Super-Twisting Sliding Mode Control for Fixed-Time Stabilization of Fractional-Order Wind Power System

Xiaomin Tian, Zhitao Hu, Liping Chen

Abstract—**This paper presents a fixed-time stabilization project based on a super-twisting SMC (sliding mode control) method. The scheme applied in fractional-order wind power system can ensure the state trajectories converge to equilibrium in limit time. Super-twisting algorithm can effectively conquer the singularity issue and significantly obviate the chattering phenomenon. Furthermore, the influence of external disturbance is considered to design a robust controller. For demonstrate the fixed-time stability of the controlled system, the fractional-order and integer-order Lyapunov theory are applied to verify the stability of reaching phase and sliding phase, respectively. Simulation results imply that the proposed control strategy can improve the robustness of the fractional-order wind power system.**

Index Terms—**super-twisting algorithm, fractional-order system, sliding-mode control, fixed-time stability.**

I. Introduction

RECENTLY, fractional-order calculus as an important mathematical tool has been widely used to model the phenomena or processes with memory and hereditary characteristics[1-4]. There are research results have proven that fractional-order controllers are more robust and effective against system uncertainties than integer-order controllers[5-8]. At present, applications of fractional-order systems have been reported in many areas, for instance, a fractional-order dynamic model is employed to show the electrical characteristics of fuel cells [9]; A fractional-order capacitor with order within (1, 2) is studied in [10]; In addition, fractional-order derivatives are utilized to depict the cardiac tissue-electrode interface in [11], and so on.

Permanent magnet synchronous generator(PMSG) is the key device of wind power system for electromechanical energy conversion, it's stability will profoundly affect the safe operation of wind power systems. Now, many literatures about the robust control of PMSG have been reported. For instance, Zhu investigated the adaptive impulsive control scheme for the chaotic-oscillation PMSG [12]; Through design a robust nonlinear control scheme, a maximum power of PMSG has been obtained by Cheikh [13]; Seker proposed a robust backstepping approach, which can ensure the generator velocity tracking error uniformly draw near to a small bound [14]; A fresh robust nonsingular terminal sliding mode control program was gave in [15], to get high precision position tracking of permanent magnet linear synchronous motor; A robust amplitude control set model predictive current control scheme is designed by Zhang, it can enhance the prediction accuracy and control precision for permanent magnet synchronous motor [16], and so on.

However, the abovementioned research results are only focus on the integer-order PMSG model, the fractional-order PMSG model is rarely involved. Meanwhile, the fixed-time convergence of system's state trajectories is more meaningful than system's asymptotic stability. Actually, fixed-time stability imply the optimality in settling time, and fixed-time control method has good robustness and anti-interference characteristics. Subsequently, in view of these advantages of fixed-time stability, it is very essential to realize the stabilization of fractional-order system in fixed time.

SMC technology are extensively used in the robust control of nonlinear system, for example, a sliding mode control method is used in [17], that a class of uncertain delayed fractional-order reaction-diffusion memristor neural networks are researched; Zhou designed a periodic delayed sliding mode surfaces in [18], which can achieve the nonsingular prescribed-time control; Tu used a dual-layer sliding mode scheme in [19], so a class of nonlinear systems with actuator faults and disturbances is studied; A finite-time sliding mode control method is investigated in [20], then the nonlinear robotic systems with unknown dynamics are discussed, and so forth.

As we all known that traditional sliding mode control has a significant drawback, that is chattering phenomenon. Chattering can be considered as a cause of performance degradation or even instability, while, system's stability is a prerequisite for normal operation. On the other hand, SMC has some advantages, such as, fast response, insensitivity to parameter changes and disturbances, and simple physical implementation, and so on. So, it is challenging to improve traditional SMC performance. Super-twisting algorithm (STA) has the good points of the traditional high-order SMC, which can effectively suppress chattering. Additionally, the STA can quicken the approximation of the sliding variables approach to sliding surface then optimize the settling time.

