Reforming Antenna Engineering Pedagogy: 1D-to-2D Monopole Teaching Using Gain-to-Length Ratio and Circular Disc Monopole Demonstrations at 433 MHz for Improved Learning Outcomes in Higher Education

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Abstract—Conventional teaching omnidirectional monopole antennas relies on image theory but often lacks interpretability regarding the engineering merits of the quarter-wave principle. This study bridges the gaps among fundamental physics, applied research, and higher education by introducing a novel comprehensive performance metric—the Gain-over-Length Ratio (GoLR)—defined specifically for the antenna's radiating element. Furthermore, we extend prior work by numerically investigating the two-dimensional (2D) shaping effects on vertical monopole configurations operating at 433 MHz, a sub-gigahertz frequency widely used in low-earth orbit (LEO) satellite tracking. Under identical grounding conditions, the GoLR is employed to comprehensively evaluate and compare an optimised 1D vertical monopole and an optimised 2D circular disc monopole (CDM). Two new observations are derived and highlighted in this work. First, the CDM antenna rejects the conventional quarter-wave conjecture for the perimeter. Instead, it is the height (diameter) of the CDM that really matters, not the perimeter. Second, the CDM design exhibits a smeared GoLR profile as a function of height (diameter) compared to the 1D monopole, leading to a marginally more compact vertical profile, specifically, a 0.85 cm reduction in height relative to the 1D counterpart. However, this dimensional optimisation comes at a slight cost: a GoLR reduction of 0.04012 dBi/cm. The insights derived from this work-integrating teaching methodologies and engineering applications-mark a significant step toward fostering student-led innovations tailored antenna industry-collaborative, purpose-driven applications.

Index Terms—antenna education, circular disc monopole, antenna gain, engineering pedagogy, engineering physics, microwave engineering, monopole antenna, higher education, radiation efficiency, 433 MHz

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I. INTRODUCTION

NTENNA technology [1-5] teaching in higher Aeducation—particularly within microwave engineering [6–8] curricula—must integrate not only fundamental theory [9] and mathematical foundations [10] but also practical implementation [11] under real-world constraints [12]. Effective pedagogical delivery requires holistic curriculum design, active student engagement [13], and comparative assessment [14] within the broader radiofrequency (RF) engineering framework. For instance, while the ground-based monopole paradigm [15] is widely recognised for its simplicity and compactness compared to the dipole (both resonance principles), implementation demands further footprint minimisation [16][17] without significantly degrading antenna gain [18][19] (as mathematically illustrated in Fig. 1). This optimisation is critical to meet the increasingly stringent demands of 5G (fifth generation) [20] and post-5G [21] wireless systems.

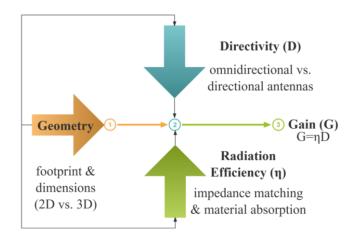


Fig. 1. Key teaching (and design) essentials of monopole antennas, featuring three-pronged approach involving geometry and footprint, directivity (D) and radiating efficiency (η), and ultimately, the comprehensive metric of gain (G).

Among the modalities listed in Fig. 1, the directivity (D) of a monopole antenna, a measure of how effectively the

antenna radiates in a specific direction compared to an isotropic source, is inherently linked to the monopole length, the configuration of the ground plane, and the operational frequency. For a monopole over a perfect ground plane (where the image theory works perfectly), the directivity (D) can be analytically approximated as Eq. (1) in a spherical coordinate with azimuthal angle θ ranging from 0 to 2π , and the zenith angle φ from 0 to π . The measure of directivity physically represents a dimensionless ratio of the peak normalized radiation pattern intensity (in a specific direction, i.e., the main beam) over the average radiation pattern (i.e., among all directions of the field detachment from the antenna into the space, more specifically, in the far field regime):

$$D = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \varphi) \sin\theta d\theta d\varphi},$$
 (1)

where $U(\theta,\phi)$ represents the radiation intensity in the direction of (θ,ϕ) . In an ideal case, a quarter-wavelength monopole exhibits its strongest radiation in the horizontal plane (azimuth), perpendicular to the monopole, with diminishing intensity along the vertical axis (dead zone). In real-world implementations, however, directivity is influenced by practical considerations, e.g., the variability in radial lengths [12], environmental factors, and fabrication tolerances [22][23]. The relationship between these parameters and the antenna's directivity (D) demonstrates a multidimensional dependency, requiring systematic exploration and optimisation.

