The Impact and Dynamic Analysis of Positive Information on Rumor Victims

Jundi Li, Yuhan Hu *

Abstract—**The propagation of misinformation adversely affects public health and psychosocial well-being, while compromising social stability and developmental processes. This study introduces an SQVR model incorporating positive information dissemination within digital networks, adopting a victim-centric analytical framework. The proposed model quantifies the mitigating effects of verified information on both affected populations and rumor disseminators, establishing mathematical proof for the existence and stability of system equilibria. An optimization-based control strategy was developed to simultaneously curb rumor diffusion and minimize victimization rates. Numerical simulations validated the theoretical propositions and parameter sensitivity profiles. Comparative evaluation with the SIR epidemiological framework demonstrated the SQVR model's accelerated convergence to equilibrium. The findings substantiate that enhancing verified information dissemination effectively contains rumor epidemics, facilitates victim recovery, and alleviates resultant societal damages.**

Index Terms—**rumor victims, rumor propagation model, positive media effects, optimum control.**

I. INTRODUCTION

RUMOR is a type of news or story that lacks substantial evidence and is often widely spread by people. It can cause harm to individuals, groups, and society by confusing public opinion, leading to misunderstandings and unnecessary panic. When the coronavirus disease 2019 began breaking out in December 2019, not only did the virus itself spread rapidly, but also a series of rumors. These rumors hindered the progress of epidemic prevention and control, causing unnecessary harm and panic to the public [1]-[2]. Maintaining social security and stability is the primary condition for economic development, therefore, reducing the spread of rumors is of great significance.

Previous studies have found astonishing similarities between rumor spreading and infectious disease transmission. Thus, infectious disease models are often used to improve the study of rumor spreading [3]. Infectious disease transmission is one of the first communication problems studied by scholars, and commonly used infectious disease models include SI models, SIS models and SIR models [4], [5], [6].

In recent years, significant academic efforts have been devoted to modeling infectious disease transmission mechanisms. Isuntier et al. [7] developed a dual-population dynamic model to elucidate critical control mechanisms in cholera transmission through differential equation analysis. Concurrently, Zhang et al. [8] innovatively proposed a fractional-order SIQR epidemiological model that incorporates network propagation characteristics; their computational simulations not only predicted viral diffusion patterns but also provided mathematical foundations for enhancing cybersecurity in wireless sensor networks. Furthermore, Anggrim et al. [9] established a multi-pathway Zika virus transmission model employing optimal control theory, which quantitatively evaluated intervention efficacy through parameter sensitivity analysis and stability proofs.

While studying the spread of viruses, researchers have also studied the spread of rumors. The classic models for rumor propagation are the DK and MT models, both of which date back to the 1960s [10]-[11]. Utilizing these models, numerous scholars have conducted extensive research to refine rumor propagation models, resulting in numerous new breakthroughs. Scholars have examined various types of rumor propagation models.

Some have researched on homogeneous networks, while others have conducted research on heterogeneous networks [12]-[13]. Tong et al. [14] focused on the mechanism of public opinion dissemination in the era of social networks. They proposed innovative solutions to address the issue of traditional models overlooking emotional drivers, introducing extreme emotional mechanisms to uncover the influence of speech dissemination in high emotional states. Nsikan Nkordeh et al. [15] explored the double-edged sword effect of social media on the reading habits of Nigerian youth, revealing both its educational potential and usage challenges. Kunwar et al. [16] innovatively modeled alcoholism behavior as a "social infectious disease" and uncovered its transmission patterns and control mechanisms through nonlinear dynamics analysis.

To improve the existing theories of rumor spreading, other researchers have considered factors such as nonlinear propagation rates [17]-[18], the credibility of information [19], the attitude of the audience [20], and the effects of media reporting methods [21]. The study of these influencing factors has added new content to existing theories of rumor spreading. The above studies help us understand how rumors spread in different contexts [22], [23], [24].

In the era of the Internet, the Internet has greatly promoted cultural dissemination and information exchange, becoming a bridge for global connectivity. The spread of rumors has become more rapid, widespread, and difficult to control. Therefore, scholars have begun to study the laws, characteristics, and impact on society of rumor spreading. On the basis of the forgetting mechanism, Zhao et al. [25]studied a rumor spreading model on an online social blog platform called Live Journal, which combines forgetting mechanism and SIR model of epidemics to provide a more detailed and realistic description of the rumor spreading process, Then, Zhao et al. [26] proposed a new rumor spreading model that takes

Manuscript received February 24, 2025; revised May 28, 2025.

This work was supported by the Social Science Planning Fund of Liaoning Province China (Grant No L22AGL015), and the National Natural Science Foundation of China (Grant No. 62441309).

Jundi Li is a graduate student of School of Science, University of Science and Technology Liaoning, Anshan 114051 China (e-mail:ljundi@163.com)

Yuhan Hu* is a Professor of Department of Mathematics, University of Science and Technology Liaoning, Anshan 114051 China (Corresponding author to provide phone: +86-18041294936, e-mail:anshanhyh@163.com)

into account the mutual influence of forgetting and memory mechanisms, studying the final scale of rumor spreading and how to reduce the maximum influence of rumors under different propagation rates, inhibition rates, forgetting rates, and network averages. Liu et al. [27] proposed an automated method based on text mining and social network analysis to detect the source of rumors on Sina Weibo. Del Vicario et al. [28] explored the mechanism and influencing factors of rumor propagation on the internet by analyzing data on Twitter social networks. Pan et al. [29] proposed a rumor spreading model based on an infectious disease model, taking into account the impact of media coverage on the speed and scope of rumor spreading, and analyzed the dynamic evolution process of rumor propagation through mathematical modeling and simulation experiments. Kang et al. [30] compared the phenomenon of super transmission and asymptomatic infection in the information dissemination of COVID-19. His results indicate that authoritative super transmitters have a beneficial effect on information transmission. In contrast, asymptomatic carriers with lower levels of individual acceptance have a negative impact on information dissemination. Then, Kang et al. [31] proposed a stochastic model that considers the existence of super propagators and implicit propagators in information propagation, as well as random perturbations of model parameters. The results of this study indicate that white noise is beneficial for the dissemination of information. The intensity of disturbance is directly proportional to the fluctuation of information propagation. Controlling random parameters can effectively promote the dissemination of information. These studies provide important reference and guidance for us to better understand rumor propagation, and also help to develop effective response measures.

Grasping the mechanism of rumor spreading, developing effective rumor spreading control strategies, and reducing the harm caused by rumors have become urgent issues to be solved. In current research, some scholars have considered the laws of rumor spreading under the Internet. However, few people combine the spreading of information on the internet with the role of rumor victims. This article establishes a new rumor propagation model by comprehensively considering the roles of both in the process of rumor spreading. The model adopts the positive information dissemination effect on the internet to help rumor victims get out of trouble and effectively control rumor spreading.

The remaining organizational parts of this article are as follows: In the second section, we establish an SQVR model for rumor victims. In the third section, we introduce the basic reproduction number, the existence of model equilibrium points, and the local and global stability of equilibrium points. In the fourth section, we provide the existence of optimal control for rumor propagation and corresponding control strategies. In the fifth section, we analyze the influence of parameters and the selected optimal control variables on rumor spreading through numerical simulation. In the sixth section, we conduct sensitivity analysis on the parameters in the model. The final section is the conclusion.

A. Model description

In an open virtual community, the group size is variable at any point in time, and the total group size can be represented by $N(t)$. Using the method of warehouse model, the population is divided into four categories: susceptible to rumors (people who have never been harmed by rumors or who have never heard of rumors), the rumor spreads (those who disseminate the rumor), rumor victims (those injured by rumor) and rumor recoverers (those who recover from the rumour dilemma or who do not disseminate the rumour), are expressed as $S(t), Q(t), V(t), R(t)$. The process of SQVR rumor propagation is shown in Fig. 1. In addition, the rules governing the spread of rumors in social networks can be summarized as follows:

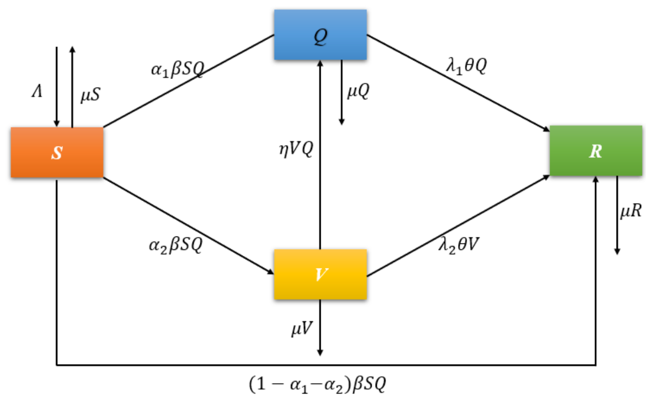


Fig. 1: SQVR rumor victim dissemination model diagram

- Assuming that individuals enter the rumor spreading system at a constant rate Λ, and considering that the ignorant, rumor spreaders, rumor victims, and rumor recoverers individuals exit the rumor spreading system for certain specific reasons, it is assumed that they all have the same migration rate μ. θ indicates the impact of positive online information on rumor spreaders and victims.
- When the rumor is ignorant, with a certain probability β when encountering rumor spreads, some ignorant individuals who hear rumor information will become spreads with a probability of α_1 and influence other audiences with a certain probability. Another group of ignorant individuals may cause psychological harm to themselves due to rumors and become victims of rumors with a probability of α_2. Some people may become immune to rumors with a probability of $(1-\alpha_1-\alpha_2)$ due to their strong willpower or lack of interest in rumors.
- If rumor victims adopt a herd mentality when encountering rumor spreaders, a portion of rumor victims will become rumor spreaders with a probability of η to influence other audiences. However, another group of rumor victims may be influenced by the positive effects on the internet, recover from difficulties after receiving inspiration, and may become rumor recoverers with a probability of λ_2.
- If rumor spreaders are influenced by the positive effects of the media, a certain proportion of rumor spreaders in λ_1 will become rumor immune individuals.

