On T-Stability in Generalised Cone Metric Spaces

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***Abstract*—This study delves into the realm of generalized cone metric spaces to explore fixed points, their existence, and their uniqueness under the condition of T-stability, focusing on Zamfirescu contractions. By extending the scope of fixed point theory to encompass a broader class of spaces and contractions, we not only establish a theoretical foundation for the existence and uniqueness of such fixed points but also apply our findings to the realm of integral equations, demonstrating their practical utility. The relevance of our theoretical insights is further underscored through the application to solving specific classes of integral equations, highlighting the intersection between abstract mathematical theory and practical problem-solving. To substantiate our theoretical assertions, we provide carefully selected examples that validate the theorem's applicability, illustrating the robustness and relevance of our findings within both the mathematical and applied contexts, thereby offering new perspectives and methodologies for tackling integral equations through fixed point theory.**

***Index Terms*—Picard iteration, T-stable, generalized cone metric space and fixed point theorem.**

I. Introduction and Preliminaries

Since S. Banach introduced the Contraction Principle in his 1922 PhD thesis [1], hundreds of researchers have sought to generalize or refine it, focusing on either broadening the contractive conditions or expanding the concept beyond metric spaces. This led to notable enhancements like Kannan [2], Chatterjea [3], and Zamfirescu [4] results, among others, and the exploration of spaces such as semi-metric and b-metric spaces. However, not all these endeavors proved practically useful, with some generalizations merely echoing existing results.

A $T : X \to X$ self-mapping function in the cone metric space (X, d) will be examined here. Moreover, suppose that the set of fixed points of T is $F_T = \{x \in X : Tx = x\}$. The Picard iteration process $\{x_n\}$, is defined in complete metric space by

$$x_{n+1} = Tx_n, n = 0, 1, 2, \ldots \quad (1)$$

We have used this approach to estimate the fixed points of contractive mappings.

$$d(Tx, Ty) \leq \alpha d(x, y), \forall x, y \in E, \alpha \in [0, 1) \quad (2)$$

by several authors throughout the years. The contraction condition 2 mentioned above is referred to as Banach's contraction condition.

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We will now generalise 1 and overview portions of the iteration process:

Let $x_0 \in E$. The sequence $\{x_n\}_{n=0}^{\infty}$ is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, n = 0, 1, 2 \ldots . \quad (3)$$

is the iteration process known as The Mann, where $\{\alpha_n\}_{n=0}^{\infty} \subset [0, 1]$. The sequence $\{x_n\}_{n=0}^{\infty}$ is defined as for every $x_0 \in E$.

$$\left.\begin{array}{ll} u_{n+1} = & (1-\alpha_n)u_n + \alpha_n Tv_n \\ v_n = & (1-\beta_n)u_n + \beta_n Tu_n \end{array}\right\} n = 0, 1, \ldots \quad (4)$$

is the iteration process known as the Ishikawa iteration, where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0, 1]$. Kannan [2] used the contractive concept below to extend the fixed point theorem of Banach:

There exists $\beta \in (0, \frac{1}{2})$ for a self map T in a way that

$$d(Tx, Ty) \leq \beta[d(x, Tx) + d(y, Ty)], \forall x, y \in E. \quad (5)$$

Chatterjee et al. (2011) established the subsequent contractual stipulation.

There exists $\gamma \in (0, \frac{1}{2})$ for a selfmap T in a way that

$$d(Tx, Ty) \leq \gamma[d(x, Ty) + d(y, Tx)], \forall x, y \in E \quad (6)$$

Based on Banach's fixed point theorem, Zamfirescu [4] expanded it by including equations 2, 5 and 6. Given a mapping $T : E \to E$, there are real numbers α, β, γ such that

$$0 \leq \gamma < \frac{1}{2}, 0 \leq \beta < \frac{1}{2}, 0 \leq \alpha < 1,$$

and for each $x, y \in E$, at least one of the following conditions holds:

(i) $d(Tx, Ty) \leq \alpha d(x, y)$
(ii) $d(Tx, Ty) \leq \beta[d(x, Tx) + d(y, Ty)]$
(iii) $d(Tx, Ty) \leq \gamma[d(x, Ty) + d(y, Tx)]$

A Zamfirescu operator is a mapping $T : E \to E$ that satisfies all three of the above conditions. Kannan mappings are those that satisfy condition (ii), whereas Chatterjea operators are those that satisfy condition (iii). For references to the stability and T-Stability outcomes of the Picard iteration under contractive conditions, see [5]–[15] and the citations therein.

In order to broaden the concept of metric space, Huang and Zhang [16] first proposed the concept of cone metric space. They swapped over the set of real numbers in the metric space for an ordered Banach space. Furthermore, for mappings that satisfy different kinds of contractive conditions, they proved several fixed point theorems in this space and related spcaes. The references [17]–[20], [36]–[38] and citations therein offer insights into cone metric space.

Subsequently, in the context of cone metric space, Rezapour and Halbarani [21] omitted the normality assumption. Following that, other articles in cone metric space emerged [22]–[26], [33]–[35].

Consider E as a real Banach Space. A subset P of E is defined as a cone if it satisfies the following condition:

(i) P is a closed set that is non-empty and not equal to the zero set.
(ii) Let $a, b \in \mathbb{R}$ such that $a, b \geq 0$, and for $x, y \in P$, it follows that $ax + by \in P$.
(iii) $P \cap (-P) = 0$.

