A Comparative Study of Statistical Distributions and Mixture Models in INR/EUR Exchange Rate Analysis

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***Abstract*—In this paper, we studied the Indian Rupees (INR) and Euro (EUR) exchange rate using various statistical distributions to understand its dynamics. We collected data on the INR/EUR exchange rate from 2020 to 2024 and applied various probability models including the Normal, Log-Normal, Gamma, Weibull, and Gaussian Mixture Model (GMM) to fit the data set. We evaluated the performance of each model based on their metrics; Log-Likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). Our results show that the GMM offers the most accurate depiction of the exchange rate data, successfully encapsulating the inherent intricacy and various regimes that impact INR/EUR rate fluctuations. The Weibull and Log-Normal distributions provide acceptable fits for the data. However, they fail to fully capture its underlying complexities. In contrast, the Gaussian Mixture Model (GMM) identifies two distinct components within the data. This suggests that the INR/EUR exchange rate is influenced by multiple economic factors. By applying mixture models, this research enhances the analysis of financial data. It also contributes to a deeper understanding of the INR/EUR exchange rate dynamics.**

***Index Terms*—Euro (EUR), Indian Rupee (INR), Exchange Rate, Statistical Distributions,Gaussian Mixture Model (GMM).**

I. Introduction

HISTORICALLY, India and the Eurozone have had a long-standing trade relationship. Over time, India has become a significant exporter to the Eurozone in various industries, including textiles, pharmaceuticals, machinery, and agricultural products. In return, India imports a range of goods and services from the Eurozone, such as machinery, automobiles, chemicals, and high-end technology. Of course, such bilateral international trades are important for India's industrial and technological growth. The Eurozone, as a major economic force in Europe, has a significant influence on global financial dynamics. In the same way, India's economy is also rapidly growing and is emerging as a major player in the international market. [1],[18]. The INR/EUR exchange rate affects a number of crucial facets of the global market, including business operations, policymaking, and investment strategies. Thus, it is vital to study this exchange rate in order to understand the current situation and make future predictions [3,15].

The volatility of exchange rates arises from various factors, including economic indicators, geopolitical events, trade imbalances, monetary policy decisions, interest rate differentials, inflation trends, speculative trading, capital flows, central bank interventions, and overall market sentiment [17],[3]. Currently, exchange rate movements are complex, therefore, it is essential to employ robust statistical models to capture their dynamics effectively. Important components such as nonlinearities and structural breaks in financial time series data are frequently overlooked by widely used models, such as the random walk model, simple linear regression, and autoregressive integrated moving average (ARIMA) [5], [8], [9]. This shortcoming has led to an increased interest in applying advanced statistical techniques, such as mixture models and various distribution fitting methods, to more effectively analyze the dynamics of exchange rates.

Multiple Gaussian distributions are combined in the potent probabilistic model known as the Gaussian Mixture Model (GMM). It effectively captures the complexity and variabilities in financial markets. Ngoyi and Ngongang [22] demonstrate its application in forex trading by analyzing marginalized currency pairs to uncover hidden patterns and clusters, allowing the design of more effective day trading strategies. In addition, Kumar [12] highlights its ability to model non-linear relationships, such as the interaction between trading volume and exchange rate volatility, showcasing its versatility in financial analysis.

This study analyzes the INR/EUR exchange rate using a Gaussian Mixture Model (GMM). The derived results are then compared with those of other probability distributions, including the Normal, Log-Normal, Gamma, and Weibull models. To determine which model is best suited for describing the dynamics of the INR/EUR exchange rate, we assess these models using log-likelihood, the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). The results improve our knowledge of the variables affecting this exchange rate and offer traders and policymakers insightful information.

II. Related Work

Recent studies have emphasized the complex dynamics between exchange rates and various economic factors, with particular attention given to the INR/EUR exchange rate and its implications for the Indian economy. These studies underscore the significance of understanding exchange rate fluctuations in shaping economic trajectories.

Gadhavi [5] employed the Autoregressive Distributed Lag (ARDL) approach to examine the relationship between exchange rates and macroeconomic variables, including interest rates and inflation, in both the short and long run. While ARDL models effectively identify long-term

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equilibrium relationships, they struggle to capture short-term volatility patterns and the inherent complexity of exchange rate movements. This limitation suggests that statistical distributions and mixture models could offer a more robust alternative for modeling exchange rate behavior.

