia and stiffness terms using integration by parts. In 2017, A. Yaghoubi [9] addressed the one-dimensional advection-diffusion equation (ADE) using a high-order finite difference approach and compare the computed results to the exact solution. In 2022, M.E. Ali [10] have implemented three different finite difference methods to solve a one-dimensional diffusion equation and analyzed their resulting computations. This investigation may assist researchers in identifying more precise and stable schemes, contributing to the improved numerical solution of diffusion-based equations.

In 2024, Y. Li et.al [11] developed based on one-dimensional nonlinear shallow water equations, with fuel consumption serving as the source term. The simulation results were validated against experimental data from continuously released n-heptane spill fires, showing strong agreement between the two sets of results. S. Mohammadiun et.al [12] presented the development of a multi-agent decision support system designed to effectively coordinate mechanical containment and recovery (MCR) operations for spilled oil, as well as the management of oily wastewater (OWM). T.H.H. Nguyen et.al [13] developed an integrated model to analyze the Sanchi oil spill event that occurred in the East China Sea in January 2018. The model utilizes results from the Advanced Research Weather Research and Forecasting model (WRF-ARW) for meteorological forecasting and the Princeton Ocean Model (POM) for hydrodynamic simulations.

Water contamination models are detailed through numerical simulations in [14] – [17]. Mathematical models for shoreline evolution with groin structures are presented in [18] – [24], with extensions incorporating the impact of wavelength on structural behavior in the system.

In this paper, we introduce a one-dimensional numerical model to simulate oil spills in a coastal bay, incorporating a delayed removal mechanism.

II. Governing Equation

Modeling oil spills presents several challenges, including the complexity of oil behavior, environmental variability, and limitations in available data. Oil behavior is intricate due to the diverse properties of oil mixtures, which can vary significantly. Additionally, ocean conditions are highly dynamic, with factors like currents, waves, and temperature changing rapidly, all of which influence the movement of oil. Furthermore, the availability of accurate data on oil characteristics, environmental conditions, and specific details of the spill is often limited, making it difficult to create precise models for predicting oil spill behavior.

This research will focus on a basic one-dimensional Eulerian oil spill model, despite the limited availability of data on oil properties, environmental conditions, and spill characteristics.

A one-dimensional dispersion-advection with removal mechanism equation is introduce be Eq.(1)

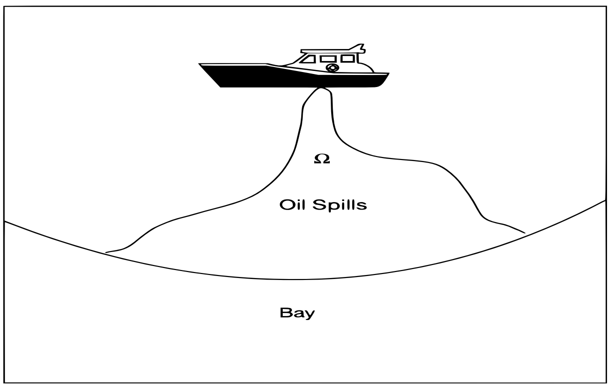

Fig. 1: Oil is spilling into a coastal bay.

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\frac{\partial^2 C}{\partial x^2}+S(t)-Q(t), \tag{1}$$

where C is the oil concentration (mg/L), u is the water flow velocity (m/hr), D is the diffusive tensor (m^2/hr), $S(t)$ is point sourse function that reoresents growing of oil spill concentration (m^3/d) and $Q(t)$ is the removal mechanism (m^3/d).

Initial condition: Due to there is some oil concentration present before it spreads throughout the entire domain,

$$C(x,0)=C_0, \tag{2}$$

where C_0 is given oil spill concentration before it's leaked.

Boundary conditions: assuming that the shoreline will absorb the spilled oil, the absorption boundary can be defined by specific conditions,

$$\frac{\partial C}{\partial x}(L,t)=C_s, \tag{3}$$

where C_s is the absorbance rate of shoreline along simulation time. Assuming that the oil spill point source concentration is represented by

$$C(0,t)=f(t) \tag{4}$$

where $f(t)$ is a interpolation function of oil spilling concentration at the point oil spilling source.

