Stability Analysis of Delay SEIQRV Mathematical Model for COVID-19

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Abstract—**This paper examines the dynamics of COVID-19 coronavirus transmissions using the SEIQRV model, taking into account the delay period between susceptible individuals contracting the infection. This study investigates the stability of the suggested delayed SEIQRV model. It is found that the endemic equilibrium point is stable when $R_0 > 1$, and the disease-free equilibrium point is locally stable when $R_0 < 1$. Numerical simulations demonstrate the stability results.**

Index Terms—**Delay, Steady-state solutions, Reproduction Number, Local Stability, Global Stability.**

I. Introduction

Human mortality has been primarily caused by infectious diseases. Mathematical models have been significant in comprehending the mechanism of spread and managing infectious diseases. Ever since, mathematical modelling has gained importance as a crucial instrument for proposing public health measures to prevent the spread and transmission of infectious diseases. In modern mathematical epidemiology, the SIR model has been extremely important [11]. Intervention strategies are essential for managing an infectious disease that has emerged and spread throughout a community or region. A new pandemic called COVID-19 recently surfaced, affecting more nations and regions globally. Various governments enforced a strict lockdown in order to contain the spread of COVID-19. These tactics have remarkably succeeded in many nations in reducing the number of contacts between susceptible and infected people in a given amount of time, which has decreased the incidence rate.

Recent COVID-19 studies have improved our understanding of the dynamics of transmission and the possible functions of different intervention strategies ([3], [5], [7], [9], [17], [18]). These strategies include providing relief and hiding the situation to slow down the pandemic's spread, reducing access to the best medical care to protect the most vulnerable individuals, bringing the number of infectious cases down to a minimum, enforcing lockdowns in areas where there are disproportionately high rates of infection, containing suspected cases at home, and isolating families living together. An Omicron variant model with a variable population size was created by some authors ([6], [12], [13], [14]).

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Many infectious diseases, like COVID-19, have delayed dynamics because of incubation times and associated phenomena. Consequently, DDE models not only provide advantages in terms of computational time and modelling but also enable a natural representation of the problem dynamics because they do away with the need for extra, hard-to-estimate compartments to account for time delays. To prevent Covid-19 infection in the host population, some authors developed delay-type models ([2], [8], [10], [15], [19], [20]). This paper proposes a mathematical model of delay for the system of ordinary differential equations. A non-linear mathematical model with a time delay is presented, which is based on the conventional model of infectious diseases. The work is interesting because it combines real-world COVID-19 data from Tamil Nadu, India, with mathematical modelling.

This research examines the dynamics of the coronavirus using mathematical modelling and analysis of the SEIQRV model, taking into consideration the delay in the conversion of susceptible individuals into infected ones. The given work is very interesting to read because it contains the mathematical modelling with real data of the COVID-19 from Chennai, Tamil Nadu, India.

The delayed SEIQRV model is presented in Section II. The reproduction number and steady-state solutions can be found in II-A. Section II-B discusses the stability analysis of equilibrium points for the proposed model. Using the real data collected from Tamil Nadu in Section III, computational simulations are performed to validate and support our theoretical findings regarding COVID-19.

II. Delayed SEIQRV Model Formulation

This section is focused on constructing the delay SEIQRV model for our problem formulation. The delayed SEIQRV model can be formulated from the integer-order model [1]. Based on the policy decisions made by the government, a set of parameters has been obtained to forecast the pandemic trend. The suitable parameters are used to formulate the Omicron-delayed SEIQRV model, in which values are described in Table I from the articles [4] and [6].

Considering the given aspects, the SEIQRV delay mathematical model is derived as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= P - \zeta_1 S - \zeta_2 S(t-\tau)I(t-\tau) + \zeta_3 R + \zeta_4 V \\
\frac{dE}{dt} &= \zeta_2 S(t-\tau)I(t-\tau) - (\zeta_1 + \zeta_5 + \zeta_6)E, \\
\frac{dI}{dt} &= \zeta_6 E - (\zeta_1 + \zeta_7 + \zeta_8 + \zeta_9)I, \qquad (1) \\
\frac{dQ}{dt} &= \zeta_5 E + \zeta_9 I - (\zeta_{10} + \zeta_{11} + \zeta_1)Q, \\
\frac{dR}{dt} &= \zeta_8 I + \zeta_{10} Q - (\zeta_1 + \zeta_3 + \zeta_{12})R,
\end{aligned}
$$

TABLE I
PARAMETERS AND THEIR DESCRIPTIONS

Parameters	Descriptions	Values
P	Rate at which humans are recruited into the population	5
ζ_1	The regular demise rate pertinent to all compartments	0.09
ζ_2	Powerful irresistible contact rate between the susceptible and infected person	0.1679
ζ_3	The rate at which the recovered compartment loses its immunities to treatment	0.0333
ζ_4	The rate at which the vaccinated compartment loses its immunities to susceptible	0.0059
ζ_5	Rate at which exposed people move to isolated class	0.3169
ζ_6	Rate at which a specific part of exposed people move to infected class	0.1858
ζ_7	The demise rate instigated by contaminations of infected people	0.0002
ζ_8	The regular recovery rates because of different components	0.1981
ζ_9	The treatment rate of the infected class	0.5864
ζ_{10}	Contact rate between infected and recovered classes	0.0505
ζ_{11}	Rate at which a specific part of isolated people gets vaccination	0.1695
ζ_{12}	Rate at which a specific part of recovered people receives vaccination	0.0197

$$\frac{dV}{dt} = \zeta_{11}Q + \zeta_{12}R - (\zeta_1 + \zeta_4)V$$

Subject to initial conditions: $S(\psi) = S_0,\ \psi \in [-\tau, 0]$, $E(0) = E_0,\ I(\psi) = I_0, \psi \in [-\tau, 0], Q(0) = Q_0, R(0) = R_0^0, V(0) = V_0$.

The system of equations can be written as

$$\begin{aligned}
\frac{dS}{dt} &= P - \zeta_1 S - \zeta_2 S(t-\tau)I(t-\tau) + \zeta_3 R + \zeta_4 V \\
\frac{dE}{dt} &= \zeta_2 S(t-\tau)I(t-\tau) - (\zeta_{22})E, \\
\frac{dI}{dt} &= \zeta_6 E - (\zeta_{33})I, \\
\frac{dQ}{dt} &= kE + \zeta_9 I - (\zeta_{44})Q, \\
\frac{dR}{dt} &= \zeta_8 I + \zeta_{10} Q - (\zeta_{55})R, \\
\frac{dV}{dt} &= \zeta_{11}Q + \zeta_{12}R - (\zeta_{66})V
\end{aligned} \tag{2}$$

where $\zeta_{22} = \zeta_1 + \zeta_5 + \zeta_6$, $\zeta_{33} = \zeta_1 + \zeta_7 + \zeta_8 + \zeta_9$, $\zeta_{44} = \zeta_{10} + \zeta_{11} + \zeta_1$, $\xi_4 = \zeta_1$, $\zeta_{55} = \zeta_1 + \zeta_3 + \zeta_{12}$ and $\zeta_{66} = \zeta_1 + \zeta_4$.

It is considered that the disease has an incubation time of the virus $\tau > 0$ transferred from a susceptible period to an incubation period. The incubation period is the delay time that passes between being susceptible and showing symptoms of the virus.

A. Steady State Solutions of the delayed SEIQRV model

The system (2) is found static, i.e., the solutions that are time-independent are obtained. The steady-state solutions in the infection-free state, when $I = 0$, are given by

$$E_q^0 = (S^0, E^0, I^0, Q^0, R^0, V^0) = \Big(\frac{P}{\zeta_1}, 0, 0, 0, 0, 0\Big). \tag{3}$$

