Mathematical Modeling and Analysis of Fast Food Consumption Dynamics

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Abstract—**This research presents a continuous mathematical model, called PLSCQ, aimed at simulating the behavior of fast-food consumers influenced by public and private addiction treatment centers. The analysis focuses on the model's key features, including the basic reproduction number R_0, which assesses the potential spread of fast-food addiction within a population. A sensitivity analysis highlights the parameters with the most significant impact on R_0. Stability analysis shows that the system is asymptotically stable, both locally and globally, at the consumption-free equilibrium E_0 when $R_0 \leq 1$, indicating controlled consumption levels. Conversely, when $R_0 > 1$, a new equilibrium E^* with ongoing consumption emerges, where the system remains asymptotically stable. This model provides valuable insights into the influence of addiction treatment centers on fast-food consumption dynamics and identifies key factors for effective addiction management.**

Index Terms—**Mathematical Model, Fast Food, Optimal Control, Lyapunov function.**

I. INTRODUCTION

FOOD consumption has significantly increased, largely driven by the convenience and popularity of fast foods, particularly among younger generations influenced by modern, fast-paced lifestyles. Despite their accessibility, these foods pose addictive tendencies and serious health risks, which many families and children tend to overlook.According to the World Health Organization [1], over 1.9 billion adults were overweight, with 650 million classified as obese, marking a nearly threefold increase in global obesity rates since 1975. Additionally, approximately 41 million children under the age of five and more than 340 million individuals aged 5 to 19 are overweight or obese. Rapid increases have been observed in urban areas of low- and middle-income countries, particularly in Africa and Asia. Currently, obesity-related deaths surpass those caused by undernutrition worldwide, although this trend varies by region [2].In the United States, 39.8% of adults were classified as obese during the 2015-2016 period, which equates to approximately 93.3 million people. Obesity was more prevalent among individuals aged 40-59 (42.8%) than those aged 20-39 (35.7%). However, there was no significant difference in obesity rates between older adults and younger groups. Although obesity rates showed a significant increase from 1999-2000 to 2015-2016, the rise between 2013-2014 and 2015-2016 was not statistically significant for either adults or youth [2]. In 2014, Morocco reported that approximately 7 million people were overweight or obese, a trend that aligns with the global rise in obesity rates [4].The Food and Agriculture Organization (FAO) highlights particularly high obesity rates in certain Pacific island nations, where fast food consumption is widespread [5]. For instance, more than 30% of the populations in Fiji and American Samoa are overweight. In ten other Pacific islands, more than half of the population is affected. Globally, around 2.6 billion people are overweight, and obesity rates increased from 11.7% in 2012 to 13.2% in 2016. In response, the FAO encourages governments to promote local diets, especially in schools, as this would not only improve public health but also stimulate local economies [6].This study employs an advanced mathematical model to analyze fast food consumption behaviors, integrating several factors: a continuous-time model, a variable representing the number of overweight individuals, a mortality rate (δ_1) linked to excessive fast food consumption, and another rate (δ_2) associated with obesity-related diseases. Numerous social and psychological studies have also addressed these issues [7].For example, Aldila [7] applied an ecological framework to study university students' eating behaviors and observed a rise in obesity rates along with negative health effects. Chunyoung Oh [8] analyzed obesity dynamics using a mathematical model, showing that when the reproduction number R_0 exceeds 1, obesity becomes persistent. Simulations suggest that limiting social interactions between overweight individuals can enhance the effectiveness of educational programs, which should be tailored to the local context.N. H. Shah [9] developed a model examining the impact of obesity on infertility, demonstrating that women consuming a calorie-rich diet have a 17% increased risk of infertility. This research highlights the importance of maintaining a healthy weight through regular physical activity.Other studies, such as those by D. Aldila et al. [7], have tested the effectiveness of intervention programs aimed at modifying the eating behaviors of overweight and obese individuals. These studies indicate that well-designed interventions [10], [11], [12], based on optimal control and educational strategies, can significantly reduce obesity rates and their associated consequences. The $PLSCQ$ model categorizes fast-food consumers into five compartments:

- Potential consumers of fast food ($\mathbf{P}$),
- Moderate consumers of fast food ($\mathbf{L}$),
- Excessive consumers of fast food ($\mathbf{S}$),
- Obese individuals ($\mathbf{C}$),
- Individuals who have quit fast food ($\mathbf{Q}$).

This study aims to identify the most effective strategies for reducing the number of obese individuals and excessive fast-food consumers, while increasing the proportion of individuals who successfully lose weight in a healthy

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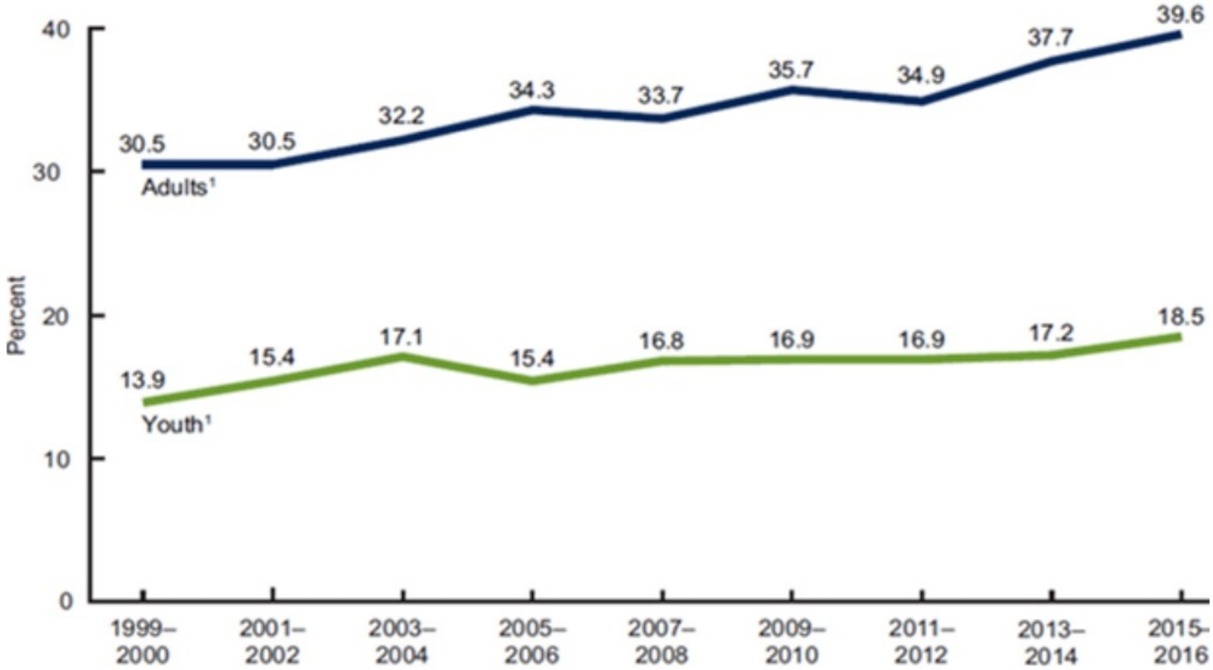

Fig. 1: Trends in the Age-Adjusted Obesity Prevalence in Adults 20and Over and Youth 2-19 Years in the United States: 1999–2000 to2015–2016 [3]

manner or prevent obesity. Our objectives are to pinpoint the key factors contributing to these issues and to design both preventive and corrective strategies. To achieve these goals, we propose three main control strategies: promoting a balanced diet, raising awareness about the risks of excessive fast-food consumption, and providing appropriate support for overweight individuals. Section 2 explores the interactions between different types of fast-food consumers. The optimal control problem for the model is addressed in Sections 3 and 4, where we discuss the existence of optimal controls and their evaluation using the Pontryagin Maximum Principle. Numerical simulations are presented in Section 5, with the conclusion in Section 6.