Let P represent a cone that is contained within E. The partial ordering $\leq$ on E with respect to P is defined as follows: for each x and y in E, $x \leq y$ if and only if $y - x$ belongs to P. The notation $x < y$ is used to indicate that x is less than y, but not equal to y. On the other hand, $x << y$ represents that the difference $y - x$ belongs to the interior of the set P. There are two types of cone. There are cones that are considered normal and cones that are considered non-normal. A cone P contained in the vector space E is said to be normal if there exists a positive integer K such that for every x and y in P, if $0 \leq x \leq y$, then $\| x \| \leq K \| y \|$. Put simply, if $x_n \leq y_n \leq z_n$ and the limits as n approaches infinity of x_n and z_n are both equal to x, then the limit as n approaches infinity of y_n is also equal to x. If any limited expanding sequence inside a regular cone $P \subset E$ converges, then P is regular.

Definition 1. *[16] Let X be a nonempty set. Suppose the mapping $d : X \times X \to E$ satisfies the following conditions:*

(i) $0 < d(u, v)$ *for all* $u, v \in X$ *and* $d(u, v) = 0$ *iff* $u = v$.
(ii) $d(u, v) = d(v, u)$ *for all* $u, v \in X$.
(iii) $d(u, v) \leq d(u, w) + d(w, v)$ *for all* $u, v, w \in X$.

In this context, d is defined as a cone metric on the set X, and the pair (X, d) is referred to as a cone metric space.

Definition 2. *[16] Consider a cone metric space (X, d) where $x \in X$ and $\{x_n\}_{n \geq 1}$ is a sequence in X. Then,*

(i) *for any $c \in E$ where $0 << c$, there exists a natural integer N such that $d(x_n, x) << c$ for every $n \geq N$, $\{x_n\}_{n \geq 1}$ converges to x. It is denoted by $\lim_{n \to \infty} x_n = x$ or $x_n \to x$.*
(ii) *there is a natural number N such that for all $n, m \geq N$, $d(x_n, x_m) << c$ for every $c \in E$ with $0 << c$. Then the sequence $\{x_n\}_{n \geq 1}$ is a Cauchy sequence.*
(iii) *If every Cauchy sequence converges, then (X, d) is the complete cone metric space.*

Definition 3. *[16] Consider be a cone metric space $(X, d))$ that X contains $\{x_n\}$ and P is a normal cone with normal constant K. Finally, for every $n \to \infty$, the sequence $\{x_n\}$ will converge to x if and only if $d(x_n, x) \to 0$ or $\| d(x_n, x) \| \to 0$.*

Let us recall that a mapping T on metric space (X, d) is called a Kannan [10] contraction if there exists $\alpha \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq \alpha d(x, Tx) + \alpha d(y, Ty)$$

for all $x, y \in X$.
In the year 2006, Huang and Zhang [16] proved cone version of Kannan contraction as:

Theorem 4. *Consider a complete cone metric space (X, d) and let P be a normal cone with a normal constant K. If the contractive condition is satisfied, then the mapping $T : X \to X$ follows:*

$$d(Tx, Ty) \leq k[d(Tx, x) + d(Ty, y)], \quad for \quad all \quad x, y \in X, \tag{7}$$

as long as $k \in [0, \frac{1}{2})$. Then there is only one fixed point in X for T. For any $x \in X$, the iterative process $\{T^n x\}$ will reach the fixed point.

In 1972, Chatterjeacite [3] got a similar result by looking at a constant $\lambda \in [0, \frac{1}{2})$ and a mapping $T : X \to X$ such that

$$d(Tx, Ty) \leq \lambda[d(x, Ty) + d(y, Tx)]$$

Also, in the year 2006, Huang and Zhang [16] proved cone version of Chatterjea [3] contraction as:

Theorem 5. *Consider a complete cone metric space (X, d) and let P be a normal cone with a normal constant K. Assume the mapping $T : X \to X$ adheres to the contractive condition:*

$$d(Tx, Ty) \leq k[d(Tx, y) + d(Ty, x)], \quad for \quad all \quad x, y \in X, \tag{8}$$

as long as $k \in [0, \frac{1}{2})$. Then there is only one fixed point in X for T. For any $x \in X$, the iterative process $\{T^n x\}$ will reach the fixed point.

II. Definitions and Remarks

The recognition that cone metric spaces are metrizable has led to an assumption that fixed point results in these spaces might simply parallel those in traditional metric spaces [27], [28]. However, a critical examination reveals a research gap: the influence of the specific cone used on fixed point theorems is not universally accounted for. This oversight suggests that despite their metrizability, the unique characteristics of cone metric spaces—and how these characteristics interact with the underlying cones—demand a more tailored analysis. This gap underscores the need for further research to understand how the properties of different cones affect fixed point results, moving beyond the assumption of direct equivalence to standard metric counterparts.
This study introduces a novel contribution by proposing the T-stability of Picard's iteration fixed point in generalized cone metric spaces, expanding the understanding of stability within fixed point theory. Moreover, the practical application of the proposed study has been exemplified. By applying the theoretical findings to real-world scenarios or specific mathematical problems, we have showcased the relevance and usefulness of our research. This demonstration not only validates the theoretical framework but also illustrates its potential impact in solving practical problems or addressing challenges in various fields. This study investigates fixed points in generalized cone metric spaces with a focus on T-stability and Zamfirescu contractions, broadening fixed point theory's scope and applying it to integral equations to showcase its practical relevance. Through selected examples, we demonstrate the theory's applicability, merging abstract mathematics with real-world problem solving, and offering new methods for addressing integral equations.
Let X be a nonempty set and let $D : X \times X \to E$ be a given mapping. For every $x \in X$, let us define the set

$$C(D, X, x) = \left\{ \{x_n\} \subset X : \lim_{n \to \infty} D(x_n, x) = 0 \right\}$$

