A Probabilistic Inventory Model for Perishable Products Fuzzy Environment Using Different Defuzzification Methods

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***Abstract*—Inventory management and the handling of the deterioration are critical in various industries worldwide. Over the years, numerous studies have examined the integration of fuzzy set theory with inventory management. Most of inventory models are based on customer demands, which makes demand a key factor in inventory optimization. In Developing sustainable inventory models that incorporate environmental considerations, in addition to traditional inventory management objectives, is becoming increasingly important. This paper proposes a probabilistic inventory model with a deterioration rate independent of time and demand uncertainty, aiming to determine the optimal inventory policy in a fuzzy environment. Furthermore, the paper introduces a fuzzy model utilizing the triangular fuzzy numbers, assuming that certain model parameters are fuzzy due to inherent imprecision. The study investigates the impact of various defuzzification methods on the optimal values of the decision variables and the associated cost function. Numerical examples are, along with graphical representations, are provided for each scenario, and sensitivity analysis is performed on various parameters to validate the proposed model.**

***Index Terms*—Fuzzy inventory model, Probabilistic uncertain demand, Deterioration, Triangular Fuzzy Number.**

I. Introduction and Literature Survey

IN numerous inventory models, fuzziness closely reflects reality, serving as a source of uncertainty. In recent years, some researchers have focused on time dependent demand rates, particularly for newly released products such as trendy clothing, electronics, and mobile phones, which initially rose gradually before stabilizing. Deterioration refers to the damage, decay, or spoilage of goods stored for future use. Given that these goods inevitably lose value over time, deterioration cannot be avoided in any business context, Kumar, Sushil, and U. S. Rajput [1].

Researchers traditionally considered demand, degradation, and other constraints in inventory models as deterministic, establishing these constraints based on available data and additional factors. Yet, due to the imprecision of currently available data and rapid shifts in market conditions, this data may be inaccurate or insufficient to establish the model's constraints. For instance, uncertainties in global economic conditions introduce variability in inventory costs. Therefore, traditional inventory models may not yield optimal outcomes for their intended purposes. This means that the ordered quantity, total cycle time, and costs may not align with actual requirements. Consequently, relying on traditional inventory model outcomes could potentially result in financial losses or reduced profits for retailers. Hence, achieving optimal outcomes for inventory models with inaccurate costs and other attributes becomes a critical concern for inventory managers. Fuzzy set theory is considered the most effective approach for managing cost distortions and other factors in inventory models in such situations. The first inventory model was developed by F. Harris [2]. Set out the concept of fuzzy set theory in inventory modeling, Zadeh, Lotfi A [3]. Considered an inventory model on decision making in fuzzy environment Bellman, R. [4]. A fuzzy inventory model on decision-making in the presence of fuzzy variables was developed by R. Jain [5]. Initially, Lee and Yao [6] utilized fuzzy principles to address the inventory model's imperfect manufacturing quantity and demand. Subsequently, numerous academics have developed a inventory models containing imprecise parameters to achieve more precise optimal solutions, utilizing the ideas of fuzzy set theory. Dutta, Anurag, et al. [40] use of the data by National Aeronautics and Space Administration to train our model.

A. Survey of sustainability

Over the past few decades, it has become increasingly apparent that our planet's habitability conditions have been gradually diminishing. Pollution from the transportation, industry, and inadequate waste management has led to a gradual deterioration in the quality of the air we breathe. There is now widespread recognition of the serious threat posed by global warming worldwide. Thus, consumers are more environmentally conscious than they used to be. Many governmental and non - governmental organizations have expressed their opinions, highlighting the importance of finding sustainable solutions to ensure the planet's thriving. One practical approach would be to impose higher taxes on businesses or factories using highly polluting manufacturing or production methods, or those with inadequate inventory management critical to production. Furthermore, companies that contribute excessive pollution during the distribution and transportation of goods should be required to pay a. From this perspective, businesses worldwide must adopt and implement inventory models that prioritize sustainability in their operations. It is crucial to devise new strategies aimed at practicing ethical behaviors that reduce their negative impact on local and global ecosystems. The objective is to encourage sustainable development over the long term to protect the environment for future generations [7].

Manuscript received May 13, 2024; revised June 14, 2025.

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Managing decreasing inventory levels while meeting of demand, minimum waste and considering environmental in impacts poses unique challenges. This underscores the need to create sustainable inventory models that integrate of the traditional inventory managementing objectives with the environmental considerations. To optimize inventory levels while the minimizing environmental impact, this study of proposes a sustainable inventory model tailored for the perishable goods in such situations. This study proposes a sustainable inventory model specifically designed for the perishable goods to optimize inventory levels while reducing environmental impact [8].

Several papers on sustainable inventory models have been published in recent years. Chaudhary, R., et al. [9] developed a sustainable inventory system under fuzziness. Taleizadeh et al. [10] addressed and examined four sustainable inventory models: without shortages, lost sales, and partial and full backorder. Mishra et al. [11] the presented an economic production quantity (EPQ) inventory model with carbon tax when the carbon emissions rate can be controlled through investment in green technology. Tiwari et al. [12] built is an inventory model for deteriorating products when some of them are of imperfect quality under carbon emissions. Mishra et al. [13] examined an EOQ inventory model with shortages and carbon emissions. Sustainable EOQ models usually include an extra cost involved to control the discharge of carbon. For example, in additionally to the ordering costs, purchasing costs, storage costs, and transportation costs, Bonney & Jaber [14] suggested a straightforward non-classical model that takes vehicle emissions and trash disposal costs into account for better results.

B. Survey of deteriorating items

Items often become unusable for their intended purposes due to degradation and decay. The rate of deterioration varies depending on the type of item, storage conditions, climate, and other factors. Specifically, various types of deterioration affect fruits, dried fruits, vegetables, dairy products, and food items. Additionally, the shelf lives of cold drinks, jewelry, health drinks, medications, radioactive materials, photographic films, electronic goods, fashionable clothing, plastic products, and textiles vary. Moreover, the cost of inventory is influenced by the deterioration of items. Thus, inventory managers encounter a challenging dilemma in minimizing the overall cost associated with managing deteriorating assets. Ghare and Schrader [15] have obtained the optimal results for retailers' inventory having constantly deteriorating items in the early days of the year 1960. Yadav, A.S., et al. [16] developed a fertilizer inventory model for TSP using Cuckoo optimization for the environment. Yadav, K.K., et al. [17] have developed an inventory model to deal with deteriorating products.

Some researchers viewed variables such as demand rate, deterioration rate, production rate, etc., as constants, while others considered them uncertain or variable. Mahapatra, Amalendu Singha, et al. [18] an inventory model with uncertain demand for deteriorating items. Kumar U. S. Rajput.[1] develop a fuzzy inventory model for deteriorating items with time-dependent and partial backlogging where demand rate, deterioration rate, and backlogging rate were considered triangular fuzzy numbers. J. S. Yao and J. Chiang [19] developed an inventory model without backorders and defuzzified the fuzzy holding cost by signed distance and centroid methods. P. K. De and A. Rawat [20] and Mahata and Debnath [42-43] developed a fuzzy inventory model without shortages using triangular fuzzy numbers. Nagamani, M., and G. Balaji.[21] developed a fuzzy inventory model with adequate shortage using the graded mean integral value method.

For deteriorating items with time-varying demand and shortages values. It discusses a production inventory model for deteriorating items. The rate of deterioration is shown as fuzzy triangular numbers together with other factors. For defuzzification using the signed distance method, centroid method, and graded mean integration method separately by Sen et al. [23]. A continuous production control inventory model for items that are deteriorating and in shortage in an uncertain environment. The signed distance method and the graded mead integration method are used to denazify the fuzzy total cost developed by Chakraborty, et al. [24]. The benefit of preservation technology with promotion and time dependent deterioration under fuzzy learning using uncertain demand Mahapatra, A. S., et al. [25]. An inventory model for a deteriorating item with trade credit policy and allowable shortages under uncertain demand Mali, Mr Vivek, et al. [26] and Deng, H. [44]. Deep Reinforcement Learning for Inventory Optimization with Non-Stationary Uncertain Demand using Dehaybe, Henri, Daniele Catanzaro, and Philippe Chevalier [27]. A proposed methodology addresses the control of perishable inventory in the face of uncertain demand. The inventory systems are managed by fuzzy logic and artificial neural networks. Fuzzy logic is used to modify the generated orders while accounting for the forecasted uncertainty, while the artificial neural network is utilized to calculate the order signal. A significant benefit is that the two-stage offline optimization procedure allows the controller behavior to be adapted to anticipated uncertainty without consuming a significant amount of computation while using Cho lodowicz, Ewelina, and Przemys law Orlowski [28].

C. Survey of Demand

Again, product demand is essential for maintaining inventory effectively. Different items experience varying demand patterns that change with consumer behavior, business cycles, and other influencing factors. The demand for items can decrease, increase, or remain constant based on customer needs. In many cases, it also hinges on variables such as time, pricing, advertising expenditures, and trade credit availability. Moreover, fluctuations in demand from market to market and cycle to cycle indicate that demand rates are often uncertain. Because it is more practical, many researchers have developed inventory models that take into consideration demand patterns with other restrictions and uncertain demand types. Kumar, B.A., and Paikray, S.K. [29] have developed an inventory model considering uncertain type demand rates.

Shortages of items during inventory cycles often occur when suppliers provide limited quantities during business cycles or when a retailer's stock point has insufficient storage capacity. Additionally, inventory shortages can also results from deterioration and variability in demand. Thus these situations, all of the customers or a few of them may choose not to wait until the next shipment of merchandise arrives. Then, the depending on the business context, different back logging rates for inventory shortages should be considered into account. Thus, shortages and backlogging also influence the inventory cost. Yadav, A.S., et al. [30] and Yadav, A.S., et al. [31] are considering shortages as partial backlogging in green inventory management.

