Anti-magic Labeling on Anti Fuzzy Path and Star Graphs

S. Suruthi, M. Shanmuga Sundari and L. Sivakami

Abstract—**A fuzzy graph is a mathematical system that associates the idea of uncertainty, or fuzziness, with traditional graphs. Classical graph theory depicts edges between vertices as binary relationships, signifying the presence or absence of a connection between them. In contrast, fuzzy graphs can represent degrees of connection or membership, providing a more realistic and flexible framework for describing real-world systems with ambiguous or imprecise relationships. The idea of fuzzy graph labeling is to extend traditional graph labeling to handle vague information associated with vertices, edges, or both. This extension allows for a more nuanced representation of relationships within a graph, recognizing that not all connections or characteristics are absolute. An anti-fuzzy graph is established when the flow of an edge goes beyond the maximum value of the vertices. Anti-fuzzy graphs find uses in diverse fields, including decision-making, pattern recognition, and network analysis. Also, extending the labeling concept in anti-fuzzy graph is very helpful to handle real-time problems. This paper aims to study the concepts of anti-magic labeling for anti-fuzzy graphs. We proposed an algorithm for anti-fuzzy labeling(AFL) in path and star graphs. We have also established anti-fuzzy edge anti-magic (AFEAM) and anti-fuzzy vertex anti-magic (AFVAM) using proposed algorithms for paths and star graphs, respectively. We derive their properties from vertex degree, strong degree, and strong edge.**

Index Terms—**Fuzzy graph, Anti-fuzzy graph, Anti-fuzzy graph labeling, Anti-magic graph**

I. Introduction

ACCORDING to Zadeh [1], fuzzy concepts can represent actual uncertainty. In addition, Rosenfeld [2] and Yeh & Bang [3] have conducted independent studies on fuzzy graph development. We can apply fuzzy graphs (FG) to solve various problems. Although it is still very young, it has a wide range of applications and is growing rapidly. Fuzzy graphs have also seen exponential growth in mathematics, science, network technology, and so on. In [4], Akram defines the anti-fuzzy graph (AFG) as a new concept. AFG is an exciting research topic because there are not many studies on AFG. In [5], Muthuraj et al. discussed the degrees of AFG and characterized regular and irregular AFG. Sanggoor & Alwan [6] discuss operations on regular and irregular AFGs. Kalaivani [7] has discussed completeness in the AFG. Seethalakshmi [8] explained some operations on AFG. Trisanti et al. [9] incorporated different types of AFG products. Nusantara et al. [10] discussed the regularity of vertex and edge in AFG. In [11], the authors presented the methodology of AFGs and incorporated the regularity and strength of these graphs. Nusantara et al. [12] postulated the idea of line graphs in AFGs and discussed the isomorphism between AFGs and their relevant line graphs. Jingwen et al. [13] proposed a procedure for adjacent vertex reducible edge labeling for some special graphs.

Hartsfield et al. [14] provided anti-magic labeling in 1994. Baca et al. [15] developed the concept of EAM for some families of graphs. Chang et al. [16] and Latchoumanane et al. [17] illustrated the anti-magic labeling of regular and product regular graphs. Anjali Yadav and Minirani [18] explored the application of distance anti-magic labeling in specific graphs for surveillance or security systems. Ranjith et al. [19] established the concept of seating arrangement with certain conditions using the sum-signed graph labeling method.

Sobha et al. [20], Jamil et al. [21], and Nagoorgani et al. [22] have studied the magic concept in FGs. FGs are called magic if they have the same value for all pairs. Sobha et al. [20] demonstrated the magic labeling of FGs for butterfly graphs, pan graphs, wheels, bulls, helms, and fan graphs. Nagoorgani et al. [22] showed FGs with magic labels for paths, cycles, and stars. Ameenal Bibi and Devi [23] investigated anti-magic concept in FGs. Thirisangu et al. [24] and Sujatha et al. [25, 26] studied anti-magic idea on star graphs and triangular FGs. Brata et al. [27] investigated the concept of magic labeling for AFG. Oktaviani et al. [28] demonstrated the existence of m-magic, both the anti-fuzzy path and the anti-fuzzy bi-polar path.

