A Study on Equipment Renewal Strategy of Manufacturing Enterprises Based on the Hotelling Model

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***Abstract*—Insufficient equipment renewal has become a key constraint on the high-quality transformation of China's manufacturing industry. To address this challenge, an evolutionary game model based on the Hotelling spatial competition framework is constructed to investigate the strategic decisions of manufacturing firms regarding equipment renewal. The model identifies the mechanisms through which variations in government subsidy intensity influence firms' equipment renewal strategies under different levels of market competition. The analysis reveals that government subsidies significantly incentivize equipment renewal investment, but this effect only emerges when subsidy intensity exceeds a specific threshold. Moreover, intensified market competition weakens the incentive effect of subsidies, particularly for firms with severe financing constraints. Although synergistic effects can enhance firms' willingness to renew equipment by reducing costs, their marginal benefits diminish in highly competitive environments. These findings provide theoretical insights for refining subsidy policies and developing differentiated policy strategies tailored to market competition intensity and firm-specific characteristics.**

***Index Terms*—Government subsidy，Equipment renewal，Hotelling，Game theory**

I. INTRODUCTION

OVER the past four decades since the initiation of reform and opening-up policies, the accelerated growth of China's manufacturing sector has emerged as a cornerstone of the country's economic growth. However, it remains "scale without sophistication," predominantly occupying mid- to low-tier positions of the global manufacturing value chain. There is an urgent need to accelerate the pace of transformation and upgrading to achieve high-quality development and transition from large-scale to capability-driven manufacturing. Equipment constitutes a pivotal element in technological capital formation and directly reflects the technological level and operational efficiency of manufacturing firms. Equipment renewal improves production efficiency and product quality. Additionally, it facilitates technological innovation and organizational transformation, ultimately enhancing firms' overall competitiveness. Furthermore, in modern manufacturing systems, equipment renewal generates synergistic effects by integrating multiple technologies to optimize production processes in a complementary manner, while leveraging technology spillovers, supply chain coordination, and strategic market interactions to reshape inter-firm competitive-cooperative equilibria, thereby simultaneously enhancing operational efficiency, innovation outcomes, and profit-sharing mechanisms. While the adoption of new technologies and equipment can effectively boost productivity, such investments are often capital-intensive, path-dependent, and subject to the uncertainties associated with technological progress. Consequently, insufficient equipment renewal has emerged as a critical constraint impeding the quality improvement and upgrading of the manufacturing sector. In February 2024, the Fourth Meeting of the Central Financial and Economic Affairs Commission emphasized that "accelerating product renewal is an important measure to promote high-quality development, and it is necessary to encourage and guide a new round of large-scale equipment renewal and consumer goods replacement." This policy directive provides strategic guidance and institutional support for advancing the manufacturing industry. As both a supplier and consumer of various types of equipment, the long-term and stable development of China's manufacturing sector depends heavily on the continuous renewal and upgrading of equipment and technology, as well as on product innovation. This policy directive resonates with existing debates on industrial upgrading. As documented in literature[1], investments in machinery and equipment exhibit particularly strong stimulating effect on economic growth. However, firms often face high initial costs when introducing new technologies and equipment. Financing constraints, in turn, reduce their capacity or willingness to undertake equipment renewal. To alleviate these financial pressures and lower the investment threshold, governments frequently intervene using various policy instruments, among which government subsidies are among the most common and effective tools.

To alleviate financial constraints and reduce investment barriers, governments typically implement various policy interventions to mitigate firms' investment risks, among which subsidies are one of the most common and effective instruments. A substantial body of literature has investigated the economic consequences of government subsidies, which

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can be broadly categorized into two perspectives: the "promotion" view and the "inhibition" view. The "promotion" view posits that subsidies can stimulate firms' investment and innovation activities by easing financial constraints and reducing investment risks[2]-[4]. In contrast, the "inhibition" view contends that subsidies may distort market resource allocation, lead to inefficiencies, and crowd out private investment[5]. However, much of the existing literature overlooks the role of the external market environment-particularly the intensity of market competition-in shaping the actual impact of subsidies. Prior research has shown that factors such as market competition, technological demand, and the strength of knowledge diffusion and spillover effects can significantly influence the outcomes of subsidy policies[6]. Some scholars argue that in highly competitive markets, firms are more motivated to leverage subsidies to expand investment and accelerate technological upgrading in order to maintain their market positions and avoid destructive competition[7][8]. Conversely, other studies suggest that excessive competition may result in resource fragmentation, ultimately undermining the effectiveness of government subsidies[9]. Therefore, whether and how the intensity of market competition affects the actual outcomes of subsidies remains an open and insufficiently addressed question in the current literature. At the same time, it is important to recognize that equipment renewal is not merely a means of updating production capacity but also serves as a crucial channel for generating synergistic effects within firms. These effects, arising from the complementary integration of new technologies, production processes, and organizational adjustments, can further amplify the advantages of equipment renewal by enhancing operational efficiency and fostering innovation outcomes. However, existing research has largely overlooked how these synergistic effects interact with market competition and government subsidy policies to shape firms' strategic behaviors. Few studies have systematically explored the dynamic interactions among government subsidies, market competition, and synergistic effects in determining firms' equipment renewal decisions. To address this gap, the present study integrates the Hotelling spatial competition model with evolutionary game theory to construct a novel dynamic framework that captures the strategic adjustment processes of manufacturing firms under different market environments.

Against this background, this paper proposes the following research questions: First, how does the intensity of market competition influence the effect of government subsidies on firms' equipment renewal decisions? Second, what role do synergistic effects associated with equipment renewal play in this process? Third, how do government subsidies, market competition, and synergistic effects interact to shape firms' dynamic strategic behaviors? In response to these questions, this study attempts to investigate the interactive relationship among government subsidies, market competition, and synergistic effects of equipment renewal, and how these factors jointly affect manufacturing firms' investment decisions in equipment renewal. To systematically depict firms' strategic choices under different market structures, this paper introduces a Hotelling model that captures the tripartite interaction mechanism among firms, consumers, and competitors. This model, combined with evolutionary game theory, examines the dynamic evolutionary paths of manufacturing firms' equipment investment strategies, aiming to provide both theoretical support and practical insights for policymakers seeking to optimize subsidy policies and promote equipment renewal in the manufacturing sector.