Radiation efficiency (η) is a power quantification (relative to the input power) after being degraded by the antenna material's absorption and the reflection losses prior to the radiating action (i.e., the field before detachment of the antenna). For a vertical monopole (i.e., a single straight conductor mounted over a conductive ground plane) and the 2D disc monopole (same vertical setup but with a diverse radiator shape), no dielectric materials are used, i.e., only metal (ohmic) loss is consumed. With the same conductor material used (e.g., copper) for different geometry designs, impedance matching (and mismatching) arguably dictates the radiation efficiency (η) of the monopole.

Conventionally, as observed in the mainstream microwave engineering lecture notes, the η is directly given in relation to the forward reflection coefficient S₁₁. There are two-fold concerns about the status. First, the use of the scattering parameters (e.g., S₁₁) usually overlooks the specification of the linear scale or logarithmic scale (dB), which can be misleading to not only students, but also experienced engineers and academics. Second, the intermediate steps for deriving the mathematical relationship between η and S_{11} are missing. In this context, we derive this in detail from Eq. (2) to Eq. (9) below, for the sake of boosting both educational and engineering physics understanding into this subject matter. Here the radiated power is denoted as Prad, the reflected power is denoted as P_{reflect}, the material absorption is denoted as Pa. Note that Pa is zero for perfect electric conductors utilized in this work for constructing monopole antennas (no dielectrics are used). The linear scale of the reflection coefficient is represented by $|S_{11}|_{linear}$, and its more usual decibel (dB) scale is denoted as S₁₁ (dB).

$$\eta = \frac{P_{\text{rad}}}{P_{\text{in}}} = \frac{P_{\text{rad}}}{P_{\text{loss}} + P_{\text{rad}}},\tag{2}$$

$$P_{loss} = P_{reflect} + P_{a}, (3)$$

$$\eta = \frac{P_{\rm rad}}{P_{\rm reflect} + P_{\rm rad}},\tag{4}$$

$$S_{11} (dB) = 10 \log \left(\frac{P_{\text{reflect}}}{P_{\text{reflect}} + P_{\text{rad}}} \right) = 20 \log |S_{11}|_{\text{linear}}, \quad (5)$$

$$\frac{P_{\text{reflect}}}{P_{\text{reflect}} + P_{\text{rad}}} + \frac{P_{\text{rad}}}{P_{\text{reflect}} + P_{\text{rad}}} = 1, \tag{6}$$

$$\eta = \frac{P_{\text{rad}}}{P_{\text{reflect}} + P_{\text{rad}}} = 1 - \frac{P_{\text{reflect}}}{P_{\text{reflect}} + P_{\text{rad}}},$$
 (7)

$$\frac{P_{\text{reflect}}}{P_{\text{reflect}} + P_{\text{rad}}} = |S_{11}|_{\text{linear}}^2 = 10^{\frac{S_{11} \text{ (dB)}}{10}},$$
(8)

$$\eta = 1 - |S_{11}|_{linear}^2 = 1 - 10^{\frac{S_{11} (dB)}{10}}$$
 (9)

In this regard, the antenna gain (G), defined as G=\nD as per Fig. 1, is arguably dictated by the impedance-matching [24] or mismatching [25] status and the directivity (D) [15] of the vertical monopole designs (valid for both 1D and 2D). Based on these assumptions (theoretically valid), explorative learning and engineering demonstration-based teaching of quarter-wave monopole antennas in higher-education (Beijing Institute of Technology) is carried out jointly in an undergraduate elective module entitled States of the Arts in Liquid Crystals Millimeter-wave Technology for 6G (run in 2024), as well as a third-year undergraduate compulsory module entitled Frontiers and Progress of Electrical and Computer Engineering in 2025 from February to April.