Also, when infection is persistent, the steady-state solutions, i.e., $I \neq 0$, are given by

$$E_q^* = (S^*, E^*, I^*, Q^*, R^*, V^*) \tag{4}$$

where

$$\begin{aligned}
E_q^* &= (S^*, E^*, I^*, Q^*, R^*, V^*) \qquad (5)\\
&= \Big(\frac{\zeta_{22}\zeta_{33}}{\zeta_2\zeta_6}, \frac{\zeta_{33}(P - \zeta_1 S^*)}{\zeta_6(\zeta_2 S^* - \zeta_3 B - \zeta_4 C)}, \\
&\frac{P - \zeta_1 S^*}{\zeta_2 S^* - \zeta_3 B - \zeta_4 C}, \\
&\frac{\zeta_5\zeta_{33} + \zeta_6\zeta_9}{\zeta_6\zeta_{44}}\big(\frac{P - \zeta_1 S^*}{\zeta_2 S^* - \zeta_3 B - \zeta_4 C}\big), \\
&\frac{(\zeta_8\zeta_{22}\zeta_{44} + \zeta_2\zeta_5\zeta_{10}S^* + \zeta_9\zeta_{22})(P - \zeta_1 S^*)}{(\zeta_{22}\zeta_{44}\zeta_{55})(\zeta_2 S^* - \zeta_3 B - \zeta_4 C)}, \\
&\frac{\zeta_{11}A + \zeta_{12}B}{\zeta_{66}}\big(\frac{P - \zeta_1 S^*}{\zeta_2 S^* - \zeta_3 B - \zeta_4 C}\big)\Big) \qquad (6)
\end{aligned}$$

where

$$\begin{aligned}
A &= \frac{\zeta_5\zeta_{33} + \zeta_6\zeta_9}{\zeta_6\zeta_{44}} \\
B &= \frac{\zeta_8\zeta_{22}\zeta_{44} + \zeta_2\zeta_5\zeta_{10}S^* + \zeta_9\zeta_{22}}{\zeta_{22}\zeta_{44}\zeta_{55}} \\
C &= \frac{\zeta_{11}A + \zeta_{12}B}{\zeta_{66}}
\end{aligned}$$

The basic reproduction number R_0 is

$$R_0 = (GV^{-1}) = \frac{P\zeta_2\zeta_6}{\zeta_1(\zeta_1 + \zeta_5 + \zeta_6)(\zeta_1 + \zeta_7 + \zeta_8 + \zeta_9)}. \tag{7}$$

B. Stability Analysis of the Delayed SEIQRV Model

The local stability of the SEIQRV system (2) for the infection-free steady-state solution (3) is examined in the next theorem applying Rouche's theorem [16]. The reproduction number R_0 determines the result.

Theorem 2.1: The infection-free consistent state E^0 (3) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$ for the time delay $\tau = 0$.

Proof: The characteristic equation of system 2, for the equilibrium point E^0, is given by

$$\Delta(\lambda) = \left|\lambda Id_{8\times 8} - J_{00} - J_{01}e^{-\tau\lambda}\right| \tag{8}$$

where

$$J_{00} = \begin{pmatrix} -\zeta_1 & 0 & 0 & 0 & \zeta_3 & \zeta_4 \\ 0 & -\zeta_{22} & 0 & 0 & 0 & 0 \\ 0 & \zeta_6 & \zeta_{33} & 0 & 0 & 0 \\ 0 & \zeta_5 & \zeta_9 & -\zeta_{44} & 0 & 0 \\ 0 & 0 & \zeta_8 & \zeta_{10} & -\zeta_{55} & 0 \\ 0 & 0 & 0 & \zeta_{11} & \zeta_{12} & -\zeta_{66} \end{pmatrix},$$

and

$$J_{01} = \begin{pmatrix} -\zeta_2 I & 0 & -\zeta_2 S & 0 & 0 & 0 \\ \zeta_2 I & 0 & \zeta_2 S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\begin{aligned} C(\lambda) &= (\lambda+\zeta_1)(\lambda+\frac{1}{2}\big(\zeta_{22}+\zeta_{33} \\ &+\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big)) \\ &(\lambda+\frac{1}{2}\big(\zeta_{22}+\zeta_{33} \\ &-\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big)) \\ &(\lambda+\zeta_{44})(\lambda+\zeta_{55})(\lambda+\zeta_{66}). \end{aligned} \tag{9}$$

When τ = 0, the eigenvalues are $-\zeta_1$, $-\zeta_{44}$, $-\zeta_{55}$, $-\zeta_{66}$, $-\frac{1}{2}\big(\zeta_{22}+\zeta_{33}+\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big)$, $-\frac{1}{2}\big(\zeta_{22}+\zeta_{33}-\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big)$.

The given system (2) is stable when
$-\zeta_{22}-\zeta_{33}+\sqrt{\zeta_{22}^2+4\zeta_2S\zeta_6-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}<0$
or $\sqrt{\zeta_{22}^2+4\zeta_2S\zeta_6-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}<(\zeta_{22}+\zeta_{33})$
or $\zeta_{22}^2+4\zeta_2S\zeta_6-2\zeta_{22}\zeta_{33}+\zeta_{33}^2<(\zeta_{22}+\zeta_{33})^2$
or $\zeta_2P\zeta_6<\zeta_{22}\zeta_{33}$ i.e., $\frac{\zeta_2P\zeta_6}{\zeta_1\zeta_{22}\zeta_{33}}<1$ That is $R_0<1$. Clearly infection free steady state E^0 is locally asymptotically stable if $R_0<1$ when $\tau=0$. ■

Theorem 2.2: The infection-free consistent state E^0 (3) is locally asymptotically stable if $R_0<1$ and unstable if $R_0>1$ for the time delay $\tau>0$.

Proof: Let $\tau>0$. In this case, we will use Rouche's theorem to prove that all roots of the characteristic equation (8) cannot intersect the imaginary axis, i.e., the characteristic equation cannot have pure imaginary roots.

Suppose for the opposite that there exists $w\in R$ such that $\lambda=wi$ is a solution of (8).

Consider the term $wi+\frac{1}{2}\big(\zeta_{22}+\zeta_{33}-\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big)=0$
$\rightarrow wi+\frac{1}{2}\big(\zeta_{22}+\zeta_{33}\big)=\frac{1}{2}\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}$
$\rightarrow \Big(wi+\frac{1}{2}\big(\zeta_{22}+\zeta_{33}\big)\Big)^2=\frac{1}{4}\big(\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2\big)$
$\rightarrow (wi)^2+\frac{1}{4}(\zeta_{22}+\zeta_{33})^2+wi(\zeta_{22}+\zeta_{33})-\frac{1}{4}\zeta_{22}^2+\frac{1}{2}\zeta_{22}\zeta_{33}-\frac{1}{4}\zeta_{33}^2=\zeta_2\zeta_6S(\cos\tau w-i\sin\tau w)$
$\rightarrow -w^2+wi(\zeta_{22}+\zeta_{33})+\zeta_{22}\zeta_{33}=\zeta_2\zeta_6S(\cos\tau w-i\sin\tau w)$

By equating the real and imaginary part, we get
$\zeta_{22}\zeta_{33}-w^2=\zeta_2\zeta_6S\cos\tau w$, $w(\zeta_{22}+\zeta_{33})=-\zeta_2\zeta_6S\sin\tau w$

By squaring and adding these two we can get
$w^4+w^2(\zeta_{22}^2+\zeta_{33}^2)=\zeta_2^2\zeta_6^2S^2-\zeta_{22}^2\zeta_{33}^2$

If $R_0<1$, $then$ $w^4+w^2(\zeta_{22}^2+\zeta_{33}^2)<0$, which is a contradiction.

Thus the infection free consistent state E^0 is locally asymptotically stable if $R_0<1$ for $\tau>0$.

Now suppose that $R_0>1$. From the characteristic polynomial (9), it is enough to consider the term $(\lambda+\frac{1}{2}\big(\zeta_{22}+\zeta_{33}-\sqrt{\zeta_{22}^2+4\zeta_2\zeta_6Se^{-\tau\lambda}-2\zeta_{22}\zeta_{33}+\zeta_{33}^2}\big))$. It is easy to see that $C_1(0)<0$. On the other hand, $\lim_{\lambda\to+\infty}C_1(\lambda)=+\infty$. Therefore, by continuity of $C_1(\lambda)$, there is at least one positive root of the characteristic equation (9). Hence, we conclude that E^0 is unstable when $R_0>1$, for any $\tau>0$. ■

The local stability of the SEIQRV system (2) for the infection's persistent steady-state solution (4) is determined using Rouché's theorem and the Routh-Hurwitz technique in the next theorem. The result is governed by the reproduction number R_0.

Theorem 2.3: If $R_0>1$, then the endemic equilibrium point $E^*$is locally asymptotically stable for $\tau=0$.