II. Model Formulation

A. Model Description

We present a continuous model, $PLSCQ$, to represent fast-food consumption behavior in a population. This population is divided into five compartments, labeled P, L, S, C, and Q.
A graphical illustration of the proposed model is shown in Figure 2.

The mathematical formulation of the model consists of a system of non-linear differential equations:

$$\begin{cases} \frac{dP(t)}{dt} = b - \beta_1 \frac{P(t)L(t)}{N} - \mu P(t) \\ \frac{dL(t)}{dt} = \beta_1 \frac{P(t)L(t)}{N} - (\mu + \beta_2)L(t) \\ \frac{dS(t)}{dt} = \beta_2 L(t) - (\mu + \delta_1 + \alpha_1 + \alpha_2)S(t) \\ \frac{dC(t)}{dt} = \alpha_1 S(t) - (\mu + \gamma + \delta_2)C(t) \\ \frac{dQ(t)}{dt} = \alpha_2 S(t) + \gamma C(t) - \mu Q(t) \end{cases} \quad (1)$$

where $P_0 \geq 0$, $L_0 \geq 0$, $S_0 \geq 0$, $C_0 \geq 0$, and $Q_0 \geq 0$.

Fast-Food Consumption Model

Compartment P: This compartment includes individuals who are strongly inclined towards fast food consumption. The population in P grows at a rate b, largely driven by social factors such as family habits, peer groups, advertising, and television. It decreases at a rate β_1, influenced by changes in eating behaviors and natural deaths. Social events, such as year-end celebrations, weddings, or parties, play a significant role in either maintaining or adopting these eating habits. The adoption of fast food as a regular habit can be seen as a contagious process, similar to acquiring an illness, where eating behaviors spread and reinforce within social groups.
Compartment L: This group includes individuals who engage in moderate fast-food consumption, either occasionally or in a manner that does not stand out in their social environment. The population in L increases when individuals shift from minimal to moderate consumption at a rate of β_1. Conversely, it decreases when moderate consumers escalate to excessive consumption at a rate of β_2, in addition to natural deaths occurring at a rate μ.
Compartment S: This group consists of individuals who engage in excessive fast-food consumption. The population in S increases at a rate β_2 as more individuals adopt excessive consumption. The population decreases at a rate α_2 when some individuals stop consuming fast food. Additionally, the population in S diminishes at a rate α_1 due to the transition from excessive consumption to obesity (compartment C). The group also experiences a decline at a rate μ due to natural deaths, and at a rate δ_1 due to diseases associated with excessive fast-food consumption.
Compartment C: This group represents individuals who are obese. The population in C increases at a rate α_1, reflecting the transition from excessive fast-food consumption to obesity. The rate γ indicates the proportion of obese individuals who cease fast-food consumption. Additionally, the popula-

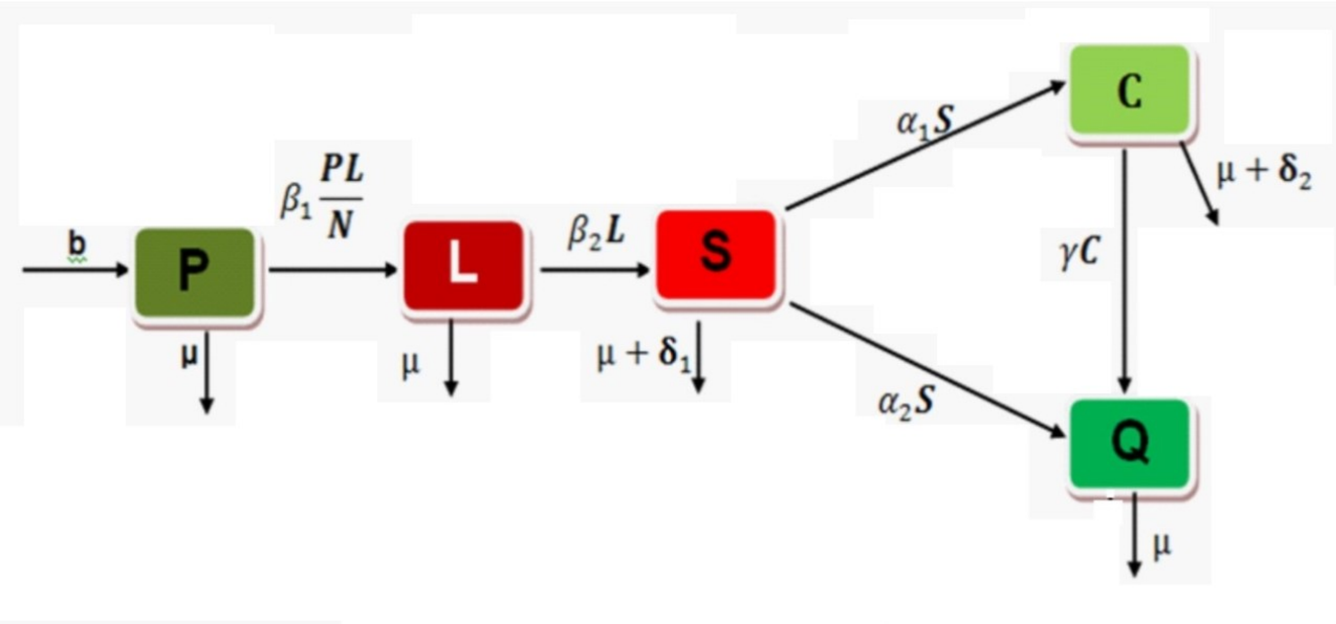

Fig. 2: Schematic diagram of the five consumer classes within the fast food consumption model(1).

tion in C declines at a combined rate of $\mu + \delta_2$, accounting for deaths due to obesity-related diseases resulting from excessive fast-food intake.

Compartment Q: This group consists of individuals who have stopped consuming fast food. The population in Q decreases at a rate μ, but increases at rates γ and α_2, reflecting the individuals who cease obesity or excessive fast-food consumption. The total population at time t, denoted by N_t, is the sum of all compartments:

$$N_t = P_t + L_t + S_t + C_t + Q_t,$$

and it is assumed to remain constant.

B. Basic Properties

1) Invariant Region: We need to show that all solutions of system (1) with positive initial data will remain positive for $t > 0$. The following lemma will prove this.

Lemma 1: All feasible solutions $P(t)$, $L(t)$, $S(t)$, $C(t)$, and $Q(t)$ of the system equation (1) are bounded within the region:

$$\Omega = \left\{ (P, L, S, C, Q) \in \mathbb{R}_+^5 : P + L + S + C + Q \leq \frac{b}{\mu} \right\} \tag{2}$$

Proof: From system equation (1),

$$\frac{dN(t)}{dt} = \frac{dP(t)}{dt} + \frac{dL(t)}{dt} + \frac{dS(t)}{dt} + \frac{dC(t)}{dt} + \frac{dQ(t)}{dt} \tag{3}$$

$$\frac{dN(t)}{dt} = b - \mu N(t) - \delta_1 S(t) - \delta_2 C(t) \tag{4}$$

This suggests that

$$\frac{dN(t)}{dt} \leq b - \mu N(t) \tag{5}$$

It follows that:

$$N(t) \leq \frac{b}{\mu} + N(0)e^{-\mu t} \tag{6}$$

where $N(0)$ is the initial value of the total population [13]. Thus,

$$\lim_{t \to +\infty} \sup N(t) \leq \frac{b}{\mu} \tag{7}$$

Then,

$$P(t) + L(t) + S(t) + C(t) + Q(t) \leq \frac{b}{\mu} \tag{8}$$

Thus, for model (1), we obtain the region defined by the set:

$$\Omega = \left\{ (P, L, S, C, Q) \in \mathbb{R}_+^5 : P + L + S + C + Q \leq \frac{b}{\mu} \right\} \tag{9}$$

This is a positively invariant set for (1); thus, the system dynamics (1) only need to be considered within the non-negative set of solutions Ω.

