Research on a New Pre-defined Time Sliding Mode Control Method for Nonlinear Systems

Wenhui Zhang, Rui Chen, Haojia Chen, Zhangping You, Yi Zhao

Abstract—**For the problem of predefined-time synchronization in nonlinear systems, a novel predefined-time sliding mode control scheme is proposed to enable rapid and stable synchronization of nonlinear systems within a predefined time. Initially, the control model of the nonlinear system is analyzed and established based on Lyapunov stability theory. Subsequently, a sliding mode surface and a predefined-time controller are designed, and a detailed analysis of the convergence properties of the controller within the predefined time is conducted to ensure that the system synchronization error converges within the predetermined timeframe. Finally, numerical simulations validate the proposed scheme, demonstrating a clear advantage in synchronization time over traditional finite-time control, fixed-time control, and conventional predefined-time control schemes, while also showcasing superior synchronization performance.**

Index Terms—**Predefined-time control scheme, Sliding mode control, Uncertain parameter, Nonlinear system**

I. Introduction

In the field of control engineering, ensuring that a system achieves its expected performance or state within a specific timeframe is a core objective. This is particularly valuable in the synchronization control of nonlinear systems, where precise control over the system reaching a synchronized state within a predetermined time is highly desirable [1]-[4]. This paper introduces a novel predefined-time sliding mode control scheme aimed at effectively synchronizing nonlinear systems by integrating sliding mode control techniques with predefined-time control strategies [20]- [22].

Traditional sliding mode control methods are widely utilized for controlling various uncertain and disturbed systems due to their robustness and precision. However, these methods typically rely on uncertainties in system response times and cannot guarantee that control objectives will be achieved within a precise, preset timeframe [11]- [16]. To overcome these limitations, researchers have introduced predefined-time control techniques, which allow control designers to directly set the maximum time for the system to reach a stable state.

Predefined-time control offers advanced stability characteristics for controlled systems by allowing for easy determination of the stability upper bounds through the adjustment of predefined-time parameters, thus providing a high degree of certainty in system behavior. In reference [5], a novel predefined-time convergent continuous control algorithm was presented for stabilizing permanent-magnet synchronous motors (PMSMs) under various disturbances. Unlike previous methods that did not allow the convergence time to be predetermined, this approach set the stabilization time a priori, enhancing predictability and robustness. The algorithm addressed three scenarios: systems without disturbances, systems with deterministic disturbances adhering to a Lipschitz condition, and systems with both deterministic disturbances and stochastic noises. Demonstrated through numerical simulations, this methodology effectively maintained predefined-time convergence without requiring exact knowledge of initial conditions or suffering from exponential control growth. In reference [6], a novel controller was introduced for stabilizing second-order vector systems within a predefined time, enhancing control predictability and system robustness. The controller utilized a variable structure approach, initially guiding system trajectories to a linear manifold and subsequently to a non-smooth manifold, both within predefined times. This dual-phase approach effectively circumvented issues of differentiability typically encountered with high-order systems in finite-time stabilization. Demonstrated through numerical simulations, including an application to a two-link planar manipulator, the controller's efficacy was validated, showcasing its practical application in precise trajectory tracking of fully actuated mechanical systems. Furthermore, In reference [7], explored Lyapunov-like conditions for predefined-time stability in dynamical systems, providing a unified framework that included existing theorems and extended to ultimate boundedness in uncertain scenarios. It introduced methods to determine stabilization timing, facilitating robust controller design for systems to achieve stability within specified timeframes. In reference [8], a prescribed-time tracking control method was presented for MIMO nonlinear systems

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with persistent uncertainties, extending the application of prescribed-time control to complex multi-input multi-output systems. This approach guaranteed precise tracking within a predetermined timeframe, regardless of initial conditions or parameter variations, significantly enhancing control robustness and predictability for critical applications such as missile interception and precision handling. In reference [9], a new predefined-time sliding mode control scheme was presented for achieving rapid synchronization of complex multi-wing chaotic systems. This method, proven through Lyapunov stability theory, enhanced the robustness of synchronization by enabling controller parameters and sliding mode surfaces to be set in advance, ensuring a shorter synchronization time compared to existing methods. It offered significant advancements in control predictability and efficiency. In reference [10], a predefined-time control approach was presented for distributed-order systems, extending fractional calculus to enable more precise modeling of complex phenomena. The control design guaranteed predefined-time convergence of solutions for distributed-order dynamical systems under any initial conditions. A robust controller and a predefined-time sliding mode design were developed to manage uncertainties within these systems. Numerical simulations validated the effectiveness of this method, demonstrating its ability to enforce rapid convergence and handle the slow dynamics typically associated with distributed-order systems.