III. A Delayed Removal Mechanism Model

An oil spill occurs when petroleum or other oil-based products are accidentally released into the environment, whether on land, in freshwater, or in the ocean. Such spills can result in severe harm to marine life, wildlife, and the ecosystem. The spill caused extensive damage to the coastline.

There are several methods for addressing oil spills, such as using containment measures like booms or barriers to stop the spread of oil. Skimmers can be used to remove oil from the water's surface. Chemical dispersants can break the oil into smaller droplets, facilitating biodegradation. Bioremediation employs microorganisms to break down the oil naturally. Another approach is burning the oil on the water's surface, though this may lead to air pollution. Manual beach cleanup helps remove oil from shorelines. Environmental restoration, such as planting mangroves, aids in repairing damaged ecosystems. Additionally, monitoring and assessing the long-term impacts on the shoreline is essential.

Delays in responding to oil spills can have severe consequences for both the environment and local economies. Quick and efficient action is crucial to reducing damage when a spill occurs. However, any delay can worsen the situation, resulting in greater environmental destruction.

Common causes of delays in oil spill response include inadequate preparedness, bureaucratic obstacles, limited resources, unfavorable weather conditions, and geographical challenges. These delays in response and the functioning of removal mechanisms can be represented by Q(t) as

$$Q(t)=\begin{cases} 0 & \text{for all} \quad 0\le t\le D_t \\ g(t) & \text{for all} \quad D_t<t\le T \end{cases}$$

where D_t is a delay in the oil spill response period of time.

IV. Numerical Techniques

In this section, we will use the finite difference method to approximate the solution of the one-dimensional advection-diffusion equation, which is a time-dependent problem. We will perform it during $0<t<T$ and on a domain that is a unoform grid: $x_j=j\Delta x$ where $j=1,2,3,\dots,L$ and $t_k=k\Delta t$ where $k=0,1,2,3,\dots,T$.

In this paper, we use an explicit forward-difference approximation for the time dericative (FT) and $4th-$central difference approximation for the space derivation (CS). We called $4th-$forward time centered space ($4th-$FTCS). We use this method for derive the governing equation Eq.(1), we have

$$\begin{aligned}&\frac{C_m^{n+1}-C_m^n}{\Delta t}+\\ &u\left(\frac{-C_{m+2}^n+8C_{m+1}^n-8C_{m-1}^n+C_{m-2}^n}{12\Delta x}\right)=\\ &D\left(\frac{-C_{m+2}^n+16C_{m+1}^n-30C_m^n+16C_{m-1}^n-C_{m-2}^n}{12\Delta x^2}\right)\\ &+S(t)-Q(t).\end{aligned} \tag{5}$$

Rearrange Eq.(5), we get

$$\begin{aligned}C_m^{n+1}=&(-\alpha-\beta)C_{m+2}^n+(8\alpha+16\beta)C_{m+1}^n\\ &+(1-30\beta)C_m^n+(-8\alpha+16\beta)C_{m-1}^n\\ &+(\alpha-\beta)C_{m-2}^n+\Delta t(S^n-Q^n),\end{aligned} \tag{6}$$

where, $\alpha=\frac{u\Delta t}{12\Delta x}$, $\beta=\frac{\Delta tD}{12\Delta x^2}$, $\Delta t=0.01$ and $\Delta x=0.25$, respectively. At the left and right boundaries of the domain, fictitious points appear. Therefore, we eliminate them by using the central space method.