Inspired by the above discussion, researching the fixed-time control of fractional-order PMSG model with super-twisting algorithm is a meaningful and valuable

Manuscript received November 2, 2024; revised July 9, 2025. This work was supported by Jiangsu Province Industry-university-research Cooperation Project under Grant BY2022637, the Education Fund Project of Jinling Institute of Technology under Grant GDJY202305, JYJG202319 and SZH202406, and the Fund Project of Ministry of Education under Grant 20230106681 and 20230106682.

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work. In order to deal with the chattering problem and realize fixed-time stability, a novel sliding mode surface is designed, and appropriate control laws are provided for the unknown bounded of external disturbance in this paper. The integer-order and fractional-order Lyapunov theory are both used to demonstrate the fixed-time stability of reaching and sliding phase.

The remaining part of this article is arranged as follows: In section 2, relevant definitions, lemmas are presented. Problem description and main results are given in section 3. Simulation results are displayed in section 4. In the end conclusion is included in section 5.

II. Fractional calculus and relevant lemmas

Fractional calculus is the general form of integer ones, it extends the contents of differentiation and integration to non-integer orders. The Caputo definition is the most frequently used definition of fractional calculus.

Definition 1 (see [21]). The Caputo fractional derivative of order α is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1<\alpha<m \\ \frac{d^m}{dt^m} f(t), & \alpha = m \end{cases} \quad (1) $$

wherein $\Gamma(\cdot)$ is the Gamma function, m is the smallest integer number that larger than α. In the remaining part of the text, for brief, we will use D^{α} to replace ${}_0D_t^{\alpha}$.

Lemma 1 (see [22]). For an integrable and continuous function $x(t)$, the following inequality is assured regarding $t \geq t_0$:

$$ \frac{1}{2} D^{\alpha} x^2(t) \leq x(t) D^{\alpha} x(t), \quad \forall \alpha \in (0,1) \quad (2) $$

Lemma 2 (see [23]). If the following system model is considered:

$$ \dot{x} = f(x), \; x(0) = x_0, \; x \in R^n, f(0) = 0 \quad (3) $$

where $f(x)$: $D \rightarrow R^n$ is continuous on an open neighborhood $D \subset R^n$. Suppose there is a continuous positive definite function $V(x)$: $D \rightarrow R$, and exists positive constants $\eta > 0$ and $0 < \gamma < 1$, such that

$$ \dot{V}(x) + \eta V^{\gamma}(x) \leq 0 \quad (4) $$

then, the system (3) is locally finite-time stable. Depending on the initial state $x(0) = x_0$, the settling time T satisfies the following inequality as

$$ T \leq \frac{V^{1-\gamma}(x_0)}{\eta(1-\gamma)} \quad (5) $$

especially, when $D = R^n$ and $V(x)$ is also radially unbounded, the state of system(3) is globally finite-time stable.

Lemma 3 (see [24]). For a fractional-order system $D^q x(t) = f(t, x(t))$, the origin is fixed-time stable if there exists a positive definite function $V(t, x(t)) \triangleq V(t)$, such that

$$ D^q V(t) \leq \frac{\lambda_1 \Gamma(1-\gamma)}{\Gamma(2-q)\Gamma(q-\gamma+1)} V^{1-q+\gamma}(t) - \frac{\lambda_2 \Gamma(1-\omega)}{\Gamma(2-q)\Gamma(q-\omega+1)} V^{1-q+\omega}(t) \quad (6) $$

with $\lambda_1 > 0$, $\lambda_2 > 0$, $1 < \gamma < q+1$ and $q-1 < \omega < q$. The settling time is estimated by

$$ T = \left[\frac{\Gamma(1+q)}{\lambda_1}\right]^{\frac{1}{q}} + \left[\frac{\Gamma(1+q)}{\lambda_2}\right]^{\frac{1}{q}} \quad (7) $$

which is independent of the initial conditions.