Proof: The characteristic equation of system 2, for the equilibrium point E^* 4 is given by

$$\Delta(\lambda)=\left|\lambda Id_{8\times8}-J_{10}-J_{11}e^{-\tau\lambda}\right|. \tag{10}$$

where the Jacobian matrices of the model at the infection persistent steady-state solution are

$$J(10)=\begin{pmatrix} -\zeta_1 & 0 & 0 & 0 & \zeta_3 & \zeta_4 \\ 0 & -\zeta_{22} & 0 & 0 & 0 & 0 \\ 0 & \zeta_6 & \zeta_{33} & 0 & 0 & 0 \\ 0 & \zeta_5 & \zeta_9 & -\zeta_{44} & 0 & 0 \\ 0 & 0 & \zeta_8 & \zeta_{10} & -\zeta_{55} & 0 \\ 0 & 0 & 0 & \zeta_{11} & \zeta_{12} & -\zeta_{66} \end{pmatrix}.$$

and

$$J(11)=\begin{pmatrix} -\zeta_2I^* & 0 & -\zeta_2S^* & 0 & 0 & 0 \\ \zeta_2I^* & 0 & \zeta_2S^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The characteristic equation is $c_1\lambda^6+c_2\lambda^5+c_3\lambda^4+c_4\lambda^3+c_5\lambda^2+c_6\lambda+c_7$,
where

$c_1=1$

$c_2=\zeta_1+\zeta_{44}+\zeta_{22}+\zeta_{33}+\zeta_2I^*e^{-\lambda\tau}+\zeta_{55}+\zeta_{66}$

$c_3=\zeta_1\zeta_{44}+\zeta_1\zeta_{22}+\zeta_{44}\zeta_{22}+\zeta_1\zeta_{33}+\zeta_{44}\zeta_{33}+\zeta_{22}\zeta_{33}+\zeta_1\zeta_{55}+\zeta_{44}\zeta_{55}+\zeta_{22}\zeta_{55}+\zeta_{33}\zeta_{55}+\zeta_1\zeta_{66}+\zeta_{44}\zeta_{66}+\zeta_{22}\zeta_{66}+\zeta_{33}\zeta_{66}+\zeta_{55}\zeta_{66}+(\zeta_{44}\zeta_2+\zeta_{22}\zeta_2+\zeta_{33}\zeta_2+\zeta_2\zeta_{55}+\zeta_2\zeta_{66})I^*e^{-\lambda\tau}-\zeta_2\zeta_6S^*e^{-\lambda\tau}$

$c_4=\zeta_1\zeta_{44}\zeta_{22}+\zeta_1\zeta_{44}\zeta_{33}+\zeta_1\zeta_{22}\zeta_{33}+\zeta_{44}\zeta_{22}\zeta_{33}+\zeta_1\zeta_{44}\zeta_{55}+\zeta_1\zeta_{22}\zeta_{55}+\zeta_{44}\zeta_{22}\zeta_{55}+\zeta_1\zeta_{33}\zeta_{55}+\zeta_{44}\zeta_{33}\zeta_{55}+\zeta_{22}\zeta_{33}\zeta_{55}+\zeta_1\zeta_{44}\zeta_{66}+\zeta_1\zeta_{22}\zeta_{66}+\zeta_{44}\zeta_{22}\zeta_{66}+\zeta_1\zeta_{33}\zeta_{66}+\zeta_{44}\zeta_{33}\zeta_{66}+\zeta_{22}\zeta_{33}\zeta_{66}+\zeta_1\zeta_{55}\zeta_{66}+\zeta_{44}\zeta_{55}\zeta_{66}+\zeta_{22}\zeta_{55}\zeta_{66}+\zeta_{33}\zeta_{55}\zeta_{66}+(\zeta_{44}\zeta_{22}\zeta_2+\zeta_{44}\zeta_{33}\zeta_2+\zeta_{22}\zeta_{33}\zeta_2+\zeta_{44}\zeta_2\zeta_{55}+\zeta_{22}\zeta_2\zeta_{55}+\zeta_{33}\zeta_2\zeta_{55}+\zeta_{44}\zeta_2\zeta_{66}+\zeta_{22}\zeta_2\zeta_{66}+\zeta_{33}\zeta_2\zeta_{66}+\zeta_2\zeta_{55}\zeta_{66})I^*e^{-\lambda\tau}-(\zeta_1\zeta_2\zeta_6+\zeta_2\zeta_{44}\zeta_6+\zeta_2\zeta_6\zeta_{55}+\zeta_2\zeta_6\zeta_{66})S^*e^{-\lambda\tau}$

$c_5=\zeta_1\zeta_{44}\zeta_{22}\zeta_{33}+\zeta_1\zeta_{44}\zeta_{22}\zeta_{55}+\zeta_1\zeta_{44}\zeta_{33}\zeta_{55}+\zeta_1\zeta_{22}\zeta_{33}\zeta_{55}+\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}+\zeta_1\zeta_{44}\zeta_{22}\zeta_{66}+\zeta_1\zeta_{44}\zeta_{33}\zeta_{66}+\zeta_1\zeta_{22}\zeta_{33}\zeta_{66}+\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{66}+\zeta_1\zeta_{44}\zeta_{55}\zeta_{66}+\zeta_1\zeta_{22}\zeta_{55}\zeta_{66}+\zeta_{44}\zeta_{22}\zeta_{55}\zeta_{66}+\zeta_1\zeta_{33}\zeta_{55}\zeta_{66}+\zeta_{44}\zeta_{33}\zeta_{55}\zeta_{66}+\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}+(\zeta_{44}\zeta_{22}\zeta_{33}\zeta_2-\zeta_3\zeta_2\zeta_6\zeta_8-\zeta_3\zeta_5\zeta_2\zeta_{10}+\zeta_{44}\zeta_{22}\zeta_2\zeta_{55}+\zeta_{44}\zeta_{33}\zeta_2\zeta_{55}+\zeta_{22}\zeta_{33}\zeta_2\zeta_{55}-\zeta_4\zeta_5\zeta_2\zeta_{11}+\zeta_{44}\zeta_{22}\zeta_2\zeta_{66}+\zeta_{44}\zeta_{33}\zeta_2\zeta_{66}+\zeta_{22}\zeta_{33}\zeta_2\zeta_{66}+\zeta_{44}\zeta_2\zeta_{55}\zeta_{66}+\zeta_{22}\zeta_2\zeta_{55}\zeta_{66}+\zeta_{33}\zeta_2\zeta_{55}\zeta_{66})I^*e^{-\lambda\tau}-(\zeta_1\zeta_2\zeta_{44}\zeta_6+\zeta_1\zeta_2\zeta_6\zeta_{55}+\zeta_2\zeta_{44}\zeta_6\zeta_{55}+\zeta_1\zeta_2\zeta_6\zeta_{66}+\zeta_2\zeta_{44}\zeta_6\zeta_{66}+\zeta_2\zeta_6\zeta_{55}\zeta_{66})S^*e^{-\lambda\tau}$

$c_6=\zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}+\zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{66}+\zeta_1\zeta_{44}\zeta_{22}\zeta_{55}\zeta_{66}+\zeta_1\zeta_{44}\zeta_{33}\zeta_{55}\zeta_{66}+\zeta_1\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}+\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}+(\zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{55}+\zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{66}+\zeta_{44}\zeta_{22}\zeta_2\zeta_{55}\zeta_{66}+\zeta_{44}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66}+\zeta_{22}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66}-\zeta_{44}\zeta_3\zeta_2\zeta_6\zeta_8-\zeta_3\zeta_{33}\zeta_5\zeta_2\zeta_{10}-\zeta_3\zeta_2\zeta_9\zeta_6\zeta_{10}$
- $\zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{11}-\zeta_4\zeta_2\zeta_9\zeta_6\zeta_{11}-\zeta_4\zeta_5\zeta_2\zeta_{55}\zeta_{11}-\zeta_4\zeta_2\zeta_6\zeta_8\zeta_{12}-\zeta_4\zeta_5\zeta_2\zeta_{10}\zeta_{12}-\zeta_3\zeta_2\zeta_6\zeta_8\zeta_{66}$
- $\zeta_3\zeta_5\zeta_2\zeta_{10}\zeta_{66})I^*e^{-\lambda\tau}-(\zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{55}+\zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{66}+\zeta_1\zeta_2\zeta_6\zeta_{55}\zeta_{66}+\zeta_2\zeta_{44}\zeta_6\zeta_{55}\zeta_{66})\ S^*e^{-\lambda\tau}$

$c_7 = \zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66} + (\zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66} - \zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{55}\zeta_{11} - \zeta_4\zeta_2\zeta_9\zeta_6\zeta_{55}\zeta_{11} - \zeta_{44}\zeta_4\zeta_2\zeta_6\zeta_8\zeta_{12} - \zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{10}\zeta_{12} - \zeta_4\zeta_2\zeta_9\zeta_6\zeta_{10}\zeta_{12} - \zeta_{44}\zeta_3\zeta_2\zeta_6\zeta_8\zeta_{66} - \zeta_3\zeta_{33}\zeta_5\zeta_2\zeta_{10}\zeta_{66} - \zeta_3\zeta_2\zeta_9\zeta_6\zeta_{10}\zeta_{66})I^*e^{-\lambda\tau} - \zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{55}\zeta_{66}S^*e^{-\lambda\tau}$