By introducing a new Gain-over-Length-Ratio (GoLR) concept, mimicking the figure-of-merit [26][27] in other engineering sectors, students and nonspecialists can develop a transferable understanding of the comprehensive performance and footprint trade-off among different antenna structures. The engineering insights newly built up in this work complement the conventionally taught classical physics-based solutions (image theory), and march toward a complete (unified) science-based solution to the key antenna paradigm (i.e. monopole) that underpins wide-ranging application fields.

II. REJECTION OF PERIMETER CONJECTURE FOR CDM

Since monopole (1D vertical radiator) and its variation of CDM (2D vertical radiator) are resonant-based antennas, the 1D monopole's principle of relying on current distributions along its length (odd multiples of a quarter wavelength) for efficient radiation is reasonably transferable to the study into CDM (2D vertical radiator) in this section.

Diving into the optimally designed CDM involves a few steps for students-led explorative learning. First, a reasonable Perimeter Conjecture on the CDM is assigned for dedicated students to validate, i.e., we deduce that the CDM antenna functions similarly (length effect) by the perimeter equaling a quarter-wavelength (i.e., an initial conjecture as depicted in Fig. 2), inferring the diameter of 5.17 cm for the CDM by design.

With this myth in mind, students carry out a series of MATLAB simulations, signal processing, and results analysis with a hands-on guide drawn from the modelling practice documented and published for a vertical monopole antenna with the same tool [12][15]. The sub-GHz ISM band 433 MHz (license-free) is taken as an example, serving various real-world scenarios, e.g., LoRa [28] and walkie-talkies [29].

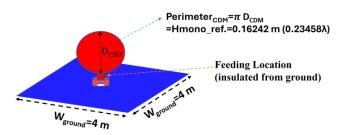


Fig. 2. Design conjecture based on vertical CDM's perimeter equaling the vertical quarter-wavelength monopole's height.

Accordingly, the scattering parameter (forward reflection coefficient S₁₁) is quantified by parameterising the designed diameter of the CDM's radiator for evaluating the impedance matching. Arguably, the quarter-wave Perimeter Conjecture serves a counterexample on the impedance match (i.e., highly mismatched), as evidenced in Fig. 3 for the return loss result of 0.08727 dB (i.e., almost all the input power is reflected) for the CDM's diameter design of 5.17 cm (corresponding to the Perimeter Conjecture of monopole) at 433 MHz.

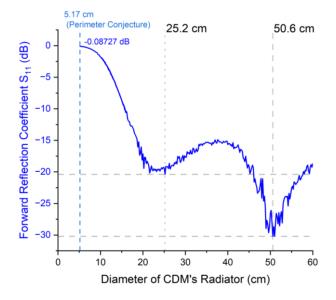


Fig. 3. MATLAB simulation results of forward reflection coefficient at 433 MHz for Circular Disc Monopole (CDM) antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the quarter-wave Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

The impedance matching status improves (i.e., return loss reduces) with the increase of the CDM radiator's diameter to 25.2 cm (i.e., its perimeter corresponding to 1.14342 wavelengths), with a return loss of 20.38337 dB reported in Fig. 3. Afterwards, the increase of the diameter leads to fluctuations in S₁₁, with the optimal impedance-matching diameter observed at 50.6 cm (i.e., the perimeter

corresponding to 2.29591 wavelengths), with the return loss dropping to 30.20624 dB for this design at 433 MHz. To evaluate the wider band performance of the three cases discussed, Figure 4 captures the frequency response of the reflection coefficients, compared among the three designs mentioned across the spectrum from 390 MHz to 476 MHz, i.e., $\pm 10\%$ of the central frequency of 433 MHz.