II. LITERATURE REVIEW

Most existing studies on equipment renewal primarily focus on the characteristics of the equipment itself, such as its economic life cycle[10]-[12]. However, relatively few have investigated the underlying drivers of equipment renewal from the perspective of firms. Equipment investment represents a crucial component of capital expenditure, directly influencing productivity, technological innovation, and long-term competitiveness. It involves fixed asset spending aimed at maintaining or expanding production capacity and includes not only the acquisition of traditional machinery but also the adoption of advanced equipment—an essential pathway for technological upgrading[13]. Firms enhance their competitiveness through equipment renewal by phasing out outdated machinery and introducing technologically advanced equipment. This process entails not only physical replacement but also improvements in production technology and efficiency [14][15].

Existing literature, both domestic and international, generally classifies the determinants of equipment investment into internal and external factors. Internal drivers include firm size[16], demand for technological innovation [17], firm performance[18]-[20], and the level of financing constraints[21][22].External factors consist of policy incentives[23][24], industry characteristics[25][26], market demand[27], and the international trade environment[28]. Among these, government subsidies remain one of the most prevalent and influential policy tools. According to the theory of market failure, government subsidies are designed to correct externalities, mitigate investment risk, and promote industrial upgrading. Literature[29] emphasize that the economic benefits associated with replacing outdated equipment are often underestimated, and that subsidies can effectively reduce the marginal cost of equipment investment. Furthermore, subsidies can signal long-term policy support, bolster firms' investment confidence, and direct firms toward specific technological trajectories, thereby accelerating the diffusion of innovation[30]. Literature[31] note that subsidies not only incentivize the subsidized firms but also increase equipment investment along their supply chains, particularly among upstream suppliers. Equipment renewal can also generate significant inter-firm synergies. Such synergies refer to interaction effects where the aggregate outcome exceeds the sum of individual contributions. Literature[32] suggests that new high-end equipment typically requires more highly skilled employees for operation, thereby fostering a complementary effect with human capital, enhancing the alignment between employee skills and technological demands, and ultimately improving productivity within manufacturing systems. Literature [33] contends that equipment modernization is a key enabler for SMEs to pursue digital transformation and complements digital capacity building, ultimately facilitating the transition to smart manufacturing. Literature[34] further suggest that modernized equipment may foster resource sharing and technical collaboration among firms, contributing to cost reduction. Similarly, literature[35] posits that advanced equipment serves as a platform for the commercialization of R&D outcomes and amplifies the marginal returns on innovation.

However, the effectiveness of government subsidies is far from uniform and is highly sensitive to the external market environment. From the perspective of industrial organization theory, the intensity of market competition can significantly influence firm behavior, thereby affecting the real impact of subsidies. In highly competitive industries, subsidies are more likely to be translated into cost reductions and R&D investment. Conversely, in less competitive environments, firms may divert subsidies toward non-productive uses, undermining their intended efficiency gains[36]. Endogenous growth theory further introduces an "inverted U-shaped" relationship between competition and innovation: while moderate competition stimulates firms to adopt new technologies and invest in R&D, excessive competition can crowd out long-term innovation investment[37][38].

In conclusion, although scholarly interest in equipment renewal has increased, there remains a relative scarcity of research examining the impact of government subsidies on firms' equipment investment behavior, particularly from the standpoint of corporate decision-making. Existing studies are predominantly centered on the technical and economic attributes of equipment itself, with limited attention to firm-level motivations and constraints. This study aims to address this gap by investigating how government subsidies influence the investment decisions of manufacturing firms regarding equipment renewal, taking into account variations in the external market environment. Given that the efficacy of subsidies may vary across different levels of market competition, this study proposes an integrated analytical framework that incorporates government subsidies, market competition, and technological synergy effects. To explore their dynamic interactions, this study adopts an evolutionary game theory approach. Furthermore, considering the critical role of consumers in shaping firm competition, the Hotelling spatial competition model is introduced to capture triadic interactions among firms, consumers, and competitors. By combining the Hotelling model with evolutionary game dynamics, this paper seeks to reveal the mechanisms underlying firms' equipment renewal strategy and to trace the evolutionary path of strategic interactions in competitive manufacturing environments.

III. An Evolutionary Game Model for Strategy Selection in Equipment Renewal by Manufacturing firms

A. Mechanism and Process Analysis of Strategy Evolution

Under the new dual circulation development paradigm, equipment renewal has become an essential lever for the transformation and upgrading of the manufacturing industry. The investment decision-making mechanisms related to equipment renewal exhibit complex and multidimensional characteristics. In principle, profit maximization remains the fundamental objective for the survival and development of enterprises, and the core purpose of equipment renewal lies in acquiring competitive advantages through technological upgrading to achieve excess returns. However, manufacturing firms frequently encounter a triple dilemma involving "willingness-capability-benefit "in the process of equipment renewal decision-making, which leads to generally insufficient willingness and relatively weak incentives for equipment renewal.

The cost-benefit balance of equipment renewal constitutes the core logic of firms' decision-making processes. At present, some manufacturing enterprises remain in a critical phase of recovery and redevelopment after experiencing operational difficulties, and they exhibit significant concerns regarding the high capital requirements, long investment cycles, and uncertain returns associated with equipment renewal. To address the insufficient willingness of manufacturing firms to undertake equipment renewal, government subsidy policies can effectively alleviate firms' liquidity constraints, significantly reduce the investment costs of equipment renewal, and consequently stimulate firms' willingness to invest in upgrading. According to Porter's theory of competitive advantage, firms can simultaneously pursue overall cost leadership and differentiation strategies through equipment renewal. On the one hand, by adopting intelligent manufacturing equipment and flexible production lines, firms can effectively lower marginal costs. On the other hand, leveraging digital manufacturing capabilities and customized production models enables firms to enhance product quality and performance, thereby achieving a differentiated competitive advantage in the market. However, the procurement of advanced equipment, integration of new production capacities, and training of technical personnel involved in the process of equipment renewal create substantial entry barriers, intensifying competitive dynamics within the industry. Based on the spatial competition theory of the Hotelling model, the position of manufacturing firms in the product or technology attribute space determines their market coverage and the degree of alignment with consumer preferences. As a critical means of shifting technological attributes, equipment renewal alters firms' competitive positions within the market space, subsequently influencing the dynamic distribution of market shares and reshaping the structure of industry competition.