To check about the stability, consider the above characteristic equation

$$\begin{aligned}
\lambda^6 &+ (e^{-\lambda\tau}I^*D_1 + C_1)\lambda^5 + ([I^*D_2 - S^*E_2]e^{-\lambda\tau} \\
&+ C_2)\lambda^4 + ([I^*D_3 - S^*E_3]e^{-\lambda\tau} + C_3)\lambda^3 \\
&+ ([I^*D_4 - S^*E_4]e^{-\lambda\tau} + C_4)\lambda^2 \\
&+ ([I^*D_5 - S^*E_5]e^{-\lambda\tau} + C_5)\lambda \\
&+ ([I^*D_6 - S^*E_6]e^{-\lambda\tau} + C_6) = 0 \qquad (11)
\end{aligned}$$

where

$D_1 = \zeta_2$,

$C_1 = \zeta_1 + \zeta_{44} + \zeta_{22} + \zeta_{33} + \zeta_{55} + \zeta_{66}$,

$D_2 = \zeta_{44}\zeta_2 + \zeta_{22}\zeta_2 + \zeta_{33}\zeta_2 + \zeta_2\zeta_{55} + \zeta_2\zeta_{66}$,

$E_2 = \zeta_2\zeta_6$,

$C_2 = \zeta_1\zeta_{44} + \zeta_1\zeta_{22} + \zeta_{44}\zeta_{22} + \zeta_1\zeta_{33} + \zeta_{44}\zeta_{33} + \zeta_{22}\zeta_{33} + \zeta_1\zeta_{55} + \zeta_{44}\zeta_{55} + \zeta_{22}\zeta_{55} + \zeta_{33}\zeta_{55} + \zeta_1\zeta_{66} + \zeta_{44}\zeta_{66} + \zeta_{22}\zeta_{66} + \zeta_{33}\zeta_{66} + \zeta_{55}\zeta_{66}$,

$D_3 = \zeta_{44}\zeta_{22}\zeta_2 + \zeta_{44}\zeta_{33}\zeta_2 + \zeta_{22}\zeta_{33}\zeta_2 + \zeta_{44}\zeta_2\zeta_{55} + \zeta_{22}\zeta_2\zeta_{55} + \zeta_{33}\zeta_2\zeta_{55} + \zeta_{44}\zeta_2\zeta_{66} + \zeta_{22}\zeta_2\zeta_{66} + \zeta_{33}\zeta_2\zeta_{66} + \zeta_2\zeta_{55}\zeta_{66}$,

$E_3 = \zeta_1\zeta_2\zeta_6 + \zeta_2\zeta_{44}\zeta_6 + \zeta_2\zeta_6\zeta_{55} + \zeta_2\zeta_6\zeta_{66}$,

$C_3 = \zeta_1\zeta_{44}\zeta_{22} + \zeta_1\zeta_{44}\zeta_{33} + \zeta_1\zeta_{22}\zeta_{33} + \zeta_{44}\zeta_{22}\zeta_{33} + \zeta_1\zeta_{44}\zeta_{55} + \zeta_1\zeta_{22}\zeta_{55} + \zeta_{44}\zeta_{22}\zeta_{55} + \zeta_1\zeta_{33}\zeta_{55} + \zeta_{44}\zeta_{33}\zeta_{55} + \zeta_{22}\zeta_{33}\zeta_{55} + \zeta_1\zeta_{44}\zeta_{66} + \zeta_1\zeta_{22}\zeta_{66} + \zeta_{44}\zeta_{22}\zeta_{66} + \zeta_1\zeta_{33}\zeta_{66} + \zeta_{44}\zeta_{33}\zeta_{66} + \zeta_{22}\zeta_{33}\zeta_{66} + \zeta_1\zeta_{55}\zeta_{66} + \zeta_{44}\zeta_{55}\zeta_{66} + \zeta_{22}\zeta_{55}\zeta_{66} + \zeta_{33}\zeta_{55}\zeta_{66}$,

$D_4 = \zeta_{44}\zeta_{22}\zeta_{33}\zeta_2 - \zeta_3\zeta_2\zeta_6\zeta_8 - \zeta_3\zeta_5\zeta_2\zeta_{10} + \zeta_{44}\zeta_{22}\zeta_2\zeta_{55} + \zeta_{44}\zeta_{33}\zeta_2\zeta_{55} + \zeta_{22}\zeta_{33}\zeta_2\zeta_{55} - \zeta_4\zeta_5\zeta_2\zeta_{11} + \zeta_{44}\zeta_{22}\zeta_2\zeta_{66} + \zeta_{44}\zeta_{33}\zeta_2\zeta_{66} + \zeta_{22}\zeta_{33}\zeta_2\zeta_{66} + \zeta_{44}\zeta_2\zeta_{55}\zeta_{66} + \zeta_{22}\zeta_2\zeta_{55}\zeta_{66} + \zeta_{33}\zeta_2\zeta_{55}\zeta_{66}$,

$E_4 = \zeta_1\zeta_2\zeta_{44}\zeta_6 + \zeta_1\zeta_2\zeta_6\zeta_{55} + \zeta_2\zeta_{44}\zeta_6\zeta_{55} + \zeta_1\zeta_2\zeta_6\zeta_{66} + \zeta_2\zeta_{44}\zeta_6\zeta_{66} + \zeta_2\zeta_6\zeta_{55}\zeta_{66}$,

$C_4 = \zeta_1\zeta_{44}\zeta_{22}\zeta_{33} + \zeta_1\zeta_{44}\zeta_{22}\zeta_{55} + \zeta_1\zeta_{44}\zeta_{33}\zeta_{55} + \zeta_1\zeta_{22}\zeta_{33}\zeta_{55} + \zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55} + \zeta_1\zeta_{44}\zeta_{22}\zeta_{66} + \zeta_1\zeta_{44}\zeta_{33}\zeta_{66} + \zeta_1\zeta_{22}\zeta_{33}\zeta_{66} + \zeta_{44}\zeta_{22}\zeta_{33}\zeta_{66} + \zeta_1\zeta_{44}\zeta_{55}\zeta_{66} + \zeta_1\zeta_{22}\zeta_{55}\zeta_{66} + \zeta_{44}\zeta_{22}\zeta_{55}\zeta_{66} + \zeta_1\zeta_{33}\zeta_{55}\zeta_{66} + \zeta_{44}\zeta_{33}\zeta_{55}\zeta_{66} + \zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}$,

$D_5 = \zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{55} + \zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{66} + \zeta_{44}\zeta_{22}\zeta_2\zeta_{55}\zeta_{66} + \zeta_{44}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66} + \zeta_{22}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66} - \zeta_{44}\zeta_3\zeta_2\zeta_6\zeta_8 - \zeta_3\zeta_{33}\zeta_5\zeta_2\zeta_{10} - \zeta_3\zeta_2\zeta_9\zeta_6\zeta_{10} - \zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{11} - \zeta_4\zeta_2\zeta_9\zeta_6\zeta_{11} - \zeta_4\zeta_5\zeta_2\zeta_{55}\zeta_{11} - \zeta_4\zeta_2\zeta_6\zeta_8\zeta_{12} - \zeta_4\zeta_5\zeta_2\zeta_{10}\zeta_{12} - \zeta_3\zeta_2\zeta_6\zeta_8\zeta_{66} - \zeta_3\zeta_5\zeta_2\zeta_{10}\zeta_{66}$,

$E_5 = \zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{55} + \zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{66} + \zeta_1\zeta_2\zeta_6\zeta_{55}\zeta_{66} + \zeta_2\zeta_{44}\zeta_6\zeta_{55}\zeta_{66}$,

$C_5 = \zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55} + \zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{66} + \zeta_1\zeta_{44}\zeta_{22}\zeta_{55}\zeta_{66} + \zeta_1\zeta_{44}\zeta_{33}\zeta_{55}\zeta_{66} + \zeta_1\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66} + \zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}$,

$D_6 = \zeta_{44}\zeta_{22}\zeta_{33}\zeta_2\zeta_{55}\zeta_{66} - \zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{55}\zeta_{11} - \zeta_4\zeta_2\zeta_9\zeta_6\zeta_{55}\zeta_{11} - \zeta_{44}\zeta_4\zeta_2\zeta_6\zeta_8\zeta_{12} - \zeta_4\zeta_{33}\zeta_5\zeta_2\zeta_{10}\zeta_{12} - \zeta_4\zeta_2\zeta_9\zeta_6\zeta_{10}\zeta_{12} - \zeta_{44}\zeta_3\zeta_2\zeta_6\zeta_8\zeta_{66} - \zeta_3\zeta_{33}\zeta_5\zeta_2\zeta_{10}\zeta_{66} - \zeta_3\zeta_2\zeta_9\zeta_6\zeta_{10}\zeta_{66}$,

$E_6 = \zeta_1\zeta_2\zeta_{44}\zeta_6\zeta_{55}\zeta_{66}$,

$C_6 = \zeta_1\zeta_{44}\zeta_{22}\zeta_{33}\zeta_{55}\zeta_{66}$.