D. Survey of probabilistic demand inventory models

A probabilistic demand strategy is the more effective in managing uncertainty, as accurately predicting customer demand for an item in the today's market environment challenging. A probabilistic inventory model with permitted payment delays was first presented by Shah, N.N., [32], who spearheaded the development of such models. Through probabilistic demand, Bhattacharjee and Sen [33] designed an inventory management system for supply chains. EOQ models are developed in the random and fuzzy-random environments considering demand to be dependent on unit cost which is a decision variable. In addition to these, the total average cost goal and constraint goal for storage area is fuzzy for the probabilistic model in a fuzzy environment by Panda, Debdulal, and Samarjit Kar [34]. Inventory model for deteriorating items with fuzzy lead time, negative exponential demand, and partially backlogged shortages and its nature due to probabilistic deterioration along with fuzzy lead time by Sen, Nabendu, and Sumit Saha [35]. Negi, Ashish, and Ompal Singh [41] use Uniform probabilistic distribution deterioration.

E. Our Contribution

The Proposed model, a probabilistic inventory model with a deterioration rate independent of time and probabilistic certain demand is developing to determine optimal inventory policies, by the taking a triangular fuzzy number in the fuzzy environment. After Using the different defuzzifications meth ods namely the Graded Mean integration method, Signed distance method, and Centroid method to find minimum optimal cost.

II. Preliminaries

A. Fuzzy Set:

If X is a universe of discourses and x be any particular element of X. The fuzzy set A defined on X is a collection of ordered pair,

$$A = \{(x, \mu_A(x)) : x \in X\}$$

where, $\mu_A(x) : X \to [0, 1]$ is called the membership function.

B. Fuzzy Number:

A fuzzy number is an extension of the real numbers,in It implies that it refers to a connected set of potential values rather than a single value with weights [36]. This is known as membership function.A convex, normalized fuzzy set of the real line is a specific instance of a fuzzy number [37]. Fuzzy number calculations enable the inclusion of uncertainty on initial conditions, parameters etc. A fuzzy number is equal to a fuzzy interval [38].

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$ is triangular if its membership function define as :

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & `a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & `a_2 \le x \le a_3` \\ 0, & `otherwise \end{cases}$$

In this way, parabolic fuzzy numbers, hexagonal fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, etc. can be defined. Several methods exist for defuzzifying fuzzy numbers. The most widely used methods for defuzzification are the centroid method, graded mean integration method, and signed distance methods.

- i Graded mean integration method representation of the triangular fuzzy number is $d_F\tilde{A} = \frac{a_1+4a_2+a_3}{6}$.
- ii Centroid method for triangular fuzzy number is $d_F\tilde{A} = \frac{a_1+a_2+a_3}{3}$.
- iii Signed distance method for triangular fuzzy number is $d_F\tilde{A} = \frac{a_1+2a_2+a_3}{4}$.

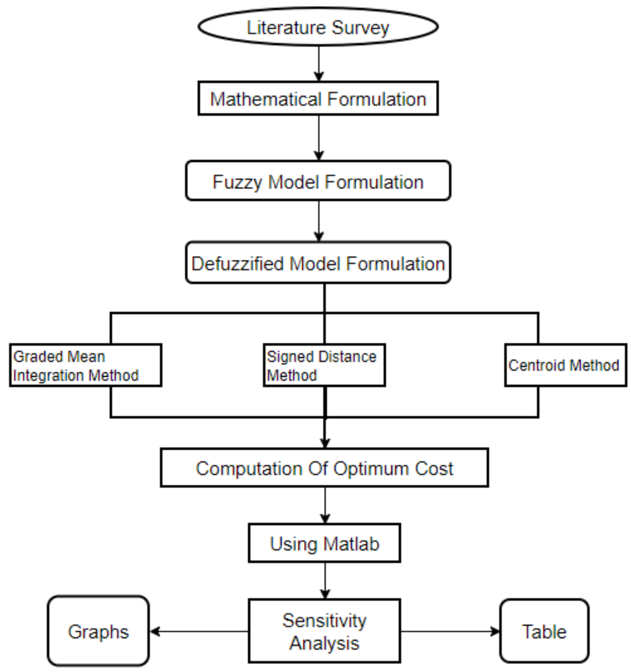

Fig. 1. Schematic diagram of proposed model

III. Assumptions

- i The inventory systems involves probabilistic of single item.
- ii Lead time is zero and shortages are allowed with complete backlogging.
- iii The Set-up cost,holding cost, deterioration rate are fuzzy.
- iv The Demand rate is $D(\rho) = D_0 + d_1\rho$, where D_0 is fixed base demand and $d_1\rho$ measure the effect of

promotional activity ρ with d_1 as a positive, constant scaling term. A Continuous random variable ϵ with expected value μ, is associated with demand $D(\rho)$ [39]. i.e. $D(\rho,\epsilon) = D(\rho) + \epsilon$ and $E(D(\rho,\epsilon)) = D(\rho) + \mu$

v The deterioration rate independent of time.

vi Replenishment is instantaneous.

IV. Notations

There are two types of models first is crisp model and second is fuzzy model. Symbols for both models are different which is represented below separately.

TABLE I
NOTATION FOR CRISP MODEL AND FUZZY MODEL

Symbol	Description
A	Set-up cost per cycle
C_h	Holding cost per unit time
θ	Deterioration rate at time t
C_d	Deterioration cost per unit time
C_p	Purchase cost per unit time
C_s	Shortage cost per unit time
$D(\rho)$	Uncertain demand rate per unit time
TAC	Total cost per unit time
t_1	Duration of production
R	Initial order quantity
M	Maximum shortage level
T	Cycle length
$I_1(t)$	Inventory level at time t, $0 \le t \le t_1$
$I_2(t)$	Inventory level at time t, $t_1 \le t \le T$
$\tilde{A}$	Fuzzy set-up cost
$\tilde{\theta}$	Fuzzy deterioration rate
$\tilde{C_h}$	Fuzzy holding cost per unit time
$\tilde{C_d}$	Fuzzy deterioration cost per unit time
$\tilde{C_p}$	Fuzzy purchase cost per unit time
$\tilde{C_s}$	Fuzzy shortage cost per unit time
$\tilde{D}(\rho)$	Fuzzy uncertain demand rate
$\widetilde{TAC}$	Fuzzy total cost per unit time
$d_F(\widetilde{TAC})$	Defuzzified value of $\widetilde{TAC}$

V. Proposed Model

This work aims to formulate and solve a probabilistic inventory model with the aforementioned assumptions considered into account. At t=0, the initial inventory level is R. It decreases in the time period $[0, t_1]$and reaches inventory level is zero at $t = t_1$. After that, complete backlogging situations occur during the interval $[t_1, T]$. The item's deterioration and demand are responsible for this depletion. This conditions is represented in fig 2.

The differential equation that describes the conditions is

$$\frac{dI_1(t)}{dt} = -\left\{D + \theta I_1(t)\right\}; \qquad 0 \le t \le t_1 \tag{1}$$

$\Rightarrow$

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D,$$

$$\frac{dI_2(t)}{dt} = -D; \qquad t_1 \le t \le T \tag{2}$$

Subject to condition, Initial Conditions $I_1(t) = R$ at $t = 0$
Boundary Conditions $I_1(t) = 0$ at $t = t_1$ and $I_2(t) = -S$ at $t = T$

Solution of equation (1) is given by

$$I_1(t) = -\frac{D}{\theta} + C_1 e^{-\theta t} \tag{3}$$

Now using Initial Conditions $I_1(t) = R$ at $t = 0$,
So $C_1 = R + \frac{D}{\theta}$.
Therefore equation (3) is

$$I_1(t) = -\frac{D}{\theta} + \left[R + \frac{D}{\theta}\right] e^{-\theta t} \tag{4}$$

Now apply Boundary Conditions $I_1(t) = 0$ at $t = t_1$, we get

$$R = -\frac{D}{\theta}\left[1 + e^{\theta t_1}\right] \tag{5}$$

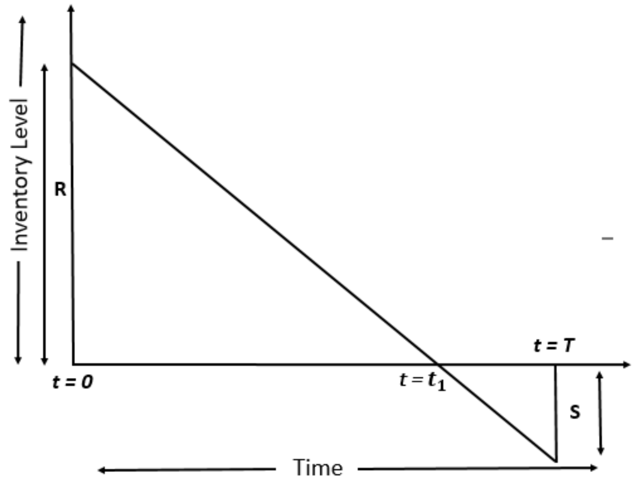

Fig. 2. Inventory situation of retailer.