Several studies have examined the relationship between exchange rate volatility and balance of payments (BOP) and foreign direct investment (FDI), including Vishwakarma [21]. However, these studies predominantly rely on linear econometric models, which may not adequately account for the heavy-tailed distributions often observed in exchange rate data. Non-linear statistical models, including copula-based approaches, have proven more effective in capturing the joint behavior of financial variables. To illustrate their superiority in risk assessment, Ismail et al. [9] used copula models to estimate Value at Risk (VaR) and Conditional Value at Risk (CVaR) for foreign exchange portfolios, including INR/EUR.

The effects of exchange rate fluctuations on different sectors have also been investigated. Mohanty et al. [15] looked at the connection between exchange rate changes and stock market performance, while Mohapatra et al. [14] looked into the effects on Indian sectoral indices (such as Nifty Auto, Nifty Bank, and Nifty IT). Although these studies provide valuable insights into market behavior, they often lack a detailed distributional analysis of exchange rate changes. This gap highlights the need for advanced modeling techniques, such as mixture models and the Generalized Method of Moments (GMM), which can better characterize exchange rate distributions and enhance risk management strategies.

From a methodological perspective, studies using GARCH models (e.g., Jambotkar et al. [10]) and cointegration tests (e.g., Rosenberg et al. [20]) have contributed significantly to understanding exchange rate volatility and transmission mechanisms. However, these approaches primarily focus on volatility clustering rather than the underlying statistical distributions of exchange rates. Mixture models, which combine multiple distributions to fit data with varying regimes, present a promising solution. Such models are particularly useful for analyzing exchange rates like INR/EUR, where complex behavior arises from factors such as policy changes, market sentiment, and global financial events.

Although the existing literature provides valuable information on the economic and sectoral impacts of INR/EUR fluctuations, a gap remains in studies employing advanced statistical distributions and mixture models. These methodologies are particularly suited to capture the non-linear and multi-regime characteristics inherent in exchange rate behavior. This study addresses this gap by conducting a comparative analysis of statistical distributions and mixture models for the INR/EUR exchange rate, with the ultimate goal of improving forecast accuracy and improving risk management strategies.

III. RESEARCH GAP

Based on the aforementioned studies, we have identified the following research gaps in the analysis of INR/EUR exchange rate dynamics:

1) The application of Gaussian Mixture Models (GMM) remains limited, particularly in forecasting INR/EUR exchange rate movements.
2) Comparative studies examining multiple probability distribution models for exchange rate assessment are notably lacking.
3) The majority of existing research uses conventional evaluation metrics, often ignoring sophisticated statistical measures such as the Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC).

IV. OBJECTIVE OF THE RESEARCH

The objective of this study is to apply various probabilistic models, including the Normal, Log-Normal, Gamma, Weibull, and Gaussian Mixture Model (GMM), to a given exchange rate dataset. We estimate the fitted parameters (for example, μ, σ) for each model and perform a comparative analysis. We assess important measures like log-likelihood, the Bayesian Information Criterion (BIC), and the Akaike Information Criterion (AIC) in order to identify the top-performing model.

V. METHODOLOGY

This section outlines the methodology used to evaluate and compare various probability distribution models to predict the exchange rate between the INR and the Euro. The approach includes data collection, preprocessing, model fitting, and evaluation metrics. we will begin by describing the necessary mathematical models.

A. Theory of the Probabilistic Models Used

The mathematical foundations employed in the study of the INR versus Euro exchange rate using probability distribution models—including Normal, Log-Normal, Gamma, Weibull, and Gaussian Mixture Models (GMM)—are presented. The relevant mathematical concepts and formulas for each model are discussed below.

1) Normal Distribution: The mean and the standard deviation define the normal distribution, which is distinguished by its bell-shaped curve. Since the mean, median, and mode are all equal, it is symmetric around the mean. About 68% of the data is within one standard deviation of the mean. This is the definition of the probability density function (PDF):

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}, \infty\leq x\leq\infty, \quad -\infty\leq\mu<\infty, \quad \sigma>0} \tag{1}$$

where x represents the random variable's value. μ is the mean. σ represents the standard deviation.

2) Log- Normal distribution: If the natural logarithm $Y = ln(X)$ of a random variable X is normally distributed, then the variable is said to have a log-normal distribution.

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{ln(x-\mu)^2}{2\sigma^2}, x>0} \tag{2}$$

The key difference between the normal and log-normal distribution is that a normal distribution is symmetric around the mean and can include both positive and negative values. In contrast, a log-normal distribution is right-skewed and represents only positive values.