2) Positivity of the Model's Solutions: .

Theorem 1: Suppose $P(0) \geq 0$, $L(0) \geq 0$, $S(0) \geq 0$, $C(0) \geq 0$, and $Q(0) \geq 0$ are non-negative initial conditions. Then, the solutions of system equation (1) remain positive for all $t > 0$.

Proof: From the first equation of system (1), we obtain:

$$\frac{dP(t)}{dt} = b - \left(\beta_1 \frac{L(t)}{N} + \mu \right) P(t) \tag{10}$$

Let us define:

$$A(t) = \beta_1 \frac{L(t)}{N} + \mu \tag{11}$$

Now, multiply equation (8) by $\exp\left(\int_0^t A(s)\, ds\right)$, yielding:

$$\begin{aligned} &\tfrac{dP(t)}{dt} \exp\left(\int_0^t A(s) ds\right) = \\ &= (b - A(t)P(t)) \exp\left(\int_0^t A(s) ds\right) \end{aligned} \tag{12}$$

This simplifies to:

$$\begin{aligned} &\exp\left(\int_0^t A(s) ds\right) \left(\tfrac{dP(t)}{dt} + A(t)P(t)\right) = \\ &= b \exp\left(\int_0^t A(s) ds\right) \end{aligned} \tag{13}$$

Finally, we obtain:

$$\frac{dP(t)}{dt} + A(t)P(t) = b \tag{14}$$

Thus, we have:

$$\frac{d}{dt}\left[P(t)\cdot\exp\left(\int_0^t A(s)\,ds\right)\right] = b\cdot\exp\left(\int_0^t A(s)\,ds\right) \tag{15}$$

By integrating from 0 to t with respect to s, we get:

$$\begin{aligned} &P(t)\cdot\exp\left(\textstyle\int_0^t A(s)ds\right) - P(0) = \\ &= b\textstyle\int_0^t \exp\left(\int_0^w A(s)ds\right)dw \end{aligned} \tag{16}$$

Multiplying equation (16) by $\exp\left(-\int_0^t A(s)\,ds\right)$, we obtain:

$$\begin{aligned} &P(t) - P(0)\exp\left(-\textstyle\int_0^t A(s)ds\right) = \\ &= b\textstyle\int_0^t \exp\left(-\int_w^t A(s)ds\right)dw \end{aligned} \tag{17}$$

Thus, we can express $P(t)$ as:

$$\begin{aligned} &P(t) = \\ &= P(0)\exp\left(-\textstyle\int_0^t A(s)ds\right) + b\textstyle\int_0^t \exp\left(-\int_w^t A(s)ds\right)dw \end{aligned} \tag{18}$$

Therefore, we obtain:

$$P(t) = P(0)\exp\left(-\int_0^t A(s)\,ds\right) +$$

$$b\int_0^t \exp\left(-\int_w^t A(s)\,ds\right)dw \geq 0$$

Thus, $P(t)$ is a positive solution. Similarly, from the second equation of system (1), we have:

$$L(t) \geq L(0)\exp\left(-\int_0^t B(s)\,ds\right) \geq 0 \tag{19}$$

where

$$B(t) = \beta_1\frac{P(t)}{N} - (\mu+\beta_2) \tag{20}$$

Furthermore, from the third, fourth, and fifth equations of system (1), we get:

$$\begin{aligned} &S(t) \geq S(0)\exp\left(-(\mu+\delta_1+\alpha_1+\alpha_2)t\right) \geq 0,\\ &C(t) \geq C(0)\exp\left(-(\mu+\gamma+\delta_2)t\right) \geq 0,\\ &Q(t) \geq Q(0)\exp\left(-(\mu+\theta)t\right) \geq 0. \end{aligned} \tag{21}$$

Therefore, we conclude that $P(0) \geq 0$, $L(0) \geq 0$, $S(0) \geq 0$, $C(0) \geq 0$, and $Q(0) \geq 0$ for $t > 0$, which completes the proof. The first three equations of system (1) are independent of C and Q. Thus, the dynamics of system (1) can be analyzed through a reduced system structure:

$$\begin{cases} \frac{dP(t)}{dt} = b - \beta_1\frac{P(t)L(t)}{N} - \mu P(t),\\ \frac{dL(t)}{dt} = \beta_1\frac{P(t)L(t)}{N} - (\mu+\beta_2)L(t),\\ \frac{dS(t)}{dt} = \beta_2 L(t) - (\mu+\delta_1+\alpha_1+\alpha_2)S(t). \end{cases} \tag{22}$$

III. Analysis of Equilibria and Their Stability

A. Equilibrium Points

We apply the standard approach for analyzing the system in equations (22). There are two equilibrium points in this model: one where there is no consumption and another where consumption is present. To identify these equilibrium points, we set the right-hand sides of equations (1)–(3) to zero.The consumption-free equilibrium occurs when there is no consumption ($L = S = 0$), and it is denoted by $E^0\left(\frac{b}{\mu},0,0\right)$. The consumption-present equilibrium, denoted $E^*(P^*,L^*,S^*)$, occurs when both $L \neq 0$ and $S \neq 0$, indicating the presence of consumers. The equilibrium values for this point are:

$$\begin{aligned} &P^* = \tfrac{b}{\mu R_0},\\ &L^* = \tfrac{b(R_0-1)}{\beta_1},\\ &S^* = \tfrac{b\beta_2(R_0-1)}{\beta_1(\mu+\delta_1+\alpha_1+\alpha_2)},\\ &R_0 = \tfrac{\beta_1}{\mu+\beta_2}. \end{aligned} \tag{23}$$

The basic reproduction number R_0 represents the average number of new consumers that a single consumer generates within a population of potential consumers. This value helps determine whether the consumption behavior will spread like an epidemic. To calculate R_0, the next-generation matrix method can be employed, as described in [14].

B. Local Stability Analysis

The stability behavior of the equilibria E^0 and E^* is now studied.

*1) **Consumption-Free Equilibrium**:* In this section, we examine the local stability of the equilibrium where there is no fast food consumption.

Theorem 2: For $R_0 < 1$ and $R_0 > 1$, the consumption-free equilibrium $E^0\left(\frac{b}{\mu},0,0\right)$ of system (22) is asymptotically unstable.