If $\tau = 0$, then by using the rule of Descartes of sign and Routh-Hurwitz stability criterion, the real parts of the complex roots are negative if $\zeta_2 I^* > 0, R_0 - 1 > 0, R_0 > 1$. Then the infection persistent steady state $(S^*, E^*, I^*, Q^*, R^*, V^*)$ is locally stable when $R_0 > 1$.

■

Theorem 2.4: If $R_0 > 1$, then the endemic equilibrium point $E^*$is locally asymptotically stable for $\tau > 0$.

Proof: If $\tau > 0$, then by using Rouche's theorem, we have to prove that all roots of the characteristic equation (11) cannot have pure imaginary roots. Suppose that there exists $w \in R$ such that $\lambda = wi$ is a solution of (11). Now equation (11) becomes

$$\begin{aligned}
w^6 &+ i(e^{-\lambda\tau}I^*D_1 + C_1)w^5 - ([I^*D_2 - S^*E_2]e^{-\lambda\tau} \\
&+ C_2)w^4 i([I^*D_3 - S^*E_3]e^{-\lambda\tau} + C_3)w^3 \\
&+ ([I^*D_4 - S^*E_4]e^{-\lambda\tau} + C_4)w^2 \\
&- i([I^*D_5 - S^*E_5]e^{-\lambda\tau} \\
&+ C_5)w - ([I^*D_6 - S^*E_6]e^{-\lambda\tau} + C_6) = 0 \qquad (12)
\end{aligned}$$

Then,

$$\begin{aligned}
w^6 &+ iG_1w^5 - G_2w^4 + iG_3w^3 + G_4w^2 - iG_5w + G_6 \\
&= (-iG_1^*w^5 + G_2^*w^4 - iG_3^*w^3 - G_4^*w^2 + iG_5^*w - G_6^*) \\
&\quad (\cos\tau w - i\sin\tau w) \qquad (13)
\end{aligned}$$

where $G_i = C_i, i = 1, 2, 3, 4, 5, 6; G_1^* = I^*D_1, G_j^* = S^*E_j^*, j = 2, 3, 4, 5, 6$.

Equating the real and imaginary parts of (13) we get

$$\begin{aligned}
w^6 &- G_2w^4 + G_4w^2 + G_6 \\
&= (G_2^*w^4 - G_4^*w^2 - G_6^*)\cos\tau w \\
&\quad + (-G_1^*w^5 - G_3^*w^3 + G_5^*w)\sin\tau w \qquad (14) \\
G_1w^5 &+ G_3w^3 - G_5w \\
&= (-G_1^*w^5 - G_3^*w^3 + G_5^*w)\cos\tau w \qquad (15) \\
&\quad - (G_2^*w^4 - G_4^*w^2 - G_6^*)\sin\tau w.
\end{aligned}$$

Squaring both equations (15), (16) and adding we get,

$$\begin{aligned}
w^{12} &+ (G_1^2 - 2G_2 - G_1^{*^2})w^{10} + (G_2^2 + 2G_4 + 2G_1G_3 \\
&+ 2G_1^*G_3^* - G_2^{*^2})w^8 + (G_3^2 + 2G_6 - 2G_2G_4 \\
&- 2G_1G_5 - 2G_1^*G_5^* - G_3^{*^2})w^6 + (G_4^2 - 2G_2G_6 \\
&- 2G_3G_5 + 2G_3^*G_5^* - G_4^{*^2})w^4 + (G_5^2 + 2G_4G_6 \\
&- G_5^{*^2})w^2 + (G_6^2 - G_6^{*^2}) = 0. \qquad (16)
\end{aligned}$$

Let $z = w^2$ in (16)

$$\begin{aligned}
z^6 &+ (G_1^2 - 2G_2 - G_1^{*^2})z^5 + (G_2^2 + 2G_4 + 2G_1G_3 \\
&+ 2G_1^*G_3^* - G_2^{*^2})z^4 + (G_3^2 + 2G_6 - 2G_2G_4 \\
&- 2G_1G_5 - 2G_1^*G_5^* - G_3^{*^2})z^3 + (G_4^2 - 2G_2G_6 \\
&- 2G_3G_5 + 2G_3^*G_5^* - G_4^{*^2})z^2 + (G_5^2 + 2G_4G_6 \\
&- G_5^{*^2})z + (G_6^2 - G_6^{*^2}) = 0. \qquad (17)
\end{aligned}$$

If $R_0 < 1$, then from equation (17) we can see that $G_6^2 - G_6^{*^2}$ is strictly negative. Thus, we can get at least one positive real root. Hence, if $R_0 > 1$, all the real parts of the roots of (11) are negative. Thus, the equilibrium position E^* is stable when $R_0 > 1$ for $\tau > 0$. ■

III. Numerical Analysis

We provide a theoretical validation for this COVID-19 pandemic data from Tamil Nadu, India. By [4], the data's source is identified. The district of Coimbatore in Tamil Nadu provided the data for this investigation. After about 30 days, the coronavirus infection seems to have stabilized, according