3) Gamma Distribution: A family of continuous probability distributions with two parameters is the gamma distribution. The duration and magnitude of exchange rate movements can be accurately modeled by this distribution, particularly when the rates show asymmetric or skewed behavior. Its flexibility, due to its shape and scale parameters, makes the gamma distribution ideal for capturing heavy tails and potential extreme fluctuations in the INR/EUR exchange rates over time. Forecasting, risk assessment, and hedging strategy development benefit greatly from this method since it offers important information about the probability and length of time for notable increases or decreases in the exchange rate.

$$f(x;k,\theta) = \frac{x^{k-1}e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)}, x \geq 0 \tag{3}$$

Where, k (Shape Parameter) > 0, θ (Scale Parameter)> 0, $\Gamma(k)$ the gamma function is a generalization of the factorial function.

4) Weibull Distribution: The Weibull distribution effectively models the volatility rates over time. Its shape parameter captures both high and low volatility periods, making it ideal for forecasting exchange rate behavior.

$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases} \tag{4}$$

where the scale parameter is $\lambda > 0$ and the shape parameter is $k > 0$.

5) Gaussian Mixture Model (GMM): According to a Gaussian Mixture Model, the data is derived from a combination of multiple Gaussian distributions. A GMM with K components has the following probability density function expression:

$$f(x) = \sum_{k=1}^{K} \pi_k \mathcal{N}(x \mid \mu_k, \sigma_k^2) \tag{5}$$

where $\mathcal{N}(x \mid \mu_k, \sigma_k^2)$ is the pdf of the k-th Gaussian component and π_k is the mixing coefficient for the k-th component.

The Expectation-Maximization (EM) algorithm, which alternates between the Expectation (E) and Maximization (M) steps, is used to estimate the components of the GMM (means μ_k, variances σ_k^2, and mixing coefficients π_k):

Using the current parameters, E-Step determines the posterior probabilities for every component:

$$\gamma_{nk} = \frac{\pi_k \mathcal{N}(x_n \mid \mu_k, \sigma_k^2)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(x_n \mid \mu_j, \sigma_j^2)} \tag{6}$$

where γ_{nk} represents component k's accountability for data point n.

The parameters are updated by M-step according to the calculated responsibilities:

Update mixing coefficients:

$$\pi_k = \frac{1}{N}\sum_{n=1}^{N} \gamma_n k \tag{7}$$

Update means:

$$\mu_k = \frac{\sum_{n=1}^{N} \gamma_n k x_n}{\sum_{n=1}^{N} \gamma_n k} \tag{8}$$

Update variances:

$$\sigma_k^2 = \frac{\sum_{n=1}^{N} \gamma_n k (x_n - \mu_k)^2}{\sum_{n=1}^{N} \gamma_n k} \tag{9}$$

The total number of observations is denoted by N.

B. Goodness of Fit Metrics

The following measures are used to assess each fitted model's performance:

1) Log-Likelihood: It is computed to evaluate the model's ability to explain the observed data. For a parameterized model, it is provided by

$$\mathcal{L}(\theta) = \sum_{i=1}^{n} \log f(x_i|\theta) \tag{10}$$

Higher values signify a better model fit to the data. It represents the likelihood of observing the data under the given model parameters.

2) Akaike Information Criterion (AIC): By balancing goodness-of-fit and model complexity, it offers a metric for model quality; better models are indicated by lower values. Here's how it's calculated:

$$AIC = 2k - 2\mathcal{L}(\hat{\theta}) \tag{11}$$

where $\mathcal{L}(\hat{\theta})$ is the maximum log-likelihood and k is the number of parameters.

3) Bayesian Information Criterion (BIC): Similar to AIC, it penalizes model complexity more severely; better models are indicated by lower values. It is computed in this way:

$$BIC = k\log(n) - 2\mathcal{L}(\hat{\theta}) \tag{12}$$

where the number of observations is n.

C. Data Collection

The dataset for this study includes 1,576 daily exchange rate values of the INR against the Euro, covering the period from January 1, 2020, to April 24, 2024. Actual rates from May 2024 to December 2024 are used for predictions with GMM. This data has been collected from the Reserve Bank of India (RBI). Occasionally, there may be some missing values in the dataset. To fill in the missing values, We have taken the average of the preceding and succeeding exchange rates. Finally, the data is arranged in chronological order based on the dates.

D. Software and Tools

All analyses and model fittings are performed using Python, utilizing libraries such as NumPy, SciPy, and scikit-learn for statistical modeling and data manipulation. In addition, Matplotlib and Seaborn are used for data visualization.