Proof. The Jacobian matrix at E^0 is:

$$J(E) = \begin{pmatrix} -\beta_1\frac{L}{N}-\mu & -\beta_1\frac{P}{N} & 0\\ \beta_1\frac{L}{N} & \beta_1\frac{P}{N}-(\mu+\beta_2) & 0\\ 0 & \beta_2 & X \end{pmatrix} \tag{24}$$
$$X = -(\mu+\delta_1+\alpha_1+\alpha_2)$$

For the consumption-free equilibrium, the Jacobian matrix is given by:

$$J(E^0) = \begin{pmatrix} -\mu & -\beta_1 & 0\\ 0 & \beta_1-(\mu+\beta_2) & 0\\ 0 & \beta_2 & X \end{pmatrix} \tag{25}$$

where $P_0 = \frac{b}{\mu} = N$.The characteristic equation of this matrix is given by $\det(J(E^0) - \lambda I_3) = 0$, where I_3 is a 3×3 identity matrix. Therefore, the eigenvalues of the characteristic equation of $J(E^0)$ are:

$$\lambda_1 = -\mu,$$
$$\lambda_2 = -(\mu+\beta_2-\beta_1) = -(\mu+\beta_2)\left(1-\frac{\beta_1}{\mu+\beta_2}\right), \tag{26}$$
$$\lambda_3 = -(\mu+\delta_1+\alpha_1+\alpha_2),$$
$$R_0 = \frac{\beta_1}{\mu+\beta_2}.$$

Consequently, each eigenvalue of the characteristic equation of $J(E^0)$ is real and negative if $R_0 < 1$. We deduce that the consumption-free equilibrium is locally asymptotically stable if $R_0 < 1$; however, it is unstable if $R_0 > 1$.

*2) **Equilibrium with Fast Food Consumption Present**:* In this section, we analyze the local stability of the equilibrium where fast food consumption is present. Assuming that at least one of the compartments with consumers is nonzero, we set $\frac{dP(t)}{dt} = 0$, $\frac{dL(t)}{dt} = 0$, and $\frac{dS(t)}{dt} = 0$ to determine the equilibrium state of the system described by equation (21). By setting the right-hand side of the system's equations to zero and solving for P^*, L^*, and S^*, we can evaluate the equilibrium of system (22). Applying this to the second equation of the system, we proceed as follows. From the initial equation in system (22), we have:

$$L^* = \frac{b(R_0-1)}{\beta_1} \tag{27}$$

Also, equation (3) in system (19) gives:

$$S^* = \frac{b\beta_2(R_0-1)}{\beta_1(\mu+\delta_1+\alpha_1+\alpha_2)} \tag{28}$$

Let us now examine the local stability using the following theorem:

Theorem 3. If $R_0 > 1$, the consumption-present equilibrium E^* is locally asymptotically stable; otherwise, it is unstable [15].

Proof. We denote $E^*(P^*, L^*, S^*)$ as the consumption-present equilibrium of system (22), with $P^* \neq 0$, $L^* \neq 0$, and $S^* \neq 0$. The Jacobian matrix at E^* is given by:

$$J(E^*) = \begin{pmatrix} -\beta_1\frac{L^*}{N}-\mu & -\beta_1\frac{P^*}{N} & 0 \\ \beta_1\frac{L^*}{N} & \beta_1\frac{P^*}{N}-(\mu+\beta_2) & 0 \\ 0 & \beta_2 & X \end{pmatrix} \tag{29}$$

where

$$P^* = \frac{b}{\mu R_0},$$
$$L^* = \frac{b(R_0-1)}{\beta_1}, \tag{30}$$
$$S^* = \frac{b\beta_2(R_0-1)}{\beta_1(\mu+\delta_1+\alpha_1+\alpha_2)}.$$

We observe that the characteristic equation $P(\lambda)$ of the Jacobian matrix $J(E^*)$ has an eigenvalue $\lambda_1 = -(\mu+\delta_1+\alpha_1+\alpha_2)$, where the real part is negative. To assess the stability of the equilibrium with fast food consumption present in model (29), we analyze the roots of the following equation $\varphi(\lambda)$:

$$\varphi(\lambda) = \lambda^2 + a_1\lambda + a_2 \tag{31}$$

where

$$a_1 = \beta_1\frac{L^*}{N}+\mu \succ 0,$$
$$a_2 = \beta_1^2\frac{P^*}{N}\frac{L^*}{N} \succ 0. \tag{32}$$

According to the Routh-Hurwitz criterion, system (22) is locally asymptotically stable if $a_1 > 0$ and $a_2 > 0$. Therefore, the equilibrium with fast food consumption present, E^*, in system (20) is locally asymptotically stable.

IV. Global Stability

A. Global Stability of the Consumption-Free Equilibrium

To establish the global asymptotic stability of the system at equilibrium, we apply Lyapunov function theory for both the consumption-free and consumption-present equilibria. We begin by demonstrating that when $R_0 \leq 1$, the consumption-free equilibrium E^0 is globally stable.

Theorem 4: If $R_0 \leq 1$, the consumption-free equilibrium E^0 is globally asymptotically stable; otherwise, it is unstable.

$$V = cL \tag{33}$$

Proof. Consider the following Lyapunov function, where c is a positive constant. The derivative of $V(P, L, S)$ with respect to t gives:

$$\frac{dV}{dt} = c\frac{dL}{dt} \tag{34}$$
$$= c\left[\beta_1\frac{P}{N}-(\mu+\beta_2)\right]L \tag{35}$$
$$= c(\mu+\beta_2)\left[\frac{\beta_1 P}{(\mu+\beta_2)N}-1\right]L \tag{36}$$
$$\leq (R_0-1)L \tag{37}$$

where $c = \frac{1}{\mu+\beta_2}$ and $R_0 = \frac{\beta_1}{\mu+\beta_2}$.

Thus, $\frac{dV}{dt} \leq 0$ if $R_0 \leq 1$. Furthermore, $\frac{dV}{dt} = 0$ if and only if $L = 0$. Hence, by LaSalle's invariance principle [16], E^0 is globally asymptotically stable.

B. Stability of the Consumption-Present Equilibrium

In this section, we analyze the global stability of E^*, the equilibrium point for the consumption-present case.

Theorem 5: The equilibrium point E^* for the consumption-present case is globally asymptotically stable if $R_0 > 1$. We define the Lyapunov function V as follows:

$$V : \Gamma \rightarrow, \quad \mathbb{R}$$
$$V(P, L) = c_1\left[P - P^*\ln\left(\frac{P}{P^*}\right)\right] + c_2\left[L - L^*\ln\left(\frac{L}{L^*}\right)\right]$$

where c_1 and c_2 are positive constants, and $\Gamma = \left\{(P, L) \in \mathbb{R}^2 \mid P > 0, L > 0\right\}$ is the feasible region. The time derivative of the Lyapunov function is given by:

$$\frac{dV(P,L)}{dt} =$$
$$= -bc_1\frac{(P-P^*)^2}{PP^*} + \frac{\beta_1}{N}(c_2-c_1)(P-P^*)(L-L^*)$$

For $c_1 = c_2 = 1$, we have:

$$\frac{dV(P,L)}{dt} = -b\frac{(P-P^*)^2}{PP^*} \leq 0 \tag{38}$$

Additionally, we obtain:

$$\frac{dV(P,L)}{dt} = 0 \Rightarrow P = P^* \tag{39}$$

Thus, by LaSalle's invariance principle [16], the consumption-present equilibrium E^* is globally asymptotically stable on Γ when $R_0 > 1$.