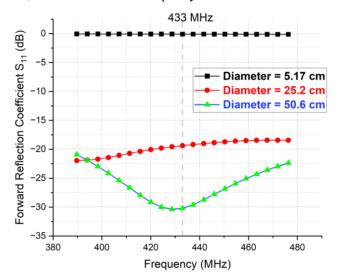


Fig. 4. MATLAB simulation results of forward reflection coefficient across 390 MHz to 476 MHz for Circular Disc Monopole (CDM) antennas with a unified ground size of 4 m by 4 m (5.77 wavelengths) and different diameters of the radiator.

On the same space parameterisation range as per Fig. 2 for the CDM radiator's diameter, the directivity results of the antenna (derived from the 3D radiation pattern) are obtained in Fig. 5. Notably, the diameter under the Perimeter Conjecture gives a maximum directivity of 6.05548 dBi. The increase of the diameter from this boundary to the 33 cm witnesses an increase of the maximum directivity to 6.82379 dBi (occurring when the perimeter reaches 1.49733 wavelengths at 433 MHz). Interestingly, for the further increase in the diameter, there is a slump in the directivity to 5.27226 dBi (for the diameter of 44.4 cm, corresponding to the perimeter of 2.01459 wavelengths at 433 MHz).

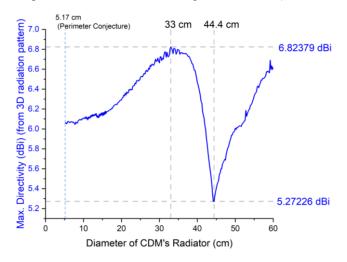


Fig. 5. MATLAB simulation results of maximum directivity at 433 MHz for Circular Disc Monopole (CDM) antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the quarter-wave Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

As per equations (2)-(9), the radiation efficiency is analytically derived for the CDM designs with the radiator's diameter parameterised from the same range as per Fig. 5, i.e., from 5.17 cm to 60 cm. The results are benchmarked and justified with those obtained by MATLAB simulation as shown in Fig. 6, i.e., agreement between theoretical and simulated results is well evidenced. Notably, there is an extremely low (nearly zero) radiation efficiency for the Perimeter Conjecture case (diameter of 5.17 cm), indicating a total reflection as analysed before in Fig. 3. With the rise in the diameter, radiation efficiency starts increasing. When the CDM's diameter of the radiator rises to 19.2 cm (perimeter corresponding to 0.87117 wavelength), the radiation efficiency starts levelling off at 98.162%. A comprehensive results presentation is graphed in Fig. 7, involving the radiation efficiency and the return loss together for ease of understanding. In summary, the results reject the presence of the quarter-wave-based Perimeter Conjecture for the CDM.

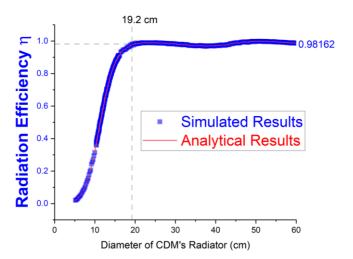


Fig. 6. MATLAB simulation results vs. analytical results of radiation efficiency at 433 MHz for Circular Disc Monopole (CDM) antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the quarter-wave Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

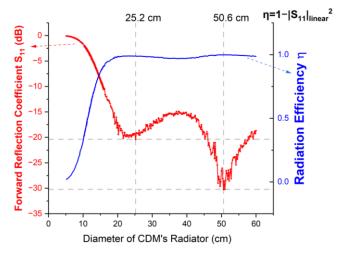


Fig. 7. MATLAB simulation results of return loss (left axis) and radiation efficiency (right axis) at 433 MHz for Circular Disc Monopole (CDM) antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the quarter-wave Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

III. 2D CDM vs. 1D Monopole and Proposal of GoLR

Summarising the results from last section and comprehending the impedance-mismatched quarter-wave Perimeter Conjecture (rejection for use) further, a generalised parameterisation of the radiator's height is conducted for both the CDM and the vertical monopole, in the format of D_{CDM} (diameter of the CDM's radiator, instead of the perimeter) for the CDM, and H_{mono} for the length or height of the 1D vertical monopole, as per Fig. 8, i.e., both antennas are with a unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter (height) parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

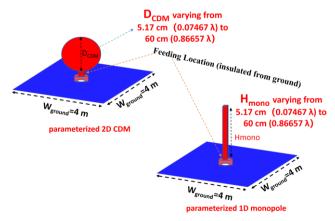


Fig. 8. Comparative parameterisation study of vertical monopole (1D) with vertical CDM (2D) in the vertical plane. The same grounding material and geometry (4 m by 4 m) are assumed for both designs.