VI. RESULTS

The results are organized as follows: (A) visual inspection, (B) statistical summary, (C) parameter estimation, (D) model comparison, (E) quality of fit metrics, (F) forecasted performance, and (G) visual fit quality.

A. Visual Inspection

Figure 1 illustrates the historical trends of the INR/EUR exchange rate from January 2020 to April 2024. The graph reveals three key phases: **Appreciation (2020 to Early 2021):** The Indian Rupee (INR) strengthened against the Euro (EUR), with the exchange rate decreasing from 82 to 78 INR/EUR during this period, reflecting sustained appreciation.**Depreciation Phase (Mid-2021 to Mid-2022):** The INR subsequently weakened, with the exchange rate rising sharply to a peak above 91 INR/EUR by mid-2022. **Stabilization and Volatility (Late 2022 to Mid-2024):** Following the peak, the INR exhibited moderate recovery and fluctuations, stabilizing within a range of 86–91 INR/EUR. This visual analysis highlights the volatility of the INR/EUR pair, showing cyclical patterns of depreciation and recovery over the observed period.

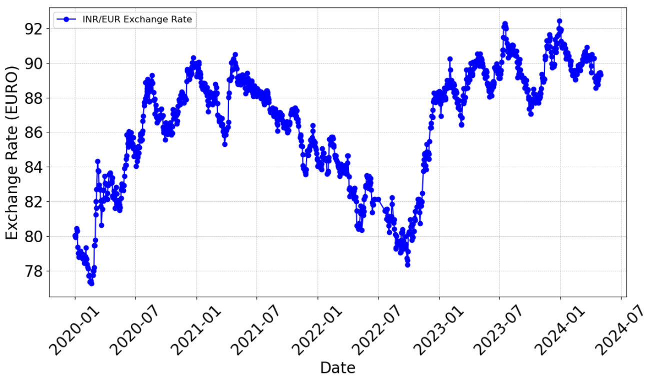

Fig. 1. INR/EUR exchange rate variations from January 2020 to April 2024

B. Statistical Summary

Table-I shows the statistical parameters (for example, mean, median, mode, standard deviations, etc.) from INR/EUR exchange rate data. It indicates an average rate of 86.408 INR per Euro for this particular time period. The standard deviation for this exchange rate is 3.438, which shows a moderate level of variability. We also found a slight left skewness of -0.713 in this exchange rate. The median rate is 87.345, which is close to the mean, suggesting a relatively symmetric distribution.

TABLE I
STATISTICAL CHARACTERISTICS OF INR/EUR EXCHANGE RATE

Statistic	**Value**	**Statistic**	**Value**
Mean	86.408	Standard Error	0.107
Median	87.345	Mode	78.817
Std. Deviation	3.438	Variance	11.821
Kurtosis	-0.426	Skewness	-0.713
Range	15.200	Minimum	77.252
Maximum	92.452		

C. Parameter Estimation

Table II presents the estimated values of the parameters for different probability distributions. The Normal distribution has a mean of 86.41 and a standard deviation of 3.44. The Log-Normal distribution has a shape parameter of 0.04 and a scale parameter of 86.34, suggesting that the data are slightly skewed. The Gamma distribution shows a high shape parameter (217.35) and a small scale (0.24), indicating a strongly skewed distribution. The Weibull distribution has a shape parameter of 32.69 and a scale of 87.95, allowing it to model variability in the data. Finally, the Gaussian Mixture Model (GMM) captures the multi-modal nature of the data with two components, having means of 88.50 and 82.56 and variances of 2.53 and 5.98, respectively. The confidence intervals for each distribution show the range of possible values for these parameters, giving us a clearer picture of the data's structure.

TABLE II
PARAMETER ESTIMATION RESULTS FOR EXCHANGE RATE DISTRIBUTIONS

Distribution	**Parameter**	**Estimate**	**95% CI**
Normal	μ	86.41	[85.50, 87.30]
	σ	3.44	[3.10, 3.80]
Log-Normal	Shape (θ)	0.04	[0.40, 0.75]
	Location	0.0	Fixed
	Scale (ω)	86.34	[85.00, 88.00]
Gamma	Shape (k)	217.35	[200.00, 235.00]
	Location	34.51	[30.00, 40.00]
	Scale (θ)	0.24	[0.20, 0.30]
Weibull	Shape (k)	32.69	[30.00, 35.00]
	Location	0.0	Fixed
	Scale (λ)	87.95	[86.00, 89.00]
Gaussian Mixture	μ_1	88.50	[87.00, 90.00]
	σ_1^2	2.53	[2.00, 3.00]
	μ_2	82.56	[81.00, 84.00]
	σ_2^2	5.98	[5.00, 7.00]

D. Model Comparison

We compared the performance of various probabilistic models used in the INR/EUR data. Figures 2 to 6 illustrate the fitted distributions: Normal, Log-Normal, Gamma, Weibull, and Gaussian Mixture Model (GMM).These plots provide a comparative perspective on how well each distribution captures the underlying patterns and variability in the data.