V. Sensitivity Analysis of R_0

Sensitivity analysis is a critical tool for identifying the parameters that significantly influence the reproduction number R_0, thereby assisting in evaluating the model's robustness to changes in parameter values. We compute the normalized forward sensitivity indices of R_0 following the methodology presented in Chitnis et al. [17]. Let

$$\Upsilon_m^{R_0} = \frac{m}{R_0} \cdot \frac{\partial R_0}{\partial m} \tag{40}$$

represent the sensitivity of R_0 with respect to a parameter m. Given that$R_0 = \frac{\beta_1}{\mu+\beta_2}$, the specific sensitivity indices are as follows:

$$\begin{aligned} \Upsilon_{\beta_1}^{R_0} &= 1, \\ \Upsilon_{\beta_2}^{R_0} &= -\tfrac{\beta_1}{\mu+\beta_2}, \\ \Upsilon_{\mu}^{R_0} &= -\tfrac{\mu}{\mu+\beta_2}. \end{aligned} \tag{41}$$

This analysis shows that the basic reproduction number R_0 is most sensitive to changes in β_1. If β_1 increases, R_0 will increase in direct proportion. Conversely, a decrease in β_1 will result in a proportional reduction of R_0. On the other hand, μ and β_2 have an inversely proportional relationship with R_0. Specifically, increases in either μ (the natural death rate of the population) or β_2 (the coefficient of transmission from L to S) will lead to a decrease in R_0, though the effect is less pronounced for β_2.Given the high sensitivity of R_0 to β_1, this suggests that controlling the effective contact rate is the most critical factor in reducing the spread of consumption. Thus, prevention measures are more effective than treatment in managing the dynamics of chronic consumption. This sensitivity analysis indicates that the focus should be on reducing β_1, as opposed to increasing the number of people receiving treatment.

TABLE I: Sensitivity indices for the model parameters.

Parameter	Description	Sensitivity Index
μ	Natural death rate	-0.68
β_1	Effective contact rate	+1
β_2	Transmission coefficient from L to S	-0.32

VI. Numerical Simulations

In this section, we present several numerical solutions to model (1) for various parameter values [18]. The Gauss-Seidel-like implicit finite-difference method (GSS1 method), developed by Gumel et al. [19] and presented in [20], was used to solve system (1). We use the following initial condition:

$$P + L + S + C + Q = 1000.$$

The parameters used are listed in Table II and Table III. Using the same parameters and different initial values given in Table II, with $R_0 = 0.43$ and $R_0 < 1$, we begin with a graphical representation of the consumption-free equilibrium E^0.
Based on these figures, we make the following observations (see Figure 3,Figure 4 ,Figure 5 ,Figure 6 and Figure 7) using various initial values for the variables P_0, L_0, S_0, C_0, and Q_0. Using the same parameters and different initial values given in Table II, with $R_0 = 0.43$ and $R_0 < 1$, we begin with a graphical representation of the consumption-free equilibrium E^0.

TABLE II: Initial Values of Infected Compartments in Model (1).

Type of Infected	Initial Values
Potential consumers of fast food $P_1(0)$	300
Potential consumers of fast food $P_2(0)$	600
Potential consumers of fast food $P_3(0)$	800
Moderate consumers of fast food $L_1(0)$	300
Moderate consumers of fast food $L_2(0)$	600
Moderate consumers of fast food $L_3(0)$	800
Excessive consumers of fast food $S_1(0)$	300
Excessive consumers of fast food $S_2(0)$	600
Excessive consumers of fast food $S_3(0)$	800
Obese patients $C_1(0)$	300
Obese patients $C_2(0)$	600
Obese patients $C_3(0)$	800
Individuals who quit from fast food $Q_1(0)$	300
Individuals who quit from fast food $Q_2(0)$	600
Individuals who quit from fast food $Q_3(0)$	800

TABLE III: Parameter Settings for Model (1).

Parameter	Description	Value
b	Birth rate or entry rate into the system	65
μ	Natural death rate	0.04
β_1	Transition rate from P to L	0.2
β_2	Transition rate from L to S	0.4
N	Total population	1000
α_1	Transition rate from S to C (obesity)	0.001
α_2	Recovery rate from excessive consumption	0.001
δ_1	Excessive fast-food intake death rate	0.07
δ_2	Death rate due to obesity	0.07
γ	Recovery rate from obesity	0.002

Based on these figures, we make the following observations (see Figure 3,Figure 4 ,Figure 5 ,Figure 6 and Figure 7) using various initial values for the variables P_0, L_0, S_0, C_0, and Q_0.

Potential Consumers of Fast Food (P)

The graph illustrates the growth of potential fast-food consumers (P) over time. Initially, there is a rapid increase in the number of consumers, which then slows as the market reaches saturation. As the curves flatten, it indicates that the number of consumers has approached its maximum capacity. Three initial values 300, 600, and 800represent different starting points. A larger initial base leads to quicker saturation, suggesting that businesses targeting larger groups may experience rapid early growth but will face slower growth as the market becomes saturated. Ultimately, this graph highlights the limit of market potential and emphasizes the importance of considering market saturation in long-term business planning (See Figure 3).

Moderate Consumers of Fast Food (L)

This graph illustrates the evolution of moderate fast-food consumers (L) over time, with initial values of 100, 300, and 500. The curves show a significant decrease, indicating a decline in fast-food consumption over time.The curves approaching zero suggest a gradual disengagement of consumers, likely due to lifestyle or health-related factors. As people become more health-conscious, consumption decreases, reflecting broader societal trends (See Figure 4).

Excessive Consumers of Fast Food (S)

This graph depicts the evolution of individuals excessively consuming fast food (S) over time, with initial values of 50, 150, and 200. The curves initially show a sharp increase,

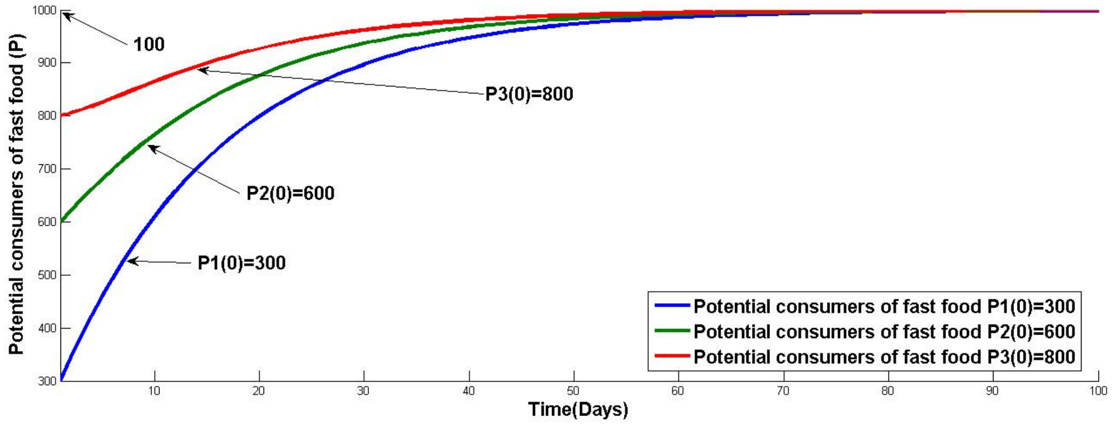

Fig. 3: When $R_0 < 1$, the consuming-free equilibrium E^0 is globally asymptotically stable.

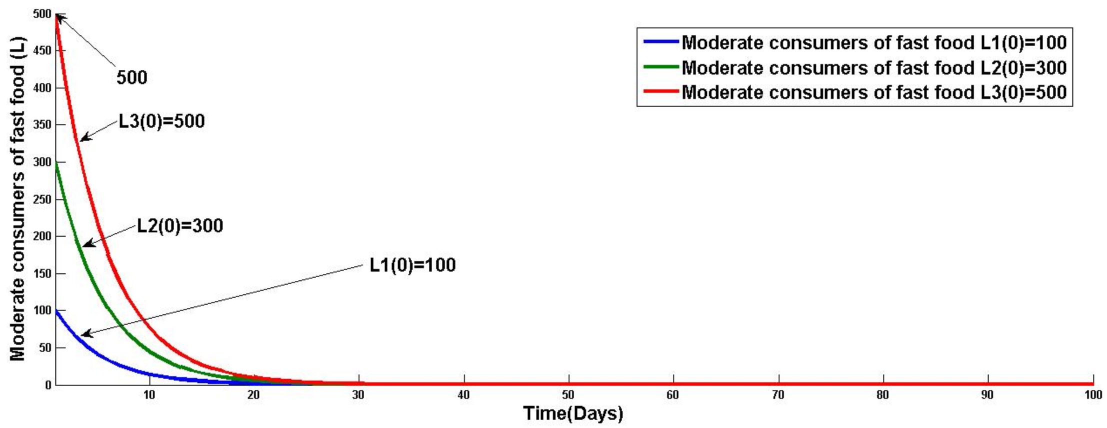

Fig. 4: When $R_0 < 1$, the consuming-free equilibrium E^0 is globally asymptotically stable.

followed by a gradual decrease. This pattern likely reflects a period of excessive consumption, followed by a decline possibly due to changes in habits or greater awareness of the negative effects of excessive fast-food intake.
This graph suggests a behavioral shift, with individuals gradually reducing their excessive consumption over time (See Figure 5).