V. Numerical Experiments

In this section, we will discuss the values of the various parameters used in this research. We assume that the quantity and source of the oil spill are known, so the value of the removal mechanism is constant. Additionally, we assume that the water velocity is gradually increasing. We will use Eq.(6) to calculate the numerical results.We will present in case 1.1-1.4

TABLE I: The paremeter is used in case 1.1

Case No.	D	u	Q
1.1.1	1.71×10^{-6}	$0.2556\lvert\sin(0.1t)\rvert$	0.0001
1.1.2	1.71×10^{-6}	$0.2556\lvert\sin(0.1t)\rvert$	10×0.0001
1.1.3	1.71×10^{-6}	$0.2556\lvert\sin(0.1t)\rvert$	20×0.0001
1.1.4	1.71×10^{-6}	$0.2556\lvert\sin(0.1t)\rvert$	40×0.0001
1.1.5	1.71×10^{-6}	$0.2556\lvert\sin(0.1t)\rvert$	80×0.0001

TABLE II: The paremeter is used in case 1.2

Case No.	D	u	Q
1.2.1	1.71×10^{-6}	$10\times0.2556\lvert\sin(0.1t)\rvert$	0.0001
1.2.2	1.71×10^{-6}	$10\times0.2556\lvert\sin(0.1t)\rvert$	10×0.0001
1.2.3	1.71×10^{-6}	$10\times0.2556\lvert\sin(0.1t)\rvert$	20×0.0001
1.2.4	1.71×10^{-6}	$10\times0.2556\lvert\sin(0.1t)\rvert$	40×0.0001
1.2.5	1.71×10^{-6}	$10\times0.2556\lvert\sin(0.1t)\rvert$	80×0.0001

TABLE III: The paremeter is used in case 1.3

Case No.	D	u	Q
1.3.1	1.71×10^{-6}	$20 \times 0.2556\lvert\sin(0.1t)\rvert$	0.0001
1.3.2	1.71×10^{-6}	$20 \times 0.2556\lvert\sin(0.1t)\rvert$	10×0.0001
1.3.3	1.71×10^{-6}	$20 \times 0.2556\lvert\sin(0.1t)\rvert$	20×0.0001
1.3.4	1.71×10^{-6}	$20 \times 0.2556\lvert\sin(0.1t)\rvert$	40×0.0001
1.3.5	1.71×10^{-6}	$20 \times 0.2556\lvert\sin(0.1t)\rvert$	80×0.0001

TABLE IV: The paremeter is used in case 1.4

Case No.	D	u	Q
1.4.1	1.71×10^{-6}	$40 \times 0.2556\lvert\sin(0.1t)\rvert$	0.0001
1.4.2	1.71×10^{-6}	$40 \times 0.2556\lvert\sin(0.1t)\rvert$	10×0.0001
1.4.3	1.71×10^{-6}	$40 \times 0.2556\lvert\sin(0.1t)\rvert$	20×0.0001
1.4.4	1.71×10^{-6}	$40 \times 0.2556\lvert\sin(0.1t)\rvert$	40×0.0001
1.4.5	1.71×10^{-6}	$40 \times 0.2556\lvert\sin(0.1t)\rvert$	80×0.0001

The numerical results of the instant removal mechanism are show in Figure 2-11. When $S(x,t) = 0.01 \times \left(\frac{L-x}{L}\right) + 0.005\lvert\sin(0.1t)\rvert$

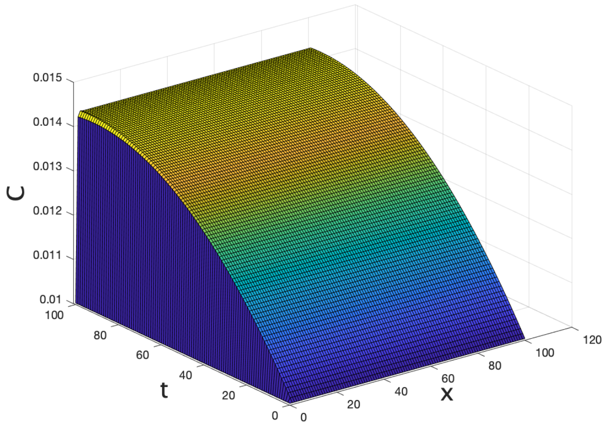

Fig. 2: Oil spill concentration when delayed removal mechanism; $Q = 0.0001$ and $u = 10 \times 0.2556\lvert\sin(0.1t)\rvert$