Solution of equation (2) is given by

$$I_2(t) = -Dt + C_2 \tag{6}$$

Now using Initial Conditions $I_2(t) = 0$ at $t = t_1$, So $C_2 = Dt_1$
Therefore equation (6) is

$$I_2(t) = D(t_1 - t) \tag{7}$$

Now apply Boundary Conditions $I_2(t) = -S$ at $t = T$, we get

$$S = -D(t_1 - T) \tag{8}$$

The components used for obtaining the total cost are given as follows:

Ordering Cost :

$$OC = A \tag{9}$$

Purchasing Cost :

$$PC = C_p(R + S)$$

$$PC = C_p\left[-\frac{D}{\theta}\left(1 + e^{\theta t_1}\right) + \left(-D(t_1 - T)\right)\right]$$

$$PC = C_p\Big[-\frac{D}{\theta}\Big(1+e^{\theta t_1}\Big) - D(t_1 - T)\Big] \quad (10)$$

Holding Cost :

$$HC = C_h\int_0^{t_1} I_1(t)dt$$

$$HC = C_h\int_0^{t_1}\Big[-\frac{D}{\theta}+\Big[R+\frac{D}{\theta}\Big]e^{-\theta t}\Big]dt$$

$$HC = C_h\Big[-\frac{D}{\theta}t_1 + \Big(\frac{D}{\theta^2}+\frac{R}{\theta}\Big)(e^{-\theta t_1}-1)\Big]$$

Put value of R From equation (5), So we get

$$HC = C_h\Big[-\frac{D}{\theta}t_1+\Big(\frac{D}{\theta^2}-\frac{D}{\theta^2}(1+e^{\theta t_1})\Big)(e^{-\theta t_1}-1)\Big] \quad (11)$$

Deterioration Cost :

$$DC = C_d\int_0^{t_1}\theta I_1(t)dt$$

$$DC = C_d\theta\int_0^{t_1}\Big[-\frac{D}{\theta}+\Big[R+\frac{D}{\theta}\Big]e^{-\theta t}\Big]dt$$

$$DC = C_d\Big[-\frac{D}{\theta}t_1 + \Big(\frac{D}{\theta^2}+\frac{R}{\theta}\Big)(e^{-\theta t_1}-1)\Big]$$

Put value of R From equation (5), So we get

$$DC = C_d\theta\Big[-\frac{D}{\theta}t_1+\Big(\frac{D}{\theta^2}-\frac{D}{\theta^2}(1+e^{\theta t_1})\Big)(e^{-\theta t_1}-1)\Big] \quad (12)$$

Shortage Cost :

$$SC = C_s\int_{t_1}^{T}[-I_2(t)]dt = -C_s\int_{t_1}^{T}D(t_1-t)dt$$

$$Sc = C_sD\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T^2}{2}\Big)\Big] \quad (13)$$

Total Cost :

$$TAC = \frac{1}{T}[OC+PC+HC+DC+SC]$$

Substitute the value of OC, PC, HC, DC and SC from equation (9), equation (10), equation (11), equation (12) and equation (13) respectively. That is

$$\begin{aligned}TAC = \frac{1}{T}\Big[A + C_p\Big[-\frac{D}{\theta}\Big(1+e^{\theta t_1}\Big)-D(t_1-T)\Big]\\ + C_h\Big[-\frac{D}{\theta}t_1+\Big(\frac{D}{\theta^2}-\frac{D}{\theta^2}(1+e^{\theta t_1})\Big)(e^{-\theta t_1}-1)\Big]\\ + C_d\theta\Big[-\frac{D}{\theta}t_1+\Big(\frac{D}{\theta^2}-\frac{D}{\theta^2}(1+e^{\theta t_1})\Big)(e^{-\theta t_1}-1)\Big]\\ + C_sD\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T^2}{2}\Big)\Big]\Big]\end{aligned} \quad (14)$$

VI. Fuzzy Model

It is not always possible to define certain parameters with precision due to environmental uncertainty, hence we fuzzify some parameters. Here we fuzzify the parameters A, C_p, C_h, C_d, C_s, θ, D. We take triangular fuzzy numbers $\tilde{A} = (A_1, A_2, A_3)$, $\tilde{C_p} = (C_{p_1}, C_{p_2}, C_{p_3})$, $\tilde{C_h} = (C_{h_1}, C_{h_2}, C_{h_3})$, $\tilde{C_d} = (C_{d_1}, C_{d_2}, C_{d_3})$, $\tilde{C_s} = (C_{s_1}, C_{s_2}, C_{s_3})$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$, $\tilde{D} = (D_1, D_2, D_3)$, Therefore

$$\begin{aligned}\widetilde{TAC} = \frac{1}{T}\Big[\tilde{A} + \tilde{C_p}\Big[-\frac{\tilde{D}}{\tilde{\theta}}\Big(1+e^{\tilde{\theta}t_1}\Big)-\tilde{D}(t_1-T)\Big)\Big]\\ + \tilde{C_h}\Big[-\frac{\tilde{D}}{\tilde{\theta}}t_1+\Big(\frac{\tilde{D}}{\tilde{\theta}^2}-\frac{\tilde{D}}{\tilde{\theta}^2}(1+e^{\tilde{\theta}t_1})\Big)(e^{-\tilde{\theta}t_1}-1)\Big]\\ + \tilde{C_d}\tilde{\theta}\Big[-\frac{\tilde{D}}{\tilde{\theta}}t_1+\Big(\frac{\tilde{D}}{\tilde{\theta}^2}-\frac{\tilde{D}}{\tilde{\theta}^2}(1+e^{\tilde{\theta}t_1})\Big)(e^{-\tilde{\theta}t_1}-1)\Big]\\ + \tilde{C_s}\tilde{D}\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T_1^2}{2}\Big)\Big]\Big]\end{aligned} \quad (15)$$

$\widetilde{TAC} = (TAC_1, TAC_2, TAC_3)$ say
Where,

$$\begin{aligned}TAC_i = \frac{1}{T}\Big[A_i + C_{p_i}\Big[-\frac{D_i}{\theta_i}\Big(1+e^{\theta_i t_1}\Big)-D_i(t_1-T)\Big]\\ + C_{h_i}\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{d_i}\theta_i\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{s_i}D_i\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T^2}{2}\Big)\Big]\Big]\end{aligned}$$

Now we find the derivatives

$$\begin{aligned}\frac{d}{dT}TAC_i = -\frac{1}{T^2}\Big[A_i + C_{p_i}\Big[-\frac{D_i}{\theta_i}\Big(1+e^{\theta_i t_1}\Big)-D_i(t_1-T)\Big]\\ + C_{h_i}\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{d_i}\theta_i\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{s_i}D_i\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T^2}{2}\Big)\Big]\Big] + \frac{1}{T}\Big[C_{p_i}D_i - C_{s_i}D_i(t_1-T)\Big]\\ i = 1,2,3\end{aligned}$$

And Now again derivative

$$\begin{aligned}\frac{d^2}{dT^2}TAC_i = \frac{2}{T^3}\Big[A_i + C_{p_i}\Big[-\frac{D_i}{\theta_i}\Big(1+e^{\theta_i t_1}\Big)-D_i(t_1-T)\Big]\\ + C_{h_i}\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{d_i}\theta_i\Big[-\frac{D_i}{\theta_i}t_1+\Big(\frac{D_i}{\theta_i^2}-\frac{D_i}{\theta_i^2}(1+e^{\theta_i t_1})\Big)(e^{-\theta_i t_1}-1)\Big]\\ + C_{s_i}D_i\Big[\frac{t_1^2}{2}-\Big(t_1T-\frac{T^2}{2}\Big)\Big]\Big] - \frac{2}{T^2}\Big[C_{p_i}D_i - C_{s_i}D_i(t_1-T)\Big]\\ + \frac{1}{T}C_{s_i}D_i \qquad i = 1,2,3\end{aligned}$$

Now find the defuzzified value of $\widetilde{TAC}$
1. Graded Mean integration method (GMIR).
2. Signed distance method (SDM)
3. Centroid method (CEM)

1. GRADED MEAN INTEGRATION METHOD

The deduzzified value is :

$$d_F\widetilde{TAC}=\frac{1}{6}(TAC_1+4TAC_2+TAC_3)$$

$$\frac{d}{dT}d_F\widetilde{TAC}=\frac{1}{6}\left(\frac{d}{dT}TAC_1+4\frac{d}{dT}TAC_2+\frac{d}{dT}TAC_3\right)$$

Where,

$$\frac{dTAC_1}{dT}=-\frac{1}{T^2}\left[A_1+C_{p_1}\left[-\frac{D_1}{\theta_1}\left(1+e^{\theta_1 t_1}\right)-D_1(t_1-T)\right]\right. + C_{h_1}\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] + C_{d_1}\theta_1\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] \left.+C_{s_1}D_1\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_1}D_1-C_{s_1}D_1(t_1-T)\right],$$

$$\frac{dTAC_2}{dT}=-\frac{1}{T^2}\left[A_2+C_{p_2}\left[-\frac{D_2}{\theta_2}\left(1+e^{\theta_2 t_1}\right)-D_2(t_1-T)\right]\right. + C_{h_2}\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] + C_{d_2}\theta_2\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] \left.+C_{s_2}D_2\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_2}D_2-C_{s_2}D_2(t_1-T)\right]$$

and

$$\frac{dTAC_3}{dT}=-\frac{1}{T^2}\left[A_3+C_{p_3}\left[-\frac{D_3}{\theta_3}\left(1+e^{\theta_3 t_1}\right)-D_3(t_1-T)\right]\right. + C_{h_3}\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] + C_{d_3}\theta_3\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] \left.+C_{s_3}D_3\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_3}D_3-C_{s_3}D_3(t_1-T)\right]$$

$$So\ \frac{d\widetilde{TAC}}{dT}d_F =\frac{1}{6}\left[-\frac{1}{T^2}\left[A_1+C_{p_1}\left[-\frac{D_1}{\theta_1}\left(1+e^{\theta_1 t_1}\right)-D_1(t_1-T)\right]\right.\right. + C_{h_1}\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] + C_{d_1}\theta_1\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] \left.+C_{s_1}D_1\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_1}D_1-C_{s_1}D_1(t_1-T)\right] + 4\left\{-\frac{1}{T^2}\left[A_2+C_{p_2}\left[-\frac{D_2}{\theta_2}\left(1+e^{\theta_2 t_1}\right)-D_2(t_1-T)\right]\right.\right. + C_{h_2}\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] + C_{d_2}\theta_2\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] \left.\left.+C_{s_2}D_2\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_2}D_2-C_{s_2}D_2(t_1-T)\right]\right\} + \left\{-\frac{1}{T^2}\left[A_3+C_{p_3}\left[-\frac{D_3}{\theta_3}\left(1+e^{\theta_3 t_1}\right)-D_3(t_1-T)\right]\right.\right. + C_{h_3}\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] + C_{d_3}\theta_3\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] \left.\left.+C_{s_3}D_3\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]+\frac{1}{T}\left[C_{p_3}D_3-C_{s_3}D_3(t_1-T)\right]\right\}$$