A. Gain and GoLR Parameterised with Diameter of CDM

The comprehensive radiation metric of gain (G) is obtained by multiplying the radiation efficiency (η) by the directivity (D) as obtained earlier. From the results shown in Fig. 9, the gain (G) largely follows the trend of radiation efficiency (as quantified and discussed in Fig. 7).

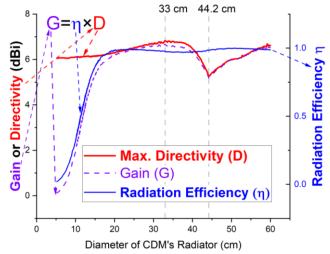


Fig. 9. MATLAB simulation results of gain and directivity (left axis) and radiation efficiency (right axis) at 433 MHz for Circular Disc Monopole (CDM) antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

To be more specific, gain reaches the maximum at CDM's radiator diameter of 33 cm, i.e., further increase of the diameter results in no enhancement but a decline of the gain, as illustrated in a local minimum of radiation efficiency as well as the gain at the radiator's diameter of 44.2 cm. To comprehensively capture this geometry-wise performance impact, the gain-over-diameter-ratio (GoDR) is proposed in this work, by taking the ratio of the antenna gain (G) over the diameter of the CDM's radiator here. The results are depicted in Fig. 10, wherein the maximal GoDR of the CDM (0.35572 dBi/cm) occurs at the CDM radiator's diameter of 15.32 cm (0.22127 wavelength), i.e., slightly shorter (lowered-profile) than the 1D quarter-wave monopole benchmark with a referenced height of 16.27 cm (0.2349 wavelength) [12][15] at 433 MHz.

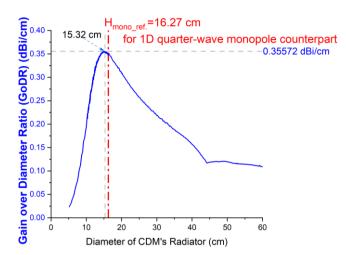


Fig. 10. MATLAB simulation results of gain-over-diameter-ratio (GoDR in dBi per cm) at 433 MHz for Circular Disc Monopole (CDM) antennas with a unified ground size of 4 m by 4 m (5.77 wavelengths) and the diameter parameterised from 5.17 cm (perimeter of 0.23458 wavelengths at 433 MHz, i.e., corresponding to the Perimeter Conjecture of monopole) to 60 cm (perimeter of 2.72242 wavelengths at 433 MHz).

B. Parameterised Radiator's Height of 1D Monopole

Interestingly, MATLAB simulation results on the 1D vertical monopole as a benchmark are reported in this section. A similar calculation procedure (parameterisation of the vertical scale from 5.17 cm to 60 cm, corresponding to 0.07467 wavelength to 0.86657 wavelength at 433 MHz) is actioned for the 1D vertical monopole (classic design). The cross-sectional size of the monopole's radiator is with a width of 0.6923 cm, corresponding to 0.0099 wavelength at 433 MHz. Note that the governing numerical method adopted in MATLAB is Method of Moments (MoM) [30].

The reflection coefficient S₁₁ and the corresponding radiation efficiency n presented in Fig. 11 exhibit two dips for low return loss, and two peaks for high radiation efficiency in the radiator's height scanning range, respectively. To be more specific, the impedance-matching point happens at the radiator's height of 16.58 cm (0.23946 wavelength), agreeing with the classic quarter-wave theorem. The second impedance-matching point occurs at 50.88 cm (0.73486 wavelength, i.e., approximated to three-quarter wavelength, the phenomenon of which well agrees with the quarter-wave theorem as well.