Dynamic competition theory further suggests that when leading firms take the lead in implementing equipment renewal and enhancing production efficiency, they can expand their market share and exert a crowding-out effect on competitors. Once a competitor's market share loss exceeds a critical threshold, it is compelled to follow suit, resulting in a typical "Red Queen effect" [39] , which accelerates the diffusion of equipment renewal strategy within the industry. Simultaneously, the market competition intensity parameter in the Hotelling model exerts a significant influence on the evolutionary process of equipment renewal decisions. The greater the intensity of market competition, the stronger the strategic interaction among firms, the more pronounced the crowding-out effect, and the faster the diffusion of equipment renewal strategy, thereby driving the continuous upgrading of the overall technological level within the industry. In this dynamic competitive environment, manufacturing firms continuously adjust their strategic choices based on changes in the returns generated by equipment renewal. If a firm adopts an equipment renewal strategy in the initial stage of strategic competition but achieves returns lower than expected, it will abandon the renewal strategy in subsequent interactions. Conversely, if equipment renewal generates excess returns, other firms will be forced to adopt similar strategies to prevent further erosion of their market shares.

In summary, equipment renewal decisions exhibit significant strategic interactivity and dynamic evolutionary characteristics. Within the framework of evolutionary game theory, manufacturing firms continuously optimize their strategic choices based on the changing costs and returns associated with equipment renewal. Over time, the proportion of firms adopting equipment renewal strategy evolves toward a stable equilibrium, displaying typical characteristics of replicator dynamics. This process not only reflects the endogenous evolutionary mechanism of equipment renewal strategy driven by market competition but also provides a theoretical foundation for subsequent dynamic behavioral analysis of equipment renewal strategy based on the Hotelling model.

B. Basic Assumptions of the Model

This paper focuses on the issue of strategic choices in equipment renewal among manufacturing enterprises. To simplify the analysis, a duopoly market framework is constructed, assuming that firms are able to produce differentiated products through equipment renewal. In existing studies, the issue of differentiated competition is primarily analyzed using two approaches: the representative consumer model and the spatial choice model. Given that this study aims to characterize both consumer preference heterogeneity and product differentiation, and the representative consumer model fails to capture the distributional characteristics of consumer preferences, this study adopts the Hotelling spatial competition model, which better reflects the heterogeneity of consumer preferences.

The equipment renewal behavior of firms constitutes a strategic interaction, operating as a game process under imperfect competition. Various game-theoretic frameworks, including the Bertrand, Cournot, Stackelberg, and Hotelling models, have been widely applied to analyze the strategic interactions among firms. Given that the equipment renewal behavior of manufacturing firms is influenced not only by competitors' strategies but also significantly constrained by consumers' preferences, this paper selects the Hotelling model as the theoretical foundation. In the original Hotelling model, the nature of a product is determined by two parameters: price and location. Consumers' "location" represents their preferred attributes, which systematically depict the interaction mechanism between firms, consumers, and competitors. Therefore, this study constructs an evolutionary game analysis framework based on the Hotelling spatial competition model, where the distribution of consumers in the product space reflects their preference differences. The position of firms in this space is viewed as the embodiment of their product differentiation strategies. The spatial model is then embedded into the evolutionary game framework to explore the driving forces behind firms' equipment renewal behavior.

Assumption 1: The oligopoly market in which the manufacturing industry operates consists of only two firms to play the game, denoted as Firm A and Firm B, and the two firms are located in any position of S_i ($i=a,b$) on the Hotelling line of the interval [0,1] , but the two positions do not overlap. Assuming that $0 \le a \le b \le 1$, $b-a=\Delta$ and Δ denotes the degree of heterogeneity of the products produced by A and B, the smaller Δ is, the smaller the difference is, implying the stronger the competition in the product market.

Assumption 2: Firms A and B are assumed to exhibit "limited rationality," meaning they make decisions aimed at maximizing their own interests. The strategy combinations available to the two firms are {Equipment renewal, No equipment renewal}. The firms' positions are represented on the Hotelling line within the interval [0,1]. A value of "1" on the Hotelling line indicates that the firm has implemented equipment renewal at that stage, while a value of "0" indicates that the firm has not carried out equipment renewal at that stage.

Assumption 3: Consumers, of unit size, are uniformly distributed along the interval. The distance between a consumer and a firm is directly proportional to the negative utility generated by the non-desirable product obtained by the consumer from purchasing that firm's product. In other words, the farther the consumer is from the firm, the higher the transport cost they must incur. Consumers located at x (where x is the point of indifference) experience a negative utility when purchasing firm A's product is $t(x-a)^2$ and also a negative utility when purchasing firm B's product is $t(x-b)^2$ .The parameter t represents the transport cost coefficient, with subsequent analyses setting t=1.

Assumption 4: Manufacturing firms that implement equipment renewal and transformation can achieve potential benefits in terms of cost reduction, quality improvement, and enhanced efficiency, among other aspects. This paper assumes that the primary benefit of equipment renewal for firms lies in the enhancement of product performance and differentiation, which, in turn, improves consumers' perception of the utility of the product unit. Other impacts are not considered in this analysis for the time being. Specifically, with the help of advanced equipment, firms can produce higher-quality products with better functionality or more closely aligned with consumers' personalized needs, making it easier to capture consumer preferences and influence their purchasing choices. For consumers in the product market, the utility derived from purchasing a unit of a product that has not been upgraded by Firm A and Firm B is denoted as U_0 (consumers are assumed to have a sufficiently large retention utility U_0 for the product ,meaning that the utility U_0 is large enough to encourage all consumers to purchase one unit of the product, ensuring full market coverage), and the utility derived from purchasing a new product produced by the equipment renewal is denoted as $U_0+\lambda_a U_0$ and $U_0+\lambda_b U_0$,where λ_a and λ_b represent the investment in equipment renewal by Firm A and Firm B, $\lambda_a,\lambda_b\in[0,1]$. Assuming Firm B faces a higher financing constraint, if both firms choose to adopt the equipment renewal, which implies that Firm A has greater financial capacity than Firm B, the willingness of Firm A to invest in equipment renewal will be greater than that of Firm B, where, $\lambda_a>\lambda_b$. Moreover, the government can regulate the size of subsidies, denoted as *S*, where （*S*>0）.