Addressing the predefined-time synchronization issue for nonlinear systems, the study first establishes a new sliding mode control framework through predefined-time stability theory, which not only maintains the high robustness characteristic of sliding mode control but also ensures that the nonlinear system achieves expected dynamic performance under strict time constraints. Subsequently, a corresponding sliding mode controller was designed, and its predefined-time stability was demonstrated using a Lyapunov function. Finally, through numerical simulation experiments, the proposed control scheme was compared with existing predefined-time, fixed-time, and finite-time control schemes, thereby validating the effectiveness and superiority of the proposed control strategy.

II. Problem Statement and Preliminaries

A. System Description

The general expression for a nonlinear multiple-input-multiple-output(MIMO) system of order n is as follows:

$$\begin{cases} \dot{x}_1 = f_1(x,t) + g_1(x,t)u_1 + d_1(x,t) \\ \dot{x}_2 = f_2(x,t) + g_2(x,t)u_2 + d_2(x,t) \\ \vdots \\ \dot{x}_n = f_n(x,t) + g_n(x,t)u_n + d_n(x,t) \\ y = x \end{cases} \tag{1}$$

Where, $x = [x_1, x_2, ..., x_n]^T$, $u = \left[u_1, u_2, \cdots, u_n\right]^T$ are state variables and control input vectors respectively; $f_i(x)$ and $g_i(x) \neq 0$, $i = 1, 2, \ldots n$ are known smooth functions; In nonlinear system (1), $d = \left[d_1, d_2, \ldots, d_n\right]^T$ represents the sum of all external disturbances and system uncertainties, And this vector d is satisfying assume 1.

Assume 1: In the system (1), all perturbations and uncertainties are constrained within a bounded range, expressed as the inequality $\left|d_i\right| \le D_i, i = 1, 2, \ldots, n$, where the upper bound $D_i, i = 1, 2, \ldots n$ is predetermined.

The tracking error is defined as $e_i = x_i - x_{di}$, $i = 1, 2, \ldots, n$ the described tracking problem is transformed into the stabilization problem of the nonlinear system corresponding to the tracking error in a predefined time [23]-[24].

$$\dot{e}_i = f_i(x) + g_i(x)u + d_i - \dot{x}_{di} \tag{2}$$

B. Key Definitions and Lemmas

Definition 1: Global finite-time stability[17]

Assume that the origin of the system (2) is globally asymptotically stable, and that any solution $e(t, e_0)$ in the system (2) can converge to the equilibrium point in a finite time.

$$\lim_{t \to T(e_0)} \|e(t)\| = \lim_{t \to T(e_0)} \|x - x_{di}\| = 0 \tag{3}$$

The origin of the error system (2) is globally finite-time stable. That is, $\|e(t)\| \equiv 0$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is called the settling time function; Then, the origin of system (2) has global finite-time stability. Therefore, system (1) can achieve global finite time synchronization.

Definition 2: Fixed time stability[18]

It is assumed that the origin of system (2) is globally finite time stable, and its settling time $T(e_0)$ is globally bounded. That is, there is a constant $T_{\max} > 0$ such that for all conditions $e_0 \in \mathbb{R}^n$, the settling time $T(e_0) \le T_{\max}$. Therefore, system (1) can achieve global fixed time synchronization.

Definition 3: Predefined time stability[19]

Assume that the origin of system (2) is fixed time stable, and its settling time $T(e_0)$ satisfies $T(e_0) \le T_c$ for all $e_0 \in \mathbb{R}^n$, where $T_c > 0$ is a constant related to the parameters of system (2), called predefined time. Therefore, the nonlinear system (1) is able to achieve globally predefined time synchronization.