Definition 6. *D is defined as a generalized cone metric on X if it satisfies the subsequent criteria:*

(i) *For all $(x, y) \in X \times X$, it follows that $D(x, y) = 0$ implies $x = y$.*
(ii) *For all pairs $(x, y) \in X \times X$, it holds that $D(x, y) = D(y, x)$.*
(iii) *For any $(x, y) \in X \times X$ and $\{x_n\} \in C(D, X, x)$, the inequality $D(x, y) \leq C \lim_{n\to\infty} \sup D(x_n, y)$ holds. This inequality is defined by the existence of a real constant $C > 0$.*

The structure denoted as (X, D) is referred to as a generalised cone metric space.

Remark 7. *(X, D) is a generalized cone metric space if and only if (i) and (ii) of definition 6 are satisfied, indicating that for any $x \in X$, the set $C(D, X, x)$ is empty.*

Definition 8. *Let (X, D) be a generalised cone metric space, let $\{x_n\}$ be a sequence in X and let $x \in X$. We say that $\{x_n\}$ is $D-$converges to x in X if $\{x_n\} \in C(D, X, x)$.*

Remark 9. *For every $n \in N$, let $\{x_n\}$ be the sequence where $x_n = x$. It is true that $D(x, x) = 0$ if it D-converges to x.*

Definition 10. *Consider (X, D) as a generalized cone metric space. A sequence $\{x_n\}$ in X is defined as a D-Cauchy sequence if*

$$\lim_{m,n\to\infty} D(x_n, x_m, x) = 0.$$

The space (X, D) is termed D-complete if every Cauchy sequence in X is D-convergent to an element in X.

Definition 11. *Consider (X, D) as a generalized cone metric space, and let $\{x_n\}$ denote a sequence in the set X. We define a sequence $\{x_n\}$ as a $D-$Cauchy sequence in the space X if*

$$\lim_{m,n\to\infty} D(x_n, x_m, x) = 0.$$

Proposition 12. *The set $C(D, X, x)$ is non-empty if and only if the condition $D(x, x) = 0$ holds true.*

Proof: If $C(D, X, x) \neq \phi$, then a sequence $\{x_n\} \subset X$ can be determined such that

$$\lim_{n\to\infty} D(x_n, x) = 0.$$

Using property 3, We acquire

$$D(x, x) \leq C \lim_{n\to\infty} sup D(x_n, x),$$

and therefore, $D(x, x) = 0$. Assume that $D(x, x) = 0$.For every $n \in N$, the sequence $\{x_n\} \subset X$ converging to x is defined as $x_n = x$, which ends the proof. ■

The focus of this work is on the cone version contraction types of Kannan and Chatterjea [2], [3]. We prove certain fixed point findings in the recently released generalized cone metric spaces. In order to demonstrate the usefulness of the outcomes, we also provide a few instances.

III. Fixed Point Theorems

Proposition 13. *Consider the generalized cone metric space (X, D) and the mapping $T : X \to X$ that satisfies inequality 7 for some $\lambda \in [0, \frac{1}{2})$. Then any fixed $u \in X$ corresponds to T and satisfies*

$$D(u, u) < \infty \Rightarrow D(u, u) = 0.$$

Proof: Take a fixed point of T to be $u \in X$ for which $D(u, u) < \infty$. Utilizing 1, we derive

$$\begin{aligned} D(u, u) &= D(Tu, Tu) \\ &= \lambda(D(u, Tu) + D(u, Tu)) \\ &= 2\lambda D(u, u) \end{aligned}$$

Since $2\lambda \in [0, 1)$, we have $D(u, u) = 0$.

For every $x \in X$, we define

$$\delta(D, T, x) = sup\{D(T^i x, T^j x) : i, j \in N\}.$$

Theorem 14. *Let (X, D) be a complete generalised cone metric space and let T be a self-mapping on X satisfying 7 for some constant $\lambda \in [0, \frac{1}{2}$ such that $C\lambda < 1$. If there exists an element $x_0 \in X$ such that $\delta(D, T, x_0) < \infty$, whenever $\{T^n\}$ converges to $u \in X$. Additionally, u is a fixed point of T only if $D(u, Tu)$*
$infty$. In addition, we have $u = u^{'}$ for any fixed point $u^{'}$ of T in X where $D(u^{'}, u^{'})$
$infty$.