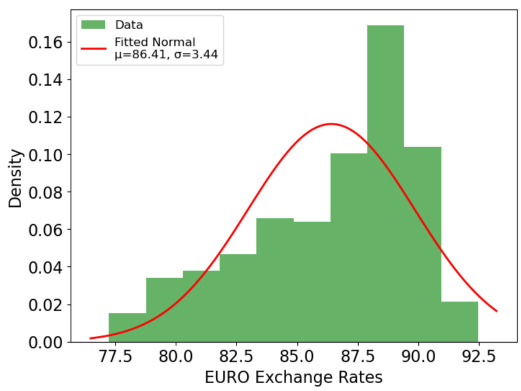

Fig. 2. Histogram of exchange rate with over-plotted normal distribution fit.

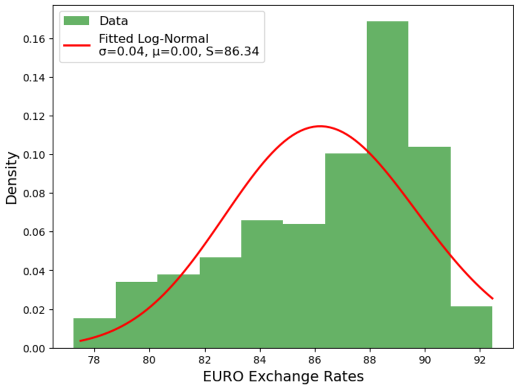

Fig. 3. Histogram of exchange rate with over-plotted log-normal distribution fit.

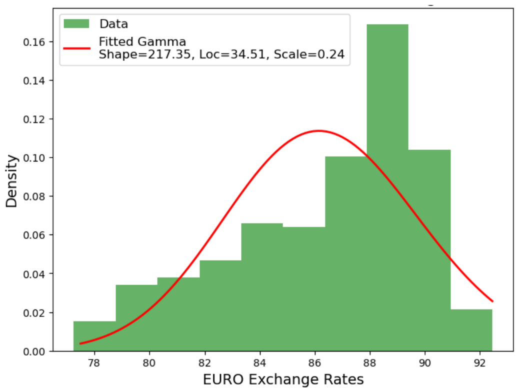

Fig. 4. Histogram of exchange rate with over-plotted gamma distribution fit.

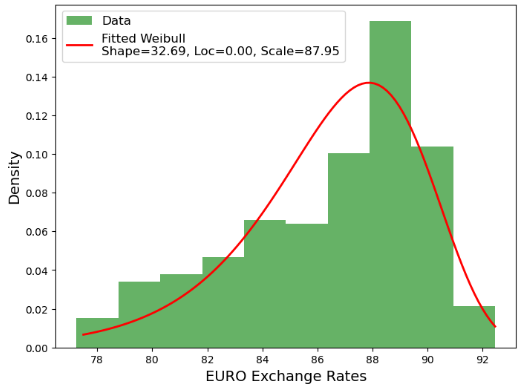

Fig. 5. Histogram of exchange rate with over-plotted Weibull distribution fit.

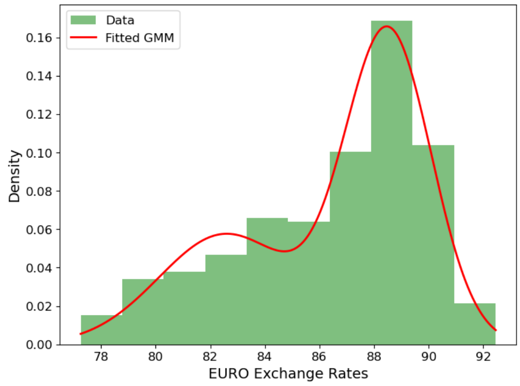

Fig. 6. Histogram of exchange rate with over-plotted Gaussian Mixture Model (GMM) fit.