And

$$\frac{d^2d_F\widetilde{TAC}}{dT^2}=\frac{1}{6}\left(\frac{d^2}{dT^2}TAC_1+4\frac{d^2}{dT^2}TAC_2+\frac{d^2}{dT^2}TAC_3\right)$$

where,

$$\frac{d^2TAC_1}{dT^2} =\frac{2}{T^3}\left[A_1+C_{p_1}\left[-\frac{D_1}{\theta_1}\left(1+e^{\theta_1 t_1}\right)-D_1(t_1-T)\right]\right. + C_{h_1}\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] + C_{d_1}\theta_1\left[-\frac{D_1}{\theta_1}t_1+\left(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\right)(e^{-\theta_1 t_1}-1)\right] \left.+C_{s_1}D_1\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]-\frac{2}{T^2}\left[C_{p_1}D_1-C_{s_1}D_1(t_1-T)\right] +\frac{1}{T}C_{s_1}D_1$$

$$\frac{d^2TAC_2}{dT^2} =\frac{2}{T^3}\left[A_2+C_{p_2}\left[-\frac{D_2}{\theta_2}\left(1+e^{\theta_2 t_1}\right)-D_2(t_1-T)\right]\right. + C_{h_2}\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] + C_{d_2}\theta_2\left[-\frac{D_2}{\theta_2}t_1+\left(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\right)(e^{-\theta_2 t_1}-1)\right] \left.+C_{s_2}D_2\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]-\frac{2}{T^2}\left[C_{p_2}D_2-C_{s_2}D_2(t_1-T)\right] +\frac{1}{T}C_{s_2}D_2$$

$$and\ \frac{d^2TAC_3}{dT^2} =\frac{2}{T^3}\left[A_3+C_{p_3}\left[-\frac{D_3}{\theta_3}\left(1+e^{\theta_3 t_1}\right)-D_3(t_1-T)\right]\right. + C_{h_3}\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] + C_{d_3}\theta_3\left[-\frac{D_3}{\theta_3}t_1+\left(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\right)(e^{-\theta_3 t_1}-1)\right] \left.+C_{s_3}D_3\left[\frac{t_1^2}{2}-\left(t_1T-\frac{T^2}{2}\right)\right]\right]-\frac{2}{T^2}\left[C_{p_3}D_3-C_{s_3}D_3(t_1-T)\right] +\frac{1}{T}C_{s_3}D_3$$

$$So\ \frac{d^2 d_F\widetilde{TAC}}{dT^2}$$
$$= \frac{1}{6}\Bigg\{\frac{2}{T^3}\Bigg[A_1 + C_{p_1}\Bigg[-\frac{D_1}{\theta_1}\Big(1+e^{\theta_1 t_1}\Big) - D_1(t_1-T)\Bigg] + C_{h_1}\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{d_1}\theta_1\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{s_1}D_1\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] - \frac{2}{T^2}\Bigg[C_{p_1}D_1 - C_{s_1}D_1(t_1-T)\Bigg] + \frac{1}{T}C_{s_1}D_1 + 4\Bigg[\frac{2}{T^3}\Bigg[A_2 + C_{p_2}\Bigg[-\frac{D_2}{\theta_2}\Big(1+e^{\theta_2 t_1}\Big) - D_2(t_1-T)\Bigg] + C_{h_2}\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{d_2}\theta_2\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{s_2}D_2\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] - \frac{2}{T^2}\Bigg[C_{p_2}D_2 - C_{s_2}D_2(t_1-T)\Bigg] + \frac{1}{T}C_{s_2}D_2\Bigg] + \frac{2}{T^3}\Bigg[A_3 + C_{p_3}\Bigg[-\frac{D_3}{\theta_3}\Big(1+e^{\theta_3 t_1}\Big) - D_3(t_1-T)\Bigg] + C_{h_3}\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] + C_{d_3}\theta_3\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] + C_{s_3}D_3\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] - \frac{2}{T^2}\Bigg[C_{p_3}D_3 - C_{s_3}D_3(t_1-T)\Bigg] + \frac{1}{T}C_{s_3}D_3\Bigg\}$$

Now $\frac{d}{dT}d_F\widetilde{TAC} = 0$

$$\implies \Big(C_{p_1}D_1 + 4C_{p_2}D_2 + C_{p_3}D_3\Big)T - \Big(2C_{s_1}D_1t_1 + 8C_{s_2}D_2t_1 + 2C_{s_3}D_3t_1\Big)T + \frac{3}{2}\Big(C_{s_1}D_1 + 4C_{s_2}D_2 + C_{s_3}D_3\Big)T^2 + C_{p_1}\Bigg[-\frac{D_1}{\theta_1}\Big(1+e^{\theta_1 t_1}\Big) - D_1(t_1-T)\Bigg] + 4C_{p_2}\Bigg[-\frac{D_2}{\theta_2}\Big(1+e^{\theta_2 t_2}\Big) - D_2(t_2-T)\Bigg] + C_{p_3}\Bigg[-D_3(t_1-T) - \frac{D_3}{\theta_3}\Big(1+e^{\theta_{13} t_1}\Big)\Bigg] = A_1 + 4A_2 + A_3 + C_{h_1}\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + 4C_{h_2}\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{h_3}\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] + C_{d_1}\theta_1\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + 4C_{d_2}\theta_2\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{d_3}\theta_3\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] - C_{s_1}D_1t_1 - 4C_{s_2}D_2t_1 - C_{s_3}D_3t_1$$

2. Signed Distance method

The deduzzified value is :

$$d_F\widetilde{TAC} = \frac{1}{4}(TAC_1 + 2TAC_2 + TAC_3)$$

$$\frac{dd_F\widetilde{TAC}}{dT} = \frac{1}{4}\Big(\frac{d}{dT}TAC_1 + 2\frac{d}{dT}TAC_2 + \frac{d}{dT}TAC_3\Big)$$

$$\frac{d}{dT}d_F\widetilde{TAC}$$
$$= \frac{1}{4}\Bigg[-\frac{1}{T^2}\Bigg[A_1 + C_{p_1}\Bigg[-\frac{D_1}{\theta_1}\Big(1+e^{\theta_1 t_1}\Big) - D_1(t_1-T)\Bigg] + C_{h_1}\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{d_1}\theta_1\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{s_1}D_1\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_{p_1}D_1 - C_{s_1}D_1(t_1-T)\Bigg] + 2\Bigg\{-\frac{1}{T^2}\Bigg[A_2 + C_{p_2}\Bigg[-\frac{D_2}{\theta_2}\Big(1+e^{\theta_2 t_1}\Big) - D_2(t_1-T)\Bigg] + C_{h_2}\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{d_2}\theta_2\Bigg[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1}-1)\Bigg] + C_{s_2}D_2\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_{p_2}D_2 - C_{s_2}D_2(t_1-T)\Bigg]\Bigg\} + \Bigg\{-\frac{1}{T^2}\Bigg[A_3 + C_{p_3}\Bigg[-\frac{D_3}{\theta_3}\Big(1+e^{\theta_3 t_1}\Big) - D_3(t_1-T)\Bigg] + C_{h_3}\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] + C_{d_3}\theta_3\Bigg[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1}-1)\Bigg] + C_{s_3}D_3\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_{p_3}D_3 - C_{s_3}D_3(t_1-T)\Bigg]\Bigg\}$$

And

$$\frac{d^2 d_F\widetilde{TAC}}{dT^2} = \frac{1}{4}\Big(\frac{d^2TAC_1}{dT^2} + 2\frac{d^2TAC_2}{dT^2} + \frac{d^2TAC_3}{dT^2}\Big)$$

$$\frac{d^2 d_F\widetilde{TAC}}{dT^2}$$
$$= \frac{1}{4}\Bigg\{\frac{2}{T^3}\Bigg[A_1 + C_{p_1}\Bigg[-\frac{D_1}{\theta_1}\Big(1+e^{\theta_1 t_1}\Big) - D_1(t_1-T)\Bigg] + C_{h_1}\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{d_1}\theta_1\Bigg[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1}-1)\Bigg] + C_{s_1}D_1\Bigg[\frac{t_1^2}{2} - \Big(t_1 T - \frac{T^2}{2}\Big)\Bigg]\Bigg] - \frac{2}{T^2}\Bigg[C_{p_1}D_1 - C_{s_1}D_1(t_1-T)\Bigg] + \frac{1}{T}C_{s_1}D_1 + 2\Bigg[\frac{2}{T^3}\Bigg[A_2 + C_{p_2}\Bigg[-\frac{D_2}{\theta_2}\Big(1+e^{\theta_2 t_1}\Big) - D_2(t_1-T)\Bigg]$$

$$+C_{h_2}\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{d_2}\theta_2\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{s_2}D_2\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]-\frac{2}{T^2}\bigg[C_{p_2}D_2-C_{s_2}D_2(t_1-T)$$
$$+\frac{1}{T}C_{s_2}D_2\bigg]+\frac{2}{T^3}\bigg[A_3+C_{p_3}\bigg[-\frac{D_3}{\theta_3}\bigg(1+e^{\theta_3 t_1}\bigg)-D_3(t_1-T)\bigg]$$
$$+C_{h_3}\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{d_3}\theta_3\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{s_3}D_3\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]-\frac{2}{T^2}\bigg[C_{p_3}D_3-C_{s_3}D_3(t_1-T)$$
$$+\frac{1}{T}C_{s_3}D_3\bigg\}$$

Now $\frac{d}{dT}d_F\widetilde{TAC}=0$

$$\implies\bigg(C_{p_1}D_1+2C_{p_2}D_2+C_{p_3}D_3\bigg)T-\bigg(2C_{s_1}D_1t_1$$
$$+4C_{s_2}D_2t_1+2C_{s_3}D_3t_1\bigg)T+\frac{3}{2}\bigg(C_{s_1}D_1+2C_{s_2}D_2$$
$$+C_{s_3}D_3\bigg)T^2+C_{p_1}\bigg[-\frac{D_1}{\theta_1}\bigg(1+e^{\theta_1 t_1}\bigg)-D_1(t_1-T)\bigg]$$
$$+2C_{p_2}\bigg[-\frac{D_2}{\theta_2}\bigg(1+e^{\theta_2 t_2}\bigg)-D_2(t_2-T)\bigg]+C_{p_3}\bigg[-D_3(t_1-T)$$
$$-\frac{D_3}{\theta_3}\bigg(1+e^{\theta_{13} t_1}\bigg)\bigg]=A_1+2A_2+A_3$$
$$+C_{h_1}\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+2C_{h_2}\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{h_3}\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{d_1}\theta_1\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+2C_{d_2}\theta_2\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{d_3}\theta_3\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$-C_{s_1}D_1t_1-2C_{s_2}D_2t_1-C_{s_3}D_3t_1$$