III. A New Pre-defined Time Sliding Mode Control Scheme for Synchronous Nonlinear Systems is Proposed

A. A New Sufficient Condition for Predefined Time Synchronization Lyapunov Functions

Theorem 1: Given a constant $T_c > 0$, if the system $\dot{x}(t) = \phi(t, x)$ has an unbounded Lyapunov function $V(x)$, the following condition holds:

$$\dot{V} \le -\frac{1}{T_c}\frac{1}{\alpha}(V^{1+\alpha} + 4V + V^{1-\alpha}) \tag{4}$$

Therefore, the system $\dot{x}(t) = \phi(t, x)$ is pre-defined time stable, where $T_c > 0$ is the pre-defined time, and $\alpha \in (0,1)$ is the parameter that can be preset. The convergence time of the system can be flexibly adjusted according to actual requirements. It follows that $T(x_0) \le T$ and $T = \sup T(x_0)$. This satisfies the condition for pre-defined time synchronization in definition 2.3, so equation (4) can be considered a sufficient condition for the new pre-defined time Lyapunov.

Derived and calculated from equation (4), the result is obtained

$$T(x_0) \le -\frac{1}{T_c}\frac{1}{\alpha}\frac{dV}{V^{1+\alpha}+4V+4V^{1-\alpha}}$$
$$= -\frac{1}{T_c}\frac{1}{\alpha}\int_{V_0}^{Vt_f}\frac{dV}{V^{1-\alpha}(V^{2\alpha}+4V^{\alpha}+4)}$$
$$= -\frac{1}{T_c}\frac{1}{\alpha}\int_{V_0}^{Vt_f}\frac{2V^{\frac{\alpha}{2}}dV^{\frac{\alpha}{2}}}{V^{2\alpha}+4V^{\alpha}+4}$$
$$= T_c\left(\frac{1}{2}-\frac{1}{2+V_0^{\alpha}}\right)$$
$$\le T_c \tag{5}$$

Where, $T(x_0) \le T$ and $T = \sup T(x_0)$.

Remark 1: Theorem 1 satisfies the predefined-time synchronization conditions outlined in definition 3 and introduces a new Lyapunov function. If the system is replaced with error system (2), the nonlinear system can achieve synchronization within the predefined time T_c .

B. Design A New Sliding Surface

To stabilize the error system (3) within a predefined time, the sliding surface can be designed as follows:

$$s_i = e_i + \int_0^t \eta_3 sign(e_i)|e_i|^{1-2\alpha}d\theta + \int_0^t \eta_2 sign(e_i)|e_i|^{1+2\alpha}d\theta$$
$$+\int_0^t \eta_4 sign(e_i)d\theta + \int_0^t \eta_1 e_i d\theta \tag{6}$$

Where, $\alpha \in (0,1)$, $\eta_1 = \frac{1}{T_{c1}}\frac{2}{\alpha}$, $\eta_2 = \frac{1}{T_{c1}}\frac{1}{\alpha}\left(\frac{1}{2}\right)^{1+\alpha}$, $\eta_3 = \frac{1}{T_{c1}}\frac{4}{\alpha}\left(\frac{1}{2}\right)^{1-\alpha}$, $\eta_4 > 0$.

Theorem 2: If the error system utilizes sliding surface (6), it will converge to zero on the sliding surface within the predefined time.

When the error system (2) is on the sliding surface, its motion will be constrained to the surface defined by $S(t) = \{e_i \mid s_i(e_i) = 0\}$, with $\dot{s}(e) = 0$ being satisfied. Therefore

$$\dot{s} = \dot{e}_i + \eta_1 e_i + \eta_2 sign(e_i)|e_i|^{1+2\alpha} + \eta_4 sign(e_i)$$
$$+\eta_3 sign(e_i)|e_i|^{1-2\alpha}$$
$$= 0 \tag{7}$$

By manipulating the formula, the following result can be derived:

$$\dot{e}_i = -\left[\eta_1 e_i + \eta_2 sign(e_i)|e_i|^{1+2\alpha} + \eta_3 sign(e_i)|e_i|^{1-2\alpha}\right.$$
$$\left.+\eta_4 sign(e_i)\right] \tag{8}$$

If

$$V = \frac{1}{2}e_i^2$$

Then

$$\dot{V} = e_i\dot{e}_i$$
$$= -e_i\left[\eta_1 e_i + \eta_2 sign(e_i)|e_i|^{1+2\alpha} + \eta_3 sign(e_i)|e_i|^{1-2\alpha}\right.$$
$$\left.+\eta_4 sign(e_i)\right]$$
$$= -\left(\eta_1 e_i^2 + \eta_2|e_i|^{2+2\alpha} + \eta_3|e_i|^{2-2\alpha} + \eta_4|e_i|\right)$$
$$\le -\left(\eta_1 e_i^2 + \eta_2|e_i|^{2+2\alpha} + \eta_3|e_i|^{2-2\alpha}\right)$$
$$\le -\frac{1}{T_c}\frac{1}{\alpha}\left(4V + V^{1+\alpha} + 4V^{1-\alpha}\right) \tag{9}$$

This satisfies theorem 1. Therefore, when the error system is on the sliding mode surface, it will converge to zero within the predefined time; hence, the nonlinear system will synchronize within the predetermined time T_{c1} .