All parameters are positive in the above analysis, resulting in a schematic diagram that considers the impact of positive media coverage on rumor victims.

TABLE I: Description of parameters in the model.

Parameter	Meaning
$S(t)$	Number of rumor ignoramus at time t.
$Q(t)$	Number of rumor spreaders at time t.
$V(t)$	Number of rumor victims at time t.
$R(t)$	Number of immunized individuals at time t.
Λ	Number of entries into the system per unit time.
θ	The degree to which victims and disseminators are affected by the dissemination of positive information on the internet.
α_1	Conversion rate from unknown to rumor spreaders.
α_2	Conversion rate from unknown to rumour victims.
β	Contact rate between ignorant and rumor spreaders.
η	Conversion rate from rumour victims to rumor spreaders.
λ_1	Conversion rate from rumor spreaders to rumour recoverers.
λ_2	Conversion rate from rumour victims to rumour recoverers.
μ	Individual mobility.

Based on these elements, we can model and analyze the process of rumor spreading and diffusion in the crowd. We constructed an SQVR model that considers the relationship between rumor victims and positive information effects on the network. The system dynamics equation is described as follows

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \beta SQ - \mu S \\ \dfrac{dQ}{dt} = \alpha_1 \beta SQ + \eta QV - \lambda_1 \theta Q - \mu Q \\ \dfrac{dV}{dt} = \alpha_2 \beta SQ - \eta QV - \lambda_2 \theta V - \mu V \\ \dfrac{dR}{dt} = (1 - \alpha_1 - \alpha_2)\beta SQ + \lambda_1 \theta Q \\ \qquad + \lambda_2 \theta V - \mu R \end{cases} \tag{1}$$

The initial conditions are met:

$$\begin{aligned} S(0) = S_0 \geq 0, Q(0) = Q_0 \geq 0, \\ V(0) = V_0 \geq 0, R(0) = R_0 \geq 0 \end{aligned} \tag{2}$$

and

$$S(t) + Q(t) + V(t) + R(t) = N(t) \tag{3}$$

II. Model analysis

For a population dynamics system, studying its basic regeneration number and equilibrium point is a prerequisite for predicting the population development trend within the system.

A. Basic regeneration number R_0

Firstly, the basic reproduction number R_0 of system (1) is obtained by the next generation matrix method.

Let $X = (Q, V, S, R)^T$

Where

$$F(X) = \begin{pmatrix} \alpha_1 \beta SQ + \eta QV \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

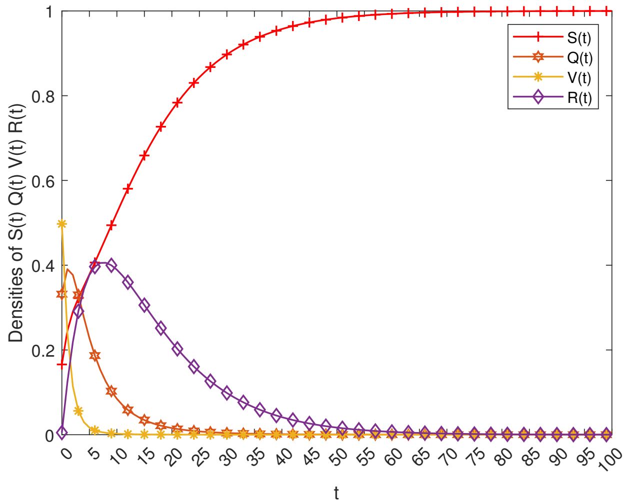


Fig. 2: Stability of the rumor free equilibrium point E_0

$$V(X) = \begin{pmatrix} \lambda_1 \theta Q + \mu Q \\ -\alpha_2 \beta SQ + \eta QV + \lambda_2 \theta V \\ -\Lambda + \beta SQ + \mu S \\ (1-\alpha_1-\alpha_2)\beta SQ - \lambda_1 \theta Q - \lambda_2 \theta V + \mu R \end{pmatrix} \tag{5}$$

By calculation

$$F = \begin{pmatrix} \alpha_1 \beta \frac{\Lambda}{\mu} & 0 \\ 0 & 0 \end{pmatrix}, V = \begin{pmatrix} \lambda_1 \theta + \mu & 0 \\ -\alpha_2 \beta \frac{\Lambda}{\mu} & \lambda_1 \theta + \mu \end{pmatrix}. \tag{6}$$

Therefore,we can obtain the inverse matrix of $V(x)$

$$V^{-1} = \begin{pmatrix} \frac{1}{\lambda_1 \theta + \mu} & 0 \\ \frac{\alpha_2 \beta \Lambda}{\mu(\lambda_1 \theta + \mu)(\lambda_2 \theta + \mu)} & \frac{1}{\lambda_2 \theta + \mu} \end{pmatrix} \tag{7}$$

The next generation matrix is

$$FV^{-1} = \begin{pmatrix} \frac{\alpha_1 \beta \Lambda}{\mu(\lambda_1 \theta + \mu)} & 0 \\ 0 & 0 \end{pmatrix} \tag{8}$$

Thus, the spectral radius of the next generation matrix FV^{-1} is R_0. The maximum value of the characteristic root of FV^{-1} is the spectral radius, so R_0 can be obtained through calculation.

$$R_0 = \frac{\alpha_1 \beta \Lambda}{\mu(\lambda_1 \theta + \mu)} \tag{9}$$

B. Equilibrium point and its stability

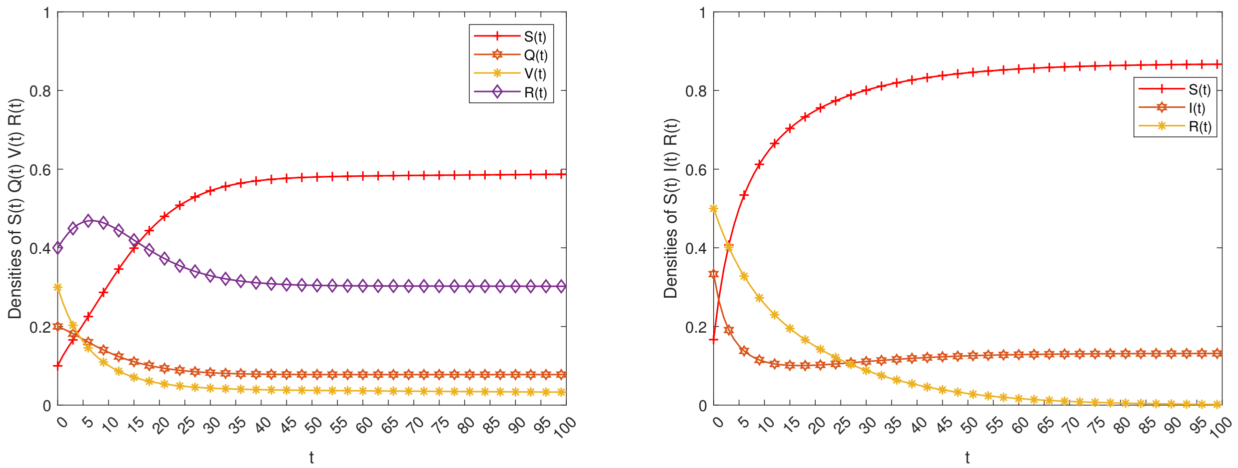
When R_0 <1, the rumor propagator disappear, and the rumor-free equilibrium point of system (1) is denoted as $E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0)$.

Theorem1: If $R_0 < 1$, the rumor-free equilibrium point of system (1) is locally asymptotically stable.

proof: The Jacobin matrix of system (1) at rumor-free equilibrium point E_0 can be written as:

$$J(E_0) = \begin{bmatrix} -\mu & -\beta S_0 & 0 & 0 \\ 0 & \alpha_1 \beta S_0 - a & 0 & 0 \\ 0 & \alpha_2 \beta S_0 & -b & 0 \\ 0 & p_1 \beta S_0 + \lambda_1 \theta & \lambda_2 \theta & -\mu \end{bmatrix} \tag{10}$$

Among them, we define $p_1\,(1 - \alpha_1 - \alpha_2)$, $a = \lambda_1 \theta + \mu$ and $b = \lambda_2 \theta + \mu$. The characteristic equation of matrix $J(E_0)$ is



(a) The stability of the equilibrium Point E^*

(b) The change of population density over time in the SIR model

Fig. 3: Density changes of $S(t), Q(t), V(t)$ and $R(t)$ under different parameters

$$|hE - J(E_0)| = 0 \tag{11}$$

then

$$\begin{vmatrix} h+\mu & \beta S_0 & 0 & 0 \\ 0 & h-\alpha_1\beta S_0 + a & 0 & 0 \\ 0 & -\alpha_2\beta S_0 & h+b & 0 \\ 0 & -p_1\beta S_0 + \lambda_1\theta & \lambda_2\theta & h+\mu \end{vmatrix} = 0 \tag{12}$$

By simplification, $J(E_0)$ has two negative eigenroots $h_1 = h_2 = -\mu < 0$ and the other eigenvalues are the characteristic roots of $|hE - J(E_0)|$, where

$$\begin{vmatrix} h-\alpha_1\beta S_0 + a & 0 \\ -\alpha_2\beta S_0 & h+b \end{vmatrix} = 0 \tag{13}$$

It is easy to get other feature roots $h_3 = -\lambda_1\theta - \mu$ <0, $h_4 = \alpha_1\beta\frac{\Lambda}{\mu} - \lambda_1\theta - \mu$ <0. If R_0 <1, thus, the rumor-free equilibrium point E_0 locally asymptotically stable.