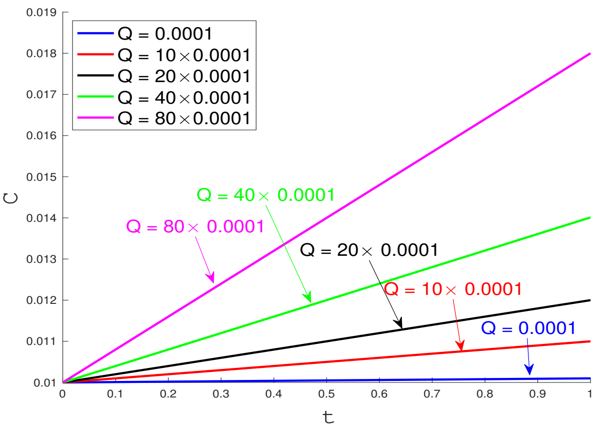

Fig. 3: Oil spill concentration of cases 1.1.1-1.1.5 with several instant removal mechanism rates.

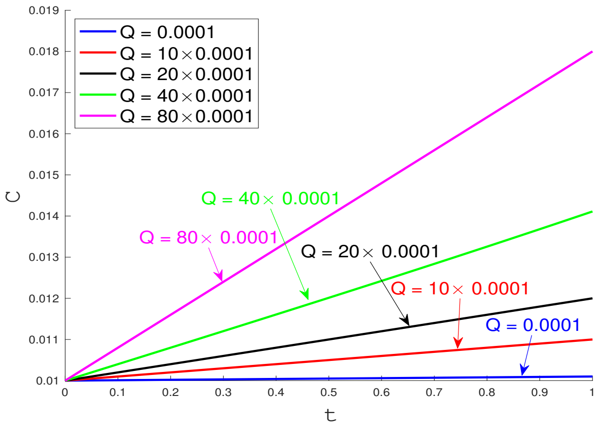

Fig. 4: Oil spill concentration of cases 1.2.1-1.2.5 with several instant removal mechanism rates.

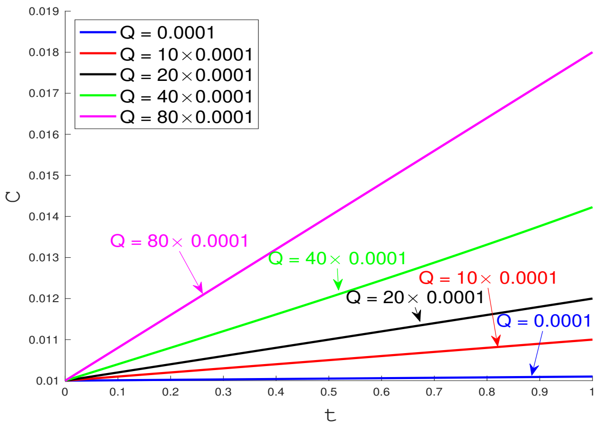

Fig. 5: Oil spill concentration of cases 1.3.1-1.3.5 with several instant removal mechanism rates.

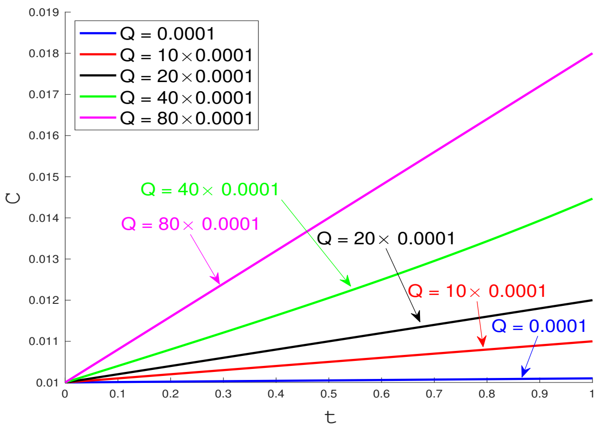

Fig. 6: Oil spill concentration of cases 1.4.1-1.4.5 with several instant removal mechanism rates.