C. A Novel Controller Design Has Been developed

To synchronize the nonlinear system within a predefined time, equation (6) has been selected as the sliding surface. Subsequently, the control input can be designed as

$$u_i = \frac{1}{g_i(x)}\left[-f_i(x) - D_i sign(s_i) + \dot{x}_{di}\right.$$
$$-(\eta_1 e_i + \eta_2 sign(e_i)|e_i|^{1+2\alpha} + \eta_3 sign(e_i)|e_i|^{1-2\alpha}$$
$$+\eta_4 sign(e_i)) - (\eta_5 s_i + \eta_6 sign(s_i)|s_i|^{1+2\alpha}$$
$$\left.+\eta_7 sign(s_i)|s_i|^{1-2\alpha} + \eta_8 sign(s_i))\right] \tag{10}$$

Where, $\alpha \in (0,1)$, $\eta_1 = \frac{1}{T_{c1}}\frac{2}{\alpha}$, $\eta_3 = \frac{1}{T_{c1}}\frac{4}{\alpha}\left(\frac{1}{2}\right)^{1-\alpha}$, $\eta_4 > 0$, $\eta_5 = \frac{1}{T_{c2}}\frac{2}{\alpha}$, $\eta_6 = \frac{1}{T_{c2}}\frac{1}{\alpha}\left(\frac{1}{2}\right)^{1+\alpha}$, $\eta_7 = \frac{1}{T_{c2}}\frac{4}{\alpha}\left(\frac{1}{2}\right)^{1-\alpha}$, $\eta_8 > 0$.

Theorem 3: If the error system employs controller (10), it will reach the sliding surface within the predefined time.

Let the Lyapunov function be $V = \frac{1}{2}s_i^2$. Deriving this yields:

$$\dot{V} = s_i\dot{s}_i$$
$$= s_i(\dot{e}_i + \eta_1 e_i + \eta_2 sign(e_i)|e_i|^{1+2\alpha}$$
$$+\eta_3 sign(e_i)|e_i|^{1-2\alpha} + \eta_4 sign(e_i))$$
$$= s_i(f_i(x) + g_i(x)u + d_i - \dot{x}_{di} + \eta_1 e_i$$
$$+\eta_2 sign(e_i)|e_i|^{1+2\alpha} + \eta_3 sign(e_i)|e_i|^{1-2\alpha} + \eta_4 sign(e_i))$$
$$= s_i(d_i - D_i sign(s_i) - (\eta_5 s_i + \eta_6 sign(s_i)|s_i|^{1+2\alpha} \tag{11}$$
$$+\eta_7 sign(s_i)|s_i|^{1-2\alpha} + \eta_8 sign(s_i)))$$
$$= s_i d_i - D_i|s_i| - (\eta_5 s_i^2 + \eta_6|s_i|^{2+2\alpha}$$
$$+\eta_7|s_i|^{2-2\alpha} + \eta_8|s_i|)$$
$$\le -\frac{1}{T_{c2}}\frac{1}{\alpha}(4V + V^{1+\alpha} + V^{1-\alpha})$$

This satisfies theorem 1. Therefore, the error system will reach the sliding surface within the predefined time T_{c2} .

Remark 2: If the system is replaced with error system (2), the nonlinear system can achieve synchronization within the predefined time T_c .

Combining theorem 1 and utilizing sliding surface (6) along with controller (10), the system error (3) converges to zero within the predefined time. This describes the custom-time sliding mode control scheme proposed in this paper. Subsequently, the superiority of the scheme was demonstrated through simulation experiments.

The flowchart of the predefined-time sliding mode control scheme is shown in Fig. 1.