Theorem2: If $R_0 < 1$, then the rumor free equilibrium point of the system (1) is globally asymptotically stable.

Proof: It is easy to know that $N^* = S^* + Q^* + V^* + R^*$ and satisfy $\frac{dN}{dt} = \Lambda - \mu S(t)$ It illustrates that:

$$\lim supN(t) \leq \frac{\Lambda}{\mu} \tag{14}$$

For $t \geq 0$. and

$$T = \Big\{(S(t), Q(t), V(t), R(t)) \in R_4^+ : S(t) + Q(t) + V(t) + R(t) \leqslant \frac{\Lambda}{\mu}\Big\} \tag{15}$$

Then, the Lyapunov function $L(t) = Q(t) + V(t) + R(t)$ can be constructed and $L^{'}(t)$ can be computed as:

$$\begin{aligned} L^{'}(t) =& \alpha_1\beta SQ + \eta QV - \lambda_1\theta Q - \mu Q + \alpha_2\beta SQ \\ & - \eta QV - \lambda_2\theta V - \mu V + (1-\alpha_1-\alpha_2)\beta SQ \\ & + \lambda_1\theta Q + \lambda_2\theta V - \mu R \\ =& (\beta S - \mu)Q - \mu(V+R) \\ \leqslant& (\frac{\Lambda\beta}{\mu} - \mu)Q - \mu(V+R) \end{aligned} \tag{16}$$

If $\Lambda\beta \leqslant \mu^2$, then $L^{'}(t) \leqslant 0$ is valid. Furthermore, $L^{'}(t) = 0$ if and only if $Q = V = R = 0$. The only solution of system (1) in Γ on which $L^{'}(t) = 0$ is E_0. Thus, by LaSalle's Invariance Principle [32], every solution of system (1) approaches E_0 as $t \to \infty$. Hence, E_0 is globally asymptotically stable.

When R_0 >1, the rumor will be spread, the rumor-existence equilibrium point of system (1) can be expressed as $E^* = (S^*, Q^*, V^*, R^*)$, which means the rumor will spread widely. The rumor-existence equilibrium E^* should satisfy:

$$\begin{cases} \Lambda - \beta S^*Q^* - \mu S^* = 0 \\ \alpha_1\beta S^*Q^* + \eta Q^*V^* - \lambda_1\theta Q^* - \mu Q^* = 0 \\ \alpha_2\beta S^*Q^* - \eta Q^*V^* - \lambda_2\theta V^* - \mu V^* = 0 \\ (1-\alpha_1-\alpha_2)\beta S^*Q^* + \lambda_1\theta Q^* + \lambda_2\theta V^* - \mu R^* = 0 \end{cases} \tag{17}$$

From the above equations, S^*, V^* and R^* can be represented by Q^*:

$$S^* = \frac{\Lambda}{(\beta Q^* + \mu)}$$

$$V^* = \frac{(\lambda_1\theta + \mu)(\beta Q^* + \mu) - \alpha_1\beta\Lambda}{\eta(\beta Q^* + \mu)}$$

$$R^* = \frac{[\Lambda(1-\alpha_1-\alpha_2)\beta + \lambda_1\theta(\beta Q^* + \mu)]Q^* + \lambda_2\theta V^*}{\mu(\beta Q^* + \mu)}$$

Where

$$Q^* = \frac{-[\eta\mu + \beta(\lambda_2\theta + \mu)(\lambda_1\theta + \mu)] + \eta\beta\Lambda(\alpha_1+\alpha_2) + \sqrt{\Delta}}{2\eta\beta(\lambda_1\theta + \mu)}$$

and

$$\Delta = \sqrt{m^2 - n^2}$$

So $Q^* > 0$, when $m > n$, where,

$$m = \eta\mu + \beta(\lambda_2\theta + \mu)(\lambda_1\theta + \mu) - \eta\beta\Lambda(\alpha_1 + \alpha_2)$$

$$n = 2\sqrt{\eta\beta(\lambda_1\theta + \mu)(\lambda_2\theta + \mu)[\mu(\lambda_1\theta + \mu) - \alpha_1\beta\Lambda]}$$

Theorem3: If R_0 >1, when conditions $2\mu + \lambda_1\theta > (\alpha_1 + \alpha_2)\beta\Lambda$ and $\alpha_1\beta > \eta\alpha_2$ are satisfied, the rumor-existence equilibrium point of system (1) is locally asymptotically stable.

Proof: The Jacobin matrix of system system (1) at equilibrium point E^* can be written as:

$$J(E^*)=\begin{bmatrix} -\beta Q^*-\mu & -\beta S^* & 0 & 0 \\ \alpha_1\beta Q^* & p_3-a & \eta Q^* & 0 \\ \alpha_2\beta Q^* & \alpha_2\beta S^*-\eta V^* & p_2 & 0 \\ p_1\beta Q^* & p_1\beta S^*+\lambda_1\theta & \lambda_2\theta & -\mu \end{bmatrix} \quad (18)$$

Among them, we define $p_2 = -\eta Q^* - b$ and $p_3 = \alpha_1\beta S^* + \eta V^*$. The characteristic equation of matrix $J(E^*)$ is

$$\begin{vmatrix} h+\beta Q^*+\mu & \beta S^* & 0 & 0 \\ -\alpha_1\beta Q^* & h-p_3+a & -\eta Q^* & 0 \\ -\alpha_2\beta Q^* & -\alpha_2\beta S^*+\eta V^* & h-p_2 & 0 \\ -p_1\beta Q^* & -p_1\beta S^*-\lambda_1\theta & -\lambda_2\theta & h+\mu \end{vmatrix}=0 \quad (19)$$

By calculation, we obtain one of the characteristic roots as $h_1 < 0$ and obtain a univariate cubic equation constructed as $a_3x^3 + a_2x^2 + a_1x + a_0 = 0$, where

$$\begin{aligned} a_3 &= 1 > 0 \\ a_2 &= (\beta+\eta)Q^* + (\lambda_2\theta+\mu) + \mu > 0 \\ a_1 &= (\beta Q^*+\mu)(\eta Q^*+\lambda_2\theta+\mu) \\ &\quad + \eta Q^*(\eta V^* - \alpha_2\beta S^*) + \alpha_1\beta^2 S^* Q^* > 0 \\ a_0 &= \eta Q^*(\eta V^* - \alpha_2\beta S^*)(\beta Q^*+\mu) \\ &\quad + [\eta(\alpha_1+\alpha_2) + \alpha_1(\lambda_2\theta+\mu)]\beta^2 S^* Q^{*2} > 0 \end{aligned} \quad (20)$$

$$\begin{aligned} a_2a_1 - a_3a_0 =& [\beta Q^* + \eta Q^* + (\lambda_2\theta+\mu)\mu] \\ & [(\beta Q^*+\mu)(\eta Q^*+\lambda_2\theta+\mu)] \\ & + \eta Q^*(\eta Q^*+\lambda_2\theta+\mu) \\ & (\eta V^* - \alpha_2\beta S^*) + \mu\alpha_1\beta^2 S^* Q^* \\ & + (\alpha_1\beta - \eta\alpha_2)\beta^2 S^* Q^{*2} \\ & > 0 \end{aligned} \quad (21)$$

When two conditions $2\mu+\lambda_1\theta > (\alpha_1+\alpha_2)\beta\Lambda$ and $\alpha_1\beta > \eta\alpha_2$ are satisfied, $a_3, a_2, a_1, a_0 > 0$ and $a_2a_1 > a_3a_0$ is true. At this time, according to Routh–Hurwitz criterion [33], the equilibrium point E^* of system (1) is locally asymptotically stable.

Theorem4: If $R_0 > 1$, the rumor-existence equilibrium point $E^* = (S^*, Q^*, V^*, R^*)$ of system (1) is globally asymptotically stable.

Proof: We construct the Lyapunov function

$$\begin{aligned} L(t) =& [(S(t)-S^*) + (Q(t)-Q^*) + \\ & (V(t)-V^*) + (R(t)-R^*)]^2 \end{aligned} \quad (22)$$

$$\begin{aligned} L(t)^{'} =& 2[(S(t)-S^*) + (Q(t)-Q^*) \\ & + (V(t)-V^*) + (R(t)-R^*)] \\ & [S(t)^{'} + Q(t)^{'} + Vt)^{'} + R(t)^{'}] \\ =& 2[(S(t)-S^*) + (Q(t)-Q^*) \\ & + (V(t)-V^*) + (R(t)-R^*)] \\ & [\Lambda - \mu S - \mu Q - \mu V - \mu R] \end{aligned} \quad (23)$$

Because of the existence of $E^* = (S^*, Q^*, V^*, R^*)$, we can know that $\Lambda - \mu S^* - \mu Q^* - \mu V^* - \mu R^* = 0$, so $\Lambda = \mu S^* - \mu Q^* - \mu V^* - \mu R^*$

Then,this equation can be computed as:

$$\begin{aligned} L(t)^{'} =& 2[(S(t)-S^*) + (Q(t)-Q^*) + (V(t)-V^*) \\ & + (R(t)-R^*)][\mu S^* + \mu Q^* + \mu V^* + \mu R^* \\ & - \mu S - \mu Q - \mu V - \mu R] \\ =& -2\mu[(S-S^*) + (Q-Q^*) + (V-V^*) \\ & + (R-R^*)]^2 \\ & < 0 \end{aligned} \quad (24)$$

When $W(t)^{'} = 0$ hold if and only $S(t) = S^*, Q(t) = Q^*, V(t) = V^*, R(t) = R^*$. Therefore, the rumor-existence equilibrium point $E^* = (S^*, Q^*, V^*, R^*)$ of system (1) is globally asymptotically stable.