AFGs may represent ambiguous or negative interactions among members in social networks. Traditional fuzzy graphs depict the extent of friendliness or affinity, while AFGs illustrate negative relationships, such as competition or hostility. Risk assessment often entails the identification of adverse causes and possible failures. AFGs may represent systems characterised by component failures or malfunctions under uncertain circumstances, particularly when these components interact in intricate ways. In fault diagnostics for engineering systems, AFGs may illustrate potential adverse interactions among components, enabling engineers to more accurately anticipate system breakdowns and reduce risks.

AFG labeling is used in chemical bonding for molecular structure representation, stability prediction, drug design, reaction pathway analysis, quantum chemistry, and nanotechnology. It facilitates the modelling of bond types, the analysis of stability, the optimisation of medicinal compounds, the examination of reaction feasibility, the interpretation of spectroscopic data, and the prediction of material characteristics.

Manuscript received February 17, 2024; revised May 05, 2025.

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AFG labeling in statistical surveys facilitates data classification, trend analysis, decision-making assistance, social impact examination, and enhancement of market research. It addresses ambiguity in replies, identifies trends, and improves policy formulation and consumer segmentation.

The purpose of this article is to prove the AFEAM and AFVAM of the path, cycle, comb, and star AFGs. We can systematically write this article as follows to ensure a clear description. The preliminary part presents AFG terminology, which includes anti-magic labeling on AFG. In the main results part, we have showed that anti-magic labels of AFGs exist for paths and stars and some properties related to degree.

II. PRELIMINARIES

A FG $G(\zeta^*, \xi^*)$ is a couple of functions $\zeta^*: V \to [0,1]$ & $\xi^*: V \times V \to [0,1]$ where $\forall\, m_i, m_j \in V$, then $\xi^*(m_i, m_j) \le \min\{\zeta^*(m_i), \zeta^*(m_j)\}$. An AFG $G(\zeta^*, \xi^*)$ is a couple of functions $\zeta^*: V \to [0,1]$ & $\xi^*: V \times V \to [0,1]$ where $\forall\, m_i, m_j \in V$, then $\xi^*(m_i, m_j) \ge \max\{\zeta^*(m_i), \zeta^*(m_j)\}$.

Example 2.1:

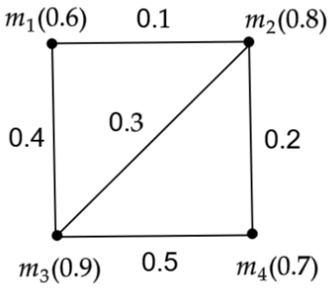

Fig 1. Fuzzy graph

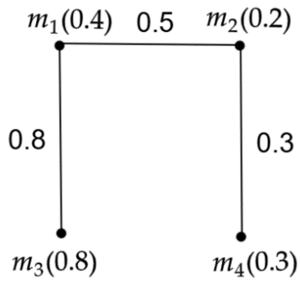

Fig 2. Anti-fuzzy graph

A graph $G(\zeta^*, \xi^*)$ is defined as fuzzy graph labeling, if $\zeta^*: V \to [0,1]$ & $\xi^*: V \times V \to [0,1]$ is bijective which the grade values on edges and vertices are distinct and $\xi^*(m_i, m_j) < \min\{\zeta^*(m_i), \zeta^*(m_j)\}\ \forall\, m_i, m_j \in V$.

Fuzzy graph labeling $G(\zeta^*, \xi^*)$ is said to be AFG labeling if $\xi^*(m_i, m_j) > \max\{\zeta^*(m_i), \zeta^*(m_j)\}$ for all $m_i, m_j \in V$. [24]

A AFG is called as regular or k-regular. If $d(m) = k$ for every $m \in V$.

An AFG $G(\zeta^*, \xi^*)$ is known as irregular, if there is a vertex that is connected only to vertices of varying degrees. [6]. For example, in Fig 3. $d(m_1) = 2.1, d(m_2) = 2.2, d(m_3) = 2.3, d(m_4) = 1.8, d(m_5) = 0.8$, all the vertices are adjacent with only distinct degree vertices so it is an irregular AFG.