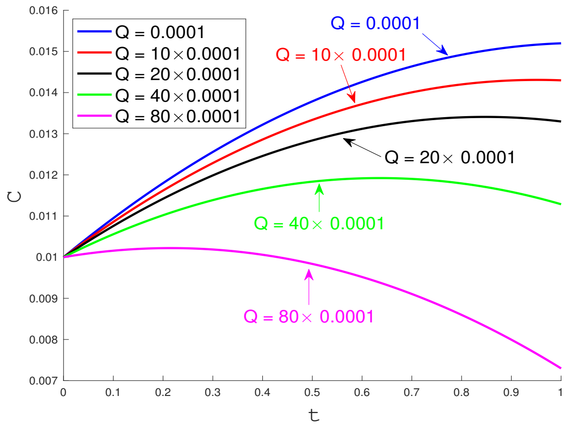

Fig. 7: Oil spill concentration with $S(x,t) = 0.01 \times \left(\frac{L-x}{L}\right) + 0.005\sin(0.1t)$ and $Q = 0.0001,\ 10\times0.0001,\ 20\times0.0001,\ 40\times0.0001$ and 80×0.0001, respectively and $u = 0.2556|\sin(0.1t)|$.

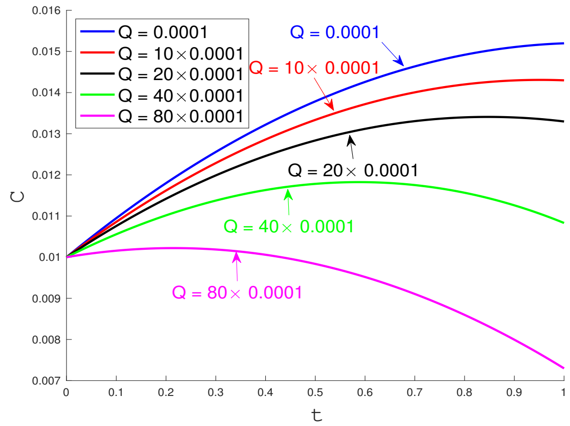

Fig. 10: Oil spill concentration with $S(x,t) = 0.01 \times \left(\frac{L-x}{L}\right) + 0.005\sin(0.1t)$ and $Q = 0.0001,\ 10\times0.0001,\ 20\times0.0001,\ 40\times0.0001$ and 80×0.0001, respectively and $u = 40\times0.2556|\sin(0.1t)|$.

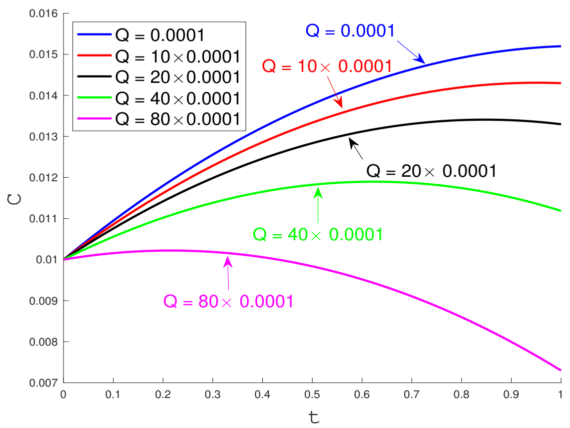

Fig. 8: Oil spill concentration with $S(x,t) = 0.01 \times \left(\frac{L-x}{L}\right) + 0.005\sin(0.1t)$ and $Q = 0.0001,\ 10\times0.0001,\ 20\times0.0001,\ 40\times0.0001$ and 80×0.0001, respectively and $u = 10\times0.2556|\sin(0.1t)|$.

Fig. 11: The comparison of the concentration of oil between the $2nd-$order FTCS and the $4th-$order FTCS in the case of an instant removal mechanism when $u = 40\times0.2556|\sin(0.1t)|$.