III. Problems related to rumor victim optimal control

On the basis of the rumor model established above, three control goals are proposed, and in the control process, the goal is to control the number of rumor victims and rumor spreaders under the condition of consuming minimal resources. Within the limited control time, we aim to reduce the number of spreaders, increase the number of immunizers, and prevent more people from being harmed by rumors and increase the number of people recovering from rumors. Therefore, the corresponding optimal control problem is proposed for the model. Considering that the three proportionality constants β, η and θ in the mind of the model have the greatest influence on the rumor propagation model constructed in this paper. Therefore, β, η and θ are chosen to transform into control variables $\beta(t), \eta(t)$ and $\theta(t)$. Control variables $\beta(t)$ are used to control the proportion of ignorant groups and rumor propagators contacted, control variables $\eta(t)$ are used to control the proportion of victims of rumors who become rumor propagators, and control variables $\theta(t)$ is used to control the consumption of positive network information resources, methods such as publicizing the harm of rumor dissemination and refuting rumors are adopted through official channels or authoritative figures to improve the public's ability to judge rumors. In this way, people who have not been exposed to rumor information will consciously identify rumor information when they are exposed to rumor information, thereby reducing the harm caused by rumors to them and reducing the number of rumor spreaders. Find the optimal control strategy with the smallest objective function in the control time interval. Therefore, the following dual-effect objective functional with discount coefficient is proposed

$$J(\beta, \eta, \theta) = \int_0^{t_f} e^{-\alpha t}[Q(t) + V(t) + z]dt \quad (25)$$

In formula (25), we define

$$z = \frac{c_1}{2}\beta^2(t) + \frac{c_2}{2}\eta^2(t) + \frac{c_3}{2}\theta^2(t)$$

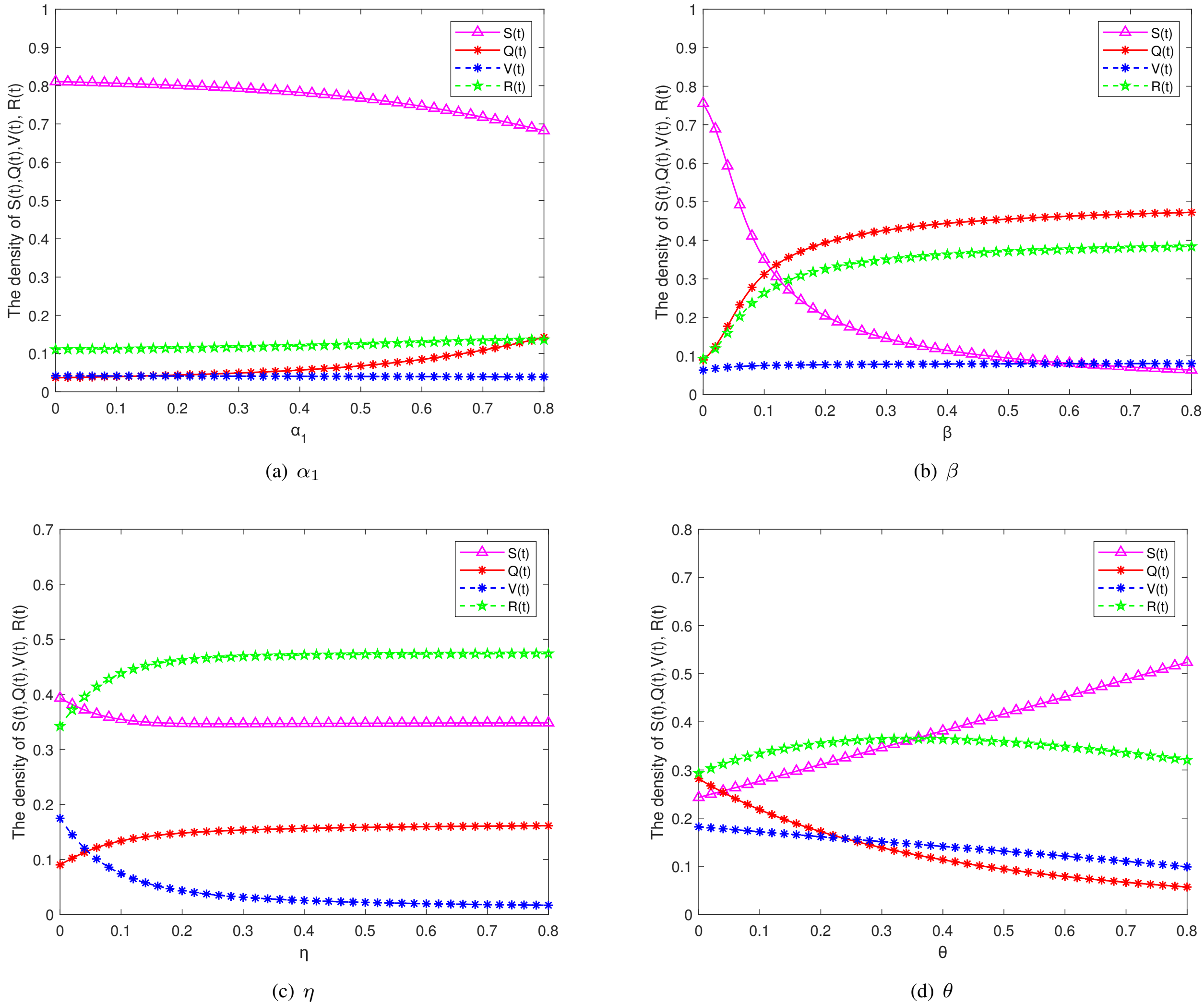


Fig. 4: Density changes of $S(t), Q(t), V(t)$ and $R(t)$ under different parameters

The variables within the target functional satisfy the following equation

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \beta(t)SQ - \mu S \\ \dfrac{dQ}{dt} = [\alpha_1\beta(t)S + \eta(t)V - \lambda_1\theta(t) - \mu]Q \\ \dfrac{dV}{dt} = \alpha_2\beta(t)SQ - \eta(t)QV - \lambda_2\theta(t)V - \mu V \\ \dfrac{dR}{dt} = (1 - \alpha_1 - \alpha_2)\,\beta(t)SQ \\ \qquad + (\lambda_1 Q + \lambda_2 V)\theta(t) - \mu R \end{cases} \tag{26}$$

The initial conditions for system (26) are satisfied:

$$\begin{aligned} &S(0){=}S_0 \geq 0, Q(0) = Q_0 \geq 0, \\ &V(0) = V_0 \geq 0, R(0) = R_0 \geq 0 \end{aligned} \tag{27}$$

Where

$$\beta(t), \eta(t), \theta(t) \in U \tag{28}$$

and

$$\begin{aligned} U \triangleq &[\beta, \eta, \theta)\,|\beta(t), \eta(t), \theta(t)\ measurable, \\ &[0 \leq \beta(t), \eta(t), \theta(t) \leq 1, \forall t \in [0, t_f]] \end{aligned} \tag{29}$$

U is the control allowable set. t_f is the end of the controlled time interval, assuming that the starting point of the controlled time interval is zero moment. The positive parameter $c_i, (i = 1, 2, 3)$ is the weight factor, which reflects the strength and importance of the three different control measures.

A. Existence of optimal control.

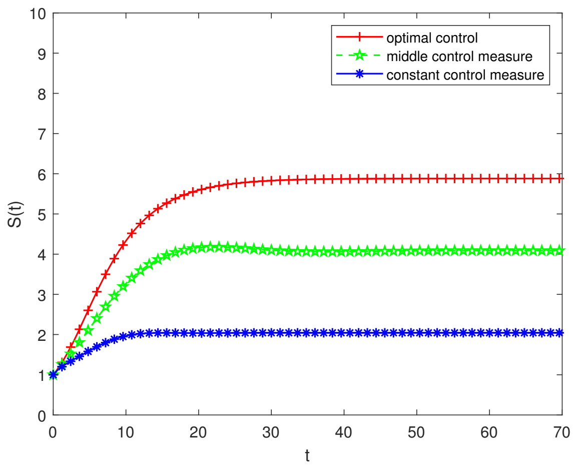
Theorem5: In the case that system (29) and the initial condition (27) are met, there is an optimal control

$$u^{*}{=}\ (\beta^{*}{,}\eta^{*}{,}\theta^{*}) \in \mathrm{U} s.t. J(\beta^{*}{,}\eta^{*}{,}\theta^{*})$$

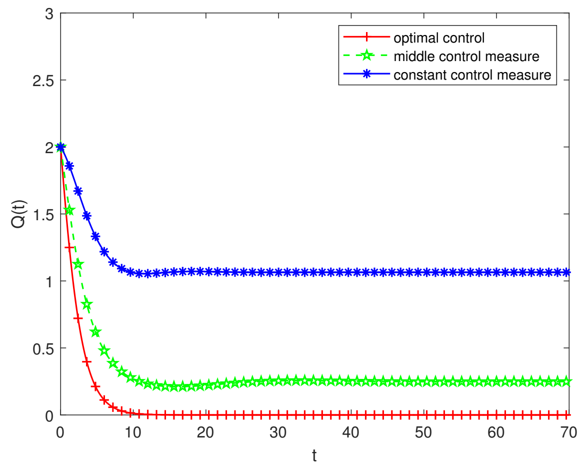
$$= \min \mathrm{J}(\beta(t){,}\eta(t){,}\theta(t), \mu_i(t) \in U)$$