Lemma 4 (see [25]). If z_1, z_2, ..., $z_n > 0$, $r > 1$ and $0 < l \leq 1$, then the following inequalities hold

$$ \sum_{i=1}^{n} z_i^r \geq n^{1-r} \Big(\sum_{i=1}^{n} z_i \Big)^r, \; \sum_{i=1}^{n} z_i^l \geq \Big(\sum_{i=1}^{n} z_i \Big)^l \quad (8) $$

Lemma 5 (see [26]). If $f(t) \in C^1([t_0, +\infty), R)$ is a continuously differentiable function, then

$$ D^q |f(t)| \leq sgn(f(t)) D^q f(t), \; 0 < q < 1 \quad (9) $$

where sgn$(\cdot)$ is the sign function.

III. System description and main results

As the core device in wind power systems, permanent magnet synchronous generator (PMSG) mainly responsible for electromechanical energy conversion, and its stability will profoundly affect the safe operation of wind power systems. Consequently, the key control issue of wind power is how to maintain the stable operation of PMSG with the externa disturbance.

In view of the merits of fractional calculus, this paper investigates the fractional-order mathematics model of PMSG, it is depicted as follows

$$ \begin{aligned} D^{\alpha} x_1 &= -x_1 + x_2 x_3 + \widetilde{u}_d \\ D^{\alpha} x_2 &= -x_2 - x_1 x_3 + \theta x_3 + \widetilde{u}_q \\ D^{\alpha} x_3 &= -\widetilde{T}_L + \sigma(x_2 - x_3) \end{aligned} \quad (10) $$

where $\widetilde{u}_d, \widetilde{u}_q, \widetilde{T}_L$ are transformed d-axis and q-axis voltages, as well as the rotor input torque, respectively. θ and σ are system's structure parameters. $\alpha \in (0, 2)$ is system fractional order, when $\alpha = 0.98$, $\theta = 110$, $\sigma = 16$, $\widetilde{u}_d = -0.45$, $\widetilde{u}_q = 0.68$, $\widetilde{T}_L = 2.5$, then system (10) behave chaotically, when the initial condition $x_0 = [1, 3, 2]^T$, the strange attractors and state trajectories are displayed in Figure 1 and Figure 2, respectively.

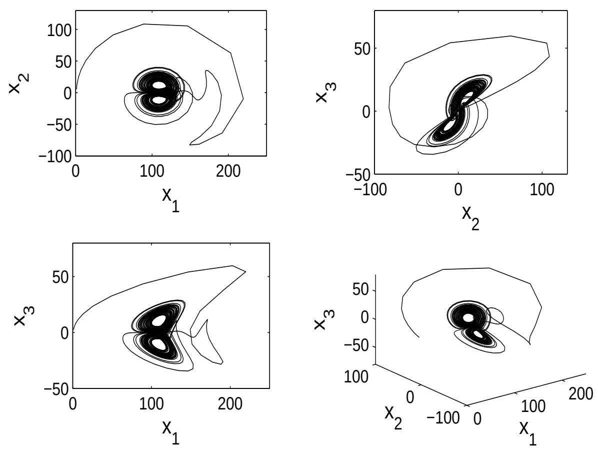

Figure 1. The strange attractors of PMSG system (10)