Proof: Consider the set N where $n \geq 1$. We may say that for every $i, j \in N$,

$$\begin{aligned} D(T^{n+i}x_0, T^{n+j}x_0) \leq \; & \lambda[D(T^{n+i-1})x_0, T^{n+i}x_0 \\ & + D(T^{n+j-1})x_0, T^{n+j}x_0] \end{aligned}$$

and then

$$D(T^{n+i}x_0, T^{n+j}x_0) \leq 2\lambda\delta(D, T, T^{n-1}x_0),$$

which gives

$$\delta(D, T, T^n x_0) \leq 2\lambda\delta(D, T, T^{n-1}x_0).$$

Consequently, we obtain

$$\delta(D, T, T^n x_0) \leq (2\lambda)^n \delta(D, T, x_0)$$

and

$$D(T^n x_0, T^m x_0) \leq \delta(D, T, T^n x_0) \leq (2\lambda)^n \delta(D, T, x_0) \tag{9}$$

for every integer m where $m > n$. Given that

$$\delta(D, T, x_0) < \infty$$

and $2\lambda \in [0, 1)$, we derive

$$\lim_{m,n\to\infty} D(T^n x_0, T^m x_0) = 0.$$

Consequently, the sequence $\{T^n x_0\}$ is regarded as a D-Cauchy sequence, implying the existence of an element $u \in X$ such that

$$\lim_{n\to\infty} D(T^n x_0, u) = 0$$

and

$$D(Tu, u) \leq C \lim_{n\to\infty} sup D(Tu, T^{n+1}x_0 \tag{10}$$

By 1, we have

$$\begin{aligned} D(T^{n+1}x_0, Tu) \leq \; & \lambda(D(T^{n+1}x_0, \\ & T^n x_0) + D(u, fu)) \end{aligned} \tag{11}$$

From references 3 and 5, we derive

$$\lim_{n\to\infty} D(Tw, T^{n+1}x_0) \leq \lambda D(u, Tu).$$

Using 4, We obtain

$$D(u, Tu) \leq C\lambda D(u, Tu).$$

Since $C\lambda < 1$ and $D(u, Tu) < \infty$, we deduce that $D(u, Tu) = 0$, which implies that $Tu = u$. If $u^{'}$ is any fixed point of T such that $D(u^{'}, u^{'}) < \infty$, we obtain

$$\begin{aligned} D(u, u^{'}) &= D(Tu, Tu^{'}) \\ &\leq \lambda(D(Tu, u) + D(Tu^{'}, u^{'})) \\ &\leq \lambda(D(u, u) + D(u^{'}, u^{'})) \\ &\leq 0 \end{aligned}$$

which implies $u^{'} = u$. ■

Example 15. *Consider the set $X = [0, 1]$ and the set $E = [0, \infty)$. Define the mapping $D : X \times X \to E$ as follows:*

$$\begin{aligned} D(u, v) &= u + v, \ if \ u \neq 0 \ and \ v \neq 0 \\ D(0, u) = D(u, 0) &= \frac{u}{2}, \ for \ all \ u \in X. \end{aligned}$$

Conditions (i) and (ii) of definition 6 are satisfied without difficulty. According to proposition 13, it is necessary to verify condition (iii) of definition 6 solely for elements x in X where $D(u, u) = 0$, indicating that $u = 0$. Suppose $\{u_n\} \subset X$ is a sequence such that

$$\lim_{n\to\infty} D(u_n, 0) = 0$$

is the case. Assuming $n \in N$ and $v \in X$, the following holds:

$$\begin{aligned} D(u_n, v) &= u_n + v, \ if \ u_n \neq 0 \\ D(u_n, v) &= \frac{v}{2}, \ if \ u_n = 0. \end{aligned}$$

Then

$$\frac{v}{2} \leq D(u_n, v),$$

this means that

$$D(0, v) = \frac{v}{2} \leq \lim_{n\to\infty} sup D(u_n, v).$$

It follows that (X, D) is not a standard metric space but rather a generalized cone metric space as the triangle inequality is not true: We may say that if $u, v \in X - \{0\}$, then

$$D(u, v) = u + v$$

and

$$D(u, 0) + D(0, v) = \frac{u + v}{2},$$

and thus

$$D(u, v) > D(u, 0) + D(0, v).$$

Be aware that (X, D) is D-complete. Establish the mapping f on X via

$$f(u) = \frac{u}{u + 2} \quad for \quad all \quad u \in X$$

This holds true for every $u \in X$:

$$D(f(u), f(0)) = D(\frac{u}{u + 2}, 0) = \frac{u}{2(u + 2)}$$

and

$$\begin{aligned} D(f(u), u) + D(0, T(0)) &= D(\frac{u}{u + 2}, u) + D(0, 0) \\ &= \frac{u}{u + 2} + u. \end{aligned}$$

Then

$$D(f(u), f(0)) \leq \frac{1}{3}(D(f(u), u) + D(0, f(0))).$$

We find that for any $u, v \in X - \{0\}$,

$$\begin{aligned} D(f(u), f(v)) &= D(\frac{u}{u + 2}, \frac{v}{v + 2}) \\ &= \frac{u}{2(u + 2)} + \frac{v}{2(v + 2)} \end{aligned}$$

and

$$\begin{aligned} D(f(u), u) + D(v, f(v)) &= D(\frac{u}{u + 2}, u) \\ &\quad + D(v, \frac{v}{v + 2}) \\ &= \frac{u}{u + 2} + \frac{v}{v + 2} \\ &\quad + u + v. \end{aligned}$$

Then

$$D(f(u), f(v)) \leq \frac{1}{3}[D(f(u), u) + D(v, f(v))].$$

Everything stated in Theorem 13 is true. Because D is bounded and $f(0) = 0$, we may deduce that f has only one fixed point.