An AFG $G(\zeta^*, \xi^*)$ is known as neighbourly irregular AFG if each pair of neighbouring vertices in G have different degrees. [6]. For example, in Fig 2. $d(m_1) = 1.2, d(m_2) = 0.8, d(m_3) = 0.8, d(m_4) = 0.4$, here each pair of neighbouring vertices have distinct degrees so it is neighbourly irregular AFG.

An AFG $G(\zeta^*, \xi^*)$ is known as highly irregular if each vertex of G is adjacent to vertices to different degrees.[6]

Let $G(\zeta^*, \xi^*)$ be an AFG. The total degree of a vertex $m_i \in V$ is denoted by $td(m_i) = \sum_{m_i \neq m_j, m_j \in V} \xi^*(m_i, m_j) + \zeta^*(m_i) = d(m_i) + \zeta^*(m_i)$.[6]. For example, in Fig 3. $td(m_1) = 2.6, td(m_2) = 2.8, td(m_3) = 3.0, td(m_4) = 2.7, td(m_5) = 1.6$.

An AFG $G(\zeta^*, \xi^*)$ is claimed to be totally irregular AFG, if there is a vertex which is attached to vertices with different total degrees. [6] If every two adjacent vertices of an AFG $G(\zeta^*, \xi^*)$ have different total degree, then G is said to be a neighbourly total irregular AFG. [6]

A path P_n is said to be an FG path if $\xi^*(m_i, m_{i+1}) > 0, 0 \le i \le n$. An FG path P_n is said to be a AFG path if $\xi^*(m_i, m_{i+1}) \ge \max(\zeta^*(m_i), \zeta^*(m_{i+1}))$. [11]

An FG star consists pair of node sets V and U with $|V| = 1$ and $|U| > 1, \exists \in \xi^*(m, m_i) > 0$ and $\xi^*(m_i, m_{i+1}) = 0$, $1 \le i \le n$. It is denoted by $S_{1,n}$. [10]. An FG star $S_{1,n}$ is known as AFG star if $\xi^*(m, m_i) \ge \max(\zeta^*(m), \zeta^*(m_i))$.

Example 2.2.

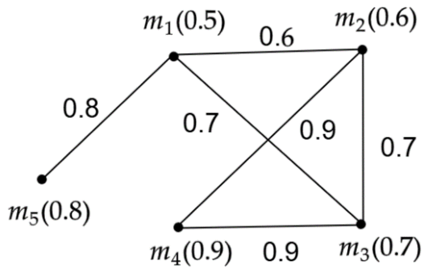

Fig 3. Highly irregular and Totally irregular AFG

Example 2.3.

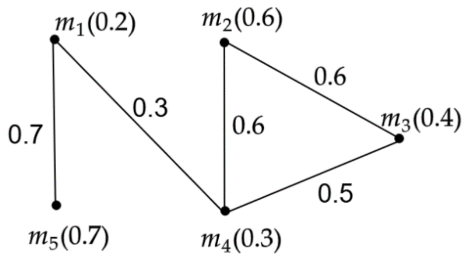

Fig 4. Neighbourly total irregular AFG

III. ANTI-FUZZY ANTI-MAGIC GRAPH

An AFG is called AFEAM labeling if $\zeta^*(u) + \xi^*(u, v) + \zeta^*(v)\ \forall\ u, v \in V$ are all distinct.

An AFG is called AFVAM labeling in a graph G is a 1-1 correspondence, in which for any two different vertices v and w, the weight of m_p is differ from weight of m_q.

An AFG that permits AFEAM labelling is referred to as the AFEAM graph. An AFG that permits AFVAM labelling is referred to as the AFVAM graph.

This section demonstrates the AFEAM and AFVAM of the path and star graph through a technique based on the theorem.

Anti-fuzzy Labeled Path graph P_n:

Algorithm 1

AFL of edges and vertices of P_n with $n \geq 2$ and $z \in (0,1]$

1. **Input:** The values provided for n along with z.
2. There is a maximum of n+1 vertices.
3. Vertex label is,
   for $i = 1$ to $n + 1$
   {
   $\zeta^*(m_i) = iz$ where $1 \leq i \leq n + 1$
   }
4. Edge label is,
   for $i = 1$ to $n + 1$
   if $1 \leq i \leq n$
   {
   $\xi^*(m_i, m_{i+1}) = n + i + 1$
   }
5. **Output:** Give the labels to indicate vertices and edges.