Proof: To prove the existence of optimal control, only the following five conditions need to be verified:

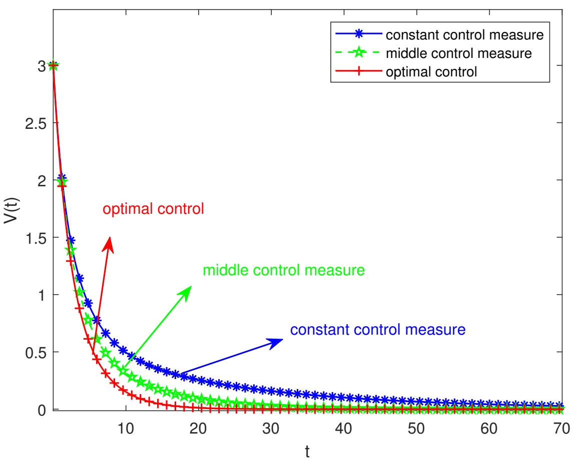
- Both control and state variables are non-null.
- The control allowable set U is a convex set and a closed set.
- The integrable function in the objective functional is convex in the allowable set U.
- The right end of the state system is a linear bounded function about control variables and state variables.



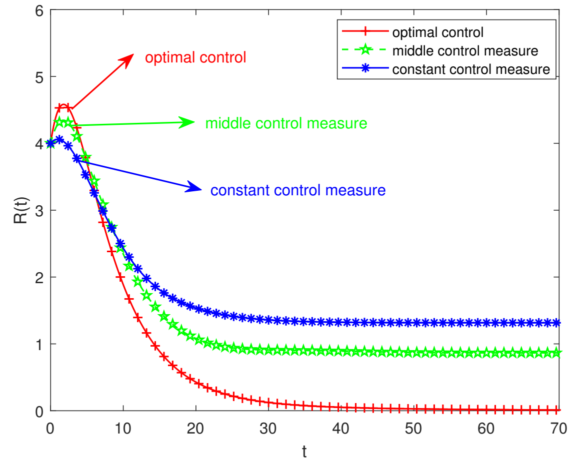
(a) The influence of control variable $\beta(t)$ on compartment $S(t)$



(b) The influence of control variable $\beta(t)$ on compartment $Q(t)$



(c) The influence of control variable $\beta(t)$ on compartment $V(t)$



(d) The influence of control variable $\beta(t)$ on compartment $R(t)$

Fig. 5: The density of $S(t), Q(t), V(t), R(t)$ in the control variable $\beta(t)$ changes over time

- There is a normal number $d_1, d_2 > 0, \xi > 1$. Such that the integrable function in the target functional

$$L(t, \beta, \eta, \theta) \triangleq e^{-\alpha t}[Q(t) + V(t) + z]$$

is satisfied

$$L(t, \beta, \eta, \theta) \geq d_1(|\beta|^2 + |\eta|^2 + |\theta|^2)^{\frac{\xi}{2}} - d_2$$

The first three conditions mentioned above are clearly valid, and only the next two conditions need to be verified. As mentioned earlier, all four state variables have an upper bound N, and the following inequality can be obtained:

$$\begin{cases} S(t)' \leq \Lambda \\ Q(t)' \leq \alpha_1\beta(t)SQ + \eta(t)QV \\ V(t)' \leq \alpha_2\beta(t)SQ \\ R(t)' \leq (1 - \alpha_1 - \alpha_2)\beta(t)SQ \\ \quad\quad + (\lambda_1 Q + \lambda_2 V)\theta(t) \end{cases} \tag{30}$$

Thus, the fourth condition is also true. For the last condition

$$L(t; \beta; \eta; \theta) \geq d_1(|\beta|^2 + |\eta|^2 + +|\theta|^2)^{\frac{\xi}{2}} - d_2 \tag{31}$$

Let $d_1 = \frac{e^{-\alpha t}}{2} \min\{c_1, c_2, c_3\}, \forall d_2 \in R^+$, $\xi = 2$. Then the last condition is satisfied. Therefore, the optimal control can be realized.

B. Optimal control strategy.

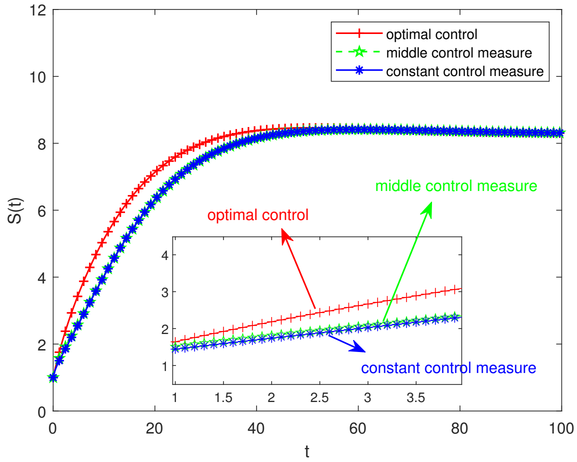
The optimal control expression defines an enlarged Hamiltonian with a penalty term as follows:

$$\begin{aligned} L = \frac{e^{-\alpha t}}{2}&\big[Q(t) + V(t) + z\big] + \delta_1\big[\Lambda - (\beta(t)Q + \mu)S\big] \\ &+ \delta_2\big[\alpha_1\beta(t)S + \eta(t)V - \lambda_1\theta(t) - \mu\big]Q \\ &+ \delta_3\big[\alpha_2\beta(t)SQ - (\eta(t)Q + \lambda_2\theta(t) + \mu)V\big] \\ &+ \delta_4\big[(1 - \alpha_1 - \alpha_2)\beta(t)SQ + (\lambda_1 Q + \lambda_2 V)\theta(t) - \mu R\big] \\ &- \omega_{11}\beta(t) - \omega_{12}(1 - \beta(t)) \\ &- \omega_{21}\eta(t) - \omega_{22}(1 - \eta(t)) \\ &- \omega_{31}\theta(t) - \omega_{32}(1 - \theta(t)) \end{aligned} \tag{32}$$

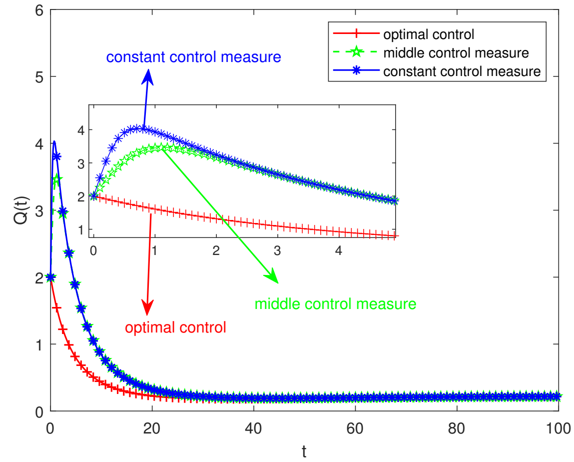
where

$\omega_{ij} \geq 0 (i = 1, 2, 3; j = 1, 2)$ is a penalty operator and satisfies

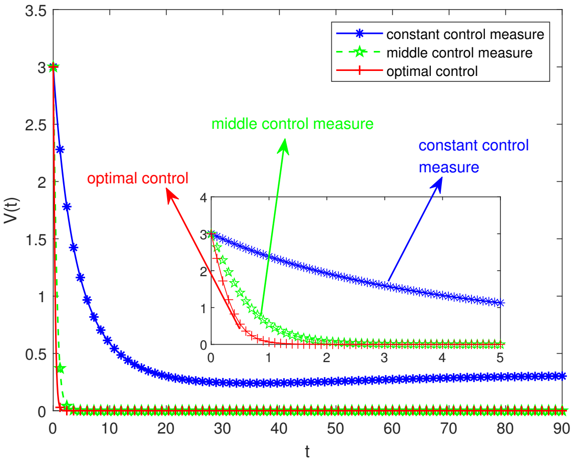
$$\omega_{11}\beta(t) = \omega_{12}(\mathrm{t})(1 - \beta(t)) = 0 \tag{33}$$



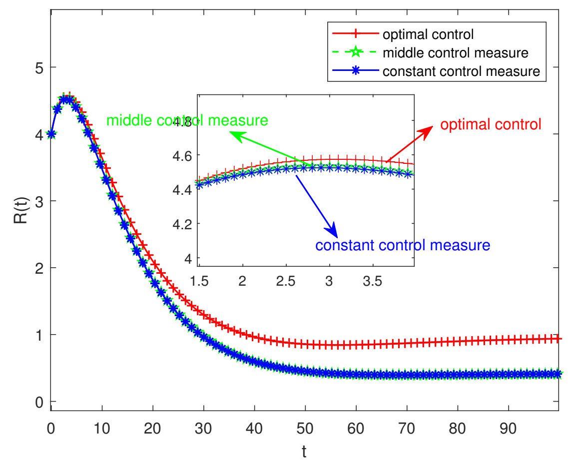
(a) The influence of control variable $\eta(t)$ on compartment $S(t)$



(b) The influence of control variable $\eta(t)$ on compartment $Q(t)$



(c) The influence of control variable $\eta(t)$ on compartment $V(t)$



(d) The influence of control variable $\eta(t)$ on compartment $R(t)$

Fig. 6: The density of $S(t), Q(t), V(t), R(t)$ in the control variable η changes over time

.

$$\omega_{21}\eta(t)=\omega_{22}(\mathrm{t})(1-\eta(t))=0 \tag{34}$$

$$\omega_{31}\theta(t)=\omega_{32}(\mathrm{t})(1-\theta(t))=0 \tag{35}$$

where in system (33) is at optimal control β^*, system (34) is at optimal control η^* and system (35) is at optimal control θ^*.