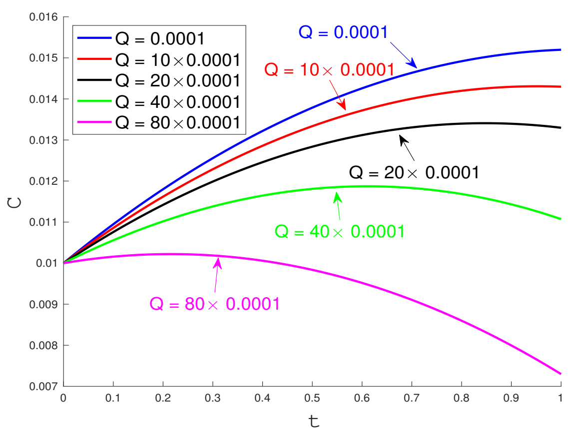

Fig. 9: Oil spill concentration with $S(x,t) = 0.01 \times \left(\frac{L-x}{L}\right) + 0.005\sin(0.1t)$ and $Q = 0.0001,\ 10\times0.0001,\ 20\times0.0001,\ 40\times0.0001$ and 80×0.0001, respectively and $u = 20\times0.2556|\sin(0.1t)|$.

TABLE V: The convergence and divergence values of this simulation where $u = 0.2556|\sin(0.1t)|$

Δx	Δt	α	β	$2nd-$FTCS	$4th-$FTCS
25	1	0.0852	2.28×10^{-10}	Stable	Stable
2.5	1	0.000852	2.28×10^{-8}	Stable	Stable
0.25	1	0.0852	2.28×10^{-6}	Stable	Stable
0.25	0.1	0.00852	2.28×10^{-7}	Stable	Stable
0.25	0.01	0.000852	2.28×10^{-8}	Stable	Stable
0.025	1	0.852	2.28×10^{-4}	Stable	Unstable
0.25	10	0.0852	2.28×10^{-6}	Stable	Unstable
0.25	20	0.0852	2.28×10^{-6}	Stable	Unstable
0.25	100	0.0852	2.28×10^{-6}	Unstable	Unstable

VI. Dissussion

All of the graphs that were shown in the previous section will be thoroughly explained in this section, which will divide our investigation into two basic scenarios. Let's assume that the removal mechanism maintains uniformity in Case 1. Their parameters for this assumption are provided in Table 1. It adds the influence of a point source, which is introduced in Case 2, and treats it as a function of both x and t. For Case 1, we conduct a numerical analysis using a fixed value of $D = 1.71 \times 10^{-6}$. The values of $Q(t)$ in this case vary incrementally as $0.0001,\ 10 \times 0.0001,\ 20 \times 0.0001,\ 40 \times 0.0001$ and 80×0.0001. Similarly, the water flow velocity u is modeled as $0.2556|\sin(0.1t)|$ followed by incremental values of $10 \times 0.2556|\sin(0.1t)|,\ 20 \times 0.2556|\sin(0.1t)|$ and $40 \times 0.2556|\sin(0.1t)|$ respectively. Figures 3-6 present these values graphically. These figures illustrate that the values utilized in the $4th-$order FTCS method exhibit different behaviors compared to those of the $2nd-$order FTCS method. The graphs make it evident that as the magnitude of the removal mechanism increases, the concentration of oil correspondingly decreases. This shows the impact of a consistent removal mechanism on the diminishing concentration over time. In Case 2, we add a point source term to the equation, making it a function of both x and t. For this scenario, we adopt a slightly different diffusion coefficient, with $D = 1.71 \times 10^{-6}$, and use the same range of water flow velocity values $u = 0.2556|\sin(0.1t)|,\ 10 \times 0.2556|\sin(0.1t)|,\ 20 \times 0.2556|\sin(0.1t)|$ and $40 \times 0.2556|\sin(0.1t)|$, respectively. Figures 7-10 illustrate the results for this case. From these figures, we observe that with an increase in the values of the removal mechanism, there is a corresponding decrease in the concentration of oil. When the removal mechanism is strong enough, the oil concentration approaches zero, highlighting the effectiveness of increased removal in reducing oil concentration. Finally, in Figure 11, we compare the accuracy and behavior of the $2nd-$order FTCS and $4th-$order FTCS methods. While the $4th-$order FTCS method offers greater accuracy, Table V indicates that it exhibits faster divergence compared to the $2nd-$order FTCS method. This comparison underlines that although higher-order methods may provide better precision, they may also introduce higher instability or divergence in certain conditions. In order to obtain dependable findings in real-world applications, the method selection process thus requires finding a balance between accuracy and stability.