Theorem 3.1. Let $P_n: (\zeta^*, \xi^*)$ be the AFL path graph for all $n \geq 2$. Then P_n admits AFEAM labeling.

Proof. Let $z \in (0,1]$ so that

$$z = \begin{cases} \frac{1}{10^2}, & 1 < n \leq 49 \\ \frac{1}{10^3}, & 49 < n \leq 499 \\ \frac{1}{10^{k+3}}, & 49 + r < n \leq 49 + s \\ & for\ k = 1,2,3,\cdots \end{cases}$$

where

$$r = \sum_{\substack{t=1 \\ 1\leq j\leq k}}^{j} (45 \times 10^t)$$

and

$$s = \sum_{\substack{t=1 \\ 1\leq j\leq k+1}}^{j} (45 \times 10^t)$$

Defining membership values for vertex and edge using Algorithm 1.

$\zeta^*: V \to [0,1] \ni \zeta^*(m_i) = iz$ where $1 \leq i \leq n + 1$

$\xi^*: V \times V \to [0,1]$ such that

$\xi^*(m_i, m_{i+1}) = n + i + 1, 1 \leq i \leq n$

We discuss EAM labeling of path graph,

$$\begin{aligned} Am_0(P_n) &= \zeta^*(m_i) + \xi^*(m_i, m_{i+1}) + \zeta^*(m_{i+1}), \\ &\quad 1 \leq i \leq n \\ &= iz + (n + i + 1)z + (i + 1)z \\ &= (n + 3i + 2)z \end{aligned}$$

From the above, it is evident that path graph P_n admits AFEAM labeling.

Theorem 3.2. Let $P_n: (\zeta^*, \xi^*)$ be the AFL path graph for all $n \geq 2$. Then P_n admits AFVAM labeling.

Proof. Given $P_n: (\zeta^*, \xi^*)$ be the AFL path graph.

To prove that AFL path graph $P_n: (\zeta^*, \xi^*)$ meets the requirement of AFVAM labeling.

This reveals that for any two vertices m_p and m_q in $S_{1,n}$, the weight of m_p is differ from weight of m_q. Let $z \in (0,1]$ so that

$$z = \begin{cases} \frac{1}{10^2}, & 1 < n \leq 49 \\ \frac{1}{10^3}, & 49 < n \leq 499 \\ \frac{1}{10^{k+3}}, & 49 + r < n \leq 49 + s \\ & for\ k = 1,2,3,\cdots \end{cases}$$

where

$$r = \sum_{\substack{t=1 \\ 1\leq j\leq k}}^{j} (45 \times 10^t)$$

and

$$r = \sum_{\substack{t=1 \\ 1\leq j\leq k+1}}^{j} (45 \times 10^t)$$

Defining membership values for vertex and edge using Algorithm 1.

$$\zeta^*: V \to [0,1] \ni \zeta^*(m_i) = iz \text{ where } 1 \leq i \leq n + 1 \tag{3.1}$$

$\xi^*: V \times V \to [0,1]$ such that

$$\xi^*(m_i, m_{i+1}) = n + i + 1, 1 \leq i \leq n \tag{3.2}$$

Also, the total of the edge labels incident at

$$m_i = Wt(m_i) = \sum_{u\in N(m_i)} \xi^*(u, m_i) \tag{3.3}$$

Where $N(m_i)$ be the neighbourhood vertices of m_i for all $i = 1$ to $n + 1$ and $Wt(m_i)$be the weight of m_i. From equation (3.1), (3.2) and (3.3) for any pair of vertices m_p and m_q with $p \neq q$, $Wt(m_p)$ and $Wt(m_q)$ have different values.

Thus, a path graph P_n admits AFVAM labeling.

Example 3.1. Fig 5 is illustrating the AFEAM and AFVAM path with path length 4.

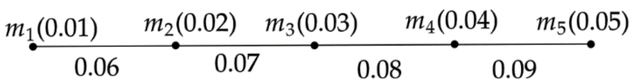

Fig 5. P_4 − AFEAM and AFVAM path graph

Illustration 3.1. Verifying the AFEAM of path graph using Fig 5.