3. By Centroid Method

The deduzzified value is :

$$d_F\widetilde{TAC}=\frac{1}{3}(TAC_1+TAC_2+TAC_3)$$
$$\frac{d}{dT}d_F\widetilde{TAC}=\frac{1}{3}\bigg(\frac{d}{dT}TAC_1+\frac{d}{dT}TAC_2+\frac{d}{dT}TAC_3\bigg)$$

$$\frac{d}{dT}d_F\widetilde{TAC}$$
$$=\frac{1}{3}\bigg[-\frac{1}{T^2}\bigg[A_1+C_{p_1}\bigg[-\frac{D_1}{\theta_1}\bigg(1+e^{\theta_1 t_1}\bigg)-D_1(t_1-T)\bigg]$$
$$+C_{h_1}\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+C_{d_1}\theta_1\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+C_{s_1}D_1\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]+\frac{1}{T}\bigg[C_{p_1}D_1-C_{s_1}D_1(t_1-T)\bigg]$$
$$+\bigg\{-\frac{1}{T^2}\bigg[A_2+C_{p_2}\bigg[-\frac{D_2}{\theta_2}\bigg(1+e^{\theta_2 t_1}\bigg)-D_2(t_1-T)\bigg]$$
$$+C_{h_2}\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{d_2}\theta_2\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{s_2}D_2\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]+\frac{1}{T}\bigg[C_{p_2}D_2-C_{s_2}D_2(t_1-T)\bigg]\bigg\}$$
$$+\bigg\{-\frac{1}{T^2}\bigg[A_3+C_{p_3}\bigg[-\frac{D_3}{\theta_3}\bigg(1+e^{\theta_3 t_1}\bigg)-D_3(t_1-T)\bigg]$$
$$+C_{h_3}\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{d_3}\theta_3\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{s_3}D_3\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]+\frac{1}{T}\bigg[C_{p_3}D_3-C_{s_3}D_3(t_1-T)\bigg]\bigg\}$$

And

$$\frac{d^2d_F\widetilde{TAC}}{dT^2}=\frac{1}{3}\bigg(\frac{d^2}{dT^2}TAC_1+\frac{d^2}{dT^2}TAC_2+\frac{d^2}{dT^2}TAC_3\bigg)$$

$$\frac{d^2}{dT^2}d_F\widetilde{TAC}$$
$$=\frac{1}{3}\bigg\{\frac{2}{T^3}\bigg[A_1+C_{p_1}\bigg[-\frac{D_1}{\theta_1}\bigg(1+e^{\theta_1 t_1}\bigg)-D_1(t_1-T)\bigg]$$
$$+C_{h_1}\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+C_{d_1}\theta_1\bigg[-\frac{D_1}{\theta_1}t_1+\bigg(\frac{D_1}{\theta_1^2}-\frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\bigg)(e^{-\theta_1 t_1}-1)\bigg]$$
$$+C_{s_1}D_1\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]-\frac{2}{T^2}\bigg[C_{p_1}D_1-C_{s_1}D_1(t_1-T)$$
$$+\frac{1}{T}C_{s_1}D_1+\bigg[\frac{2}{T^3}\bigg[A_2+C_{p_2}\bigg[-\frac{D_2}{\theta_2}\bigg(1+e^{\theta_2 t_1}\bigg)-D_2(t_1-T)\bigg]$$
$$+C_{h_2}\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{d_2}\theta_2\bigg[-\frac{D_2}{\theta_2}t_1+\bigg(\frac{D_2}{\theta_2^2}-\frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\bigg)(e^{-\theta_2 t_1}-1)\bigg]$$
$$+C_{s_2}D_2\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]-\frac{2}{T^2}\bigg[C_{p_2}D_2-C_{s_2}D_2(t_1-T)$$
$$+\frac{1}{T}C_{s_2}D_2\bigg]+\frac{2}{T^3}\bigg[A_3+C_{p_3}\bigg[-\frac{D_3}{\theta_3}\bigg(1+e^{\theta_3 t_1}\bigg)-D_3(t_1-T)\bigg]$$
$$+C_{h_3}\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{d_3}\theta_3\bigg[-\frac{D_3}{\theta_3}t_1+\bigg(\frac{D_3}{\theta_3^2}-\frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\bigg)(e^{-\theta_3 t_1}-1)\bigg]$$
$$+C_{s_3}D_3\bigg[\frac{t_1^2}{2}-\bigg(t_1T-\frac{T^2}{2}\bigg)\bigg]\bigg]\bigg]-\frac{2}{T^2}\bigg[C_{p_3}D_3-C_{s_3}D_3(t_1-T)$$
$$+\frac{1}{T}C_{s_3}D_3\bigg\}$$

Now $\frac{d}{dT} d_F \widetilde{TAC} = 0$

$$\implies \Big(C_{p_1}D_1 + C_{p_2}D_2 + C_{p_3}D_3\Big)T - \Big(2C_{s_1}D_1t_1 + 2C_{s_2}D_2t_1 + 2C_{s_3}D_3t_1\Big)T + \frac{3}{2}\Big(C_{s_1}D_1 + C_{s_2}D_2 + C_{s_3}D_3\Big)T^2 + C_{p_1}\Big[-\frac{D_1}{\theta_1}\Big(1+e^{\theta_1 t_1}\Big) - D_1(t_1 - T)\Big] + C_{p_2}\Big[-\frac{D_2}{\theta_2}\Big(1+e^{\theta_2 t_2}\Big) - D_2(t_2 - T)\Big] + C_{p_3}\Big[-D_3(t_1 - T) - \frac{D_3}{\theta_3}\Big(1+e^{\theta_{13} t_1}\Big)\Big] = A_1 + A_2 + A_3$$

$$+ C_{h_1}\Big[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1} - 1)\Big]$$
$$+ C_{h_2}\Big[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1} - 1)\Big]$$
$$+ C_{h_3}\Big[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1} - 1)\Big]$$
$$+ C_{d_1}\theta_1\Big[-\frac{D_1}{\theta_1}t_1 + \Big(\frac{D_1}{\theta_1^2} - \frac{D_1}{\theta_1^2}(1+e^{\theta_1 t_1})\Big)(e^{-\theta_1 t_1} - 1)\Big]$$
$$+ C_{d_2}\theta_2\Big[-\frac{D_2}{\theta_2}t_1 + \Big(\frac{D_2}{\theta_2^2} - \frac{D_2}{\theta_2^2}(1+e^{\theta_2 t_1})\Big)(e^{-\theta_2 t_1} - 1)\Big]$$
$$+ C_{d_3}\theta_3\Big[-\frac{D_3}{\theta_3}t_1 + \Big(\frac{D_3}{\theta_3^2} - \frac{D_3}{\theta_3^2}(1+e^{\theta_3 t_1})\Big)(e^{-\theta_3 t_1} - 1)\Big]$$
$$- C_{s_1}D_1t_1 - C_{s_2}D_2t_1 - C_{s_3}D_3t_1$$

Let $\frac{\widetilde{D}}{\widetilde{P}} < 1$ i.e. $\frac{\widetilde{D_1}}{P_1} < 1$, $\frac{\widetilde{D_2}}{P_2} < 1$, $\frac{\widetilde{D_3}}{P_3} < 1$,
then in each case T exists and clearly for this value of T, we see that $\frac{d^2}{dT^2} d_F \widetilde{TAC} > 0$.
Hence defuzzified value of total cost per unit time.
i.e. $d_F\widetilde{TAC}$ is minimum. And minimum value is obtained by substitute the value T in $d_F\widetilde{TAC}$ for the respective cases.

VII. Solution Procedure

A algorithm is developed and coded in MATLAB to solve the proposed model. The proposed algorithm looks like this.
Algorithm :
Step-1 : Start
Step-2 : Set fuzzy variables: $\tilde{A}$, $\tilde{C_p}$, $\tilde{C_h}$, $\tilde{C_d}$, $\tilde{C_s}$, $\tilde{\theta}$, $\tilde{D}$
Step-3 : For T∈ [0,1] (Cycle length)
Step-4 : Evaluate Total Average cost: TAC
Step-5 : Set TAC_1,TAC_2,TAC_3
Step-6 : Evaluate: TAC_{gm},TAC_{sd},TAC_{cen}
Step-7 : Find the optimal cost : min$\{TAC_{gm}\}$, min$\{TAC_{sd}\}$,min$\{TAC_{cen}\}$
Step-8 : End loop

VIII. Result and Discussion

To perform a numerical and graphical analysis of the model, the following numerical values of the parameters in the proper units are taken into consideration.

$\tilde{A} = (50, 56, 62)$, $\tilde{C_p} = (48, 52, 56)$, $\tilde{C_h} = (6, 9, 12)$, $\tilde{C_d} = (1.2, 1.6, 2.0)$, $\tilde{C_s} = (1.12, 1.18, 1.24)$, $\tilde{\theta} = (0.006, 0.011, 0.016)$, $\mu = 10$, $\tilde{D} = (450, 510, 570)$
The output of the given model using MATLAB software under different defuzzification method are given below $TAC_{gm} = 172.5328$, T = 0.6800; $TAC_{sd} = 154.7554$, T = 0.7500; and $TAC_{cen} = 136.3928$, T = 0.8500.