Assumption 5: Assuming that the basic cost of producing products for both firms A and B is C_0 , the cost of equipment renewal for Firm A is denoted as C_a and the cost of equipment renewal for firm B is denoted as C_b. If both implement equipment renewal, the synergistic effects resulting from the inter-firm sharing of equipment resources, technological cooperation, and exchange of experiences can reduce costs and improve the overall efficiency of technological innovation and application[33]. In this scenario, the equipment renewal investment costs borne by the two manufacturing firms are denoted as $C_a^{'}$、$C_b^{'}$, and due to the synergistic effect of both parties to undertake the equipment renewal will reduce the cost of more trial-and-error , making $C_i^{'}<C_i(i=a,b)$.

C. Hotelling Competition Model Construction

Model 1 Firms A and B both choose to invest in equipment renewal, and the prices of their products are P_a^1 、P_b^1 , respectively, and the net surplus of consumers at *x* who buy units of products from firms A and B is as follows.

$$\begin{cases} U_a^{1,1}=U_0+U_\lambda-P_a^1-(x-a)^2 \\ U_b^{1,1}=U_0+U_\beta-P_b^1-(x-b)^2 \end{cases} \tag{1}$$

Where, $U_\lambda=\lambda_a U_0(0<\lambda_a<1)$, $U_\beta=\lambda_b U_0(0<\lambda_b<1)$

Let the consumer located at x^*（$x^*\in[0,1]$） be indifferent between purchasing from the two firms, implying that the consumer at point x^* receives the same utility from purchasing a unit of product from either firm. Then, x^* needs to satisfy the following condition:

$$U_0+U_\lambda-P_a^1-(x-a)^2=U_0+U_\beta-P_b^1-(x-b)^2 \tag{2}$$

Solution:

$$x^*=-\frac{-a^2+b^2+\lambda_a U_0-\lambda_b U_0-P_a^1+P_b^1}{2(a-b)} \tag{3}$$

Consumer demand for the products of the two firms, located at x^* , is as follows:

$$\begin{cases} D_a=\int_0^{x^*}1dx=-a-\dfrac{-a^2+b^2+\lambda_a U_0-\lambda_b U_0-P_a^1+P_b^1}{2(a-b)} \\ D_b=\int_{x^*}^1 1dx=b+\dfrac{-a^2+b^2+\lambda_a U_0-\lambda_b U_0-P_a^1+P_b^1}{2(a-b)} \end{cases} \tag{4}$$

Firm A and Firm B's production profit function $\pi_i(i=a,b)$ is

$$\pi_i^{1,1}=(P_i^1-c)D_i(i=a,b) \tag{5}$$

Solved according to the first order condition for profit π_i maximization:

$$\begin{cases} \pi_a^{1,1}=-\dfrac{(3(a-b)^2+\lambda_a U_0-\lambda_b U_0)^2}{18(a-b)} \\ \pi_b^{1,1}=-\dfrac{(3(a-b)^2-\lambda_a U_0+\lambda_b U_0)^2}{18(a-b)} \end{cases} \tag{6}$$

Firms A、B all choose to undertake the equipment renewal and received government subsidies, the total revenue of the firms $R_i(i=a,b)$ are as follows:

$$\begin{cases} R_a^{1,1}=\pi_a^{1,1}+S-C_a^{'}=S-C_a^{'}-\dfrac{(3(a-b)^2+\lambda_a U_0-\lambda_b U_0)^2}{18(a-b)} \\ R_b^{1,1}=\pi_b^{1,1}+S-C_b^{'}=S-C_b^{'}-\dfrac{(3(a-b)^2-\lambda_a U_0+\lambda_b U_0)^2}{18(a-b)} \end{cases} \tag{7}$$

Model 2 Firm A chooses to invest in equipment renewal and Firm B does not. The prices of their products are P_a^1、P_b^0 , and the net surplus of consumers located at *x* who buy units of Firm A and Firm B is as follows:

$$\begin{cases} U_a^{1,0}=U_0+U_\lambda-P_a^1-(x-a)^2 \\ U_b^{1,0}=U_0-P_b^0-(x-b)^2 \end{cases} \tag{8}$$

Then the demand for the products of the two firms by consumers at x^* is as follows:

$$\begin{cases} D_a=\int_0^{x^*}1dx=-a-\dfrac{-a^2+b^2+\lambda_a U_0-P_a^1+P_b^0}{2(a-b)} \\ D_b=\int_{x^*}^1 1dx=b+\dfrac{-a^2+b^2+\lambda_a U_0-P_a^1+P_b^0}{2(a-b)} \end{cases} \tag{9}$$

Correspondingly, the production profit functions of firms A and B are as follows:

$$\begin{cases} \pi_a^{1,0}=-\dfrac{(3(a-b)^2+\lambda_a U_0)^2}{18(a-b)} \\ \pi_b^{1,0}=-\dfrac{(3(a-b)^2-\lambda_a U_0)^2}{18(a-b)} \end{cases} \tag{10}$$

The total revenues of the firms A and B are as follows:

$$\begin{cases} R_a^{1,0}=\pi_a^{1,0}+S-C_a=S-C_a-\dfrac{(3(a-b)^2+\lambda_a U_0)^2}{18(a-b)} \\ R_b^{1,0}=\pi_b^{1,0}=-\dfrac{(3(a-b)^2-\lambda_a U_0)^2}{18(a-b)} \end{cases} \tag{11}$$

Model 3 Firm A chooses not to invest in equipment renewal and firm B invests in equipment renewal. The prices of their products are P_a^0 、P_b^1 , and the net surplus of consumers at *x* who buy units from firm A and firm B are

$$\begin{cases} U_a^{0,1}=U_0-P_a^0-(x-a)^2 \\ U_b^{0,1}=U_0+U_\beta-P_b^1-(x-b)^2 \end{cases} \tag{12}$$

Consumer demand for the products of the two firms, located at x^* , is as follows

$$\begin{cases} D_a=\int_0^{x^*}1dx=\dfrac{a^2-b^2+\lambda_b U_0+P_a^0-P_b^1}{2(a-b)}-a \\ D_b=\int_{x^*}^1 1dx=b-\dfrac{a^2-b^2+\lambda_b U_0+P_a^0-P_b^1}{2(a-b)} \end{cases} \tag{13}$$

Correspondingly, the production profit functions of firms A and B are as follows

$$\begin{cases} \pi_a^{0,1}=-\dfrac{(3(a-b)^2-\lambda_b U_0)^2}{18(a-b)} \\ \pi_b^{0,1}=-\dfrac{(3(a-b)^2+\lambda_b U_0)^2}{18(a-b)} \end{cases} \tag{14}$$