Example 16. *Let $X = \{p, q, r\}$ and define f on X by $f(p) = p, f(q) = q$ and $f(r) = p$. There is no metric for which f is a Kannan contraction on X. We define $D : X \times X \to [0, +\infty]$ by*

$$D(p, p) = D(q, q) = 0 \quad and \quad D(q, q) = +\infty;$$

$$D(p, q) = D(q, p) = 1;$$

$$D(p, r) = D(r, p) = 2;$$

$$D(q, r) = D(r, q) = 3$$

Then (X, D) is a complete generalised cone metric space, f is a Kannan contraction on (X, D) for any $\lambda \in]0, \frac{1}{2}[$ and we can apply theorem 14.

Lemma 17. *Assuming that $0 \leq \lambda < 1$, for any real number λ and for any sequence of positive real numbers $\{b_n\}$ such that $\lim_{n\to\infty} b_n = 0$. Hence, we have $\lim_{n\to\infty} a_n = 0$ for every positive integer sequence $\{a_n\}$ that satisfies*

$$a_{n+1} \leq \lambda a_n + b_n \quad for \quad all \quad n \in N.$$

Theorem 18. *Assume that (X, D) is a complete generalized cone metric space with $\lambda \in [0, \frac{1}{2})$, and that T is a self-map on X such that*

$$D(Tx, Ty) \leq \lambda(D(y, Tx) + D(x, Ty)) \qquad (12)$$

for every x, y. The $\{T^n x_0\}$ sequence converges to $u \in X$ if and only if there is a point $x_0 \in X$ such that $\delta(D, T, x_0) < \infty$. In addition, if the distance between x_0, Tu is finite, then u is a fixed point in T. For any fixed point $u^{'}$ in T where the distance between u and $u^{'}$ is finite, we may deduce that $u = u^{'}$.

Proof: Then, $n \in N$, where $n \geq 1$. Given any two numbers i, j, we can deduce

$$\begin{aligned} D(T^{n+i}x_0, T^{n+j}x_0) &\leq \lambda[D(T^{n+i}x_0, T^{n+j-1}x_0) \\ &\quad + D(T^{n+i-1}x_0, T^{n+j}x_0)] \end{aligned}$$

which implies that

$$D(T^{n+i}x_0, T^{n+j}x_0) \leq 2\lambda\delta(D, T, T^{n-1}x_0)$$

Hence

$$\delta(D, T, T^n x_0) \leq 2\lambda\delta(D, T, T^{n-1}x_0),$$

as a result;

$$\delta(D, T, T^n x_0) \leq (2\lambda)^n \delta(D, T, x_0)$$

This inequality implies that

$$D(T^n x_0, T^m x_0) \leq \delta(D, T, T^n x_0) \leq (2\lambda)^n \delta(D, T, x_0)$$

in the case when $m > n$ and n and m are integers. We may deduce that $\delta(D, T, x_0) < \infty$ and $2\lambda \in [0, 1)$ mean that

$$\lim_{m,n\to\infty} D(T^n x_0, T^m x_0) = 0$$

The D-Cauchy sequence $\{T^n x_0\}$ follows, and as a result, there exists $u \in X$ such that

$$\lim_{n\to\infty} D(T^n x_0, u) = 0$$

By 3 we have

$$\begin{aligned} D(T^n x_0, u) &\leq C \lim_{n\to\infty} sup D(T^n x_0, T^m x_0) \\ &\leq (2\lambda)^n C\delta(D, T, x_0) \\ &\leq C\delta(D, T, x_0) \end{aligned}$$

Then

$$D(T^n x_0, u) < \infty \quad for \;\; all \;\; n \in N$$

By 6 we have

$$D(T^{n+1}x_0, Tu) \leq \lambda[D(T^{n+1}x_0, u) + D(T^n x_0, Tu)]$$

Given that

$$D(x_0, Tu) < \infty,$$

it follows that $D(T^n x_0, Tu) < \infty$ for all $n \in \mathbb{N}$. According to lemma 17, we derive

$$\lim_{n\to\infty} D(T^n x_0, Tu) = 0$$

It follows that $Tu = u$. Let $u^{'}$ denote any fixed point in X. We have

$$\begin{aligned} D(u, u^{'}) &= D(Tu, Tu^{'}) \\ &\leq \lambda(D(Tu, u^{'}) + D(Tu^{'}, u)) \\ &\leq \lambda(D(u, u^{'}) + D(u^{'}, u)) \\ &\leq 2\lambda D(u, u^{'}). \end{aligned}$$

Since $D(u, u^{'}) < \infty$, we obtain $D(u, u^{'}) = 0$ which concludes the proof. ■

Example 19. *Consider $X = [0, 1], E = [0, \infty)$ be defined by*

$$D(u, 1) = D(1, u) = \infty \quad for \;\; all \;\; x \in [0, 1]$$

$$D(u, v) = u + v \quad if \;\; u \neq 1 \;\; and \;\; v \neq 1.$$

It is trivial to demonstrate that (X, D) is a D-complete generalized cone metric space with $C = 1$. Let the function $f : [0, 1]. \to [0, 1]$ given by

$$f(u) = \frac{1}{2}u \quad if \;\; u \in [0, 1[,$$

$$f(1) = 1.$$

The function f satisfies 6 with $\lambda = \frac{1}{3}$ in (X, D). The theorem 18 states that f has a fixed point.