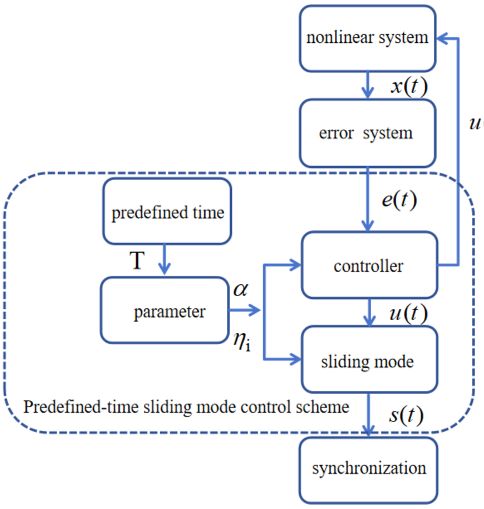

Fig. 1. Flow chart of predefined time sliding mode control scheme.

IV. Numerical simulation

To verify the effectiveness of the predefined-time sliding mode control method designed in this paper, the following nonlinear system is considered:

$$\begin{cases} \dot{x}_1 = \sin(x_1) + u_1 + d_1 \\ \dot{x}_2 = \cos(x_2) + 2\sin(x_1 - x_2) + u_2 + d_2 \\ y = x \end{cases} \tag{12}$$

Where, $f_1(x,t) = \sin(x_1)$, $g_1(x,t) = 1$, $f_2(x,t) = \cos(x_2) + 2\sin(x_1 - x_2)$, $g_2(x,t) = 1$. $d_1 = \sin(t)$ 、 $d_2 = \sin(t)$ represents a bounded external disturbance. The initial state of the system is $x_1 = 4.2$, $x_2 = 2.1$, $x_{d1} = x_{d2} = \frac{\pi}{4}(1 - e^{-2t})$.

A. Comparative experiments of different stabilization time schemes

The purpose of this experiment is to compare three different stabilization time sliding mode synchronization schemes, highlighting the advantages of the predefined time scheme. All schemes are applied to the nonlinear system (12).

Experimental group 1 employs the sliding mode surface (13) and controller (14) as the finite-time sliding mode synchronization scheme. [25]

$$s_i = e_i + \int_0^t \left(\omega_1 e_i + \omega_2 \, \mathrm{si\,gn}(e_i)\right) d\tau \tag{13}$$

$$u_i = \frac{1}{g_i(x)}\Big[-f_i(x) - D_i sign(s_i) + \dot{x}_{di} - \left(\omega_1 e_i + \omega_2 sign(e_i)\right) - \left(\omega_3 s_i + \omega_4 sign(s_i)\right)\Big] \tag{14}$$

Where, ω_1, ω_2, ω_3, $\omega_4 > 0$. (Fig. 2)

Experimental group 2 employs sliding mode surface (15) and controller (16) as the fixed-time sliding mode synchronization scheme. [26]

$$s_i = e_i + \int_0^t \left(\kappa_1 |e_i|^\beta \, \mathrm{si\,gn}(e_i) + \kappa_2 |e_i|^\gamma \, \mathrm{si\,gn}(e_i)\right) d\tau \tag{15}$$

$$u_i = \frac{1}{g_i(x)}\Big[-f_i(x) - D_i sign(s_i) + \dot{x}_{di} - \left(\kappa_1 |e_i|^\beta \, \mathrm{si\,gn}(e_i) + \kappa_2 |e_i|^\gamma \, \mathrm{si\,gn}(e_i)\right) - \left(\kappa_3 |s_i|^\beta \, \mathrm{si\,gn}(s_i) + \kappa_4 |s_i|^\gamma \, \mathrm{si\,gn}(s_i)\right)\Big] \tag{16}$$

Where, $0 < \beta < 1 < r$, κ_1, κ_2, κ_3, $\kappa_4 > 0$. (Fig. 3)

The control group employs sliding mode surface (6) and controller (10) as the predefined-time sliding mode synchronization scheme, as proposed in this paper. (Fig. 4)

To ensure the effectiveness of the comparative experiments, we strive to standardize the parameters of similar projects. Given the ability to define predefined time schemes in advance, we aim to achieve synchronization within 0.2s. Parameters $T = T_c = 0.1$, $\alpha = \beta = 0.5$, and $\gamma = 1.5$ are set accordingly, and the approximation terms for the three schemes are determined as $\omega_1 = \eta_1 = \omega_3 = \eta_5 = 10$, $\kappa_1 = \eta_2 = \kappa_3 = \eta_6 = 40\sqrt[4]{2^3}$, $\kappa_2 = \eta_3 = \kappa_4 = \eta_7 = \frac{40}{\sqrt[4]{2^5}}$, $\omega_2 = \eta_4 = \omega_4 = \eta_8 = 5$.