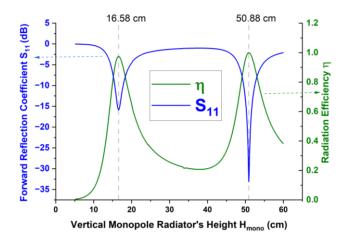


Fig. 11. MATLAB simulation results of return loss and radiation efficiency at 433 MHz for 1D vertical Monopole antennas with a unified ground size of 4 m by 4 m (5.77 wavelengths) and the radiator's height parameterised from 5.17 cm (0.07467 wavelength) to 60 cm (0.86657 wavelength) at 433 MHz.

Accordingly, the directivity and gain are obtained in Fig. 12, and the GoLR is quantified for the 1D vertical monopole in Fig. 13.

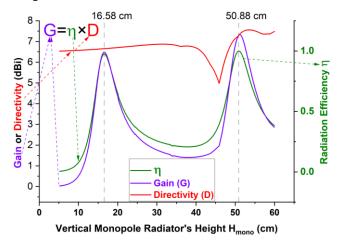


Fig. 12. MATLAB simulation results of gain and directivity (left axis) and radiation efficiency (right axis) at 433 MHz for 1D vertical Monopole antennas with unified ground size of 4 m by 4 m (5.77 wavelengths) and the radiator's height parameterised from 5.17 cm (0.07467 wavelength) to 60 cm (0.86657 wavelength) at 433 MHz.

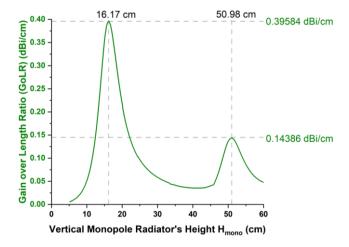


Fig. 13. MATLAB simulation results of gain-over-length-ratio (GoLR in dBi per cm) at 433 MHz for 1D vertical Monopole antennas with a unified ground size of 4 m by 4 m (5.77 wavelengths) and the radiator's height parameterised from 5.17 cm (0.07467 wavelength) to 60 cm (0.86657 wavelength) at 433 MHz.

Similarly to the CDM case, the trend of G follows closely with radiation efficiency. The geometry with the maximum GoLR of 0.39584 dBi/cm occurs at the radiator's height of 16.17 cm (corresponding to 0.23354 wavelength, i.e., near the quarter-wavelength), while the second optimum GoLR of 0.14386 dBi/cm happens at the monopole height of 50.98 cm (i.e., three-quarter wavelength). This convincingly explains the benefit of designing a quarter-wavelength one for both compact and decent performance in antenna gain.

C. Optimised 2D CDM vs. Optimised 1D Monopole

The generalised metric, gain-over-length-ratio (GoLR) is formulated and adopted for benchmarking between the 2D CDM and the 1D monopole antennas. For the 2D CDM, the GoLR is equivalent to the results obtained earlier on GoDR (gain-over-diameter-ratio), wherein the diameter equals the length for the vertical projection. The two topologies under comparison are presented in Fig. 14, with the optimal geometry sizes denoted for the CDM and 1D monopole, respectively. The results of GoLR comparison between the CDM and the 1D monopole antennas are presented in Fig. 15.

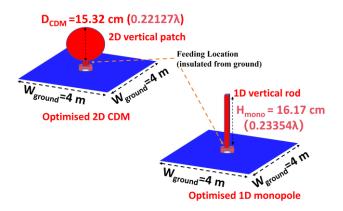


Fig. 14. Overarching comparison between the optimised 1D monopole (vertical, bottom right in the figure) and the optimised 2D CDM design model (top left in the figure). The same grounding material and geometry (4 m by 4 m) are assumed for both designs.