The total revenues of the firms A and B are as follows:

$$\begin{cases} R_a^{0,1} = \pi_a^{0,1} = -\dfrac{(3(a-b)^2 - \lambda_b U_0)^2}{18(a-b)} \\ R_b^{0,1} = \pi_b^{0,1} + S - C_b = S - C_b - \dfrac{(3(a-b)^2 + \lambda_b U_0)^2}{18(a-b)} \end{cases} \quad (15)$$

Model 4 Firms A and B do not invest in equipment renewal, and the prices of their products are P_a^0 、 P_b^0 respectively, and the net surplus of consumers at x who buy units of firm A and firm B are as follows

$$\begin{cases} U_a^{0,0} = U_0 - P_a^0 - (x-a)^2 \\ U_b^{0,0} = U_0 - P_b^0 - (x-b)^2 \end{cases} \quad (16)$$

Then the demand for the products of the two firms by consumers at x^* is

$$\begin{cases} D_a = \int_0^{x^*} 1dx = -a - \dfrac{-a^2 + b^2 - P_a^0 + P_b^0}{2(a-b)} \\ D_b = \int_{x^*}^1 1dx = b + \dfrac{-a^2 + b^2 - P_a^0 + P_b^0}{2(a-b)} \end{cases} \quad (17)$$

Correspondingly, the production profit functions of firms A and B are as follows

$$\begin{cases} \pi_a^{0,0} = -\dfrac{(a-b)^3}{2} \\ \pi_b^{0,0} = -\dfrac{(a-b)^3}{2} \end{cases} \quad (18)$$

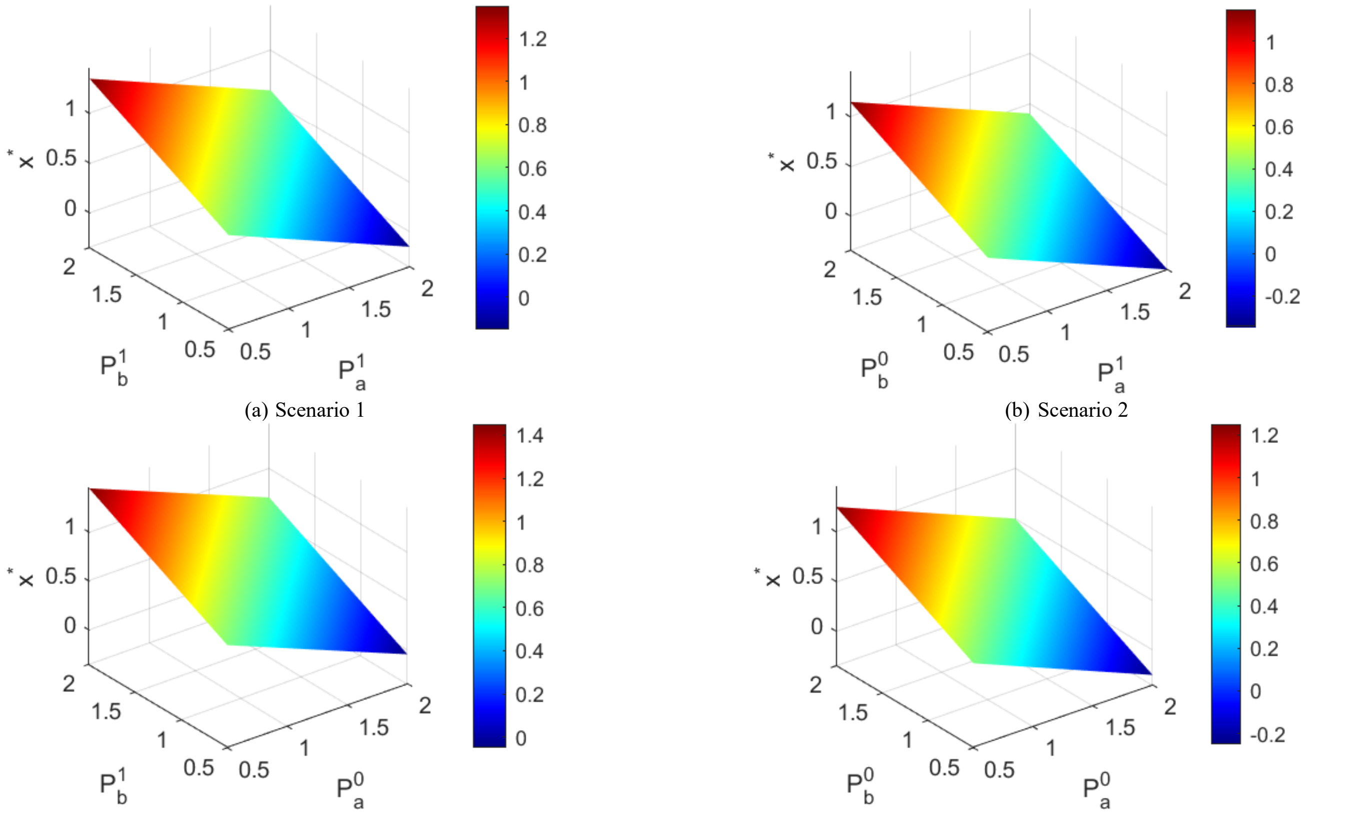
The total revenues of firms A and B are as follows

$$\begin{cases} R_a^{0,0} = \pi_a^{0,0} = -\dfrac{(a-b)^3}{2} \\ R_b^{0,0} = \pi_b^{0,0} = -\dfrac{(a-b)^3}{2} \end{cases} \quad (19)$$

To further characterize the impact of equipment renewal behavior on market space division, this study constructs a three-dimensional surface graph of indifference point x^* with respect to product price changes based on four different equipment renewal combination scenarios, as shown in Figure 1. Among them, Figures (a) Scenario1, (b) Scenario2, (c) Scenario3, and (d) Scenario4 correspond to the cases of Model 1, Model 2, Model 3, and Model 4, respectively.