$n = 4, z = 0.01$

$$Am_0(P_n) = \zeta^*(m_i) + \xi^*(m_i, m_{i+1}) + \zeta^*(m_{i+1}),\ 1 \le i \le n$$
$$= (n + 3i + 2)z, 1 \le i \le n$$

$i = 1,$
$$Am_0(P_4) = \zeta^*(m_1) + \xi^*(m_1, m_2) + \zeta^*(m_2)$$
$$Am_0(P_4) = (4 + 3(1) + 2)0.01 = 0.09$$
$i = 2,$
$$Am_0(P_4) = \zeta^*(m_2) + \xi^*(m_2, m_3) + \zeta^*(m_3)$$
$$Am_0(P_4) = (4 + 3(2) + 2)0.01 = 0.12$$
$i = 3,$
$$Am_0(P_4) = \zeta^*(m_3) + \xi^*(m_3, m_4) + \zeta^*(m_4)$$
$$Am_0(P_4) = (4 + 3(3) + 2)0.01 = 0.15$$

$i = 4,$
$$Am_0(P_4) = \zeta^*(m_4) + \xi^*(m_4, m_5) + \zeta^*(m_5)$$
$$Am_0(P_4) = (4 + 3(4) + 2)0.01 = 0.18$$

Therefore, in all the three cases $Am_0(P_4)$ are different and unique.
Thus P_4 is AFEAM path graph.

Illustration 3.2. Verifying the AFVAM of path graph using Fig 5.

$n = 4, z = 0.01$
for the vertex m_1, $Am_0(P_4) = \xi^*(m_1, m_2) = 0.06$

for the vertex m_2, $Am_0(P_4) = \xi^*(m_1, m_2) + \xi^*(m_2, m_3) = 0.06 + 0.07 = 0.13$

for the vertex m_3, $Am_0(P_4) = \xi^*(m_2, m_3) + \xi^*(m_3, m_4) = 0.07 + 0.08 = 0.15$

for the vertex m_4, $Am_0(P_4) = \xi^*(m_3, m_4) + \xi^*(m_4, m_5) = 0.08 + 0.09 = 0.17$

for the vertex m_1, $Am_0(P_4) = \xi^*(m_4, m_5) = 0.09$

Therefore, weight of all the vertex $Am_0(P_4)$ are different and unique.
Thus P_4 is AFVAM path graph.

Anti-fuzzy Labeled Star graph $S_{1,n}$:

Algorithm 2

AFL of edges and vertices of $S_{1,n}$ with $n \ge 2$ and $z \in (0,1]$

1. **Input:** The values provided for n along with z.
2. There is a maximum of n+1 vertices.
3. Vertex label is,
   for $i = 1$ to $n + 1$
   {
   $\zeta^*(m_i) = iz$ where $1 \le i \le n$
   }
   if $n \ge 2$
   $\zeta^*(m_{n+1}) = (n + 1)z$
4. Edge label is,
   for $i = 1$ to $n + 1$
   if $1 \le i \le n$
   {
   $\xi^*(m_i, m_{i+1}) = (n + i + 1)z$
   }
6. **Output:** Give the labels to indicate vertices and edges.

Theorem 3.3. Let $S_{1,n} : (\zeta^*, \xi^*)$ be the AFL star graph for all $n \ge 2$. Then $S_{1,n}$ admits AFEAM labeling.
Proof. Let $z \in (0,1]$ so that

$$z = \begin{cases} \frac{1}{10^2}, & 1 < n \le 49 \\ \frac{1}{10^3}, & 49 < n \le 499 \\ \frac{1}{10^{k+3}}, & 49 + r < n \le 49 + s \\ & for\ k = 1,2,3, \cdots \end{cases}$$

where

$$r = \sum_{\substack{t=1 \\ 1 \le j \le k}}^{j} (45 \times 10^t)$$

and

$$r = \sum_{\substack{t=1 \\ 1 \le j \le k+1}}^{j} (45 \times 10^t)$$

Defining membership values for vertex and edge using Algorithm 2.