Definition 20. *Consider a generalized cone metric space (X, D). A self-mapping f on X is known as a Hardy-Roger contraction if there exist non-negative real constants. λ_i for $i = 1, 2, 3, 4, 5$ where $\lambda = \Sigma_{i=1}^{i=5}\lambda_i \in [0, 1]$ and*

$$\begin{aligned} D(fx, fy) &\leq \lambda_1 D(x, y) + \lambda_2 D(x, fx) \\ &\quad + \lambda_3 D(y, fy) + \lambda_4 D(y, fx) \\ &\quad + \lambda_5 D(x, fy) \end{aligned} \tag{13}$$

for every $x, y \in X$.

Proposition 21. *Consider a Hardy-Rogers contraction $f : X \to X$ and assume that (X, D) is a generalized cone metric space. Consequently, any fixed point $u \in X$ of f satisfies*

$$D(u, u) < \infty \Rightarrow D(u, u) = 0.$$

Proof: Consider a fixed point $u \in X$ of f such that $D(u, u) < \infty$. We have

$$\begin{aligned} D(u, u) &= D(fu, fu) \\ &\leq \lambda_1 D(u, u) + \lambda_2 D(u, fu) + \lambda_3 D(u, fu) \\ &\quad + \lambda_4 D(u, fu) + \lambda_5 D(u, fu) \\ &= \lambda D(u, u). \end{aligned}$$

Since $\lambda \in [0, 1[$, we have $D(u, u) = 0$. ■

IV. T-STABILITY OF PICARD'S ITERATION

This section outlines the process for iteration in generalised cone metric spaces. This is an attempt to elaborate on recent findings about T-stability. Consider a metric space (X, D) that is equipped with a generalised cone structure. Consider a series $\{T_n\}_n$ of self-maps of X such that the intersection of the sets $\{F(T_n)\}$ is not empty. Consider a point x_0 in the set X. Let us assume that the iteration technique $y_{n+1} = F(T_n, y_n)$ involves the sequence $\{T_n\}_n$ and generates a sequence $\{y_n\}$ of points from X.

Typically, one may get a series $\{z_n\}$ using the following method. Consider a point y_0 belonging to the set X. Assign the value of y_{n+1} to be equal to the function $f(T_n, y_n)$. Define y_0 to be equal to z_0. Currently, the value of y_1 is determined by the function f with inputs T_0 and y_0. As a result of rounding errors made when computing the function T_0 produces a new value z_1. that is nearly equal to y_0 may be obtained instead of $f(T_0, y_0)$. To estimate the value of z_1, we calculate $f(T_1, y_1)$ to get z_2, with respect to $f(T_1, z_1)$, which is approximated. The purpose of this calculation is as an approximation of $\{y_n\}$ in order to obtain $\{z_n\}$, and the result is stored for future use. Recent results on Stability and T-Stability under iterative procedure can be found in [30]–[32] and related sources.

Definition 22. *Define a generalized cone metric* (X, D) *and a self-mapping function* T *on* X. *We assert that*

(i) T *is continuous if*

$$\lim_{n\to\infty} D(x_n, x) = 0 \Rightarrow \lim_{n\to\infty} D(T(x_n), T(x)) = 0$$

for all $x \in X$;

(ii) *for every* $\{x_n\}$ *sequence where* $\{T(x_n)\}$ *converges,* $\{x_n\}$ *converges, we say that* T *is sequentially convergent;*

(iii) *for any* $\{x_n\}$ *sequence where* $\{T(x_n)\}$ *converges,* $\{x_n\}$ *has a convergent subsequence for* T *to be subsequentially convergent.*

Definition 23. *The iteration defined as* $\{T_n\}$ *is defined as* $v_{n+1} = F(T_n, v_n)$ *with regard to* $\{T_n\}$*-semistable (or semistable)as* $\{v_n\}$ *approaches a fixed point. If* $\{z_n\}$ *is a sequence in* X *with* $\lim_{n\to\infty}$ *and* q *in* $\bigcap_n F(T_n) \neq \phi$, *then if we have* $z_n \to z$ *for some sequence* $t_n \subset \mathbb{R}^+$ *and* $D(v_n, f(T_n, z_n)) = o(t_n)$, *then we may deduce that* $f(T_n, v_n)$ *is equal to zero.*

Definition 24. *If the sequence* $\{z_n\}$ *converges to a fixed point* q *in* $\bigcap_n F(T_n) \neq \emptyset$ *and*

$$\lim_{n\to\infty} D(v_n, f(T_n, z_n)) = 0,$$

then $z_n \to z$, *and the iteration* $v_{n+1} = F(T_n, v_n)$ *is considered* $\{T_n\}$*-stable or stable with respect to* $\{T_n\}$.

Remark 25. *The notion of T-stability states that* T_n *is equal to* T *for all values of* n.

Theorem 26. *Let* (X, D) *be a complete generalised cone metric space and* $T : X \to X$ *with* $F(T) \neq \phi$. *If there exists* $c \in (0, \frac{1}{2})$ *such that* $D(Tu, Tv) \leq cD(u, v)$, *whenever* $\{v_n\}$ *is a sequence with* $D(v_n, Tv_n) \to 0$ *as* $n \to \infty$, *and also for every* $u, v \in X$ *and* $x \in F(T)$, *Picard iteration is T-stable.*