By employing the Hotelling model, this study computes the consumer indifference point x^* across various equipment renewal strategy combinations and illustrates the corresponding three-dimensional surface plot, as shown in Figure 1. The X-axis and Y-axis represent the product prices of firms A and B, respectively, while the Z-axis indicates the position of the consumer indifference point x^* . The color gradient, ranging from cool to warm, reflects the variation in the values of the indifference point. As shown in Figure 1, the positions of the indifference point exhibit considerable differences across the four equipment renewal scenarios. In Scenario 1, where both Firms A and B renew their equipment, symmetrical improvements in product utility are achieved, resulting in a relatively stable response of the indifference point x^* to price changes. In Scenario 2, where Firm A renews its equipment while Firm B does not, the indifference point shifts toward Firm B, indicating a distinct consumer preference for Firm A. Scenario 3 presents the opposite situation, with Firm B updating its equipment while Firm A does not, causing the indifference point x^* to move toward Firm A. In Scenario 4, where neither firm renews its equipment, market division is solely driven by price differences, and the position of the indifference point x^* remains relatively stable and approximately symmetrical.

In summary, equipment renewal behavior has a significant impact on consumer market preferences. Firms' equipment renewal decisions, together with their pricing



(a) Scenario 1 (b) Scenario 2

(c) Scenario 3 (d) Scenario 4

Fig. 1 Surface Plots of Indifference Point Variations under Different Equipment Renewal Scenarios strategies, jointly shape the division of market space, thereby providing a solid foundation for the subsequent simulation of evolutionary dynamics.

D.Construction of Evolutionary Game Dynamic Equations for Equipment Renewal

Combining the four scenarios above, manufacturing firms A and B choose the payment matrix for equipment renewal investments, as shown in Table 1 below.

Suppose that the probabilities of firm A choosing the equipment renewal and no equipment renewal are x and $1-x$, respectively, and the probabilities of firm B choosing the equipment renewal and no equipment renewal are y and $1-y$, respectively, $(0 \le x, y \le 1)$. Then, from Table 1:

TABLE I
GAME PAYMENT MATRIX

		Firm B	
		Equipment renewal	No equipment renewal
Firm A	Equipment renewal	($R_a^{1,1}, R_b^{1,1}$)	($R_a^{1,0}, R_b^{1,0}$)
	No equipment renewal	($R_a^{0,1}, R_b^{0,1}$)	($R_a^{0,0}, R_b^{0,0}$)

The revenue from firm A's choice to invest in equipment renewal is:

$$E_a^1 = (y-1)(C_a - S + \frac{(3(a-b)^2 + \lambda_a U_0)^2}{18(a-b)}) - y(C_a^{'} - S + \frac{(3(a-b)^2 + \lambda_a U_0 - \lambda_b U_0)^2}{18(a-b)}) \quad (20)$$

The revenue firm A's choice not to invest in equipment renewal is

$$E_a^0 = (y-1)\frac{(a-b)^3}{2} - y\frac{(3(a-b)^2 - \lambda_b U_0)^2}{18(a-b)} \quad (21)$$

The revenue from firm B's choice to invest in equipment renewal is:

$$E_b^1 = (x-1)(C_b - S + \frac{(3(a-b)^2 + \lambda_b U_0)^2}{18(a-b)}) - x(C_b^{'} - S + \frac{(3(a-b)^2 - \lambda_a U_0 + \lambda_b U_0)^2}{18(a-b)}) \quad (22)$$

The revenue firm B's choice not to invest in equipment renewal is

$$E_b^0 = (x-1)\frac{(a-b)^3}{2} - x\frac{(3(a-b)^2 - \lambda_a U_0)^2}{18(a-b)} \quad (23)$$

According to evolutionary game theory, the replicator dynamic equation of Firm A for the equipment renewal strategy can be expressed as follows:

$$dx = x(x-1) \times (-S + C_a - C_a y + C_a^{'} y + \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0 - 2\lambda_b U_0 y)^2}{18(a-b)}) \quad (24)$$

Similarly, the replicator dynamic equation of firm B as

$$dy = y(y-1) \times (-S + C_b - C_b x + C_b^{'} x + \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0 - 2\lambda_a U_0 x)^2}{18(a-b)}) \quad (25)$$

To examine the stability of the system's equilibrium points, this study adopts the local stability analysis method proposed by Friedman to test the properties of equilibrium points. Based on the characteristics of the Jacobian matrix of the replicator dynamic system in evolutionary game theory, the stability of each equilibrium point is determined. The specific approach is as follows: if the equilibrium point Jacobian matrix determinant $DetJ>0$, and the trace $TrJ<0$, then it can be judged that the corresponding equilibrium point has the nature of asymptotic stability called *ESS* point; if $DetJ>0$, and $TrJ<0$, then it can be judged that the corresponding equilibrium point is unstable; if $DetJ>0$ and $TrJ<0$ or when it is not certain, then the corresponding equilibrium point can be judged as a saddle point. Therefore, the Jacobi matrix is obtained by replicating the system of dynamic equations as: $J = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where:

$$a_{11} = (1-2x)(S - C_a + C_a y - C_a^{'} y - \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0 - 2\lambda_b U_0 y)}{18(a-b)})$$

$$a_{12} = -x(x-1)(C_a - C_a^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)})$$

$$a_{21} = -y(y-1)(C_b - C_b^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)})$$

$$a_{22} = (1-2y)(S - C_b + C_b x - C_b^{'} x - \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0 - 2\lambda_a U_0 x)}{18(a-b)})$$

The equilibrium point of the game is $E_1(0,0)$, $E_2(0,1)$, $E_3(1,0)$, $E_4(1,1)$, $E_5(x_0, y_0)$ from $dx/dt = 0$ and $dy/dt = 0$.Where,

$$x_0 = \frac{-S + C_b + \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0)}{18(a-b)}}{C_b - C_b^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}},$$

$$y_0 = \frac{-S + C_a + \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0)}{18(a-b)}}{C_a - C_a^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}}$$

, and at

$$0 < \frac{-S + C_b + \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0)}{18(a-b)}}{C_b - C_b^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}}, \frac{-S + C_a + \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0)}{18(a-b)}}{C_a - C_a^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}} < 1$$

, $E_5(x_0, y_0)$ is also equilibrium point.