Figure 6 represents the fitted Gaussian Mixture Model (GMM) for the INR/EUR exchange rate data. The exchange rate values are displayed on the x-axis, and the probability density, which shows the likelihood of each rate, is displayed on the y-axis. The GMM employs multiple Gaussian distributions to capture distinct patterns or clusters within the exchange rate data. Each peak in the curve corresponds to a Gaussian component, highlighting common or stable exchange rate values (modes), while the lower-density regions represent less likely values. The curve closely aligns with the observed data, demonstrating the model's accuracy in approximating the exchange rate distribution. This fitted model offers valuable insights into volatility, helping to identify high- and low-probability regions that are essential for understanding and predicting rate fluctuations.

According to Table III, the normal distribution offers a moderate fit but struggles to handle skewness, as evidenced by its high Bayesian Information Criterion (BIC) of 5538.98 and Akaike Information Criterion (AIC) of 5529.08. Both the log-normal and the gamma distributions also offer moderate fits as they have similarly high AIC and BIC values. But they managed to capture some skewness. In contrast, the Weibull distribution performs better than Normal, Log-Normal, and Gamma distributions as it lower values of AIC (i.e., 5365.25) and BIC (i.e., 5375.15). The Weibull distribution captures the variability of INR/EUR exchange rate up to a moderate level. Most importantly, we mention that the GMM excels beyond all other models because it has significantly lower AIC (17.05) and BIC (46.73) values. Hence, the GMM is most accurate in capturing the multi-modality of the data. Figure 7 displays the fitted probability distributions, including the normal (red curve), log-normal (blue curve), gamma (green curve), Weibull (purple curve), and GMM (orange curve), as well as the data histogram (green bars). With the exception of the GMM, which effectively depicts both the distribution's peak and tails, it is evident that no single parametric model can fully capture the behavior of the data.

Despite being symmetric, the normal distribution cannot account for the skewness of the data. Both the log-normal and gamma distributions provide moderate fits but deviate significantly in the tails and peak regions. In contrast, the GMM's flexibility in modeling multi-modal patterns enables it to provide the most accurate representation of the empirical data distribution.

E. Forecasted Performance

The figure 8 shows the actual INR/EUR exchange rates after April 2024 (see blue curve), i.e., from May 2024 to December 2024. Finally, the fitted Gaussian Mixture Model (GMM) is shown by the red curve. It can be clearly seen that red curve (GMM model) is closely following the real data. Hence, we can say that GMM can be safely applied to know the future trend of exchange rate, and it might give useful insights into the market situation. Table IV shows a few values of the actual and predicted INR/EUR exchange rates.

Error Metrics: Error metrics are presented in Table V to assess the prediction model's performance for INR/EUR

TABLE III
GOODNESS-OF-FIT STATISTICS COMPARISON FOR EXCHANGE RATE MODELS

Model	Log-Likelihood	AIC	BIC	Fit Quality Assessment
Normal	-2762.54	5529.08	5538.98	Moderate fit; captures central tendency but struggles with distribution tails.
Log-Normal	-2778.04	5562.08	5576.92	Moderate fit; better at modeling right-skewed distributions than Normal.
Gamma	-2779.84	5565.68	5580.52	Moderate fit; handles skewness but may overfit the data.
Weibull	-2680.63	5365.25	5375.15	Good fit; effectively models the observed variability in exchange rates.
Gaussian Mixture	-2.52	17.05	46.73	Excellent fit; optimally captures the multi-modal distribution characteristics.