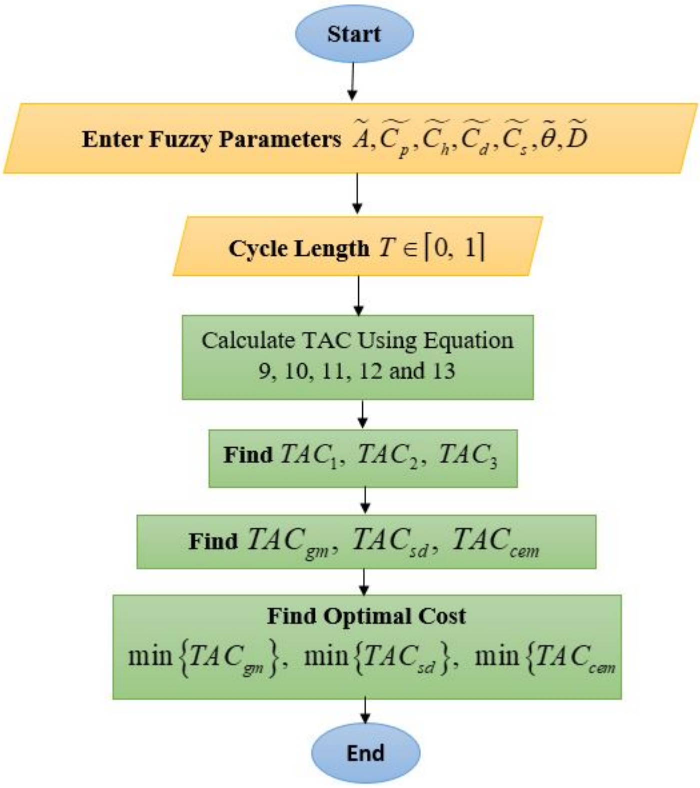

Fig. 3. Step of solution procedure

IX. Sestivity Analysis

The above result illustrates that, when the centroid method of defuzzification is applied, the total cost is minimum with a corresponding value of T. On the other hand, when the graded mean integration value is used, T is the maximum and has a corresponding total cost. The cost functions are plotted against T in the following figure.

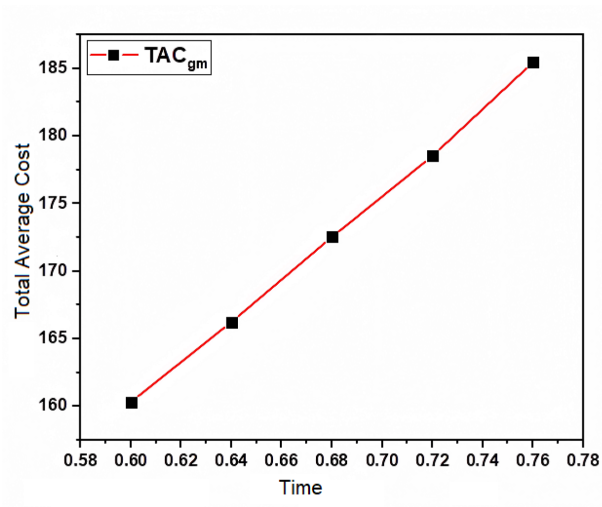

Fig. 4. Sensitivite Analysis on $\tilde{A}$ Using GMIR method: Time vs. Total avarge cost

TABLE II
SENSITIVITY ON $\tilde{A}$

% Change	$\tilde{A}$	Graded Mean Method		Signed Distance Method		Centroid Method	
		TAC_{gm}	Time (yrs)	TAC_{sd}	Time (yrs)	TAC_{cen}	Time (yrs)
+20 %	(60,67.2,74.4)	185.4627	0.7600	165.2675	0.8300	148.2565	0.9300
+10%	(55,61.6,68.2)	178.5368	0.7200	158.6358	0.7900	142.4682	0.8900
0 %	(50,56,62)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(45,50.4,55.8)	166.2461	0.6400	148.4230	0.7200	128.6421	0.8100
-20 %	(40,44.8,49.6)	160.3204	0.6000	139.1057	0.6800	122.2489	0.7700

TABLE III
SENSITIVITY ON $\tilde{C_p}$

% Change	$\tilde{C_p}$	Graded Mean Method		Signed Distance Method		Centroid Method	
		TAC_{gm}	Time (yrs)	TAC_{sd}	Time (yrs)	TAC_{cen}	Time (yrs)
+20 %	(57.6,62.4,67.2)	177.2203	0.7600	162.5536	0.8300	144.5462	0.9300
+10%	(52.8,57.2,61.6)	174.3708	0.7200	157.4321	0.7900	140.5405	0.8900
0 %	(48,52,56)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(43.2,46.8,50.4)	169.3402	0.6400	151.2502	0.7200	131.9752	0.8100
-20 %	(38.4,41.6,44.8)	167.5423	0.6000	146.4623	0.6800	126.4561	0.7700

TABLE IV
SENSITIVITY ON $\tilde{C_h}$

% Change	$\tilde{C_h}$	Graded Mean Method		Signed Distance Method		Centroid Method	
		TAC_{gm}	Time (yrs)	TAC_{sd}	Time (yrs)	TAC_{cen}	Time (yrs)
+20 %	(7.2,10.8,14.4)	188.5421	0.6200	171.4652	0.6900	149.6792	0.7800
+10%	(6.6,9.9,13.2)	180.4536	0.6500	163.4521	0.7200	143.5479	0.8200
0 %	(6,9,12)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(5.4,8.1,10.8)	164.1054	0.7100	146.4687	0.7900	129.5482	0.9000
-20 %	(4.8,7.2,9.6)	156.4652	0.7400	139.4657	0.8400	122.5794	0.9500

TABLE V
SENSITIVITY ON $\tilde{C_d}$

% Change	$\tilde{C_d}$	Graded Mean Method		Signed Distance Method		Centroid Method	
		TAC_{gm}	Time (yrs)	TAC_{sd}	Time (yrs)	TAC_{cen}	Time (yrs)
+20 %	(1.44,1.92,2.4)	172.5551	0.6800	154.7714	0.7500	136.3956	0.8500
+10%	(1.32,1.76,2.2)	172.5436	0.6800	154.7634	0.7500	136.3942	0.8500
0 %	(1.2,1.6,2.0)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(1.08,1.44,1.8)	172.5214	0.6800	154.7474	0.7500	136.3914	0.8500
-20 %	(0.96,1.28,1.6)	172.5101	0.6800	154.7394	0.7500	136.3900	0.8500

TABLE VI
SENSITIVITY ON $\tilde{C}_s$

% Change	$\tilde{C}_s$	Graded Mean Method TAC_{gm}	Graded Mean Method Time (yrs)	Signed Distance Method TAC_{sd}	Signed Distance Method Time (yrs)	Centroid Method TAC_{cen}	Centroid Method Time (yrs)
+20 %	(1.34,1.40,1.48)	172.5428	0.6800	154.7634	0.7500	136.3936	0.8500
+10%	(1.23,1.29,1.36)	172.5378	0.6800	154.7594	0.7500	136.3932	0.8500
0 %	(1.12,1.18,1.24)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(1.01,1.07,1.12)	172.5278	0.6800	154.7514	0.7500	136.3924	0.8500
-20 %	(0.90,0.96,1.0)	172.5228	0.6800	154.7374	0.7500	136.3920	0.8500

TABLE VII
SENSITIVITY ON $\tilde{\theta}$

% Change	$\tilde{\theta}$	Graded Mean Method TAC_{gm}	Graded Mean Method Time (yrs)	Signed Distance Method TAC_{sd}	Signed Distance Method Time (yrs)	Centroid Method TAC_{cen}	Centroid Method Time (yrs)
+20 %	(0.0072,0.0132,0.0192)	172.5556	0.6800	154.7694	0.7500	136.3954	0.8500
+10%	(0.0066,0.0121,0.0176)	172.5442	0.6800	154.7524	0.7500	136.3941	0.8500
0 %	(0.006,0.011,0.016)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(0.0054,0.0099,0.0144)	172.5213	0.6800	154.7485	0.7500	136.3913	0.8500
-20 %	(0.0048,0.0088,0.0128)	172.5098	0.6800	154.7415	0.7500	136.3898	0.8500

TABLE VIII
SENSITIVITY ON $\tilde{D}$

% Change	$\tilde{D}$	Graded Mean Method TAC_{gm}	Graded Mean Method Time (yrs)	Signed Distance Method TAC_{sd}	Signed Distance Method Time (yrs)	Centroid Method TAC_{cen}	Centroid Method Time (yrs)
+20 %	(540, 612, 684)	-256.4568	1.0000	-286.0238	1.0000	-342.4658	1.0000
+10%	(495, 561, 627)	-6.4592	1.0000	-45.7568	1.0000	-67.7849	1.0000
0 %	(450, 510, 570)	172.5328	0.6800	154.7554	0.7500	136.3928	0.8500
-10%	(405, 459, 513)	247.4583	0.4700	245.7458	0.4900	236.3789	0.5100
-20 %	(360, 408, 456)	295.7952	0.3900	285.4589	1.4100	295.7459	0.4200

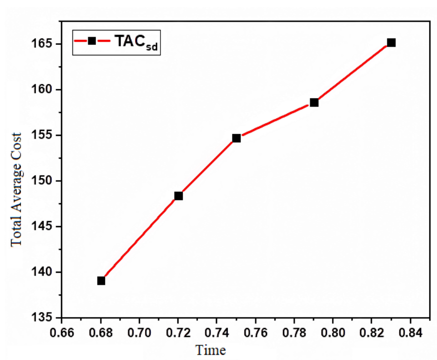

Fig. 5. Sensitivity Analysis of $\tilde{A}$ Using SDM Method: Time vs. Total Average Cost

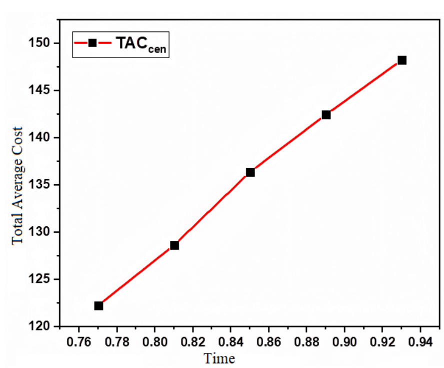

Fig. 6. Sensitivite Analysis on $\tilde{A}$ Using CEM method: Time vs. Total avarge cost

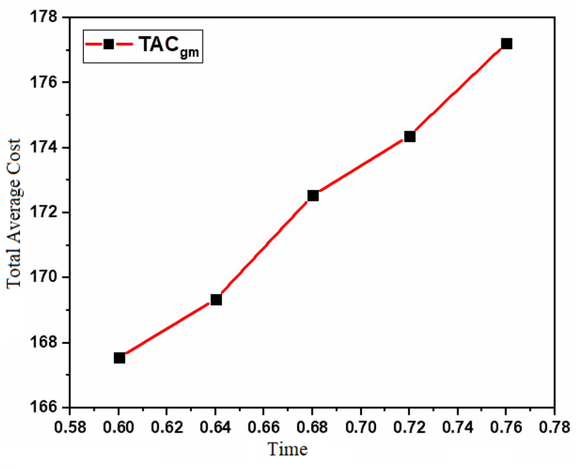

Fig. 7. Sensitivite Analysis on $\tilde{C_p}$ Using GMIR method: Time vs. Total avarge cost