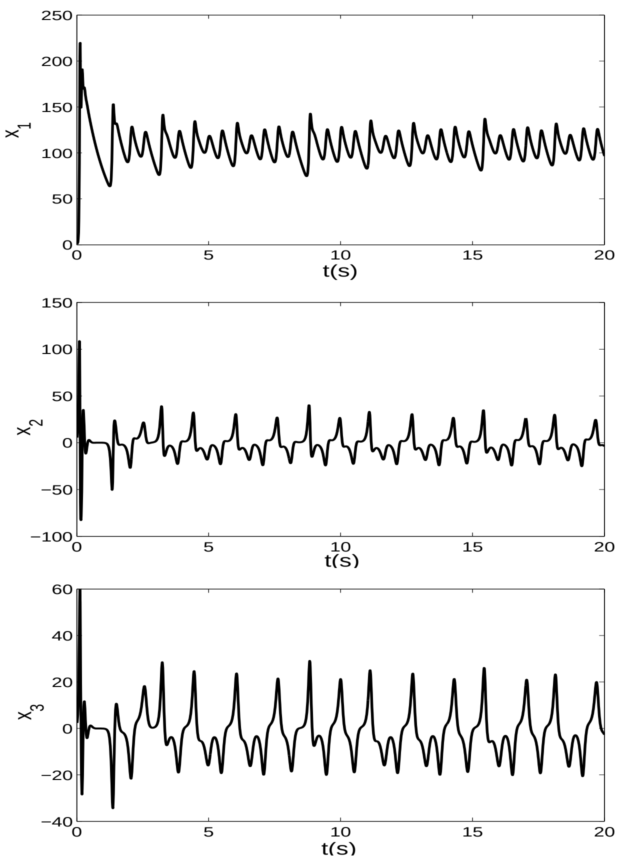

Figure 2. The state trajectories of PMSG system (10)

Considering the effect of controller, the controlled system described as follows

$$\begin{aligned} D^{\alpha}x_1 &= -x_1 + x_2x_3 + \widetilde{u}_d + d_1(t) + u_1(t) \\ D^{\alpha}x_2 &= -x_2 - x_1x_3 + \theta x_3 + \widetilde{u}_q + d_2(t) + u_2(t) \\ D^{\alpha}x_3 &= -\widetilde{T}_L + \sigma(x_2 - x_3) + d_3(t) + u_3(t) \end{aligned} \tag{11}$$

where $d(t) = [d_1(t), d_2(t), d_3(t)]^T$ represents bounded external disturbance, which satisfied $|\dot{d}_i(t)| \le \epsilon$. $u(t) = [u_1(t), u_2(t), u_3(t)]^T$ denotes fixed-time controller to be designed in the next content.

When the system's parameters are selected as eq.(10), the system has three equilibrium, that are $E_1(-0.451, 0.152, -0.005)$, $E_2(108.95, -10.382, -10.538)$, $E_3(109.05, 10.541, 10.386)$. All of them are unstable saddle point. Now, we taking the equilibrium point $E_i^*(x_1^*, x_2^*, x_3^*)$, $i = 1, 2, 3$ as the control objective, thus, defining the tracking error as $e_1 = x_1 - x_1^*, e_2 = x_2 - x_2^*$, $e_3 = x_3 - x_3^*$. The error dynamics model described as follows

$$\begin{aligned} D^{\alpha}e_1 &= -e_1 - x_1^* + (e_3 + x_3^*)(e_2 + x_2^*) + \widetilde{u}_d + d_1(t) + u_1(t) \\ D^{\alpha}e_2 &= -e_2 - x_2^* - (e_3 + x_3^*)(e_1 + x_1^*) + \theta(e_3 + x_3^*) + \widetilde{u}_q \\ &\quad + d_2(t) + u_2(t) \\ D^{\alpha}e_3 &= -\widetilde{T}_L + \sigma(e_2 + x_2^* - e_3 - x_3^*) + d_3(t) + u_3(t) \end{aligned} \tag{12}$$

In this paper, our goal is to design a super-twisting sliding mode control strategy to stabilize the error system (12) in limit time. Generally, two steps are contained in the design of a fixed-time sliding mode controller. The first step is to establish a sliding surface, the sliding variable can converge to origin in limit time. The second step is to design a controller to guarantee the occurrence of the sliding motion.

Now, we put forward a novel fractional-order integral type sliding surface as follows

$$s_i = D^{\alpha-1}e_i + \int_0^t (k_1 e_i |e_i|^{\beta_1} + k_2 e_i |e_i|^{\beta_2}) d\tau \tag{13}$$

where $i = 1, 2, 3$. $k_1, k_2 > 0$ are positive real numbers, $1 - \alpha < \beta_1 < 1$, $-1 < \beta_2 < 0$.