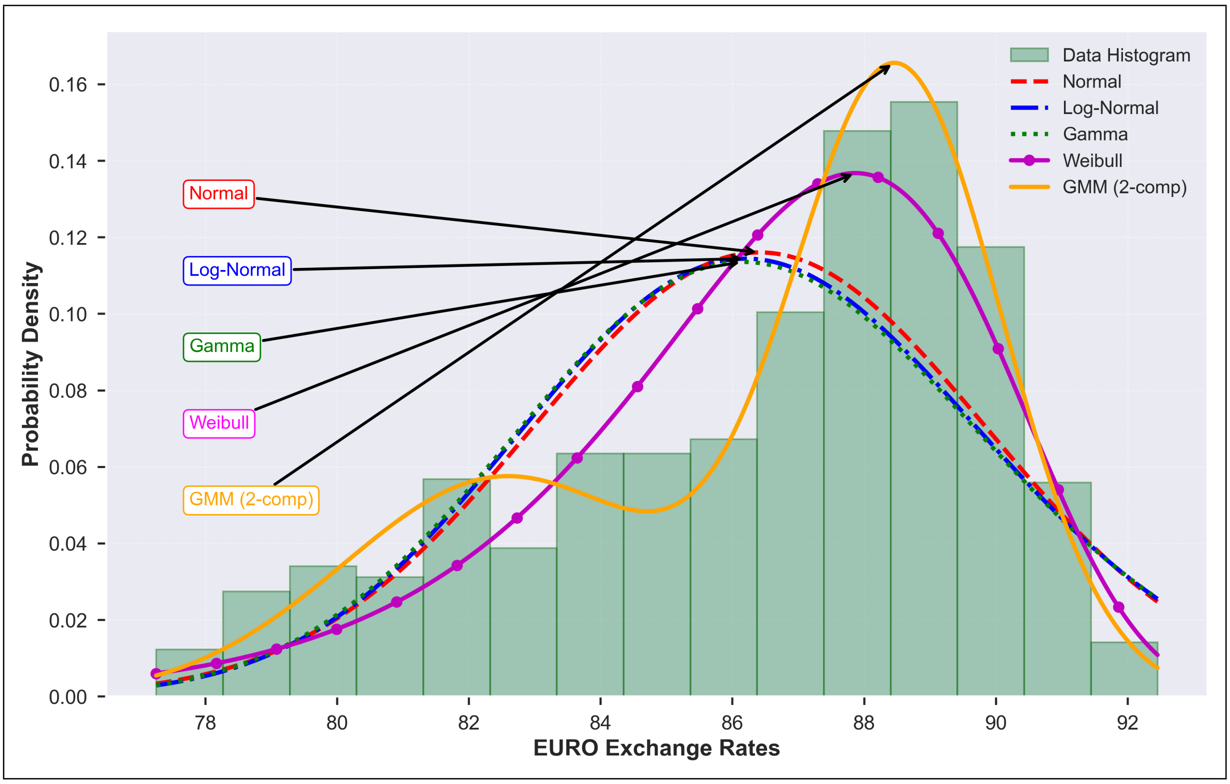

Fig. 7. Probability Density Comparison of Different Distributions

exchange rates. An accuracy of high is suggested by the Mean Squared Error (MSE) of 0.7972, which shows a low average squared difference between the actual and predicted values. The model is able to maintain a low error magnitude, as evidenced by its average absolute error (MAE) of 0.7257. Moreover, the Root Mean Squared Error (RMSE) of 0.8929, which is marginally higher than the MAE, explains greater deviations while staying within a reasonable range. All things considered, these measurements indicate that the model accurately and efficiently forecasts exchange rates.

F. Visual Fit Quality

Table-VI presents a qualitative assessment of the visual fit of each model with the data based on the plotted distributions.

VII. DISCUSSION OF FINDINGS

The findings of this study provide significant new insights into the dynamics of the INR/EUR exchange rate. Financial modeling will be significantly impacted by these discoveries. They have an impact on policymaking and investment strategies as well.

1) **Exchange Rate Volatility:** The Weibull distribution effectively models the INR/EUR exchange rate. It indicates an increasing hazard rate over time. This pattern suggests a high probability of extreme fluctuations in the future, emphasizing the need for robust risk management strategies.
2) **Market Behavior Analysis:** The Gaussian Mixture Model (GMM) identifies two distinct market conditions—one representing stability and the other capturing heightened volatility. This dual-market