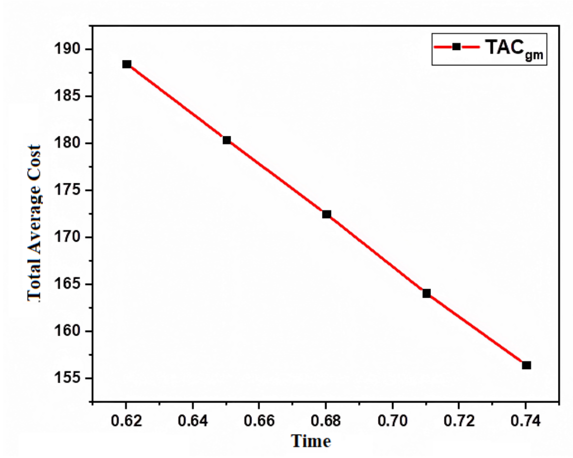

Fig. 10. Sensitivite Analysis on $\tilde{C_h}$ Using GMIR method: Time vs. Total avarge cost

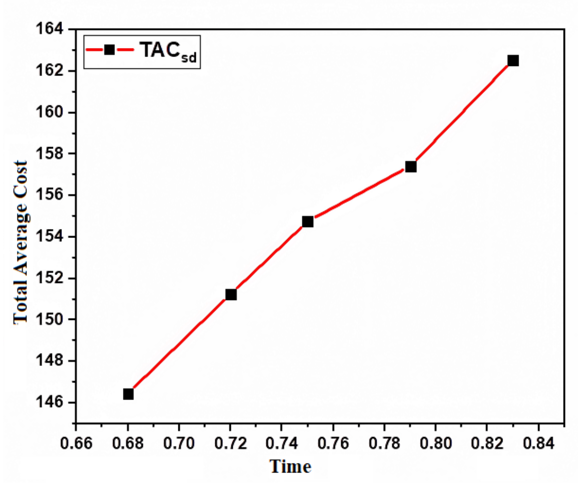

Fig. 8. Sensitivite Analysis on $\tilde{C_p}$ Using SDM method: Between Time vs. Total avarge cost

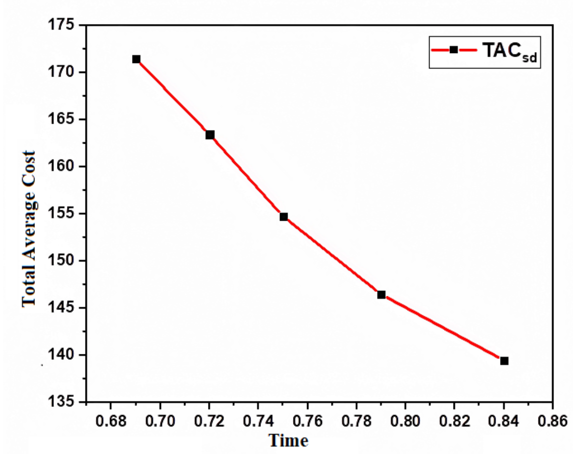

Fig. 11. Sensitivite Analysis on $\tilde{C_h}$ Using SDM method: Time vs. Total avarge cost

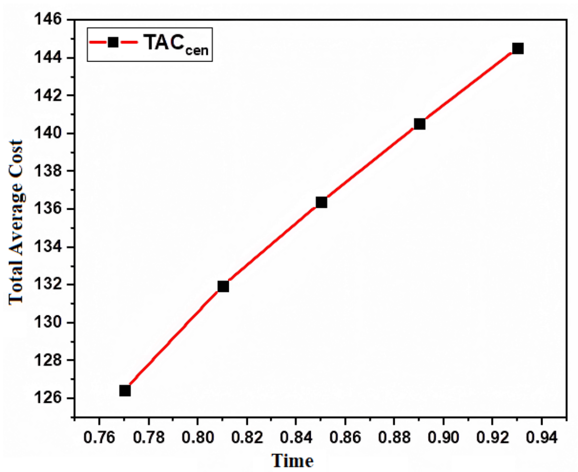

Fig. 9. Sensitivite Analysis on $\tilde{C_p}$ Using CEM method: Time vs. Total avarge cost

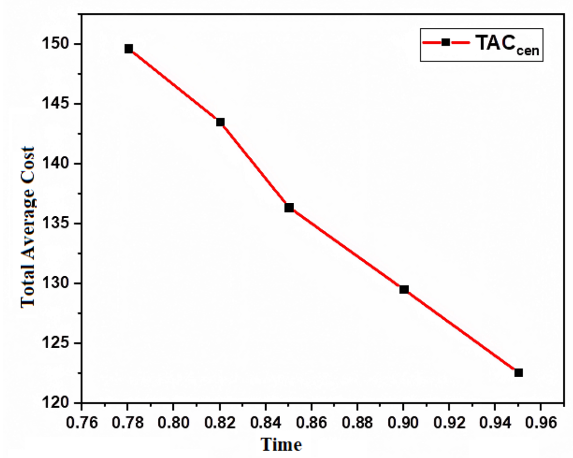

Fig. 12. Sensitivite Analysis on $\tilde{C_h}$ Using CEM method: Time vs. Total avarge cost

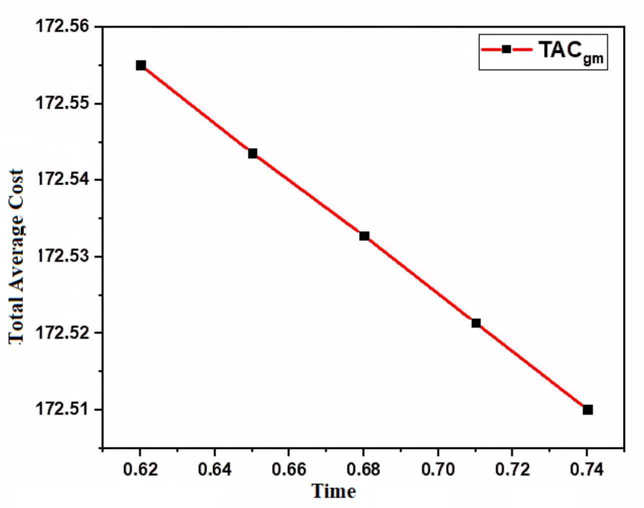

Fig. 13. Sensitivite Analysis on $\tilde{C}_d$ Using GMIR method: Time vs. Total avarge cost

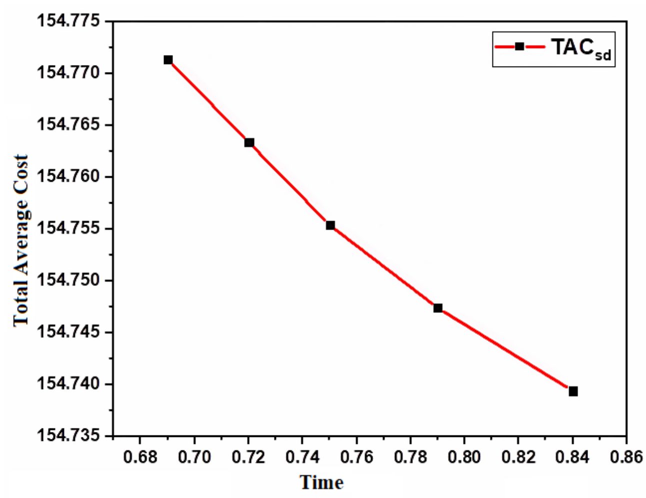

Fig. 14. Sensitivite Analysis on $\tilde{C}_d$ Using SDM method: Time vs. Total avarge cost

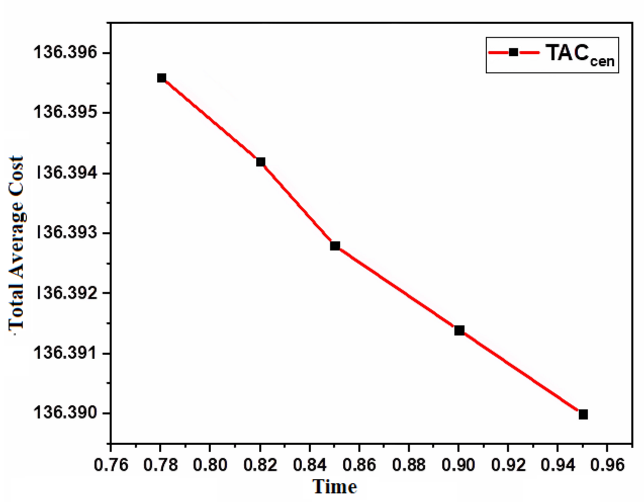

Fig. 15. Sensitivite Analysis on $\tilde{C}_d$ Using CEM method: Time vs. Total avarge cost

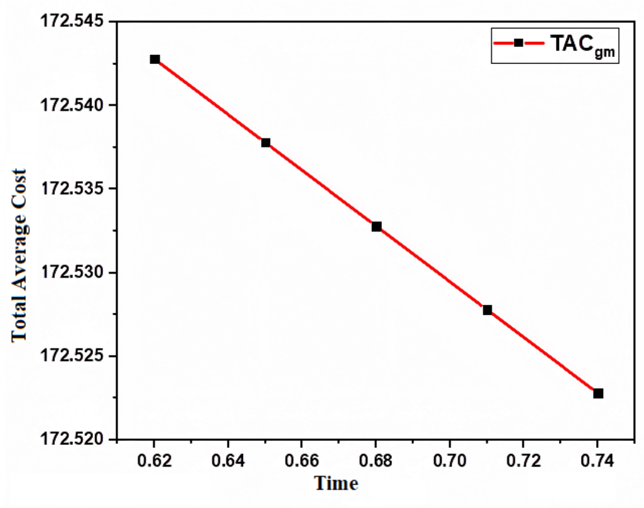

Fig. 16. Sensitivite Analysis on $\tilde{C}_s$ Using GMIR method: Time vs. Total avarge cost