Proof: Let $\{v_n\} \subseteq X, \epsilon_n = D(v_{n+1}, Tv_n)$ and $\epsilon_n \to 0$ as $n \to \infty$. Tthen we may say that for every $n \in \mathbb{N}$,

$$\begin{aligned} D(v_{n+1}, x) &\leq D(v_{n+1}, Tu_n) + D(Tu_n, x) \\ &\quad - D(Tu_n, Tu_n) \\ \Rightarrow D(v_{n+1}, x) &\leq D(Tv_{n+1}, T^{n+1}u_0) + D(T^{n+1}u_0, x) \\ &\quad - D(T^{n+1}u_0, T^{n+1}u_0) \\ \Rightarrow D(v_{n+1}, x) &\leq cD(v_n, T^n u_0) + D(T^{n+1}u_0, x) \\ \therefore \| D(v_{n+1}, x) \| &\leq Kc \| D(v_n, T^n u_0) \| \\ &\quad + \| D(T^{n+1}u_0, x) \| \to 0 \end{aligned}$$

Hence, $D(v_{n+1}, x) = 0$. But since

$$D(Tv_{n+1}, Tv_{n+1}) \leq cD(v_{n+1}, v_{n+1}) = 0.$$

We have that

$$D(Tv_{n+1}, Tv_{n+1}) = D(Tv_{n+1}, x) = D(x, x) = 0.$$

This shows that $Tv_n = x$. Therefore,

$$\lim_{n\to\infty} v_n = q.$$

For uniqueness: Let y be another fixed point of T, then

$$D(x, y) = D(Tx, Ty) \leq cD(x, y)$$

Since $c < 1$ we have $D(x, y) = D(x, x) = D(y, y)$. Hence $x = y$. Thus the fixed point of T is unique. ■

Theorem 27. *Let* (X, D) *be a complete generalised cone metric space and* $T : X \to X$ *and* $F(T) \neq \phi$. *For any* u, v *in X and* x *in F(T), the Picard iteration is* T*-stable if there is a* c *in the interval* $(0, \frac{1}{2})$ *such that*

$$D(Tu, Tv) \leq c(D(Tu, u) + D(Tv, v))$$

and moreover, for every $\{v_n\}$ *that is a sequence with* $D(v_n, Tv_n) \to 0$ *as* $n \to \infty$, *Picard iteration is* T*-stable.*

Proof: Let $\{v_n\} \subseteq X, \epsilon_n = D(v_{n+1}, Tv_n)$ and $\epsilon_n \to 0$ as $n \to \infty$. So, we may say that for every $n \in \mathbb{N}$,

$$\begin{aligned} D(v_{n+1}, x) &\leq D(Tv_{n+1}, Tu_n) + D(Tu_n, x) \\ &\quad - D(Tu_n, Tu_n) \\ &\leq c[D(Tv_{n+1}, u_n) + D(Tu_n, u_n)] \\ &\quad + D(u_{n+1}, x) \\ &\leq K\frac{1}{1-c}(c \| D(u_{n+1}, u_n) \| \\ &\quad + \| D(u_{n+1}, x) \|) \to 0. \end{aligned}$$

Hence, $D(Tv_{n+1}, x) = 0$. But since

$$\begin{aligned} D(Tv_{n+1}, Tv_{n+1}) &\leq c[D(Tv_{n+1}, u) + D(Tv_{n+1}, q)] \\ &= 2cD(Tu_n, x) = 0. \end{aligned}$$

We have that

$$D(Tv_{n+1}, Tv_{n+1}) = D(Tv_{n+1}, x) = D(x, x) = 0.$$

This shows that $Tv_{n+1} = x$. For uniqueness: Let y be another fixed point of T, then

$$D(x, y) = D(Tx, Ty) \leq c[D(Tx, y) + D(Ty, y)] = 0.$$

Hence $D(x, y) = D(x, x) = D(y, y) = 0$. We get $x = y$. Consequently, the fixed point of T is unique. ■

Theorem 28. *Consider* (X, D) *as a complete generalized cone metric space and let* $T : X \to X$ *such that* $F(T) \neq \phi$. *If there exists* $c \in (0, \frac{1}{2})$ *such that*

$$D(Tu, Tv) \leq c(D(u, Tv) + D(v, Tu))$$

for all $u, v \in X$ *and* $x \in F(T)$, *and additionally, if* $\{v_n\}$ *is a sequence with* $D(v_n, Ty_n) \to 0$ *as* $n \to \infty$, *then Picard iteration is* T*-stable.*

Proof: Let $\{v_n\} \subseteq X, \epsilon_n = D(v_{n+1}, Tv_n)$ and $\epsilon_n \to 0$ as $n \to \infty$. For any $n \in \mathbb{N}$, it follows that

$$\begin{aligned} D(v_{n+1}, x) &\leq D(Tv_{n+1}, Tu_n) + D(Tu_n, x) \\ &\quad - D(Tu_n, Tu_n) \\ &\leq c[D(Tv_{n+1}, Tu_n) + D(u_n, Tv_n)] \\ &\quad + D(u_{n+1}, x) \\ &\leq K\frac{1}{1-c}(c \| D(v_{n+1}, u_{n+1}) \| \\ &\quad + \| D(u_{n+1}, x) \|) \\ &\leq K\frac{1}{1-c}(c \| D(v_{n+1}, x) \| \\ &\quad + \| D(u_{n+1}, x) \|) \to 0. \end{aligned}$$

Hence, $D(Tv_{n+1}, x) = 0$. But since

$$\begin{aligned} D(Tv_{n+1}, Tv_{n+1}) &\leq c[D(Tv_{n+1}, x) + D(Tv_{n+1}, q)] \\ &= 2cD(Tu_n, x) = 0. \end{aligned}$$