The results of this study are not just theoretical; they hold significant implications for real-world situations, particularly in environmental management and disaster response. Consider, for instance, an oil spill in a river or coastal area. The ability to model the diffusion and removal of pollutants accurately can inform strategies to minimize environmental damage and protect aquatic ecosystems. By aligning our findings with practical scenarios, this study bridges the gap between computational analysis and actionable solutions, emphasizing the role of advanced numerical methods in tackling real-world environmental challenges.

Therefore, the concentration of oil in marine environments will be substantially diminished if oil spill removal is conducted promptly and efficiently.

VII. Conclusion

In this study, we construct a one-dimensional mathematical model to examine the complex dynamics of an oil spill in a coastal bay, with a particular emphasis on the effects of delayed removal mechanisms. We introduce a governing equation that characterizes the behavior of oil spills under these delayed conditions and define the initial and boundary conditions to realistically reflect the coastal bay environment. To incorporate the delays in removal, a mathematical framework is proposed, and solutions are approximated using the finite difference approach, specifically the $4th-$order forward time-centered space (4th-FTCS) method. The equation's right boundary is approximated using a centered space method, ensuring accuracy in the boundary handling. The simulations explore two primary scenarios: one involving instant removal mechanisms and another focused on delayed removal mechanisms. In the instant removal scenario, we test a range of simple average removal rates alongside water flow currents to observe basic removal dynamics. In the delayed removal scenario, we model more realistic oil spill events, taking into account both the oil spill source rates and the varying rates of removal over the entire simulation period. The results illustrate that as the effectiveness of the removal mechanism increases, the concentration of oil correspondingly decreases. Moreover, a comparative analysis between the $2nd-$FTCS and $4th-$order FTCS methods demonstrates that the $4th-$FTCS method achieves greater accuracy but diverges at a faster rate compared to the $2nd-$FTCS method. This comparison focuses attention on the compromise between accuracy and stability in numerical methods for modeling oil spills, demonstrating how important it is to select the most efficient technique depending on specific simulation settings and purposes.

The additional section highlights the practical implications of the study in real-world scenarios, particularly in environmental management and disaster response. In Case 1, the findings demonstrate the effectiveness of consistent removal mechanisms, such as scheduled cleanups or absorbent materials, in gradually reducing pollutant concentrations. Case 2 reflects situations where pollutants are continuously introduced, emphasizing the importance of addressing both the source and removal strategies to mitigate long-term impacts. Furthermore, the comparison of $2nd$-order and $4th$-order FTCS methods underscores the trade-off between accuracy and stability in computational modeling, offering valuable insights for industries and agencies using simulations for decision-making.

This study not only advances the understanding of the dynamics of oil spill concentration over time, but also underscores the critical importance of timely intervention in mitigating environmental damage. By offering a more precise prediction of the consequences of delayed removal mechanisms, this research supports informed decision-making for disaster response teams and policymakers. The findings can be instrumental in refining oil spill management strategies, potentially shaping the development of more effective response protocols, optimizing resource allocation, and enhancing public awareness initiatives. Ultimately, this study provides a valuable framework for strengthening environmental protection efforts, reducing the long-term impacts of oil spills, and ensuring the sustainability of coastal ecosystems.

ACKNOWLEDGMENT

Teerat Kasamwan sincerely thanks Assoc. Prof. Dr. Nopparat Pochai for his invaluable guidance, support, and encouragement throughout this research. His expertise and direction were essential to the successful completion of this study.

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