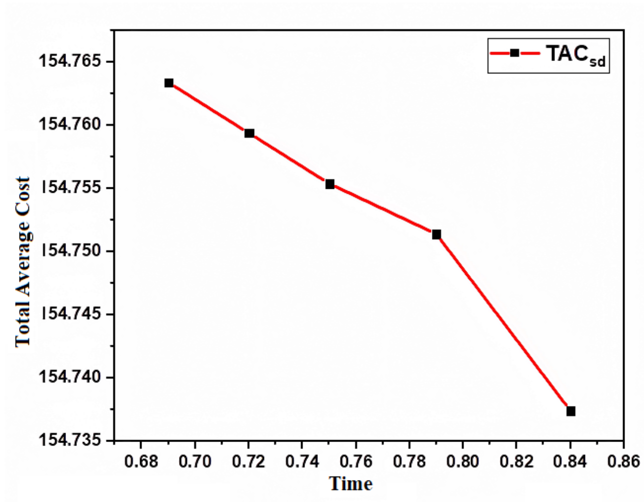

Fig. 17. Sensitivite Analysis on $\tilde{C}_s$ Using SDM method: Time vs. Total avarge cost

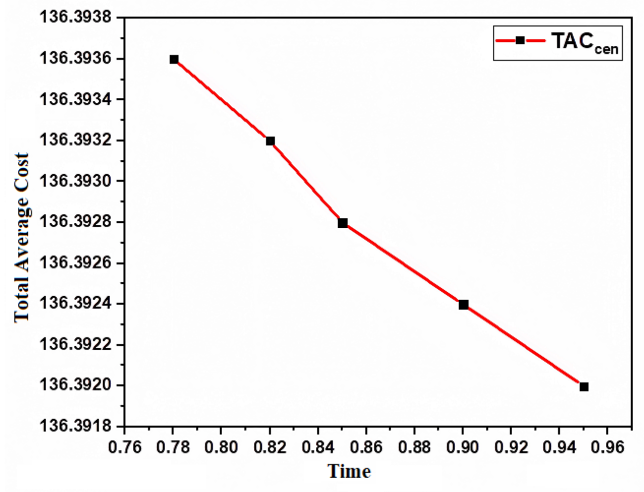

Fig. 18. Sensitivite Analysis on $\tilde{C}_s$ Using CEM method: Time vs. Total avarge cost

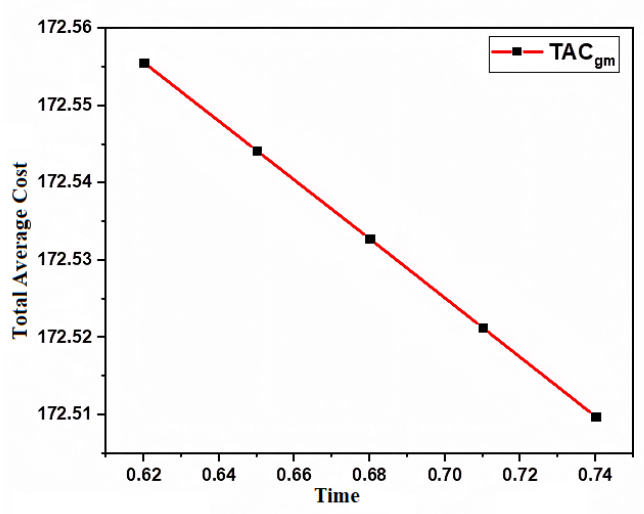

Fig. 19. Sensitivite Analysis on $\tilde{\theta}$ Using GMIR method: Time vs. Total avarge cost

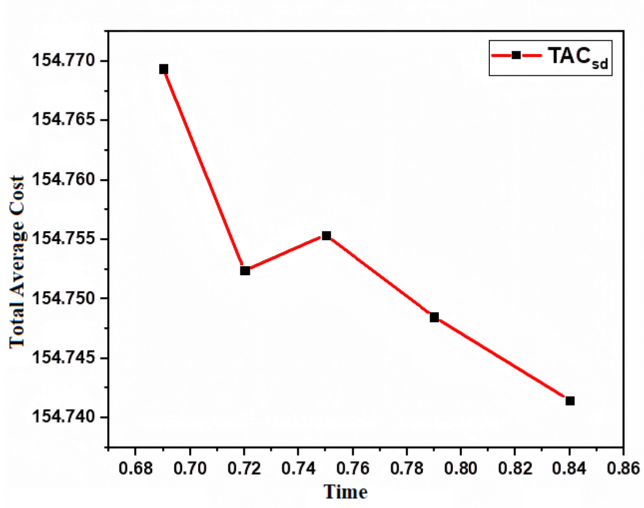

Fig. 20. Sensitivite Analysis on $\tilde{\theta}$ Using SDM method: Time vs. Total avarge cost

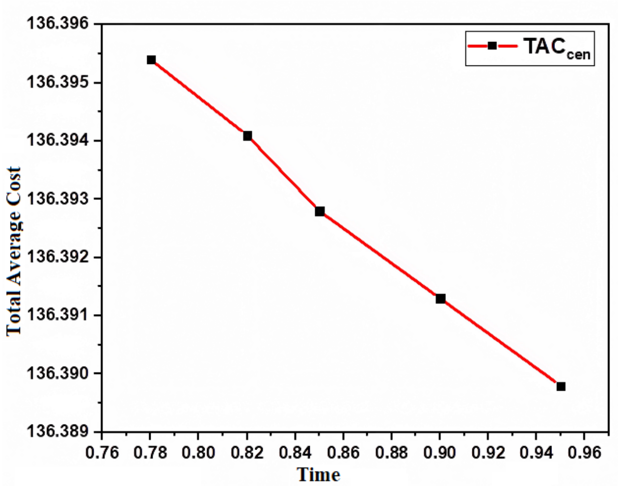

Fig. 21. Sensitivite Analysis on $\tilde{\theta}$ Using CEM method: Time vs. Total avarge cost

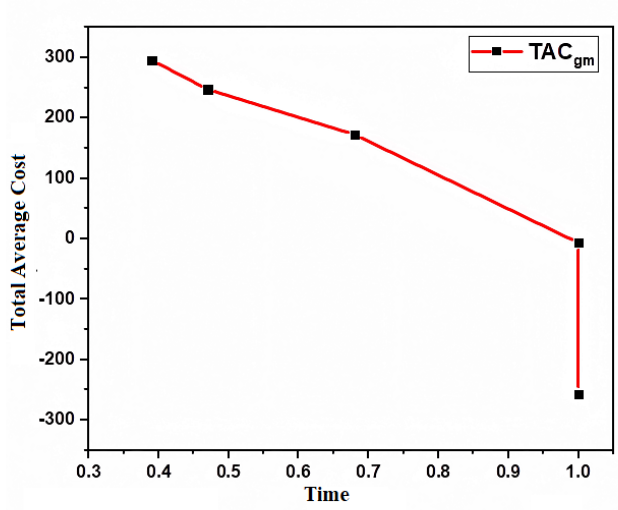

Fig. 22. Sensitivite Analysis on $\tilde{D}$ Using GMIR method : Time vs. Total avarge cost

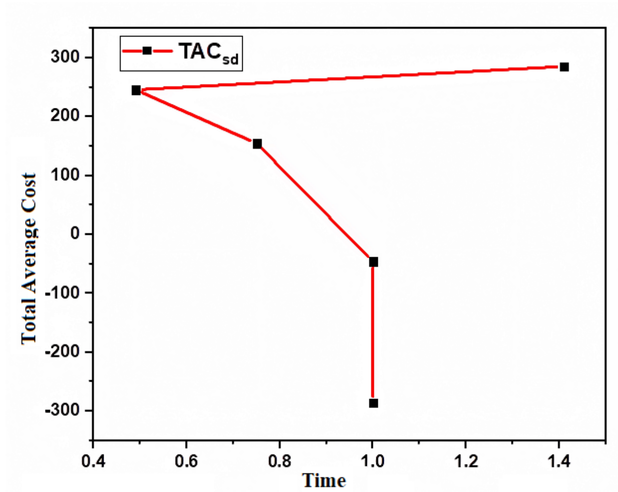

Fig. 23. Sensitivite Analysis on $\tilde{D}$ Using SDM method : Time vs. Total avarge cost

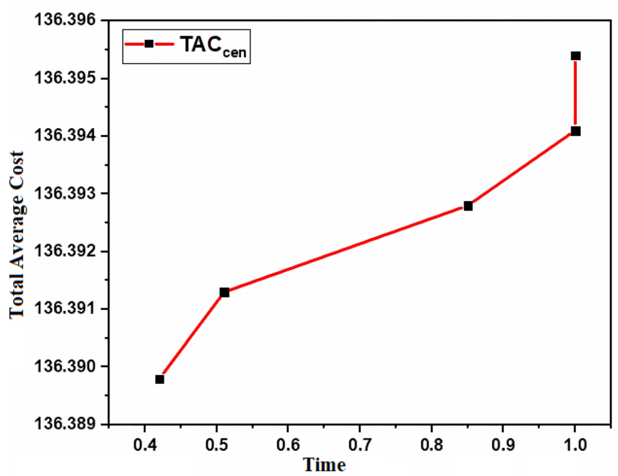

Fig. 24. Sensitivite Analysis on $\tilde{D}$ Using CEM method : Time vs. Total avarge cost

X. Conclusion

The aim of this paper is to determine an optimal inventory policy in a fuzzy environment by analyzing a probabilistic inventory model with a time-independent deterioration rate, complete backlog, and probabilistic uncertain demand. The paper further develops corresponding models in a fuzzy environment using triangular fuzzy numbers to represent fuzzy parameters. Three defuzzification methods—namely the Centroid method, the Graded Mean Integration method, and the Signed Distance method—are employed to handle the fuzziness in the model. The primary objective of the model is to minimize total costs. Numerical examples are provided to illustrate the proposed model, and the convexity of the cost function is examined. Additionally, the sensitivity of the model to changes in parameter values is investigated through sensitivity analysis. Both the models and defuzzified results are solved, and the sensitivity results are presented graphically and in tabular form. The graphical representations further enhance our understanding of the scenario.

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