Obese Patients (C)

The graph displays the evolution of the number of obese patients (C) over time, with initial values of 45, 10, and 5. The curves indicate a marked decrease, suggesting a decline in the number of obese patients, likely due to reduced excessive fast-food consumption. This could signal an improvement in public health, driven by better dietary habits and healthier lifestyles. The graph suggests a positive trend in public health, with fewer obese patients, which may reflect the impact of increased awareness and efforts to manage dietary habits (See Figure 6).

Individuals Who Quit Fast Food (Q)

This graph represents the number of individuals who have quit consuming fast food (Q) over time, with initial values of 60, 30, and 4. The curves show an exponential decrease, indicating a higher abandonment rate at the beginning, followed by a phase of stabilization. This suggests that people are gradually stopping their fast-food consumption, and the rapid initial decline slows over time as the market stabilizes. The graph highlights a phase of significant behavioral change followed by a plateau as fewer people quit fast food (See Figure 7). These graphs illustrate the different phases of fast-food consumption, from initial interest to eventual abandonment. Thus, the solution curves toward the equilibrium point $E^0(P_0, 0, 0, 0, 0)$ when $R_0 < 1$, indicating that model (1) is globally asymptotically stable.They provide insights into both excessive and moderate consumption patterns and highlight the effects of fast food on obesity.
This data is crucial for studying consumption trends and assessing the potential impact of increased awareness about healthier eating habits. In summary, these graphs serve as a valuable tool for understanding the evolution of eating behaviors and promoting a more balanced and health-conscious lifestyle.
We use the parameters listed in Table IV and Table V. Additionally, we start with a graphical representation of the consumption-present equilibrium E^*, using the same parameters but with different initial values given in Table IV and Table V. With $R_0 = 3.65$ and $R_0 > 1$, the following observations were obtained from the figures, based on varying initial values of P_0, L_0, S_0, C_0, and Q_0 (See Figure 8, Figure 9 , Figure 10, Figure 11 and Figure 12).

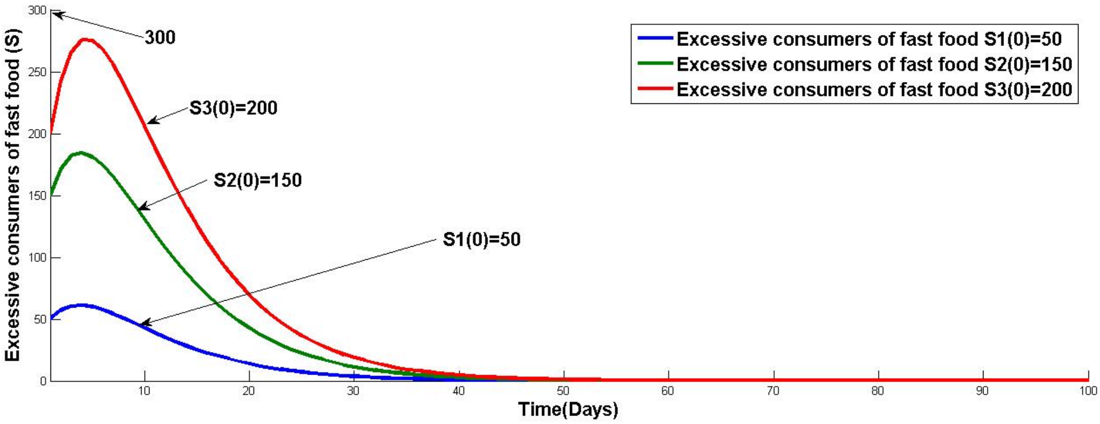

Fig. 5: When $R_0 < 1$, The consuming-free equilibrium E^0 is globally asymptotically stable.

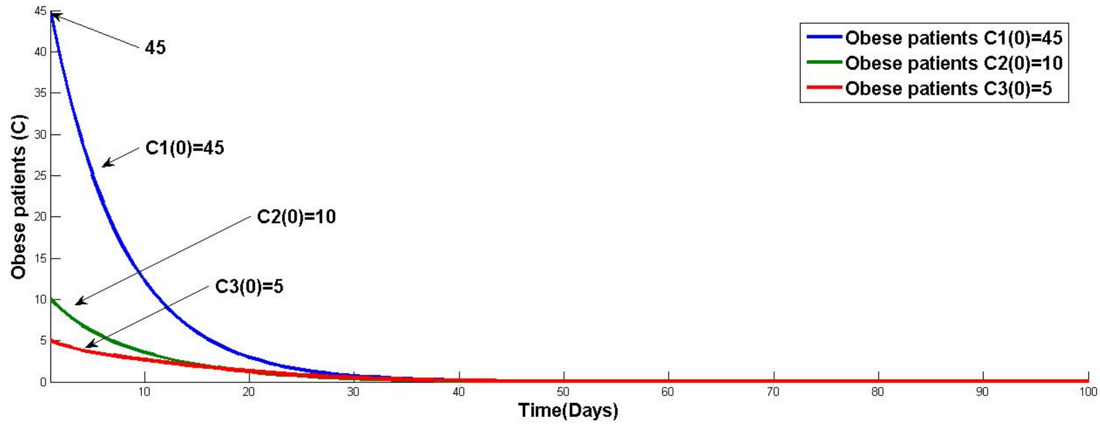

Fig. 6: When $R_0 < 1$, The consuming-free equilibrium E^0 is globally asymptotically stable.

TABLE IV: Paramètres académiques utilisés dans le modèle (1).

Type of Infected	Initial Values
Potential consumers of fast food $P_1(0)$	200
Potential consumers of fast food $P_2(0)$	300
Potential consumers of fast food $P_3(0)$	600
Moderate consumers of fast food $L_1(0)$	300
Moderate consumers of fast food $L_2(0)$	400
Moderate consumers of fast food $L_3(0)$	500
Excessive consumers of fast food $S_1(0)$	100
Excessive consumers of fast food $S_2(0)$	300
Excessive consumers of fast food $S_3(0)$	500
Obese patients $C_1(0)$	45
Obese patients $C_2(0)$	5
Obese patients $C_3(0)$	10
Individuals who quit from fast food $Q_1(0)$	60
Individuals who quit from fast food $Q_2(0)$	30
Individuals who quit from fast food $Q_3(0)$	5

TABLE V: Parameter Settings.