The trace $TrJ<0$ and determinant $DetJ>0$ of the Jacobian equation are respectively:

$$trJ = a_{11} + a_{22} = (1-2x)(S - C_a + C_a y - C_a^{'} y - \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0 - 2\lambda_b U_0 y)}{18(a-b)}) + (1-2y)(S - C_b + C_b x - C_b^{'} x - \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0 - 2\lambda_a U_0 x)}{18(a-b)}) \quad (26)$$

$$detJ = a_{11}a_{22} - a_{12}a_{21} = (1-2x)(S - C_a + C_a y - C_a^{'} y - \frac{\lambda_a U_0 (6(a-b)^2 + \lambda_a U_0 - 2\lambda_b U_0 y)}{18(a-b)}) \times (1-2y)(S - C_b + C_b x - C_b^{'} x - \frac{\lambda_b U_0 (6(a-b)^2 + \lambda_b U_0 - 2\lambda_a U_0 x)}{18(a-b)}) - x(x-1)(C_a - C_a^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}) \times y(y-1)(C_b - C_b^{'} + \frac{2\lambda_a \lambda_b U_0^2}{18(a-b)}) \quad (27)$$

E. Analysis of Strategy Evolution Path and Equilibrium Trend

TABLE II
DETERMINANTS AND TRACES OF THE JACOBIAN MATRIX OF EQUILIBRIUM POINTS FOR EQUIPMENT RENEWAL STRATEGIES

balance point	$DetJ$	TrJ
(0,0)	$S-C_a-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0)}{18(a-b)}\times S-C_b-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0)}{18(a-b)}$	$S-C_a-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0)}{18(a-b)}+S-C_b-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0)}{18(a-b)}$
(0,1)	$(S-C_a^{'}-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0-2\lambda_b U_0)}{18(a-b)})\times[-(S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})]$	$(S-C_a^{'}-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0-2\lambda_b U_0)}{18(a-b)})+[-(S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})]$
(1,0)	$-(S-C_a-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0)}{18(a-b)})\times S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})$	$-(S-C_a-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0)}{18(a-b)})+S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})$
(1,1)	$-(S-C_a^{'}-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0-2\lambda_b U_0)}{18(a-b)})\times[-(S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})]$	$-(S-C_a^{'}-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0-2\lambda_b U_0)}{18(a-b)})+[-(S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})]$
(x_0,y_0)	$x_0(x_0-1)(C_a-C_a^{'}+\frac{2\lambda_a\lambda_b U_0^{\ 2}}{18(a-b)})\times y_0(y_0-1)(C_b-C_b^{'}+\frac{2\lambda_a\lambda_b U_0^{\ 2}}{18(a-b)})$	0

Drawing on the initial state of the equipment renewal strategy selection mechanism and the evolutionary phase diagram presented in Table 3, the evolutionary trajectories of the strategy combinations adopted by firms A and B are analyzed separately. For the purpose of analysis, let

$$-(S-C_a^{'}-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0-2\lambda_b U_0)}{18(a-b)})=1,$$

$$-(S-C_b^{'}-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0-2\lambda_a U_0)}{18(a-b)})=2,$$

$$S-C_a-\frac{\lambda_a U_0(6(a-b)^2+\lambda_a U_0)}{18(a-b)}=3,$$

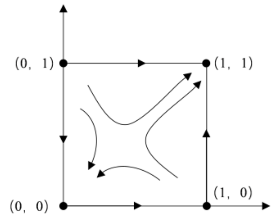
$$S-C_b-\frac{\lambda_b U_0(6(a-b)^2+\lambda_b U_0)}{18(a-b)}=4.$$

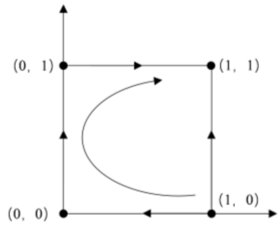
TABLE III
LOCAL STABILITY OF EQUILIBRIUM POINTS ①–⑮ UNDER THE INITIAL STATES OF EQUIPMENT RENEWAL STRATEGY

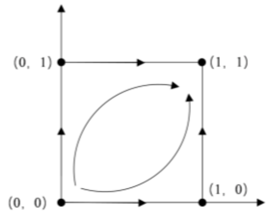
balance point	①			②			③		
	1 < 0, 2 < 0, 3 < 0, 4 < 0			1 < 0, 2 < 0,3 < 0, 4 > 0			1 < 0, 2 < 0, 3 > 0, 4 > 0		
	$DetJ$	TrJ	Stability	$DetJ$	TrJ	Stability	$DetJ$	TrJ	Stability
$E_1(0,0)$	+	−	*ESS*	−	±	saddle point (math.)	+	+	instability
$E_2(0,1)$	+	+	instability	−	±	saddle point (math.)	−	±	saddle point (math.)
$E_3(1,0)$	+	−	instability	+	+	instability	−	±	saddle point (math.)
$E_4(1,1)$	+	−	*ESS*	+	−	*ESS*	+	−	*ESS*
$E_5(x_0,y_0)$			saddle point (math.)			non-existent			non-existent

TABLE III (CONTINUED)
LOCAL STABILITY OF EQUILIBRIUM POINTS ①–⑮ UNDER THE INITIAL STATES OF EQUIPMENT RENEWAL STRATEGY

Evolutionary phase diagram

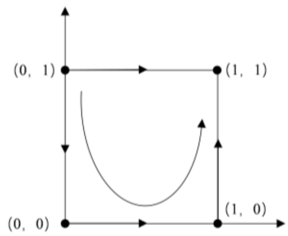


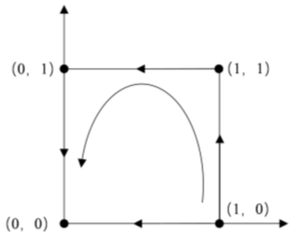


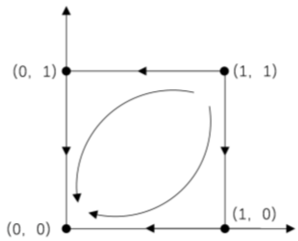


	④			⑤			⑥		
balance point	1 < 0, 2 < 0, 3 > 0, 4 <			1 > 0, 2 < 0, 3 < 0, 4 < 0			1 > 0, 2 > 0, 3 < 0, 4 < 0		
	DetJ	*TrJ*	Stability	*DetJ*	*TrJ*	Stability	*DetJ*	*TrJ*	Stability
$E_1(0,0)$	−	±	saddle point (math.)	+	−	*ESS*	+	−	*ESS*
$E_2(0,1)$	+	+	instability	−	±	saddle point (math.)	−	±	saddle point (math.)
$E_3(1,0)$	−	±	saddle point (math.)	+	+	instability	−	±	saddle point (math.)
$E_4(1,1)$	+	−	*ESS*	−	±	saddle point (math.)	+	+	instability
$E_5(x_0, y_0)$			non-existent			non-existent			non-existent