In case the system trajectories arrived in sliding surface, the following equations are satisfied

$$s_i = 0, \quad \dot{s}_i = 0 \tag{14}$$

that is, taking the time derivative of the fractional-order sliding surface (13), we have the expected sliding mode dynamics as follows

$$D^{\alpha}e_i = -k_1 e_i |e_i|^{\beta_1} - k_2 e_i |e_i|^{\beta_2} \tag{15}$$

Theorem 1 *Considering the sliding mode dynamics (15), the system is stable and the state trajectories converge to equilibrium in fixed time.*

Proof. From the eq.(15), if $e_i \to 0$ in a given time imply that the state trajectories can tends to equilibrium in a fixed time, so we just need to verify the finite-time stability of error system (15).

Choosing the following Lyapunov function for error system (15),

$$V_1(t) = \sum_{i=1}^{3} |e_i| \tag{16}$$

Taking the α-order differentiation of eq.(16), and according to lemma 4 and 5, it yields

$$\begin{aligned} D^{\alpha}V_1 &\le \sum_{i=1}^{3} sgn(e_i) D^{\alpha} e_i \\ &= -\sum_{i=1}^{3} k_1 |e_i|^{\beta_1+1} - \sum_{i=1}^{3} k_2 |e_i|^{\beta_2+1} \\ &\le -3^{-\beta_1} k_1 \Big(\sum_{i=1}^{3} |e_i|\Big)^{1+\beta_1} - 3^{-\beta_2} k_2 \Big(\sum_{i=1}^{3} |e_i|\Big)^{1+\beta_2} \end{aligned} \tag{17}$$

further, we have

$$D^{\alpha}V_1 = -3^{-\beta_1} k_1 V_1^{1+\beta_1} - 3^{-\beta_2} k_2 V_1^{1+\beta_2} \tag{18}$$

choosing $\gamma = \alpha + \beta_1$, $\omega = \alpha + \beta_2$, and $\lambda_1 = \frac{-3^{-\beta_1} k_1 \Gamma(2-\alpha)\Gamma(\alpha-\gamma+1)}{\Gamma(1-\gamma)}$, $\lambda_2 = \frac{3^{-\beta_2} k_2 \Gamma(2-\alpha)\Gamma(\alpha-\omega+1)}{\Gamma(1-\omega)}$. Then eq.(18) can be rewritten as

$$\begin{aligned} D^{\alpha}V_1 = & \frac{\lambda_1 \Gamma(1-\gamma)}{\Gamma(2-\alpha)\Gamma(\alpha-\gamma+1)} V_1^{1-\alpha+\gamma} \\ & - \frac{\lambda_2 \Gamma(1-\omega)}{\Gamma(2-\alpha)\Gamma(\alpha-\omega+1)} V_1^{1-\alpha+\omega} \end{aligned} \tag{19}$$

according to lemma 3, it is obviously that the error e_i will converge to 0 in a fixed time $T_1 = \left[\frac{-\Gamma(1+\alpha)\Gamma(1-\alpha-\beta_1)}{3^{-\beta_1} k_1 \Gamma(2-\alpha)\Gamma(1-\beta_1)}\right]^{\frac{1}{\alpha}} + \left[\frac{\Gamma(1+\alpha)\Gamma(1-\alpha-\beta_2)}{3^{-\beta_2} k_2 \Gamma(2-\alpha)\Gamma(1-\beta_2)}\right]^{\frac{1}{\alpha}}$. The proof is completed.