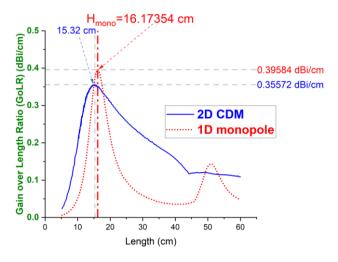


Fig. 15. MATLAB simulation results of gain-over-length-ratio (GoLR in dBi per cm) at 433 MHz for 1D vertical Monopole antennas vs. 2D CDM with unified ground size of 4 m by 4 m (5.77 wavelengths) and the radiator's height (diameter) parameterised from 5.17 cm (0.07467 wavelength) to 60 cm (0.86657 wavelength) at 433 MHz. Smearing effect of GoLR vs. antenna size (diameter) of the 2D CDM is observed.

In the current phase of this work, both the 2D CDM and 1D vertical monopole (classic) antennas demonstrate consistent observations on the benefits of one-quarter-wave operation. Interestingly, the optimum GoLR of the 2D CDM is 0.35572 dBi/cm, which happens at the critical length of 15.32 cm (0.22127 wavelength). The 1D vertical monopole design reports a slightly higher GoLR (maximum value) of 0.39584 dBi/cm occurring at the critical length of 16.17 cm (corresponding to 0.23354 wavelength).

In other words, the 2D CDM antenna design arguably presents a smeared profile of the GoLR vs. length when compared to the 1D monopole counterpart. This smearing effect leads to a slightly compact design on the height (i.e., the 2D CDM with a lower profile by 0.85 cm compared with the 1D monopole), albeit at the cost of a minor degradation of 0.04012 dBi/cm in the GoLR of interest.

Note that the footprint saving of 0.85 cm (5.55% of the 2D CDM antenna's height) as obtained at 433 MHz (ultrahigh frequency band) can lead to a substantial difference (contribution in miniaturisation) for antennas operating at extremely low frequency (ELF) and very low frequency (VLF) spectra (from 3 Hz to 30 kHz), the wavelength of which ranges from 10 km to 100000 km. The transferable size reduction can be predicted in a range from 138.75 m to 1387.5 km.

Even for a monopole antenna for Medium-wave Broadcasting service, the height reduction (if actioning the CDM design) can be significant in the urban deployment for aesthetic considerations, by way of illustration, a 639 kHz broadcasting medium wave antenna (tower) as picturized in Fig. 16. The quarter-wavelength of 117.37 m in height (operated in air) could be reduced to 110.86 m as per the CDM's smearing implication on GoLR.



Fig. 16. Photograph of a 200 kW medium-wave 639 kHz antenna in Fangshan District, Beijing for broadcasting radio signals over long distances.

IV. DISCUSSIONS AND FUTURE EDUCATIONAL SCOPES ON MONOPOLE CONFIGURATION UPGRADE FOR UNCONVENTIONAL APPLICATIONS

Akin to the phase-shift-to-insertion-loss ratio, indexed as figure-of-merit (FoM) and widely employed in phase-tunable RF devices [31], the GoLR index newly proposed in this work arguably corroborates the quarter-wave principle of the monopole antenna for the most effective radiation in a

compact manner. From the lens of GoLR (i.e., aiming for high gain and compactness at the same time), the proof-of-principle CDM in this work is slightly less performant than its predecessor, i.e., the 433 MHz vertical quarter-wave monopole as reported in [15]. However, the CDM reaches its optimum GoLR at a slightly smaller size (0.22127 wavelength) than the monopole one (0.23354 wavelength), i.e., it has its merit of reducing the height of the antenna (albeit not significant), i.e., towards low-profile operations (albeit not yet eligible for competing with ultra-low-profile PCB counterparts, e.g., patch antennas). In addition to the antenna directivity, impedance matching, radiating efficiency, gain, gain-over-length ratio (GoLR) and radiation pattern that have been quantified in this work, bandwidth coverage and manufacturability will be assessed in future work for producing well-executed designs.