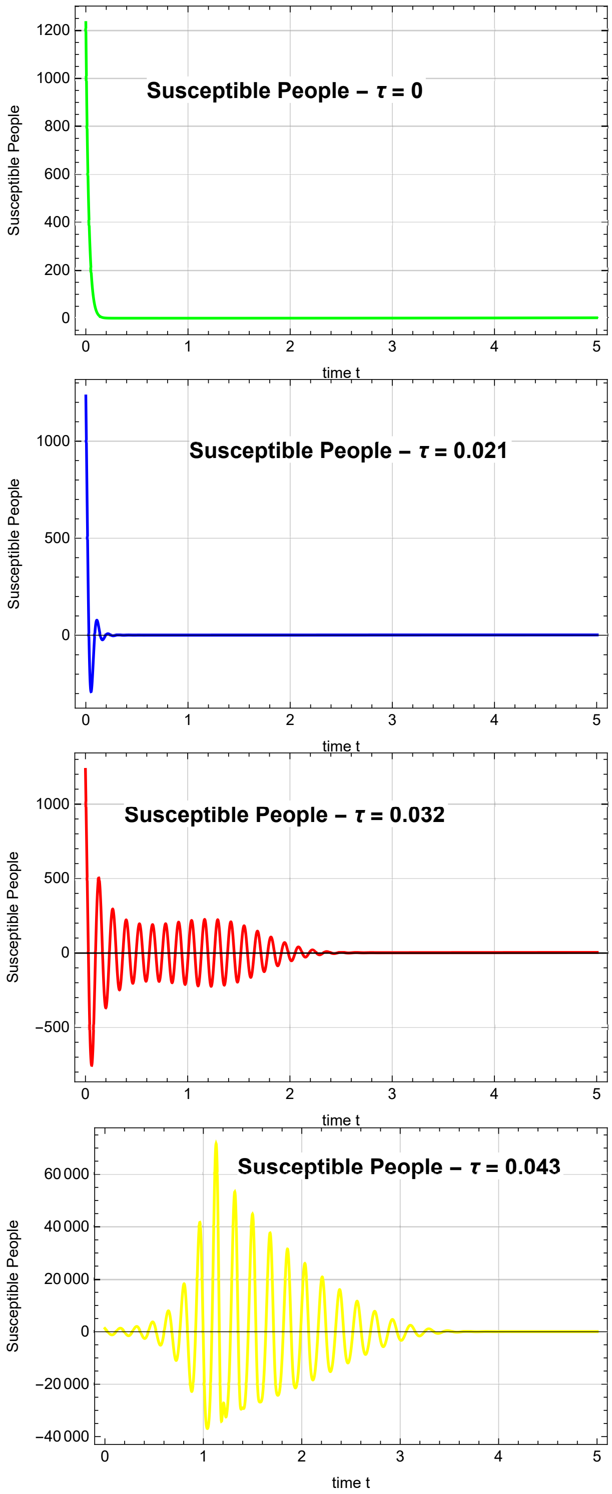

Fig. 1. Susceptible people S(t) with various τ for SEIQRV against time t

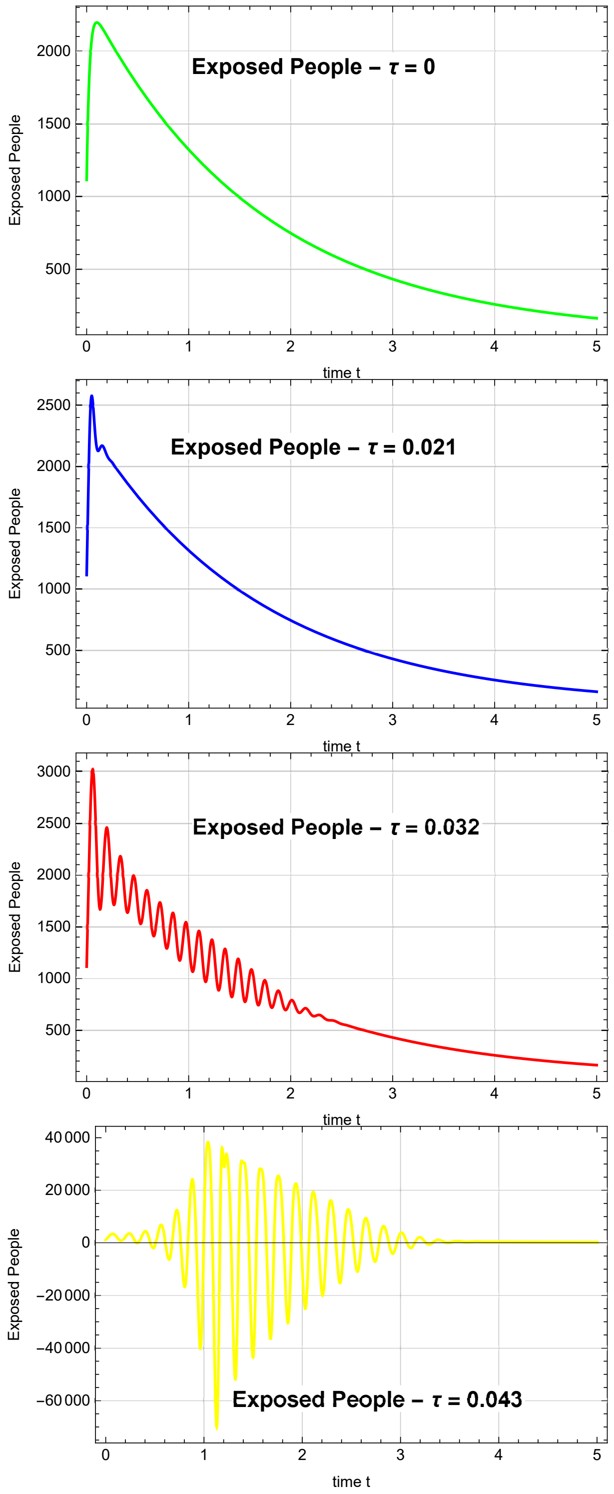

Fig. 2. Exposed people E(t) with various τ for SEIQRV against time t

to the data and our mathematical models. The table contains a list of the values for the parameters and variables I.

The Susceptible individual curves for the system SEIQRV is depicted with different values of τ = 0.021, 0.032, 0.043 in Figure 1.

As seen in Figure 2, a decrease in the exposed population results in a corresponding decrease in the population of other compartments; conversely, an increase in the exposed population causes a rise in the population of all related compartments.

The coronavirus infection rate may be reduced, as shown in Figures 3. Figure 3 illustrates how quickly the coronavirus spread after it was identified and how the government's widespread vaccination campaigns and quarantines stopped the variant's spread to a safe level. The SEIQRV model is able to moderately control the rate of increase in the number of infected individuals by incorporating additional compartments from earlier models. Those who received the COVID-19 vaccination were able to prevent the spread of the SARS-CoV-2 coronavirus. The infection rate is demonstrated in this figure with various values of τ (= 0.021, 0.032, 0.043).