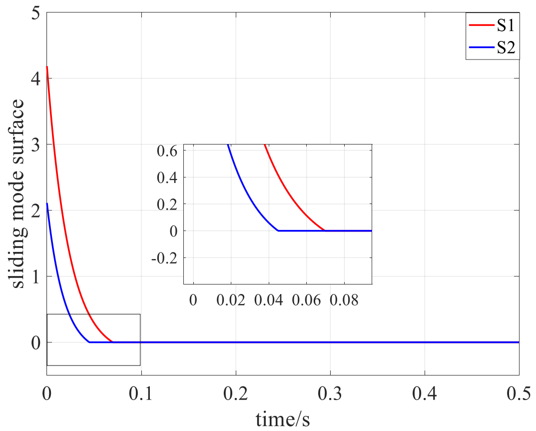

Fig. 2. Sliding mode surface in finite-time control scheme

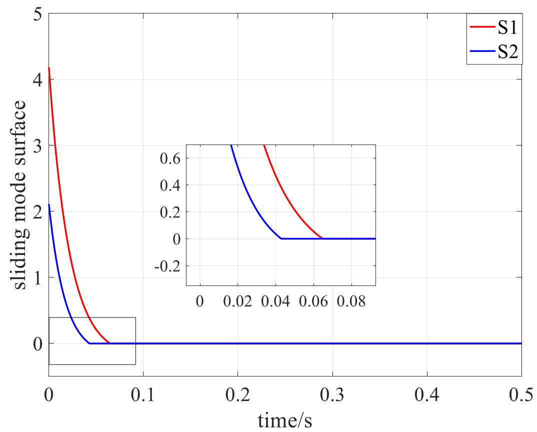

Fig. 3. Sliding mode surface in fixed-time control scheme

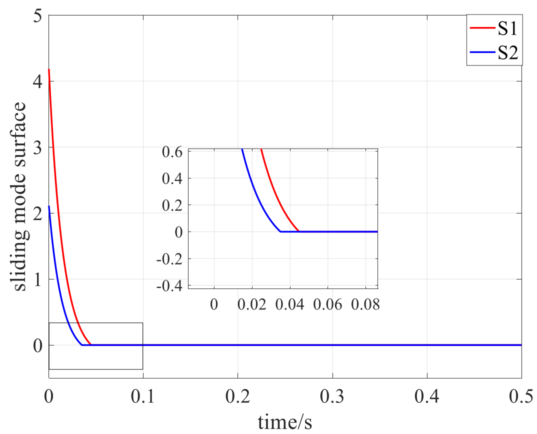

Fig. 4. Sliding mode surface in predefined-time control scheme

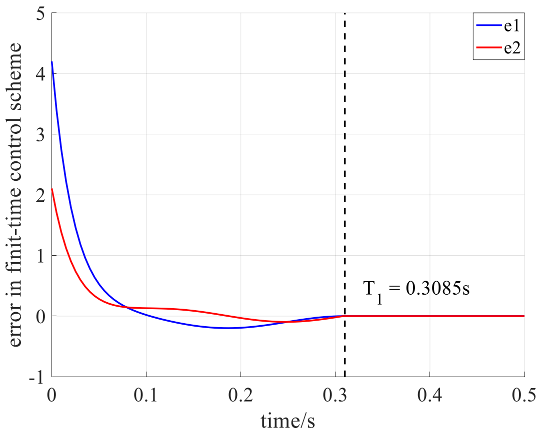

Fig. 5. Error in finite-time control scheme

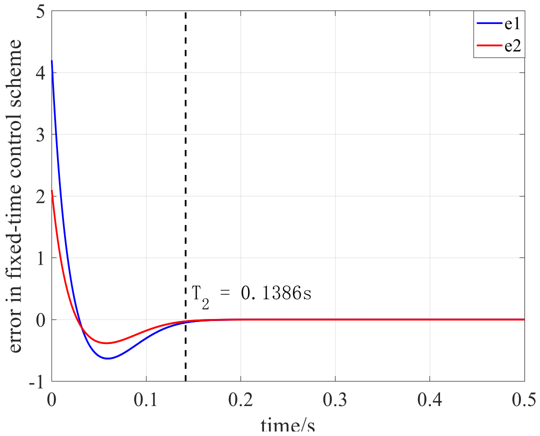

Fig. 6. Error in fixed-time control scheme

The experimental results are shown in Fig. 5, 6, and 7. The finite-time sliding mode control scheme proposed by experimental group 1 achieves a convergence time of $T_1 = 0.3085s$, while the fixed-time sliding mode control scheme proposed by experimental group 2 achieves a convergence time of $T_2 = 0.1386s$. The predefined-time sliding mode control scheme proposed by the control group achieves a convergence time of $T_3 = 0.0695s$. These results indicate that the predefined-time sliding mode control scheme proposed in this paper exhibits a faster convergence speed.