Parameter	Description	Value
b	Birth rate or entry rate into the system	65
μ	Natural death rate	0.04
β_1	Transition rate from P to L	0.75
β_2	Transition rate from L to S	0.4
N	Total population	1000
α_1	Transition rate from S to C (obesity)	0.001
α_2	Recovery rate from excessive consumption	0.001
δ_1	Fast-food overconsumption mortality rate	0.07
δ_2	Obesity-related mortality rate	0.07
γ	Recovery rate from obesity	0.002

Potential Consumers of Fast Food (P)

This graph illustrates the evolution of the number of individuals interested in fast food consumption (P) over time, with initial values of 200, 300, and 600. Interpretation: The curves show a sharp initial decline, indicating a rapid decrease in the interest of potential consumers, likely due to market saturation or initial doubts about fast food. After this drop, the curves stabilize, suggesting that the market reaches a more constant level of interest. Conclusion: The fast-food market may experience an initial peak in interest that quickly diminishes, probably due to competition or growing consumer awareness, and then stabilize at a regular demand level (See Figure 8).

Moderate Consumers of Fast Food (L)

This graph depicts the evolution of moderate fast-food consumers (L) over time, with initial values of 300, 400, and 500. Interpretation: Each curve peaks early, suggesting an initial increase in moderate consumption, likely as new consumers adapt to a balanced eating pattern. After the peak, the curves gradually decline and stabilize, implying that moderate consumers eventually find a consistent level of fast-food consumption. Conclusion: This graph indicates that moderate consumers reach a balanced consumption pattern after an initial adjustment phase, ultimately maintaining a

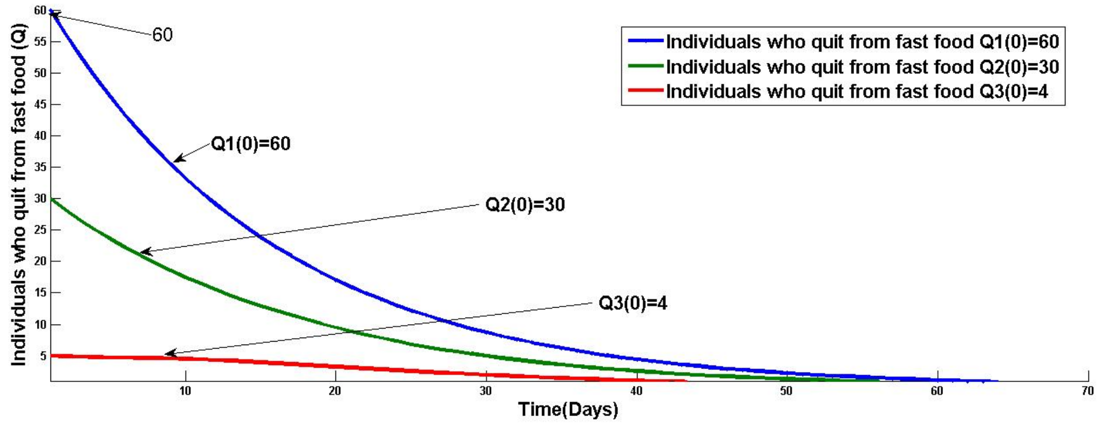

Fig. 7: When $R_0 < 1$, the consuming-free equilibrium E^0 is globally asymptotically stable.

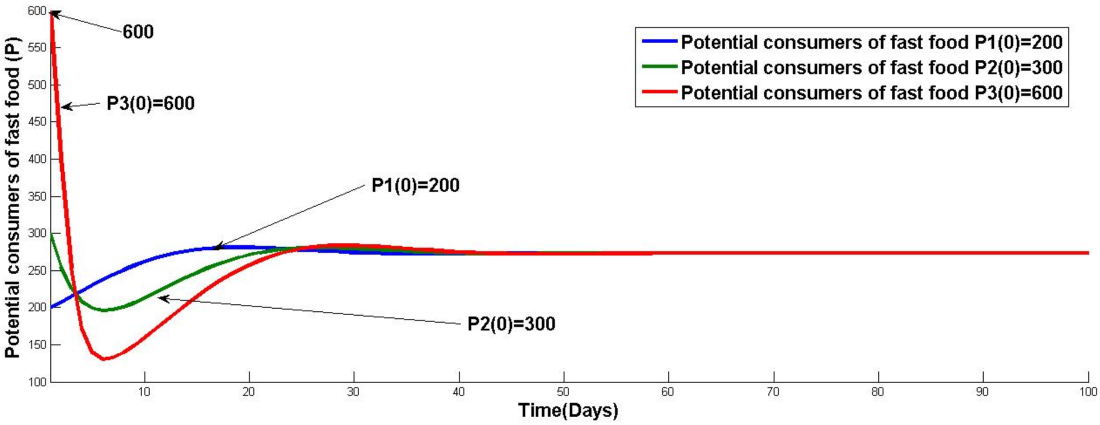

Fig. 8: When $R_0 > 1$, the consumption-present equilibrium E^* is globally asymptotically stable.

stable and moderate level of fast-food intake (See Figure 9).

Excessive Consumers of Fast Food (S)

This graph represents the number of excessive fast-food consumers (S) over time, starting with initial values of 100, 300, and 500. Interpretation: The curves show a rapid rise to a peak, reflecting a period of high fast-food consumption. After reaching the peak, the numbers decline, eventually stabilizing at a lower level, suggesting that some consumers reduce their excessive consumption or shift to lower levels of intake. Conclusion: This graph indicates that although some individuals initially consume fast food excessively, their consumption tends to decrease over time, possibly due to a shift toward healthier eating habits or a natural reduction in fast-food appeal (See Figure 10).

Obese Patients (C)

This graph tracks the number of obese patients (C) over time, with starting values of 45, 10, and 5. Interpretation: The sharp decline in each curve suggests a significant reduction in obesity, likely due to healthier eating choices or reduced consumption of fast food. As the curves level off, it indicates that the number of obese patients stabilizes at a lower level, reflecting an overall health improvement. Conclusion: This trend suggests a positive shift in health, with decreasing obesity rates correlating to reduced fast-food consumption. This indicates a transition toward more health-conscious behaviors (See Figure 11).

Individuals Who Quit Fast Food (Q)

This graph represents the number of people who have stopped consuming fast food (Q), starting with initial values of 60, 30, and 5. Interpretation: A sharp decline at the beginning of each curve suggests that many people quit fast food quickly, likely due to health concerns or dissatisfaction. Over time, the curves stabilize, indicating that the abandonment rate slows down, likely because only the most loyal or regular consumers remain. Conclusion: This trend highlights an initial wave of abandonment among less engaged consumers, followed by a stabilization that leaves a core group of regular consumers continuing their fast-food habits (See Figure 12). These revised graphs depict the progression of fast-food consumption trends across various consumer categories: Potential Consumers (P): Initially fluctuate but stabilize at a consistent level of interest. Excessive Consumers (S): Experience an initial surge in consumption, followed by a period of moderation. Individuals Who Quit Fast Food (Q): Exhibit a high initial dropout rate, which levels off as only the most committed consumers remain. Moderate Consumers (L): Reach a balanced level of consumption after a phase of growth. Obese Patients (C): Show a decline in numbers, likely due to the adoption of

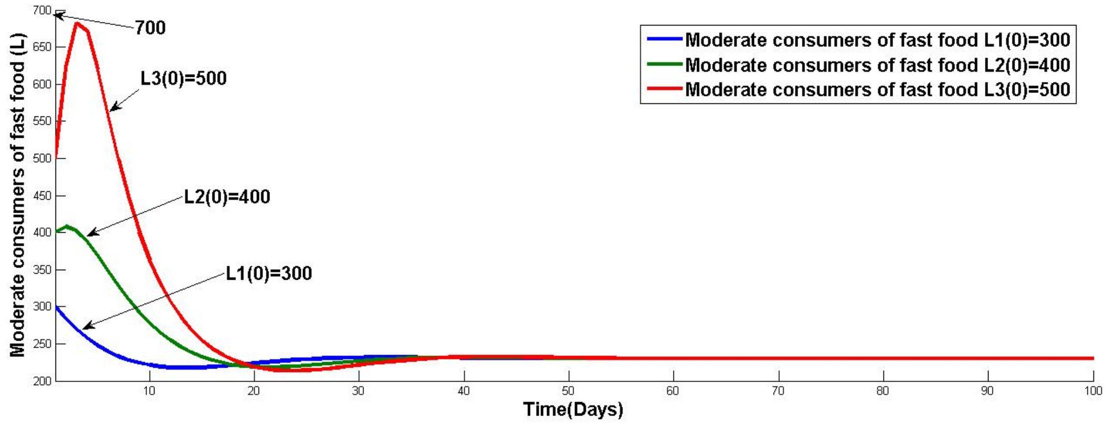

Fig. 9: When $R_0 > 1$, the consumption-present equilibrium E^* is globally asymptotically stable.