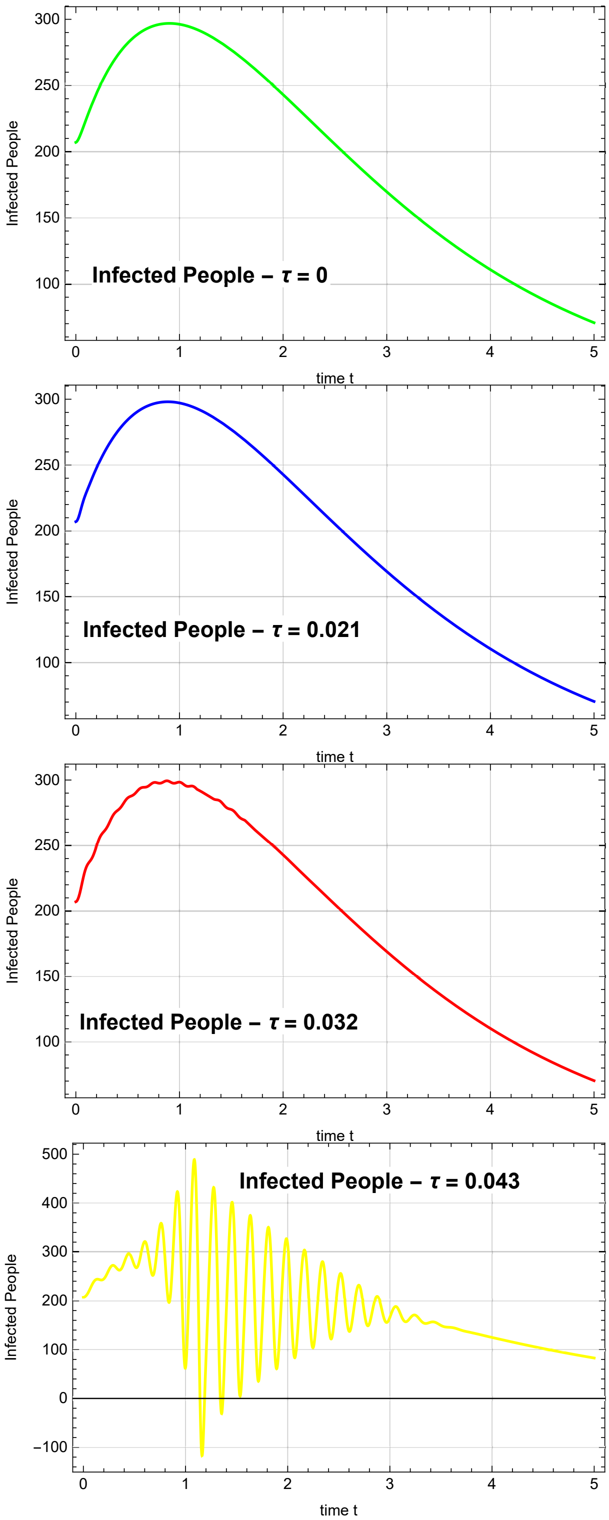

Fig. 3. Infected people I(t) with various τ for SEIQRV against time t

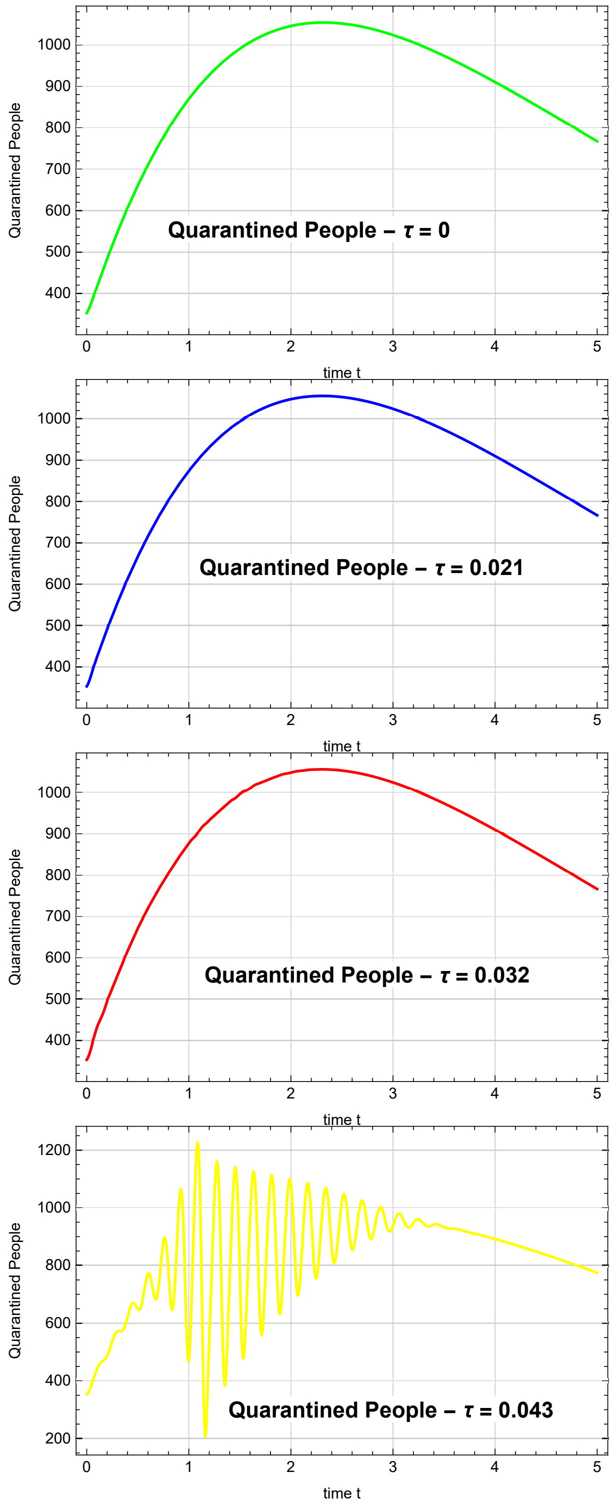

Fig. 4. Quarantined people Q(t) with various τ for SEIQRV against time t

Figure 4 shows the individual level that is quarantined at time t. The outbreak of the disease was stopped when the government imposed a strict quarantine on the state of Tamil Nadu, allowing things to return to normal.

Figure 5 shows the increase in recovered rates for both systems in Tamil Nadu. The SEIQRV system reaches stability by matching standard rates to both recovered and infected rates.

Figure 6 shows how the number of vaccinations is rapidly increasing. The system's infection rate consequently dropped dramatically, and it stabilized. These figures illustrate the importance of vaccination for the Omicron virus control strategy.

Figure 7 shows the effect of delayed SEIQRV model construction as a decline in reservoir individuals over time t. The stability of the SEIQRV mathematical model for the state of Tamil Nadu at different delay values is displayed in Figure 7. As Figure 7 illustrates, the infection rate for the SEIQRV system declines after a quick spread over an extended period of time. Within a few days, these systems

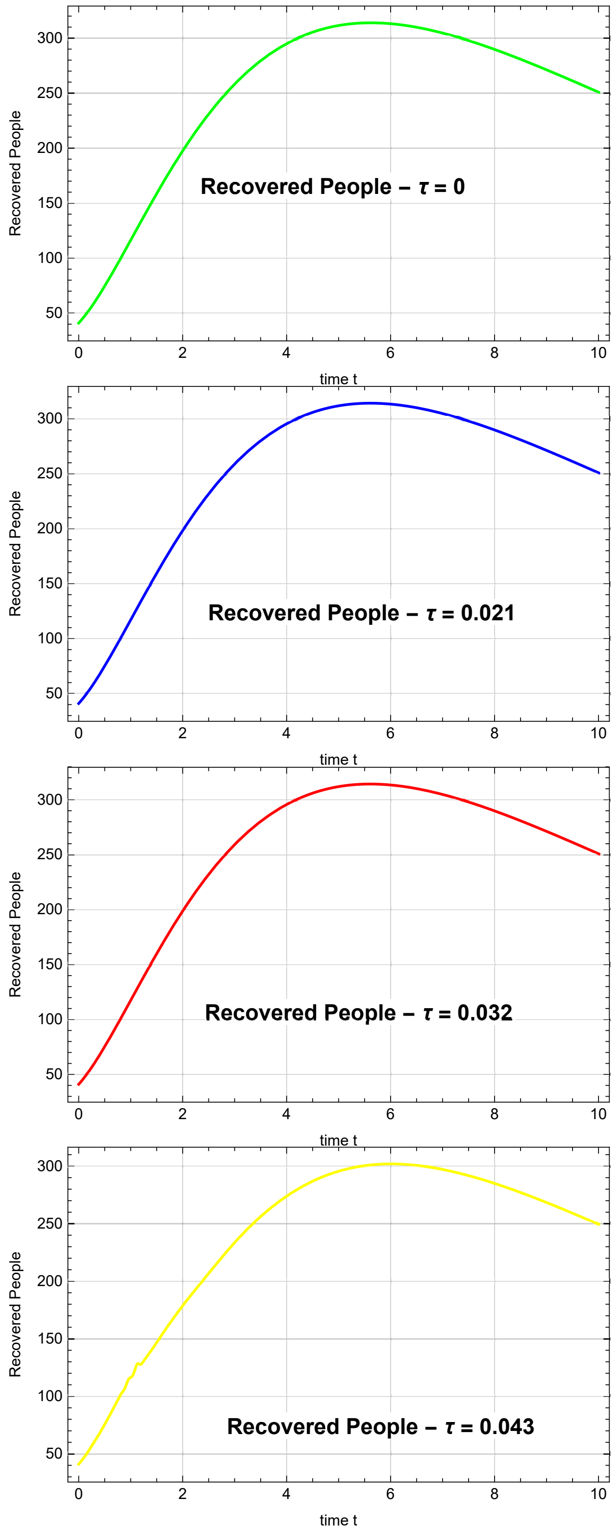

Fig. 5. Recovered people R(t) with various τ for SEIQRV against time t

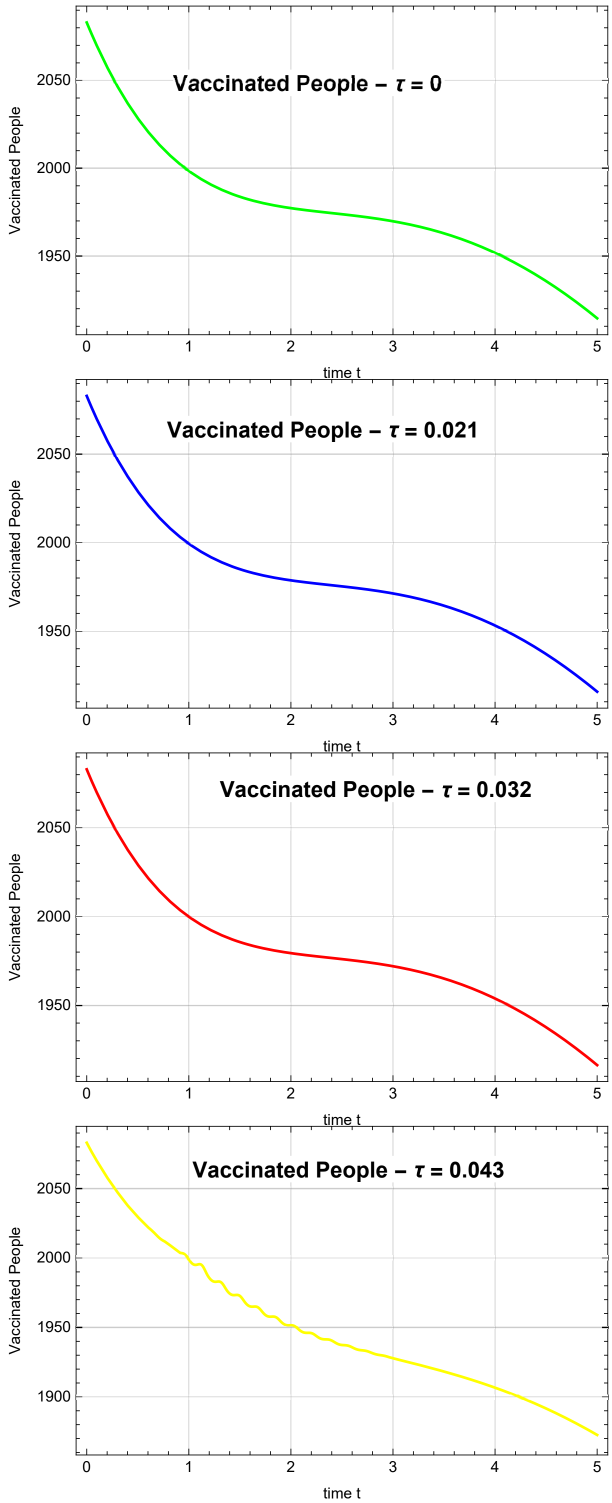

Fig. 6. Vaccinated people V(t) with various τ for SEIQRV against time t

contain the infection and prevent it from spreading.