Combining the results from the three experimental groups, the composition of synchronization errors is analyzed as shown in Fig. 8.

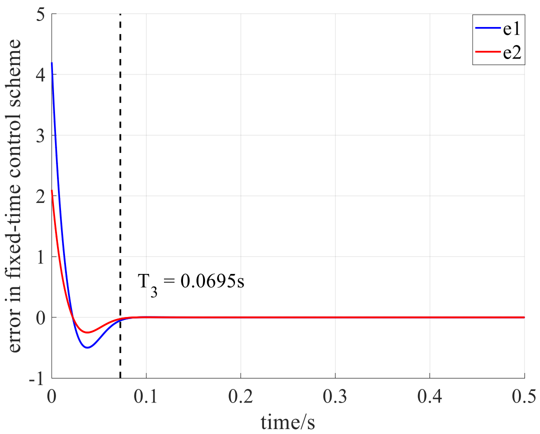

Fig. 7. Error in predefined-time control scheme

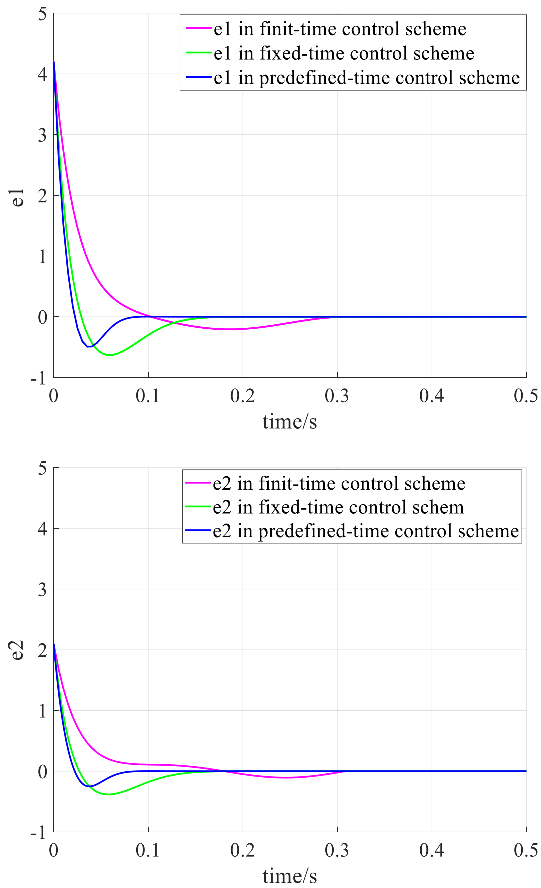

Fig. 8. Evolution of error systems under different control schemes

B. Comparative test of different predetermined time schemes

The experiment compared the traditional predefined-time sliding mode synchronization scheme with the new predefined-time sliding mode synchronization scheme.

Theorem 4: [27]Given a constant $T > 0$, if the system $\dot{x}(t) = \phi(t, x)$ has an unbounded Lyapunov function $V(x)$, the following condition holds:

$$\dot{V} \le -\frac{\pi}{\alpha T}(V^{1-\frac{\alpha}{2}} + V^{1+\frac{\alpha}{2}}) \tag{17}$$

Therefore, the system $\dot{x}(t) = \phi(t, x)$ is pre-defined time stable, where $T > 0$ is the pre-defined time, $\alpha \in (0,1)$.