Theorem6: The optimal control pair for system (26) $(\beta^*,\eta^*,\theta^*)$ and state solution $S(t), Q(t), V(t), R(t)$, there is a covariate variable $\delta_i (i = 1, 2, 3, 4)$ that satisfies the equation

$$\begin{cases} \delta_1' = \delta_1\mu + (\delta_1-\delta_4)\beta(t)Q + (\delta_4-\delta_2)\alpha_1\beta(t)Q \\ \qquad + (\delta_4-\delta_3)\alpha_2\beta(t)Q \\ \delta_2' = -e^{-\alpha t} + (\delta_1-\delta_4)\beta(t)S \\ \qquad + (\delta_4-\delta_2)\alpha_1\beta(t)S \\ \qquad + (\delta_4-\delta_3)\alpha_2\beta(t)S + (\delta_3-\delta_2)\eta(t)V \\ \qquad - \delta_4\lambda_2\theta(t) + \delta_2(\lambda_1\theta(t)+\mu) \\ \delta_3' = -e^{-\alpha t} + (\delta_3-\delta_2)\eta(t)Q + \delta_3(\lambda_1\theta(t)+\mu) \\ \qquad - \delta_4\lambda_2\theta(t) \\ \delta_4' = \delta_4\mu \end{cases} \tag{36}$$

And cross-sectional conditions $\delta_i(t_f), (i = 1, 2, 3, 4)$, what is more, the optimal control pair expression

$$\begin{cases} \beta(t)^* = \min\left(1, \max\left(0, \frac{1}{c_1 e^{-\alpha t}}\left[(\delta_1-\delta_4)Q \right.\right.\right. \\ \qquad \left.\left.\left. +(\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q\right]\right)\right) \\ \eta(t)^* = \min\left(1, \max\left(0, \frac{1}{c_2 e^{-\alpha t}}(\delta_3-\delta_2)QV\right)\right) \\ \theta(t)^* = \min\left(1, \max\left(0, \frac{1}{c_3 e^{-\alpha t}}\left[(\delta_2-\delta_4)\lambda_1 Q \right.\right.\right. \\ \qquad \left.\left.\left. +(\delta_3-\delta_4)\lambda_2 V\right]\right)\right) \end{cases} \tag{37}$$

Proof: According to Pontryagin's Maximum principle, the derivative of Hamiltonian operator for each state variable can be used to calculate the covariate system. The covariate system is

$${\delta_1}' = -\frac{\partial L}{\partial S}, {\delta_2}' = -\frac{\partial L}{\partial Q}, {\delta_3}' = -\frac{\partial L}{\partial V}, {\delta_4}' = -\frac{\partial L}{\partial R} \tag{38}$$

The conditions for determining the solution of covariate system are

$$\delta_i(t_f)=0, i=1,2,3,4 \tag{39}$$

Then, take the derivative of the Hamiltonian operator L with respect to the state variable a and set it to zero, i.e

$$\begin{cases} \dfrac{\partial L}{\partial \beta} = c_1 e^{-\alpha t}\beta + \big[(\delta_4-\delta_1)-(\delta_2-\delta_4)\alpha_1 \\ \qquad + (\delta_3-\delta_4)\alpha_2\big]SQ - \omega_{11} + \omega_{12} = 0 \\ \dfrac{\partial L}{\partial \eta} = c_2 e^{-\alpha t}\eta - (\delta_2-\delta_3)QV - \omega_{21} + \omega_{22} = 0 \\ \dfrac{\partial L}{\partial \theta} = c_3 e^{-\alpha t}\theta - (\delta_4-\delta_2)\lambda_1 Q + (\delta_4-\delta_3)\lambda_2 V \\ \qquad - \omega_{31} + \omega_{32} = 0 \end{cases} \tag{40}$$

Find the optimal control expression from it

$$\begin{cases} \beta^* = \dfrac{1}{c_1 e^{-\alpha t}}\big[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q \\ \qquad + (\delta_4-\delta_3)\alpha_2 Q + \omega_{11} - \omega_{12}\big] \\ \eta^* = \dfrac{1}{c_2 e^{-\alpha t}}\big[(\delta_3-\delta_2)QV + \omega_{21} - \omega_{22}\big] \\ \theta^* = \dfrac{1}{c_3 e^{-\alpha t}}\big[(\delta_2-\delta_4)\lambda_1 Q + (\delta_3-\delta_4)\lambda_2 V \\ \qquad + \omega_{31} - \omega_{32}\big] \end{cases} \tag{41}$$

First of all, consider β^*, to obtain the final expression of the optimal control without penalty terms, consider the following three situations.

- On set $\{t \,|0 < \beta^*(t) < 1\}$, let $\omega_{11}(t) = \omega_{12}(t) = 0$, thus the optimal control is

$$\beta^* = \frac{1}{c_1 e^{-\alpha t}}[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q] \tag{42}$$

- On set $\{t \,|\beta^*(t)=1\}$,let $\omega_{11}(t) = 0$,therefore

$$1 = \beta^* = \frac{1}{c_1 e^{-\alpha t}}[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q - \omega_{12}(t)] \tag{43}$$

- On set $\{t \,|\beta^*(t)=0\}$, let $\omega_{12}(t) = 0$, therefore

$$0 = \beta^* = \frac{1}{c_1 e^{-\alpha t}}[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q + \omega_{11}(t)] \tag{44}$$

This indicates that $\frac{1}{c_1 e^{-\alpha t}}[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q \le 0$, because of $\omega_{11}(t) \ge 0$.

In summary, the optimal control expression for A is:

$$\beta^* = \min\left\{1, \max\left\{0, \frac{1}{c_1 e^{-\alpha t}}\big[(\delta_1-\delta_4)Q + (\delta_4-\delta_2)\alpha_1 Q + (\delta_4-\delta_3)\alpha_2 Q\big]\right\}\right\} \tag{45}$$

Similarly, the optimal control expressions for η^* and θ^* are obtained:

$$\begin{aligned} \eta^* &= \min\left\{1, \max\left\{0, \frac{1}{c_2 e^{-\alpha t}}\big[(\delta_3-\delta_4)QV\big]\right\}\right\} \\ \theta^* &= \min\left\{1, \max\left\{0, \frac{1}{c_3 e^{-\alpha t}}\big[(\delta_2-\delta_4)\lambda_1 Q + (\delta_3-\delta_4)\lambda_2 V\big]\right\}\right\} \end{aligned} \tag{46}$$

Any pair of the best control must satisfy the best control system, therefore, the optimal control system is

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \beta(t)^* SQ - \mu S \\ \dfrac{dQ}{dt} = \alpha_1\beta(t)^* SQ + \eta(t)^* QV - \lambda_1\theta(t)^* Q - \mu Q \\ \dfrac{dV}{dt} = \alpha_2\beta(t)^* SQ - \eta(t)^* QV - \lambda_2\theta(t)^* V - \mu V \\ \dfrac{dR}{dt} = (1-\alpha_1-\alpha_2)\beta(t)^* SQ + \lambda_1\theta(t)^* Q \\ \qquad + \lambda_2\theta(t)^* V - \mu R \\ \dfrac{d\delta_1}{dt} = \delta_1\mu + (\delta_1-\delta_4)\beta(t)^* Q + (\delta_4-\delta_2)\alpha_1\beta(t)^* Q \\ \qquad + (\delta_4-\delta_3)\alpha_2\beta(t)^* Q \\ \dfrac{d\delta_2}{dt} = -e^{-\alpha t} + (\delta_1-\delta_4)\beta(t)^* S \\ \qquad + (\delta_4-\delta_2)\alpha_1\beta(t)^* S + (\delta_4-\delta_3)\alpha_2\beta(t)^* S \\ \qquad + (\delta_3-\delta_2)\eta(t)^* V - \delta_4\lambda_2\theta(t)^* \\ \qquad + \delta_2(\lambda_1\theta(t)^* + \mu) \\ \dfrac{d\delta_3}{dt} = -e^{-\alpha t} + (\delta_3-\delta_2)\eta(t)^* Q \\ \qquad + \delta_3(\lambda_1\theta(t)^* + \mu) - \delta_4\lambda_2\theta(t)^* \\ \dfrac{d\delta_4}{dt} = \delta_4\mu \\ S(0) = S_0 \ge 0,\ Q(0) = Q_0 \ge 0, \\ \qquad V(0) = V_0 \ge 0,\ R(0) = R_0 \ge 0 \\ \lambda_i(t_f) = 0, \quad i = 1,2,3,4 \end{cases} \tag{47}$$

IV. NUMERICAL SIMULATION

In this section, we conducted numerical simulations to validate the established rumor propagation model and its optimal control problem. we simulated the model parameters and control scheme, and compared them with theoretical analysis results to verify the correctness of the model. In addition, we conducted sensitivity analysis to evaluate the impact of different parameters on system behavior.