Once the sliding surface has been proposed, next, we will investigate a control strategy to drive the state trajectories of the error system (12) to sliding mode surface in limit time and stay on it forever. The investigated controllers have the following form

$$\begin{aligned} u_1 &= e_1 + x_1^* - (e_3 + x_3^*)(e_2 + x_2^*) - \widetilde{u}_d - k_1 e_1|e_1|^{\beta_1} \\ &\quad -k_2 e_1|e_1|^{\beta_2} - m_1|s_1|^{\frac{1}{2}} sgn(s_1) - \int_0^t m_2 sgn(s_1) d\tau \\ u_2 &= e_2 + x_2^* + (e_3 + x_3^*)(e_1 + x_1^*) - \theta(e_3 + x_3^*) - \widetilde{u}_q \\ &\quad -k_1 e_2|e_2|^{\beta_1} - k_2 e_2|e_2|^{\beta_2} - m_1|s_2|^{\frac{1}{2}} sgn(s_2) \\ &\quad -\int_0^t m_2 sgn(s_2) d\tau \\ u_3 &= \widetilde{T}_L - \sigma(e_2 + x_2^* - e_3 - x_3^*) - k_1 e_3|e_3|^{\beta_1} - k_2 \times \\ &\quad e_3|e_3|^{\beta_2} - m_1|s_3|^{\frac{1}{2}} sgn(s_3) - \int_0^t m_2 sgn(s_3) d\tau \end{aligned} \tag{20}$$

where m_1, m_2 are positive numbers, which will be determined in the next content.

Theorem 2 *Considering the error system (12), if the system is controlled by the control law (20), then the state trajectories of system (12) will converge to $s_i = 0$ ($i = 1, 2, 3$) in fixed time.*

Proof. To prove the closed-loop stability of the controller, the first derivative of the sliding surface (13) is given as

$$\dot{s}_i = D^{\alpha} e_i + (k_1 e_i|e_i|^{\beta_1} + k_2 e_i|e_i|^{\beta_2}) \tag{21}$$

we can introduce a new tuple $[\eta_{i1}, \eta_{i2}]$ to rewrite (21) for further simplification:

$$\begin{aligned} \eta_{i1} &= s_i \\ \eta_{i2} &= -m_2 \int_0^t sgn(s_i) d\tau + d_i(t) \end{aligned} \tag{22}$$

taking the first derivative of eq.(22), we obtain

$$\begin{aligned} \dot{\eta}_{i1} &= -m_1|\eta_{i1}|^{\frac{1}{2}} sgn(\eta_{i1}) + \eta_{i2} \\ \dot{\eta}_{i2} &= -m_2 sgn(\eta_{i1}) + \dot{d}_i(t) \end{aligned} \tag{23}$$

through introducing a new state vector $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}]^T = [|\eta_{i1}|^{\frac{1}{2}} sgn(\eta_{i1}), \eta_{i2}]^T$, then (23) can be rewritten by ς_{i1} and ς_{i2} as

$$\begin{aligned} \dot{\varsigma}_{i1} &= \frac{1}{2|\varsigma_{i1}|}(-m_1\varsigma_{i1} + \varsigma_{i2}) \\ \dot{\varsigma}_{i2} &= \frac{1}{2|\varsigma_{i1}|}(-2m_2\varsigma_{i1}) + \dot{d}_i(t) \end{aligned} \tag{24}$$

as aforementioned, $\dot{d}_i(t)$ has upper bound, thus it can be descried as

$$\dot{d}_i(t) = \delta(t) sgn(\eta_{i1}) = \delta(t)\frac{\varsigma_{i1}}{|\varsigma_{i1}|} \tag{25}$$

where $\delta(t)$ is a bounded function and satisfies $0 < \delta(t) < \epsilon$, eq.(24) can be rewritten as

$$\begin{bmatrix} \dot{\varsigma}_{i1} \\ \dot{\varsigma}_{i2} \end{bmatrix} = \frac{1}{2|\varsigma_{i1}|}\begin{bmatrix} -m_1 & 1 \\ -2m_2 + 2\delta(t) & 0 \end{bmatrix}\begin{bmatrix} \varsigma_{i1} \\ \varsigma_{i2} \end{bmatrix} \tag{26}$$

If the new state vector $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}]^T \to 0$ in fixed time, then η_{i1} and η_{i2} will tend to zero in given time, that is sliding surface s_i can arrive at origin in limit time.