$\zeta^*: V \to [0,1] \ni \zeta^*(m_i) = iz$ where $1 \le i \le n$

$\zeta^*(m_{n+1}) = (n + 1)z, n \ge 2$

$\xi^*: V \times V \to [0,1]$ such that
$\xi^*(m_i, m_{n+1}) = (n + i + 1)z, 1 \le i \le n$

We discuss EAM labeling of star graph,
$$Am_0(S_{1,n}) = \zeta^*(m_i) + \xi^*(m_i, m_{n+1}) + \zeta^*(m_{n+1}), 1 \le i \le n$$
$$= iz + (n + i + 1)z + (n + 1)z$$
$$= (2n + 2i + 2)z$$
From the above, it is evident that the AFL star graph $S_{1,n}$ admits AFEAM labeling.

Theorem 3.4. Let $S_{1,n}: (\zeta^*, \xi^*)$ be the AFL star graph for all $n \ge 2$. Then $S_{1,n}$ admits AFVAM labeling.
Proof. Given $S_{1,n}: (\zeta^*, \xi^*)$ be the AFL star graph.

To prove that AFL star graph $S_{1,n}: (\zeta^*, \xi^*)$ meets the requirement of AFVAM labeling.

This reveals that for any two vertices m_p and m_q in $S_{1,n}$, the weight of m_p is differ from weight of m_q.
Let $z \to (0,1]$ so that

$$z = \begin{cases} \frac{1}{10^2}, & 1 < n \le 49 \\ \frac{1}{10^3}, & 49 < n \le 499 \\ \frac{1}{10^{k+3}}, & 49 + r < n \le 49 + s \\ & for\ k = 1,2,3,\cdots \end{cases}$$

where

$$r = \sum_{\substack{t=1 \\ 1 \le j \le k}}^{j} (45 \times 10^t)$$

and

$$r = \sum_{\substack{t=1 \\ 1 \le j \le k+1}}^{j} (45 \times 10^t)$$

Defining membership values for vertex and edge using Algorithm 2.

$\zeta^*: V \to [0,1] \ni \zeta^*(m_i) = iz$ where $1 \le i \le n+1$ (3.4)

$\xi^*: V \times V \to [0,1]$ such that

$\xi^*(m_i, m_{i+1}) = n + i + 1, 1 \le i \le n$ (3.5)

Also, the total of the edge labels incident at

$m_i = Wt(m_i) = \sum_{u \in N(m_i)} \xi^*(u, m_i)$ (3.6)

Where $N(m_i)$ be the neighbourhood vertices of m_i for all $i = 1$ to $n + 1$ and $Wt(m_i)$be the weight of m_i. From equation (3.4), (3.5) and (3.6) for any two vertices m_p and m_q with $p \ne q$, $Wt(m_p)$ and $Wt(m_q)$ have different values.

Thus, a star graph $S_{1,n}$ admits AFVAM labeling.

Example 3.2. Fig 6 is illustrating the AFEAM and AFVAM star with 5 pendant edges.

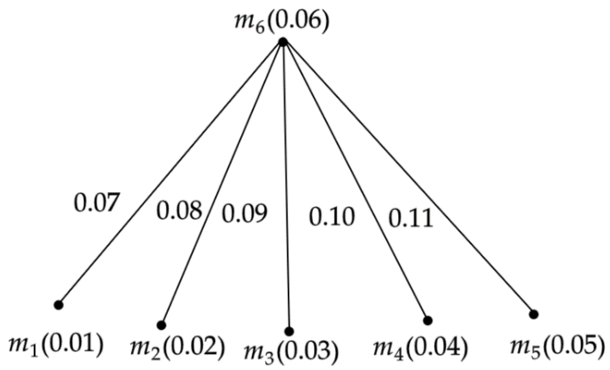

Fig 6. $S_{1,5}$ – AFEAM and AFVAM star graph

Illustration 3.3. Verifying the AFEAM of star graph using Fig 6.