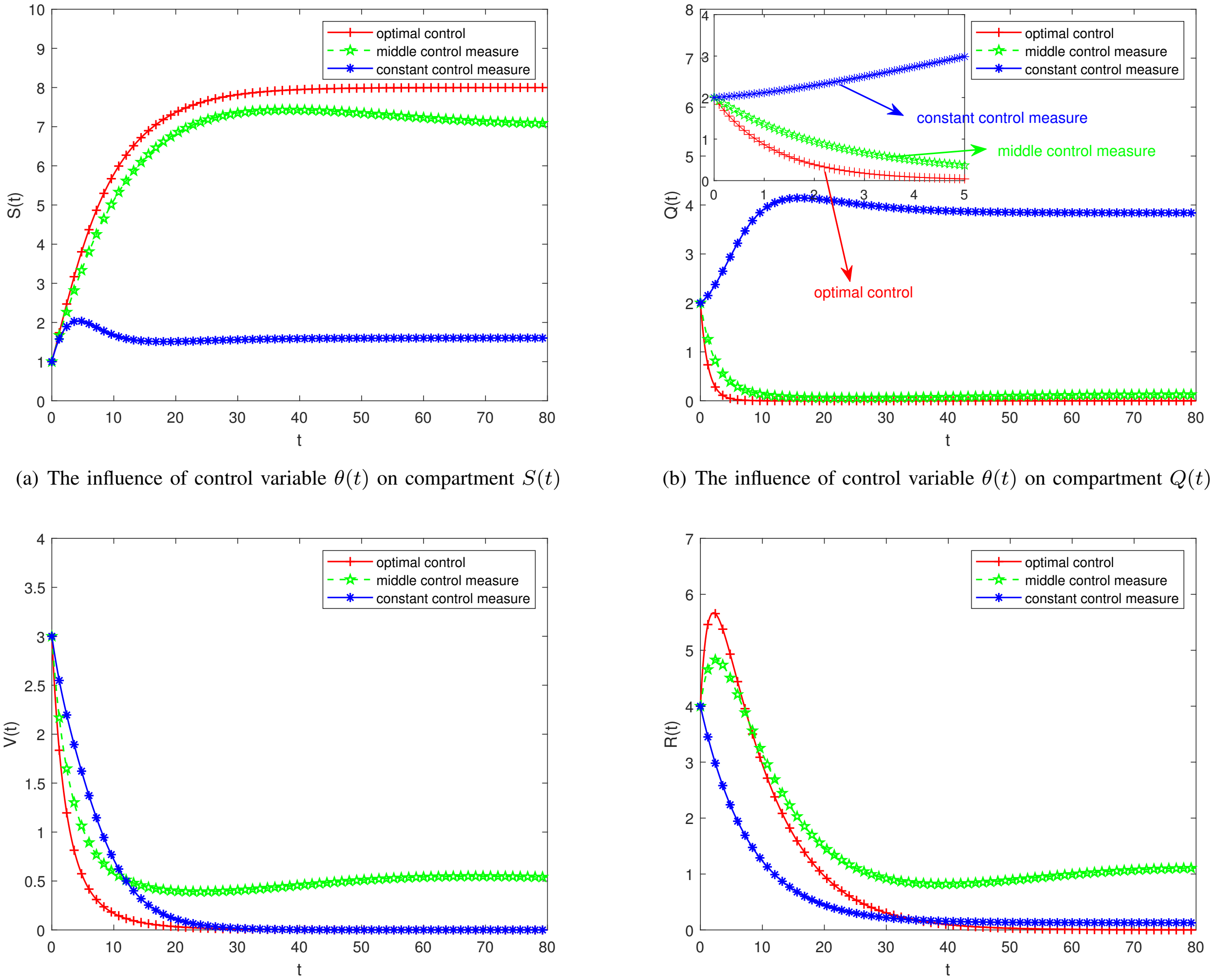
A. Numerical simulation of system stability

Let $\Lambda = 1, \beta = 0.1, \mu = 0.11, \alpha_1 = 0.15, \eta = 0.2, \lambda_1 = 0.22, \theta = 0.6, \alpha_2 = 0.15, \lambda_2 = 0.22$. It was calculated that $R_0 = 0.5635 < 1$. Then the equilibrium point E_0 for no rumour transmission is stable under different initial conditions. It can be seen from Fig. 2 that the stability of E_0 remains unchanged under different initial value conditions.

B. Comparison of results between SIR model and SQVR model

Let $\Lambda = 1, \beta = 0.278, \mu = 0.17, \alpha_1 = 0.2, \eta = 0.019, \lambda_1 = 0.112, \theta = 0.29, \alpha_2 = 0.2, \lambda_2 = 0.25$. It was calculated that $R_0 = 1.6153 > 1$. The rumour is then that the equilibrium point E^* is stable under different initial conditions. It can be seen from Fig. 3(a) that the stability of E^* remains unchanged under different initial value conditions.

By comparing Fig. 3(a) and Fig. 3(b), it can be seen that while maintaining the original parameters unchanged. Although the number of propagators in the SIR model initially



(a) The influence of control variable $\theta(t)$ on compartment $S(t)$

(b) The influence of control variable $\theta(t)$ on compartment $Q(t)$

(c) The influence of control variable $\theta(t)$ on compartment $V(t)$

(d) The influence of control variable $\theta(t)$ on compartment $R(t)$

Fig. 7: The density of $S(t), Q(t), V(t), R(t)$ in the control variable θ changes over time

.

decreases with time, it slightly increases after reaching a certain point in time. The rumor immune system ultimately disappears in this model. After adding the impact of rumor victims and positive online information on disseminators and victims in this article, the scale of the rumor outbreak (the peak of the spreader curve) has significantly decreased and reached stability, and the time of the rumor outbreak has also been delayed. In addition, when the rumor fades its heat and reaches a stable state, the number of rumor victims is effectively controlled, and the number of immunized individuals in the population is also stabilized.

C. Numerical simulation of optimization system

In Fig. 4, we observe that the parameter β, η and θ the impact on rumor spreaders and victims is significant. Therefore, controlling the number of rumor spreaders is one of the effective ways to reduce the number of rumor victims. Meanwhile, the flow of positive information on the internet can significantly reduce the number of rumor victims and disseminators. Therefore, in the next section, we choose to control β, η and θ. To achieve the goal of reducing the spread of rumors and the number of victims.

Fig. 5 shows the density of $S(t), Q(t), V(t)$ and $R(t)$ in the control variables β the change over time. The simulation results show that using the optimal control strategy shown in Fig. 5 can achieve the optimal density of $S(t), Q(t), V(t)$ and $R(t)$. In controlling variables β In this case, the number of rumor spreaders and victims rapidly decreases, indicating that the control variable β it is effective. Therefore, in practical life, measures should be taken to control the contact rate between ignorant individuals and disseminators.

Fig. 6 shows the density of $S(t), Q(t), V(t)$ and $R(t)$ in the control variables η the change over time. The simulation results show that using the optimal control strategy shown in Fig. 6 can achieve the optimal density of $S(t), Q(t), V(t)$ and $R(t)$, effectively reducing the number of rumor spreaders and victims, and control variables β the difference is that in controlling variables η. In this case, the number of rumor spreaders and victims rapidly decreases but eventually approaches zero, indicating that the control variable η is effective. Therefore, in practical life, measures should be taken to help rumor victims escape the current predicament.

As demonstrated in Fig. 7, Fig. It shows the density of $S(t), Q(t), V(t)$ and $R(t)$ in the control variables θ. the proposed optimal control strategy effectively regu-

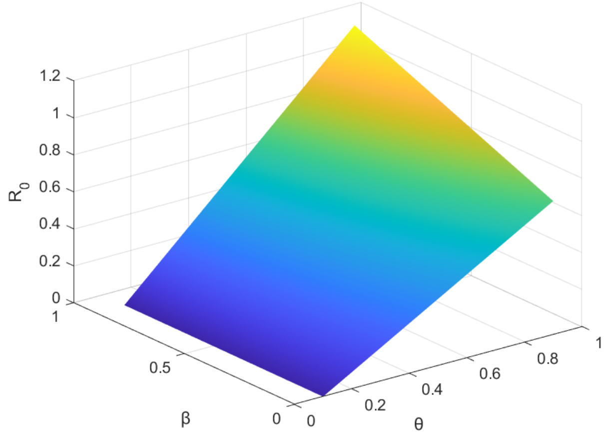


Fig. 8: The sensitivity analysis of the basic reproduction number R_0

lates the dynamic trajectories of population densities for $S(t), Q(t), V(t)$ and $R(t)$ throughout the time horizon. The simulation results show that increasing the promotion of positive information on the internet can effectively reduce the number of rumor spreaders, not only reducing the harm of rumors to the audience, but also helping rumor victims recover good mental health as soon as possible.

V. Sensitivity analysis

By calculating R_0, consider the parameters separately β and θ, we can obtain the partial derivative of R_0:

$$\frac{\partial R_0}{\partial \beta} = \frac{\alpha_1 \Lambda}{\mu(\lambda_1 \theta + \mu)} > 0 \tag{48}$$

$$\frac{\partial R_0}{\partial \theta} = -\frac{\lambda_1 \alpha_1 \beta \Lambda}{\mu(\lambda_1 \theta + \mu)^2} < 0 \tag{49}$$

From (48), it can be seen that if β as the value of increases, the number of basic regeneration numbers R_0 increases, resulting in an increase in the number of rumor spreaders and rumor victims. So we need to control β contact rate.

From (49), the basic regeneration number follows θ. The increase and decrease in the size of rumors and the number of rumor victims indicate that expanding the dissemination of positive information on the internet is beneficial for controlling the number of rumor spread and effectively helping rumor victims out of their predicament.