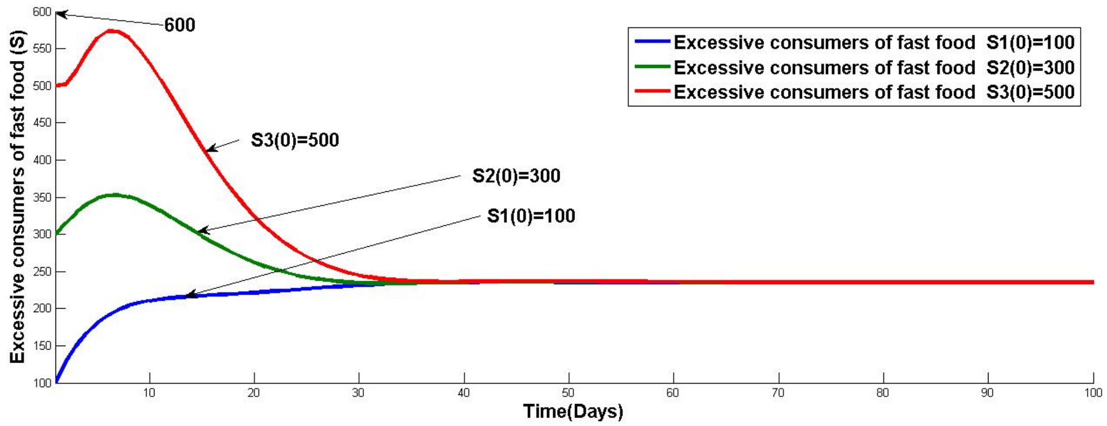

Fig. 10: When $R_0 > 1$, the consumption-present equilibrium E^* is globally asymptotically stable.

healthier choices over time. These observations highlight the lifecycle of fast-food consumption, where initial enthusiasm diminishes as consumers moderate their habits and health outcomes improve. This information could be valuable for stakeholders in the fast-food industry as well as public health policymakers.

VII. Conclusion

In this study, we developed the continuous mathematical model $PLSCQ$ to capture the dynamics of fast food consumption, considering the influence of both private and public addiction treatment centers. We analyzed the model's behavior and derived the basic reproduction number, $R_0 = \frac{\beta_1}{\mu+\beta_2}$,a key metric for understanding the system's dynamics. A sensitivity analysis was conducted to identify the parameters that most significantly impact R_0, shedding light on the factors that affect the model's outcomes.We then assessed the stability of the system using nonlinear stability analysis. Our findings show that the consumer-free equilibrium, E^0, is stable when $R_0 \leq 1$, indicating that the number of consumers will decrease under this condition. Conversely, when $R_0 > 1$, the consumer equilibrium, E^*, is stable, meaning consumption will persist in the population.Through the use of Lyapunov functions, we confirmed that E^0 is globally stable when $R_0 \leq 1$, ensuring that the system will eventually reach a state of no consumers. Similarly, when $R_0 > 1$, we demonstrated that E^* is globally stable, meaning the system will stabilize at the consumer equilibrium.In conclusion, the $PLSCQ$ model provides a useful tool for understanding and predicting fast food consumption behavior, while also evaluating the impact of addiction treatment interventions. The insights from this model can inform public health policies aimed at reducing fast food consumption and promoting healthier eating habits.

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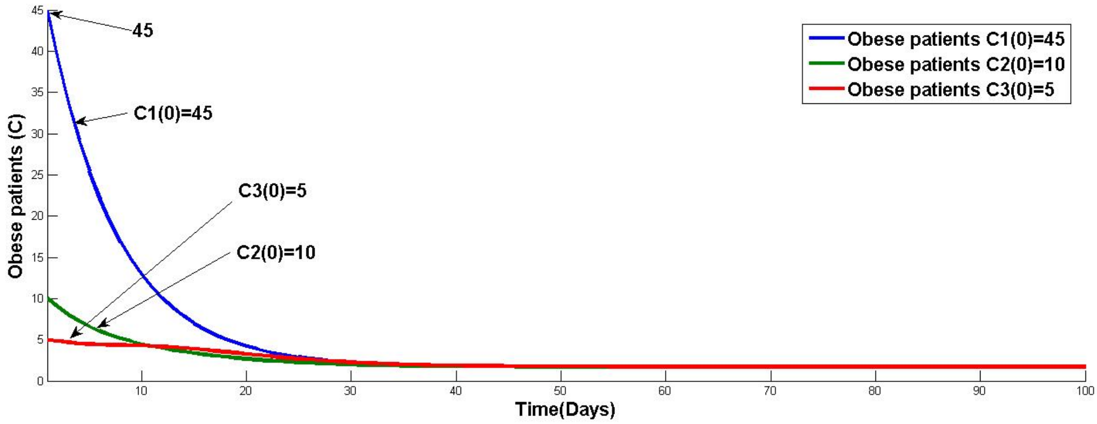

Fig. 11: When $R_0 > 1$, the consumption-present equilibrium E^* is globally asymptotically stable.

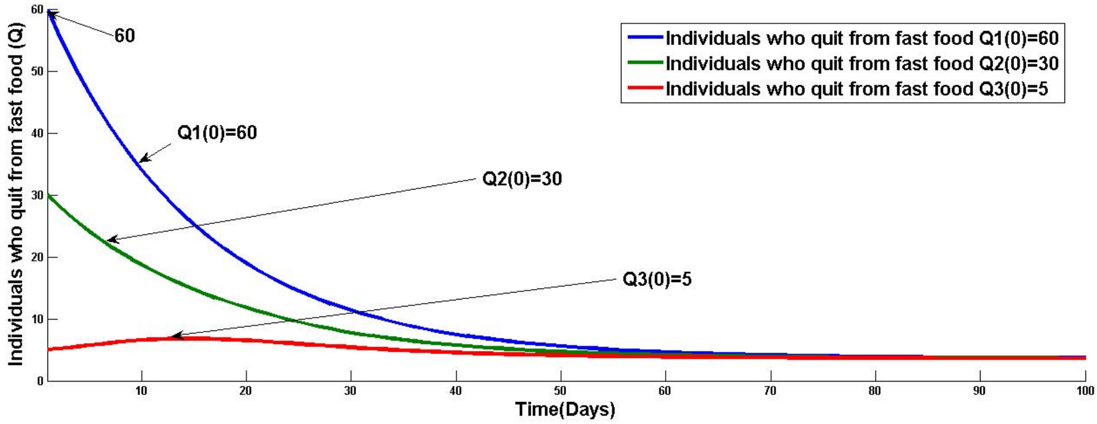

Fig. 12: When $R_0 > 1$, the consumption-present equilibrium E^* is globally asymptotically stable.

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