Selecting the Lyapunov function as

$$V_{2i}(t) = (\varsigma_{i1} - \varsigma_{i2})^2 + \varsigma_{i1}^2 = \varsigma_i^T P \varsigma_i^T \tag{27}$$

where P is a positive definite matrix and defined as

$$P = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \tag{28}$$

the first derivative of eq.(27) is

$$\begin{aligned} \dot{V}_{2i} &= 4\varsigma_{i1}\dot{\varsigma}_{i1} - 2\dot{\varsigma}_{i1}\varsigma_{i2} - 2\varsigma_{i1}\dot{\varsigma}_{i2} + 2\varsigma_{i1}\dot{\varsigma}_{i2} \\ &= 4\varsigma_{i1}\frac{1}{2|\varsigma_{i1}|}(-m_1\varsigma_{i1} + \varsigma_{i2}) - 2\frac{1}{2|\varsigma_{i1}|}(-m_1\varsigma_{i1} + \varsigma_{i2})\varsigma_{i2} \\ &\quad -2\varsigma_{i1}\frac{1}{2|\varsigma_{i1}|}(-2m_2 + 2\delta(t))\varsigma_{i1} + 2\varsigma_{i2}\frac{1}{2|\varsigma_{i1}|}(-2m_2 \\ &\quad +2\delta(t))\varsigma_{i1} \end{aligned} \tag{29}$$

considering the upper bound of $\dot{d}_i(t)$, we obtain

$$\begin{aligned} \dot{V}_{2i} &\le -\frac{1}{2|\varsigma_{i1}|}\Big(4m_1\varsigma_{i1}^2 - 4\varsigma_{i1}\varsigma_{i2} - 2m_1\varsigma_{i1}\varsigma_{i2} + 2\varsigma_{i2}^2 \\ &\quad -4m_2\varsigma_{i1}^2 + 4\epsilon\varsigma_{i1}^2 + 4m_2\varsigma_{i1}\varsigma_{i2} - 4\epsilon\varsigma_{i1}\varsigma_{i2}\Big) \\ &= -\frac{1}{2|\varsigma_{i1}|}\varsigma_i^T Q \varsigma_i \end{aligned} \tag{30}$$

where $Q = Q^T$, and it is expressed as

$$Q = \begin{bmatrix} 4m_1 - 4m_2 + 4\epsilon & -2 - m_1 + 2m_2 - 2\epsilon \\ -2 - m_1 + 2m_2 - 2\epsilon & 2 \end{bmatrix} \tag{31}$$

through setting the gain parameter $m_2 = 0.5m_1$, eq.(31) can be rewritten as

$$Q = \begin{bmatrix} 2m_1 + 4\epsilon & -2 - 2\epsilon \\ -2 - 2\epsilon & 2 \end{bmatrix} \tag{32}$$

It is obviously that when $m_1 > \frac{(2+2\epsilon)^2 - 8\epsilon}{4}$, then Q is positive definite. Meanwhile, the following inequality holds:

$$\begin{cases} \lambda_{min}(P)||\varsigma_i||_2^2 \le V_{2i}(t) \le \lambda_{max}(P)||\varsigma_i||_2^2 \\ |\varsigma_{i1}| \le ||\varsigma_i||_2 \le \sqrt{\frac{V_{2i}(t)}{\lambda_{min}(P)}} \end{cases} \tag{33}$$

further, eq.(30) can be rewritten as

$$\begin{aligned} \dot{V}_{2i} &\le \frac{-1}{2|\varsigma_{i1}|}\varsigma_i^T Q \varsigma_i \le \frac{-1}{2|\varsigma_{i1}|}\lambda_{min}(Q)||\varsigma_i||_2^2 \\ &\le \frac{-1}{2|\varsigma_{i1}|}\lambda_{min}(Q)\frac{V_{2i}}{\lambda_{max}(P)} \\ &\le -\frac{\lambda_{min}^{0.5}(P)\lambda_{min}(Q)}{2\lambda_{max}(P)}V_{2i}^{0.5} = -\mu V_{2i}^{0.5} \end{aligned} \tag{34}$$

according to lemma 2, the state trajectories of error system (12) will converge to $s_i = 0$ in a fixed time $T_2 = \frac{V_{2i}^{0.5}(t_0)}{0.5\mu}$. The proof is completed.