We plotted R_0 and control variables β and θ the three-dimensional relationship between. In the Fig. 8, in order to visually see the impact of control parameters on the number of thresholds. We plotted the three-dimensional curves of R_0, β and θ. As shown in Fig. 8, R_0 along β as the spread of rumors increases, the number of disseminators and victims will also increase. R_0 along θ reduce the spread of rumors and the number of rumor victims. This means that effective prevention can better control the spread of rumors and effectively help rumor victims escape the current predicament. Further confirms the significance of the control strategy proposed in this article.

VI. Conclusions

This study investigates rumor propagation dynamics from the perspective of affected populations. We developed an enhanced epidemiological model incorporating the influence of positive information on both rumor victims and disseminators. Through analytical methods including next-generation matrix calculation for basic reproduction number, Routh-Hurwitz stability criterion, and Lyapunov function construction, we systematically verified the existence and stability of equilibrium points. Comparative numerical simulations with classical SIR models revealed that our proposed framework demonstrates superior alignment with real-world rumor propagation patterns. The results indicate that expanding populations of rumor disseminators significantly increase victimization risks, suggesting that enhancing the dissemination rate of positive information serves as an optimal control strategy to mitigate transmission velocity and reduce victim scale. Our findings demonstrate that proactive dissemination of verified positive information through digital platforms substantially assists affected individuals in overcoming challenges while effectively containing rumor proliferation. This modeling approach not only advances understanding of novel rumor transmission mechanisms but also elevates societal awareness regarding vulnerable populations in information epidemics.

However, the current research exhibits certain limitations. The model does not account for rumor typology variations, audience demographics, credibility assessment mechanisms, or regulatory impacts from social media platforms. Future investigations should explore these multidimensional factors influencing both disseminators and victims. Furthermore, comprehensive intervention strategies should be developed to alleviate psychological burdens and facilitate rehabilitation processes for rumor-affected individuals.

References

[1] G. Jiang, S. Li, M. Li, "Dynamic rumor spreading of public opinion reversal on weibo based on a two-stage SPNR model," *Physica A: Statistical Mechanics and its Applications*, vol. 558, pp. 0378-4371, 2020.

[2] L.Zhang, K.Chen, H. Jiang, J.Zhao, "How the health rumor misleads people's perception in a public health emergency: lessons from a purchase craze during the COVID-19 outbreak in China," *International Journal of Environmental Research and Public Health*, vol. 17, p. 7213, 2020.

[3] J. Kostka, Y. A. Oswald, R. Wattenhofer, "Word of mouth: Rumor dissemination in social networks," *Structural Information and Communication Complexity*, vol. 15, pp. 185–196, 2008.

[4] D. Han, M. Sun, "An evolutionary vaccination game in the modified activity driven network by considering the closeness," *Physica A: Statistical Mechanics and its Applications*, vol. 443, pp. 49–57, 2016.

[5] P. Manshour, A. Montakhab, "Contagion spreading on complex networks with local deterministic dynamics," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, pp. 2414–2422, 2014.

[6] W. Liu, C. Liu, Z. Yang, X. Liu, Y. Zhang, Z. Wei, "Modeling the propagation of mobile malware on complex networks," *Communications in Nonlinear Science and Numerical Simulation*, vol. 37, pp. 249–264, 2016.

[7] Sukanya Isuntier, and Ekkachai Kunnawuttipreechachan, "Stability Analysis of the Vector-Host Epidemic Model for Cholera," *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists*, 14-16 March, Hong Kong, pp. 446-449, 2018.

[8] Jianke Zhang, Zhuoqing Zhang, Chang Zhou, and Xiaojue Ma, "A New SIQR Model and Residual Power Series Method in Wireless Sensor Networks," *IAENG International Journal of Computer Science*, vol.51, no. 9, pp. 1240-1249, 2024.

[9] Nursanti Anggriani, Asep Kuswandi Supriatna, Meksianis Zadrak Ndii, Khaerunisa, Rika Amelia Wahyu Suryaningrat, and Mochammad Andhika Aji Pratama, "Dynamical Analysis and Optimal Control in Zika Disease Transmission Considering Symptomatic and Asymptomatic Classes," *Engineering Letters*, vol.31, no. 1, pp. 246-254, 2023.

[10] D. J. Daley, D. G. Kendall, "Epidemics and rumours," *Nature*, vol. 204, pp. 1118, 1964.

[11] D. J. Daley, D. G. Kendall, "Stochastic rumours," *IMA Journal of Applied Mathematics*, vol. 1, pp. 42–55, 1965.

[12] L. Zhu, H. Zhao, "Dynamical behaviours and control measures of rumour spreading model with consideration of network topology," *International Journal of Systems Science*, vol. 48, pp. 2064–2078, 2017.

[13] S. Chen, H. Jiang, L. Li, J. Li, "Dynamical behaviors and optimal control of rumor propagation model with saturation incidence on heterogeneous networks," *Chaos, Solitons & Fractals*, vol. 140, p. 110206, 2020.

[14] Qiujuan Tong, Shengqi Yue, Jianke Zhang, Yihan Liu, and Zheyu Han, "A SEInIeR Cyber Public Opinion Propagation Prediction Model with Extreme Emotion Mechanism," *IAENG International Journal of Computer Science*, vol.51, no. 5, pp. 477-488, 2024.

[15] Nsikan Nkordeh, Oluyinka Oni, Felix Olowononi, and Ibinabo Bob-Manuel, " Impact of Social Media on The Reading Culture of Nigerian Youths; Is the Social Media platform providing an Alternative Source of Education?," *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science*, 25-27 October, San Francisco, USA, pp. 223-226, 2017.

[16] Laxman Bahadur Kunwar, and Vijai Shanker Verma, "Mathematical Analysis of Alcoholism with Effect of Awareness through Media in Developing Countries," *Engineering Letters*, vol. 31, no.1, pp. 295-304, 2023.

[17] L. Huo,Y. Cheng, C. Liu, F. Ding, "Dynamic analysis of rumor spreading model for considering active network nodes and nonlinear spreading rate," *Physica A: Statistical Mechanics and its Applications*, vol. 506, pp. 24–35, 2018.

[18] T. Li, Y. Guo, " Nonlinear dynamical analysis and optimal control strategies for a new rumor spreading model with comprehensive interventions," *Qualitative theory of dynamical systems,* vol. 20, pp. 1–24, 2021.

[19] A. Zubiaga, M. Liakata, R. Procter, G. Wong Sak Hoi, P. Tolmie, " Analysing how people orient to and spread rumours in social media by looking at conversational threads," *Plos One,* vol. 11, p. e0150989, 2016.

[20] Y. Hu, Q. Pan, W. Hou, M. He, "Rumor spreading model with the different attitudes towards rumors," *Physica A: Statistical Mechanics and its Applications*, vol.502, pp. 331–344, 2018.

[21] L. Bode, E. K. Vraga, "In related news, that was wrong: The correction of misinformation through related stories functionality in social media," *Journal of Communication*, vol. 65, pp. 619–638, 2015.

[22] L. Huo, T. Lin, C. Fan, C. Liu, J. Zhao, " Optimal control of a rumor propagation model with latent period in emergency event," *Advances in Difference Equations*, vol.2015, no. 54, pp. 1–19, 2015.

[23] J. Wang, L. Zhao, R. Huang, "SIRaRu rumor spreading model in complex networks," *Physica A: Statistical Mechanics and its Applications*, vol. 398, pp. 43–55, 2014.

[24] K. Hattaf, A. Lashari, Y. Louartassi, N. Yousff, "A delayed SIR epidemic model with a general incidence rate," *Electronic Journal of Qualitative Theory of Differential Equations*, vol. 2013, no. 3, pp. 1–9, 2013.

[25] L. Zhao, Q. Wang, J. Cheng, Y. Chen, J. Wang, W. Huang, "Rumor spreading model with consideration of forgetting mechanism: A case of online blogging live journal," *Physica A: Statistical Mechanics and its Applications*, vol.390, pp. 2619–2625, 2011.

[26] L. Zhao, J. Wang, Y. Chen, Q. Wang, J. Cheng, H. Cui, " SIHR rumor spreading model in social networks," *Physica A: Statistical Mechanics and its Applications*, vol. 391, pp. 2444–2453, 2012.

[27] S. Sun, H. Liu, J. He, X. Du, "Detecting event rumors on sina weibo automatically," *Web Technologies and Applications: 15th Asia-Paciffc Web Conference*, vol. 15, pp. 120–131, 2013 .

[28] M. Del Vicario, A. Bessi, F. Zollo, F. Petroni, A. Scala, G. Caldarelli,et al., "The spreading of misinformation online," *Proceedings of the national academy of Sciences*, vol. 113, no. 3, pp. 554–559, 2016.

[29] W. Pan, W. Yan, Y. Hu, R. He, L. Wu, G. Rasool, "Dynamic analysis and optimal control of rumor propagation model with reporting effect," *Advances in Mathematical Physics*, vol. 2022, pp. 1–14, 2022.

[30] S. Kang, X. Hou, Y. Hu, H. Liu, "Dynamical analysis and optimal control of the developed information transmission model," *PLOS ONE,* vol. 17, p. e0268326, 2022.

[31] S. Kang, X. Hou, Y. Hu, H. Liu, "Dynamic analysis and optimal control of a stochastic information spreading model considering super-spreader and implicit exposer with random parametric perturbations," *Frontiers in Physics*, vol. 11, p. 377, 2023.

[32] J. P. Lasalle, "The stability of dynamical systems," *Society for Industrial and Applied Mathematics*, pp. 1096-1105, 1976.

[33] J. J. Anagnost, C. A. Desoer, "An elementary proof of the routh-hurwitz stability criterion," *Circuits, systems and signal processing*, vol.710, no. 10, pp. 101–114, 1991.