IV. Conclusions

This paper developed a new delayed mathematical model for the coronavirus COVID-19. The stability of the SEIQRV model has been investigated and confirmed. It was discovered that the equilibrium points devoid of disease are unstable and will never stabilize when $R_0 > 1$, while they are locally asymptotically stable when $R_0 < 1$. Furthermore, we have discovered that stable endemic equilibrium points occur when $R_0 > 1$. Based on the known information, we may conclude that if more persons are separated, recovered, and vaccinated against the coronavirus, the host community will be safe. Scientists working in the medical field will find this study useful. Additional work can be done to generalize this with other fractional derivative models.

Authors' Contributions

All authors contributed equally to this work.

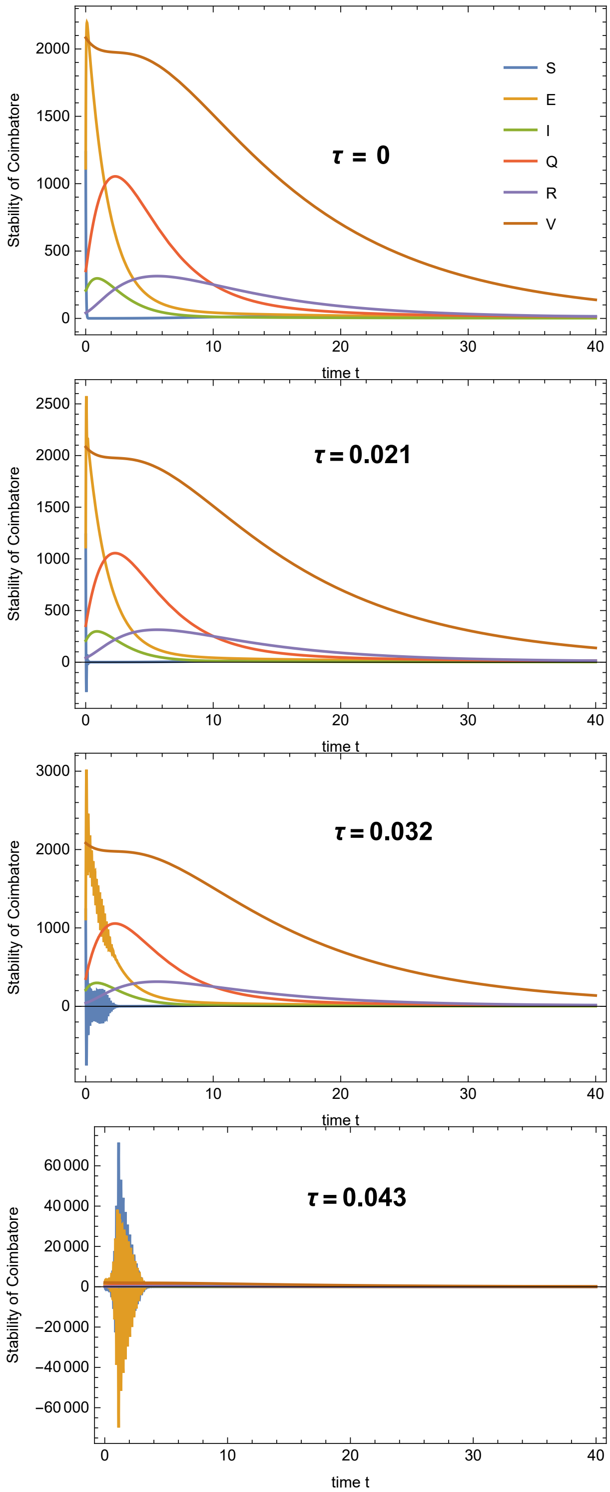

Fig. 7. SEIQRV system with various τ against time t

References

[1] S. A. R. Bavithra, S. Padmasekaran, *Epidemic SEIQRV Mathematical Model and Stability Analysis of COVID-19 Transmission Dynamics of Coronavirus*, J. Appl. Math. Inform., **41** (2023), no. 6, 1393-1407.

[2] S. Bugalia, J. P. Tripathi, and H. Wang, *Mathematical modelling of intervention and low medical resource availability with delays: Applications to COVID-19 outbreaks in Spain and Italy*, MBE, **18**, no. 5, 2021, 5865-5920. DOI:10.3934/mbe.2021295.

[3] G. E. Chatzarakis, S. Dickson, S. Padmasekaran, *A dynamic SI_qIRV Mathematical model with non-linear force of isolation, infection and cure*, Nonauton. Dyn. Syst., 9(1), 2022, 56-67, https://doi.org/10.1515/msds-2022-0145.

[4] S. Dickson, S. Padmasekaran, and G. E. Chatzarakis, *Stability Analysis Of B.1.1.529 SARS-Cov-2 Omicron Variant Mathematical Model: The Impacts Of Quarantine And Vaccination*, Nonauton. Dyn. Syst., **9**, no. 1, 2022, 290-306.

[5] S. Dickson, S. Padmasekaran, and G. E. Chatzarakis, and S. L. Panetsos, *SQIRV Model for Omicron Variant with time delay*, Aust. J. Math. Anal. Appl., **19** no. 2, Art. 16, 16pp, 2022.

[6] S. Dickson, S. Padmasekaran, and Pushpendra Kumar, *Fractional order mathematical model for B.1.1.529 SARS-Cov-2 Omicron variant with quarantine and vaccination*, Int. J. Dyn. Control., **11** (5), 2023, 2215-2231.

[7] S Dickson, S Padmasekaran, and P Kumar, *A Caputo-type fractional-order SQIRV mathematical model for Omicron variant*, Contemp. Math., **4**, no. 4, 2023, 620-636.

[8] S. Dickson, S. Padmasekaran, K. Lakshmanan, *Delayed Fractional order SEIQI_cRVW mathematical model for Omicron variant*, Int. J. Dyn. Control., 2023. https://doi.org/10.1007/s40435-023-01287-2.

[9] S. Dickson, S. Padmasekaran, Pushpendra Kumar, K. S. Nisar, and H. Marasi, A Study on the Transmission Dynamics of the Omicron Variant of COVID-19 using Nonlinear Mathematical Models, *Comput. Model. Eng. Sci.*, **139** (3), 2024, 2265-2287.

[10] H.K. Ebraheem, N. Alkhateeb, H. Badran, and E. Sultan, *Delayed dynamics of SIR model for COVID-19*, OJMSi., 9(2), 2021, 146-158.

[11] W. O. Kermack, A. G. McKendrick, A contribution to the mathematical theory of epidemics, Proc. R. Soc. Lond. A Math. Phys. Sci., **115**, 1927, 700–721.

[12] P. Kumar, S. Erturk, *A case study of COVID-19 epidemic in India via new generalised Caputo type fractional derivatives*, Math. Methods. Appl, Sci., **2021**, 2021, 1-14, DOI: 10.1002/mma.7284.

[13] P. Kumar, V. Govindaraj, and Z. A. Khan, *Some novel mathematical results on the existence and uniqueness of generalized Caputo-type initial value problems with delay*, AIMS Mathematics, **7**, no. 6, 2022, 10483-10494. DOI:10.3934/math.2022584.

[14] E. S. Kurkina and E. M. Kotsova, *Mathematical Modeling of the propagation of COVID-19 pandemic waves in the world*, Comp. math. and modelling, **32**, no. 2, 2021, 147-170, DOI: 10.23937/2474-3658/1510158.

[15] Z. Liu, P. Magal,O. Seydi, and G. Webb, *A COVID-19 epidemic model with latency period*, Infect. Dis. Model., **5**, 2020, 323-337.

[16] S.I. Niculescu, *Delay effects on stability*, Lecture notes in control and information Sciences, 269, Springer: London, UK, 2001.

[17] M. Tomochi, M. Kono, *A mathematical model for COVID-19pandemic-SIIR model: Effects of asymptomatic individuals*, J. Gen. and fam. Med., **22**, 2021, 5-14, DOI: 10.1002/jgf2.382.

[18] S. Tyagi, S. Gupta, S. Abbas, K.P. Das, and B. Riadh, *Analysis of infectious disease transmission and prediction through SEIQR epidemic model*, Nonauton. Dyn. Syst., **8** no. 1, 2021, 75-86. https://doi.org/10.1515/msds-2020-0126.

[19] C. Yang, Y. Yang,Z. Li, and L. Zhang, *Modeling and analysis of COVID-19 based on a time delay dynamic model*, MBE, **18** no. 1, 2021, 154-165.

[20] M.A. Zaitri,C. J. Silva, and D. F. Torres, *Stability analysis of delayed COVID-19 models*. Axioms, **11** no. 8, 2022, 400. https://doi.org/10.3390/axioms11080400.