$n = 5, z = 0.01$

$$Am_0(S_{1,n}) = \zeta^*(m_i) + \xi^*(m_i, m_{n+1}) + \zeta^*(m_{n+1}), \quad 1 \le i \le n$$
$$= (2n + 2i + 2)z, 1 \le i \le n$$

$i = 1,$
$Am_0(S_{1,5}) = \zeta^*(m_1) + \xi^*(m_1, m_6) + \zeta^*(m_6)$
$Am_0(S_{1,5}) = (10 + 2(1) + 2)0.01 = 0.14$

$i = 2,$
$Am_0(S_{1,5}) = \zeta^*(m_2) + \xi^*(m_2, m_6) + \zeta^*(m_6)$
$Am_0(S_{1,5}) = (10 + 2(2) + 2)0.01 = 0.16$

$i = 3,$
$Am_0(S_{1,5}) = \zeta^*(m_3) + \xi^*(m_3, m_6) + \zeta^*(m_6)$
$Am_0(S_{1,5}) = (10 + 2(3) + 2)0.01 = 0.18$

$i = 4,$
$Am_0(S_{1,5}) = \zeta^*(m_4) + \xi^*(m_4, m_6) + \zeta^*(m_6)$
$Am_0(S_{1,5}) = (10 + 2(4) + 2)0.01 = 0.20$

$i = 5,$
$Am_0(S_{1,5}) = \zeta^*(m_5) + \xi^*(m_5, m_6) + \zeta^*(m_6)$
$Am_0(S_{1,5}) = (10 + 2(5) + 2)0.01 = 0.22$

Therefore, in all the three cases $Am_0(S_{1,5})$ are different and unique.

Thus $S_{1,5}$ is AFEAM star graph.

Illustration 3.4. Verifying the AFVAM of star graph using Fig 6.

$n = 5, z = 0.01$

for the vertex m_1, $Am_0(S_{1,n}) = \xi^*(m_1, m_6) = 0.07$
for the vertex m_2, $Am_0(S_{1,n}) = \xi^*(m_2, m_6) = 0.08$
for the vertex m_3, $Am_0(S_{1,n}) = \xi^*(m_3, m_6) = 0.09$
for the vertex m_4, $Am_0(S_{1,n}) = \xi^*(m_4, m_6) = 0.10$
for the vertex m_5, $Am_0(S_{1,n}) = \xi^*(m_5, m_6) = 0.11$
for the vertex m_6, $Am_0(S_{1,n}) = \xi^*(m_1, m_6) + \xi^*(m_2, m_6) + \xi^*(m_3, m_6) + \xi^*(m_4, m_6) + \xi^*(m_5, m_6)$
$= 0.07 + 0.08 + 0.09 + 0.10 + 0.11 = 0.45$

Therefore, weight of all the vertex $Am_0(S_{1,n})$ are different and unique.

Thus $S_{1,n}$ is AFVAM star graph.

IV. FINDINGS

1. For any AFEAM and AFVAM path and star graph, $d(m_i) \ne d(m_j)$ for any pair of vertices $m_i, m_j \in V$.
2. For any AFEAM and AFVAM path and star graph, $d_s(m_i) \ne d_s(m_j)$ for any pair of vertices $m_i, m_j \in V$.
3. For any AFEAM and AFVAM path and star graph, $d_s(m_i) = d(m_i)\ \forall\ m_i \in V$.
4. In AFEAM and AFVAM path and star graph all the edges are fuzzy bridges and strong edges.
5. Every AFEAM and AFVAM path and star are irregular, neighbourly irregular and highly irregular.
6. AFEAM and AFVAM path and star graph are totally irregular.
7. Every AFEAM and AFVAM path and star graph are neighbourly total irregular.

V. CONCLUSION

The fuzzy graph provides an efficient system for describing and analysing systems defined by uncertainty and imprecision. Such tools have a wide range of applications, making them valuable for addressing complexities in real-life situations. Research in fuzzy graphs continues to refine and extend the theory to meet the demands of diverse applications. Researchers continue to explore the integration of fuzzy graph theory with other mathematical models, such as probability theory. The most challenging aspects of fuzzy graph labeling are defining appropriate membership functions, dealing with computational complexity, and developing effective visualisation techniques. As a result, the proposed article introduced AFEAM and AFVAM labeling on graphs. We discussed EAM and VAM labeling on AFGs for path and star graphs. Additionally, we have examined the properties of AFEAM and AFVAM path and star graphs. This concept can be extended to more standard graphs, such as the bi-star, coconut tree, and octopus graph, among others.

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