From theorem 1 and theorem 2, we know that the error system (12) can be stabilized in a fixed time $T_1 + T_2$.

IV. Simulation Results

In this section, simulation results are given to verify the effectiveness and feasibility of the proposed control scheme. The system parameters are given by eq.(10), the parameters of fractional-order sliding surface are $\beta_1 = 0.5$, $\beta_2 = -0.5$, $k_1 = k_2 = 1$. The controller parameters are set as $m_1 = 2$, $m_2 = 1$, and $\epsilon = 2$. The external disturbance $d(t) = [0.2\sin t, 0.2\sin t, 0.2\sin t]^T$, The PMSG system (10) has three equilibrium points, respectively are $E_1(-0.451, 0.152, -0.005)$, $E_2(108.95, -10.382, -10.538)$, $E_3(109.05, 10.541, 10.386)$. Taking E_2 as an example, when activated the controller in $t = 20s$, we can observe that the state trajectories of controlled PMSG system (11) can converge to equilibrium point E_2 in a given time, which are shown in Figure 3.

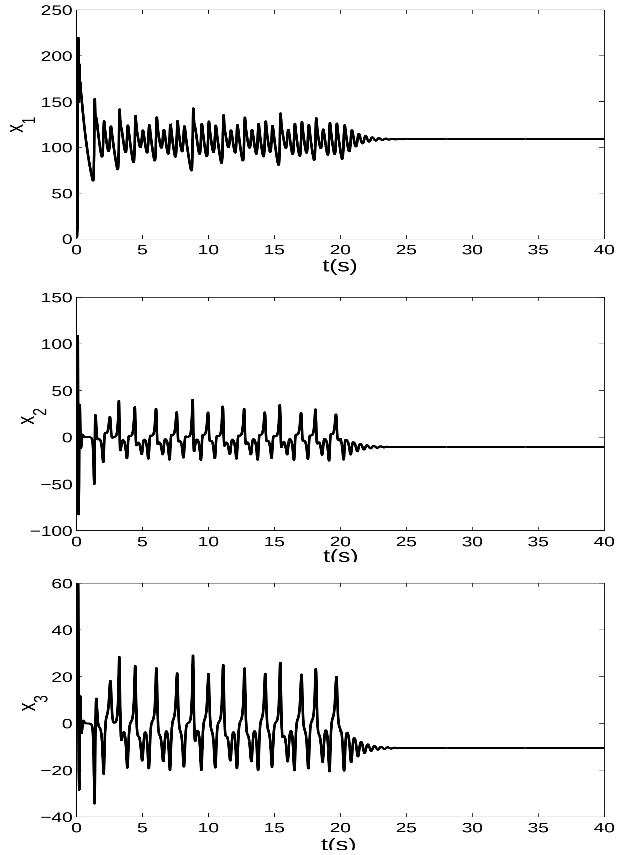

Figure 3. The state trajectories of PMSG system (11) with controller activated

The above simulation results sufficiently demonstrate that the presented super-twisting sliding mode control method is effective in stabilizing fractional-order nonlinear wind power system, the state trajectories of controlled system can converge to equilibrium points in a fixed time.

V. Conclusion

In this paper, a super-twisting SMC strategy is investigated for stabilizing fractional-order wind power system. Under the action of the designed controller, the controlled system showed good ability to resist external disturbance, and has fast convergence. To deal with the chattering phenomenon of traditional sliding mode scheme, super-twisting algorithm has been used in this paper, which can quicken the approximation of the sliding variables approach to sliding surface, and finally optimize the settling time. In order to demonstrate the stability of two stages in sliding mode control, fractional and integer Lyapunov stability theorem both are used. Simulation results confirmed the feasibility and effectiveness of the designed control project.

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