Arguably, operating as a resonant antenna, the monopole was developed a hundred years ago, with its usage spanning the general field of wireless communications and sensing. Staying updated on this mature technology through its long existence, we seek to expand the boundary by proposing an engineered index comprehending the gain and size of the antenna, i.e., gain-over-length ratio (GoLR), and apply this figure-of-merit ratio to an educational study into 433 MHz monopole antennas.

More specifically, actioning on converting vertical monopole antennas from 1D (rod) to 2D (plane), this work conducts an educational investigation into the teaching practice of circular disc monopole (CDM) benchmarked with the traditional vertical monopole, using the new index of GoLR for the integration into antenna workflows.

Building on these findings, future research shall explore new frontiers in 2D vertical monopole antenna design, e.g., by investigating the use of diverse conformal shaping for the radial-attached surfaces, including scenarios where radials are suspended in the air or affixed to non-metallic substrates. Figure 17 illustrates a typical radial rod-based discrete grounding configuration. Interestingly, the radial rods can be designed for installation and dismounting with ease, i.e., they can be bent, folded or stretched when installed for diverse environments and be conformal to diverse surfaces.

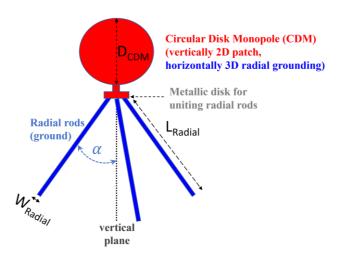


Fig. 17. Future work underway on a perturbing radial rods-based discrete grounding configuration that exemplifies its practicality in real-life applications where continuous grounding is unavailable. Scales are pictured for illustration only.

Furthermore, the radiator itself can also be made flexible and conformal to open a new door for modern applications including unconventional locations with curved edges (e.g., the interior of a submarine). Note that the submarine demands low-frequency underwater communication hardware [32] (instead of the modern high-frequency GHz communication like 4G/5G) to mitigate the significant attenuation due to sea water [33]. The early availability of 433 MHz CDM as reported in this work is envisaged to provide a custom antenna solution for maritime applications, particularly underwater communications in this regard.

Successful implementation of the GoLR concept is anticipated for low-cost low-profile installation of the quarter-wave antenna in the studied CDM topology. Application-specific scenarios for the CDM radial antenna, e.g., its potential integration into surface acoustic wave (SAW) sensing systems and in-body antennas [34][35] will be exploited. These systems often require antennas to operate on non-metallic surfaces, eliminating the need for a large continuous grounding plane typical of standard monopole designs. Multi-scale multi-physics [36–38] characterisation (with higher-order modes' prediction capability [39]) will be conducted.

Additionally, various conductive materials (not limited to copper as simulated in the current work) will be tested numerically to evaluate their impact on the CDM antenna performance. Notably, metal meshes [40][41], and transparent conductive films [42] (optically transparent and virtually invisible to the human eye) are of both research and education interest in the pursuit of various energy-efficient antenna scenarios towards a green future.

Last but not least, physical layer security-related concerns [43][44] are gaining significant attention on the roadmap to 6G communications [45][46], wherein various growing threats on hardware-based attacking scenarios [43] and software-level attacking vulnerabilities [47][48] shall be involved in the front-end components design and system integration. For the CDM device raised in this work, grounding plane attacks pose a particularly severe risk (e.g., destroying the ground plane partially or fully). Such attacks-whether involving partial or complete destruction of the ground plane—can catastrophically impair antenna functionality, translating to the loss of beam tracking in safety-critical communication and sensing applications. autonomous vehicle communications and including high-precision sensing. The resultant catastrophic beam tracking failure can lead to service disruption or even hazardous operational conditions.

To mitigate these risks, future research should explore resilient antenna designs with redundant grounding structures, real-time fault detection mechanisms, and adaptive beamforming algorithms capable of compensating for hardware degradation. Additionally, the integration of physical-layer authentication and hardware-intrinsic security features [46][50] could further safeguard CDM systems against tampering and malicious interference. As 6G networks aim to support ultra-reliable low-latency communication and mission-critical applications, addressing these security challenges at the hardware level will be imperative to ensure system robustness and user safety.

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