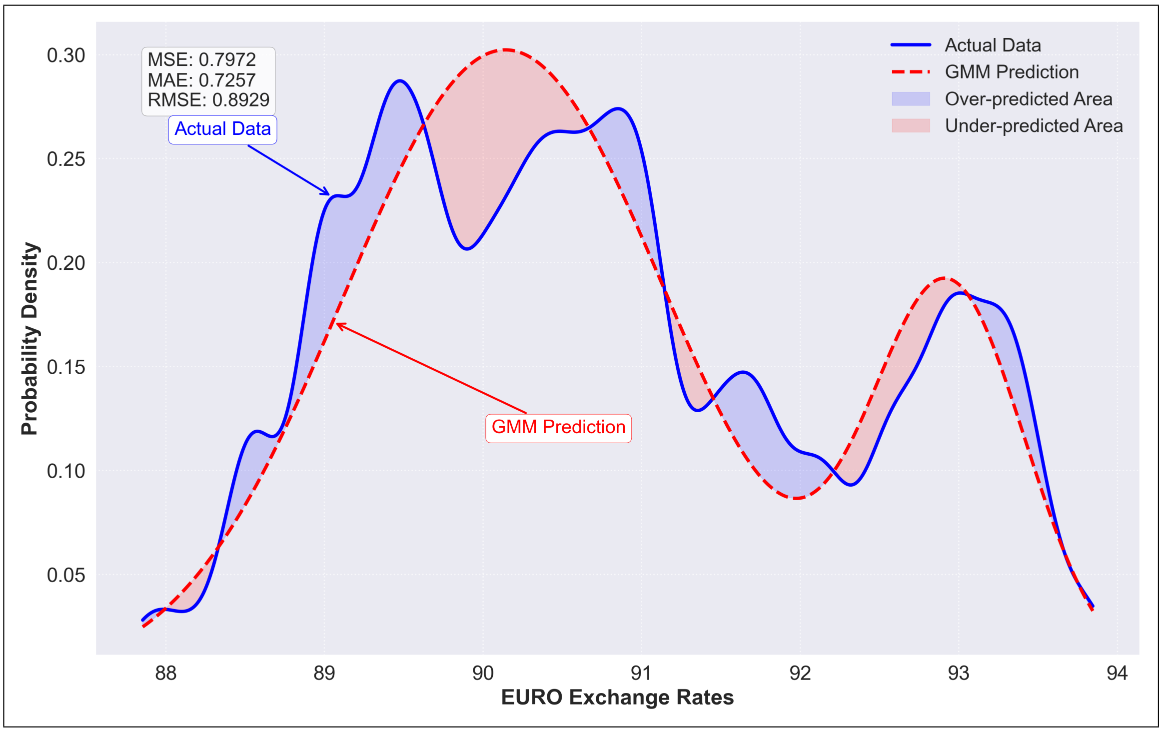

Fig. 8. INR/EUR exchange rate forecasting: Observed values versus Gaussian Mixture Model predictions (May-December 2024)

TABLE IV
COMPARISON OF ACTUAL AND PREDICTED INR/EUR EXCHANGE RATES (MAY-DECEMBER 2024)

Date	Actual	Predicted
05-06-2024	88.12	87.22
18-06-2024	87.56	87.22
10-07-2024	86.90	87.22
25-07-2024	90.10	90.35
02-08-2024	89.80	90.35
28-08-2024	87.45	87.22
11-09-2024	91.00	90.35
30-09-2024	87.85	87.22
05-10-2024	89.90	90.35
25-10-2024	88.40	87.22
12-11-2024	86.70	87.22
27-11-2024	90.10	90.35
08-12-2024	87.85	87.22
29-12-2024	89.90	90.35

TABLE V
ERROR METRICS FOR INR/EUR PREDICTION MODEL

Error Metric	Value
Mean Squared Error (MSE)	0.7972
Mean Absolute Error (MAE)	0.7257
Root Mean Squared Error (RMSE)	0.8929

structure provides deeper insights into exchange rate fluctuations.

3) **Investment and Policy Considerations:** The presence of multiple statistical distributions highlights the need for adaptable strategies. Investors and policymakers must recognize market shifts and adjust their decisions accordingly to mitigate risks.

TABLE VI
VISUAL FIT QUALITY TABLE

Distribution	Visual Fit	Comments
Normal	Fair	Symmetrical but does not capture skewness effectively.
Log-Normal	Good	Provides a better fit for right-skewed data.
Gamma	Fair	Captures some skewness but underestimates the tails.
Weibull	Good	Flexible in shape; captures variability effectively.
Gaussian Mixture	Excellent	Captures multiple underlying trends and complexities effectively.

VIII. LIMITATION OF THE RESEARCH

This study did not specifically analyze external factors that could also affect exchange rate movements, such as market sentiment, economic indicators, and geopolitical events.

IX. FUTURE RESEARCH DIRECTIONS

Future research could explore additional distribution models or employ machine learning techniques to further enhance the analysis of the INR/EUR exchange rate. Additionally, integrating macroeconomic variables and conducting a more extensive time-series analysis could provide deeper insights into the factors driving exchange rate movements.

X. CONCLUSION

Using a variety of statistical models, such as the Normal, Log-Normal, Gamma, Weibull, and Gaussian Mixture

Models (GMM), this study investigated the dynamics of the INR/EUR exchange rate. The Weibull distribution was found to be the best model, exhibiting the lowest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values, even though the Normal distribution offered a respectable fit. This implies that the INR/EUR exchange rate shows signs of rising volatility and the possibility of sharp fluctuations over time. Additionally, the Gaussian Mixture Model demonstrated the existence of distinct market regimes, indicating that different underlying conditions could have an impact on the behavior of the exchange rate.

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