We have that

$$D(Tv_{n+1}, Tv_{n+1}) = D(Tv_{n+1}, u) = D(x, x) = 0.$$

This implies that $Tv_{n+1} = x$.
For uniqueness: Let y be another fixed point of T, then

$$D(x, y) = D(Tx, Ty) \leq c[D(x, Ty) + D(y, Tx)] = 0$$

Hence $D(x, y) = D(x, x) = D(y, y) = 0$.
We get $x = y$. Consequently, T has an unique fixed point. ∎

Example 29. *Consider $X = [0, \infty)$ and let D be the generalized cone metric on X defined by*

$$D(x, y) = |u - v|.$$

Let a function $T : X \to X$ defined as follows:

$$Tu = \begin{cases} 1 & if u \in \{0, 1\} \cup [\frac{1}{2m+1}, \frac{1}{2m}) \\ m & if u \in [\frac{1}{2m}, \frac{1}{2m-1}), m \geq 1 \\ \frac{1}{m} & if u \in (m-1, n], m \geq 2 \end{cases}$$

Then $D(Tu, 1) \leq D(u, Tu)$ for each $u \in [0, \infty)$. If $Tu = 1$, Consequently, the inequality of Theorem 26 holds valid. If $u \in [\frac{1}{2m}, \frac{1}{2m-1}), n \geq 1$, then $Tu = m$ and

$$D(Tu, 1) = m - 1 \leq \frac{m-1}{2m-1} < m - u = D(u, Tu).$$

If $u \in (m-1, m]), m \geq 2$, then $Tu = \frac{1}{m}$ and

$$D(Tu, 1) = 1 - \frac{1}{m} < \frac{u-1}{m} = D(u, Tu).$$

for each $u \in X$, where $q = 1 \in F(T)$ and It is evident that the Picard iteration $u_{m+1} = Tu_m$ converges to 1 for every initial value $u_0 \in X$. Let

$$v_{2m} = \frac{1}{2m}, v_{2m+1} = \frac{1}{4m+4}, \ m \geq 1.$$

Then

$$D(v_{2m+1}, Ty_{2m}) = \frac{1}{2m} - \frac{1}{(4m+4)} = \frac{(m+2)}{[4m(m+1)]}$$

and

$$D(v_{2m+2}, Tv_{2m+1}) = 2m + 2 - 2m - 2 = 0,$$

so $D(v_{m+1}, Tv_m) \to 0$.

V. Applications

Theorem 30. *Consider $X = C[0, 1], \mathbb{R}$ where*

$$\| f \|_\infty = Sup_{0 \leq u \leq 1} \mid f(u) \mid$$

for all $f \in X$. Define $T : X \to X$ as

$$Tf(u) = \int_0^1 F(u, f(t))dt,$$

where

$$\|_\infty = Sup_{0 \leq u \leq 1} \mid f(u) \mid:$$

1) *a continuous function is $F : [0, 1] \times \mathbb{R} \to \mathbb{R}$.*
2) *for any c in the interval $[0, 1$, there is a $\mid F_v(u, v) \mid \leq c$, indicating that the partial derivative F_v of F over v exists.*
3) *for any pair $u, v \in [0, 1]$, there exists a such that $au \leq F(u, av)$, where a is a real number between 0 and 1.*

Consider $P = \{(u, v) \in \mathbb{R}^2 : u, v \geq 0\}$ be a cone and (X, D) the complete generalized cone metric space defined as $D(f, g) = (\| f - g \|_\infty, \alpha \| f - g \|_\infty)$ in cases when α is greater than or equal to zero. If the equation $0 \leq c \leq \frac{1}{2}$ holds, then Picard's iteration is T-stable.

VI. Conclusion

In this study, we embarked on an exploration of generalized cone metric spaces, with a focus on the establishment of a generalized version of Kannan, Chatterjea contraction fixed point theorems, and the T-stability of Picard's Iteration. Our investigation ventured into the nuanced domain of fixed point theory, extending the conventional boundaries to embrace a more inclusive spectrum of spaces and contraction conditions. Through a meticulous examination and application of Zamfirescu contractions, we have unfolded new theoretical landscapes that affirm the existence and uniqueness of fixed points in generalized cone metric spaces. The significance of our findings is twofold. Theoretically, we have contributed to the expansion of fixed point theory by incorporating a wider array of spaces and contractions, thus providing a solid groundwork for future mathematical inquiries. Practically, the application of our theoretical results on T-stability of Picard's Iteration with generalized cone metric to solve integral equations introduces a novel approach, bridging the gap between abstract mathematical theories and tangible problem-solving scenarios. Our study is further enriched by the inclusion of specific examples that not only substantiate the validity of our theorems by demonstrating the existence of a unique fixed point and the T-stability of Picard's iteration but also demonstrate their practical utility in solving integral equations on generalized cone metric. These examples serve as a testament to the robustness and relevance of our findings, highlighting the efficacy of fixed point theory as a powerful tool for mathematical analysis and problem solving. Our study advances fixed point theory on but is limited by its focus on generalized cone metric spaces and certain conditions, affecting its broader applicability and the range of integral equations addressed. That succinctly captures the essence of potential directions for future research, highlighting the need for exploration beyond current limitations to broaden the scope of fixed-point theory's applicability and its integration with other disciplines for enhanced theoretical understanding and practical problem-solving.

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