The experimental group used sliding mode surface (18) and controller (19) as the traditional predefined-time sliding mode synchronization scheme.

$$s_i = e_i + \int_0^t \left(d_1 e_i + d_2 sign(e_i)|e_i|^{1-\lambda} + d_3 sign(e_i)|e_i|^{1+\lambda} \right) \tag{18}$$

$$\begin{aligned} u_i = & \frac{1}{g_i(x)}[-f_i(x) - D_i sign(s_i) + \dot{x}_{di} \\ & -\left(d_1 e_i + d_2 sign(e_i)|e_i|^{1-\lambda} + d_3 sign(e_i)|e_i|^{1+\lambda} \right) \\ & -\left(d_5 s_i + d_6 sign(s_i)|s_i|^{1-\lambda} + d_7 sign(s_i)|s_i|^{1+\lambda} \right)] \end{aligned} \tag{19}$$

Where, $d_1, d_5 > 0$, $d_2 = \frac{\pi}{T_{d_1}\lambda 2^{1-\frac{\lambda}{2}}}$, $d_6 = \frac{\pi}{T_{d_2}\lambda 2^{1-\frac{\lambda}{2}}}$,

$d_3 = \frac{\pi}{T_{d_1}\lambda 2^{1+\frac{\lambda}{2}}}$, $d_7 = \frac{\pi}{T_{d_2}\lambda 2^{1+\frac{\lambda}{2}}}$, $\lambda \in (0,1)$.

Remark 3: Since sliding surface (18) meets the conditions of theorem 4, it is defined as the traditional predefined-time sliding surface. Additionally, system (1) is operating on this sliding surface, $\dot{s}(e) = 0$, therefore,

$$\dot{e}_i = -\int_0^t \left(d_1 e_i + d_2 sign(e_i) |e_i|^{1-\lambda} + d_3 sign(e_i) |e_i|^{1+\lambda} \right) d\tau \qquad (20)$$

If $V = \frac{1}{2} e_i^2$,

$$\dot{V} = e_i \dot{e}_i = -e_i \left(d_1 e_i + d_2 sign(e_i) |e_i|^{1-\lambda} + d_3 sign(e_i) |e_i|^{1+\lambda} \right)$$

$$= -\left(d_1 e_i^2 + d_2 |e_i|^{2-\lambda} + d_3 |e_i|^{2+\lambda} \right) \le -\frac{\pi}{\alpha T} \left(V^{1-\frac{\lambda}{2}} + V^{1-\frac{\lambda}{2}} \right) \qquad (21)$$

In the aforementioned experiments, the parameters for each approximation term were kept consistent. The predefined time was maintained at 0.2s, $T_{d1} = T_{d2} = 0.1s$, $\alpha = \lambda = 0.5s$.

The coefficients for each approximation term will also be determined:

$\eta_1 = d_1 = \eta_5 = d_5 = 10$, $\eta_2 = d_2 = \eta_6 = d_6 = 40\sqrt[4]{2^3}$, $\eta_3 = d_3 = \eta_7 = d_7 = \frac{40}{\sqrt[4]{2^5}}$, $d_4 = d_8 = 5$.

Fig. 9 illustrates the evolution of the synchronization error system. As shown in Fig. 9, the predefined-time sliding mode control scheme proposed in this paper performs optimally while maintaining consistency in system parameters.

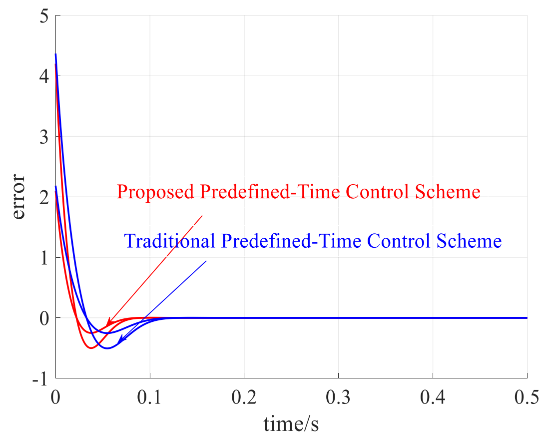

Fig. 9. Comparison of error between traditional predefined-time control scheme and proposed predefined control scheme

V. Conclusion

Based on Lyapunov stability theory and pre-defined time stability theory, the fast synchronization problem of nonlinear systems under sliding mode control studied. First, the sufficient conditions for Lyapunov function to guarantee the stability of state variables in a predefined time are presented. Secondly, according to a new sliding mode variable structure control strategy, a new sliding mode controller is designed to ensure that the state variables converge quickly to the sliding surface and ensure the pre-defined time stability. Finally, numerical simulation results show that the proposed control strategy can achieve rapid convergence of state variables and ensure the pre-defined time stability.

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