Evolutionary phase diagram

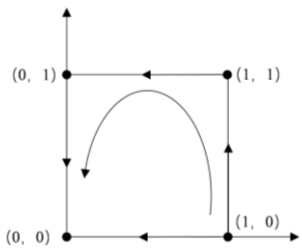


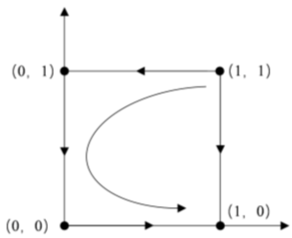




	⑦			⑧			⑨		
balance point	1 < 0, 2 > 0, 3 < 0, 4 < 0			1 < 0, 2 < 0, 3 > 0, 4 < 0			1 < 0, 2 > 0, 3 > 0, 4 < 0		
	DetJ	*TrJ*	Stability	*DetJ*	*TrJ*	Stability	*DetJ*	*TrJ*	Stability
$E_1(0,0)$	+	−	*ESS*	−	±	saddle point (math.)	−	±	saddle point (math.)
$E_2(0,1)$	+	+	instability	−	±	saddle point (math.)	+	+	instability
$E_3(1,0)$	−	±	saddle point (math.)	+	−	*ESS*	+	−	*ESS*
$E_4(1,1)$	−	±	saddle point (math.)	+	+	instability	−	±	saddle point (math.)
$E_5(x_0, y_0)$			non-existent			non-existent			non-existent

Evolutionary phase diagram





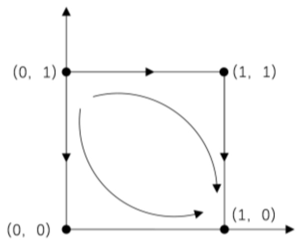
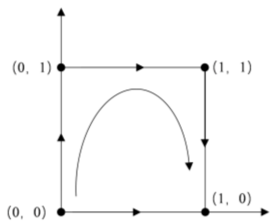
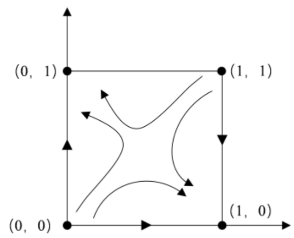
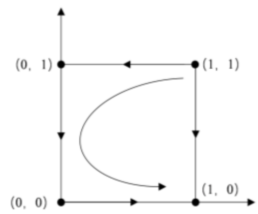
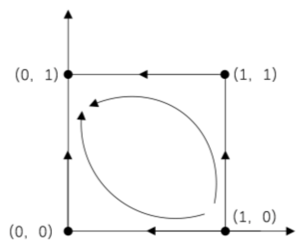
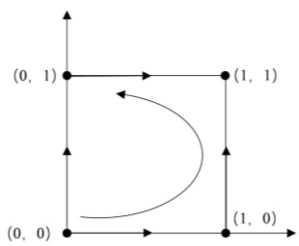
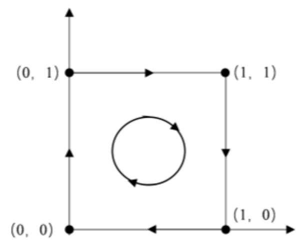


TABLE III (CONTINUED)
LOCAL STABILITY OF EQUILIBRIUM POINTS ①–⑮ UNDER THE INITIAL STATES OF EQUIPMENT RENEWAL STRATEGY

balance point	⑩			⑪			⑫		
	1 < 0, 2 > 0, 3 > 0, 4 > 0			1 > 0, 2 > 0, 3 > 0, 4 > 0			1 > 0, 2 > 0, 3 < 0, 4 > 0		
	DetJ	*TrJ*	Stability	*DetJ*	*TrJ*	Stability	*DetJ*	*TrJ*	Stability
$E_1(0,0)$	+	+	instability	+	+	instability	−	±	saddle point (math.)
$E_2(0,1)$	−	±	saddle point (math.)	+	−	*ESS*	+	−	*ESS*
$E_3(1,0)$	+	−	*ESS*	+	−	*ESS*	−	±	saddle point (math.)
$E_4(1,1)$	−	±	saddle point (math.)	+	+	instability	+	+	instability
$E_5(x_0, y_0)$			non-existent	−	0	saddle point (math.)			non-existent
Evolutionary phase diagram									

balance point	⑬			⑭			⑮		
	1 > 0, 2 < 0, 3 < 0, 4 > 0			1 > 0, 2 < 0, 3 > 0, 4 > 0			1 > 0, 2 < 0, 3 > 0, 4 < 0		
	DetJ	*TrJ*	Stability	*DetJ*	*TrJ*	Stability	*DetJ*	*TrJ*	Stability
$E_1(0,0)$	−	±	saddle point (math.)	+	+	instability	−	±	saddle point (math.)
$E_2(0,1)$	+	−	*ESS*	+	−	*ESS*	−	±	saddle point (math.)
$E_3(1,0)$	+	+	instability	−	±	saddle point (math.)	−	±	saddle point (math.)
$E_4(1,1)$	−	±	saddle point (math.)	−	±	saddle point (math.)	−	±	saddle point (math.)
$E_5(x_0, y_0)$			non-existent			non-existent			non-existent
Evolutionary phase diagram									

For the purpose of this analysis, $S_i - C_i = K_i$, $S_i - C_i^{'} = K_i^{'}$, then $-K_i$ indicate the net cost of equipment renewal, $-K_i^{'}$ indicates the net cost of collaborative renewal to the firms, $K_i < K_i^{'} < 0$, $K_i - K_i^{'} = \Delta K_i$, $-\Delta K_i$ indicates the cost savings from collaborative renewal, that is, the synergies generated by technological advances brought about by renewing the equipment, and in the following analyses $i = a,b; j = a,b$ and $i \neq j$.

Discussion I $(a-b)^2 \geq \frac{2\lambda_a U_0 - \lambda_b U_0}{6}$, when $(a-b)^2$ is relatively large and the intensity of market competition is low, the evolutionary path of the system is as follows:

(1)when $K_i^{'} > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$ 、 $K_i > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, corresponding to the initial table state in the table ③ (1 < 0, 2 < 0, 3 > 0, 4 > 0) . The system has four local equilibrium points, and the stable point is $E_4(1,1)$, implying that both firm A and firm B adopt the equipment renewal strategy, This suggests that, under a high level of government subsidy, and a moderate cost savings from collaborative renewal $-\Delta K_i$, the system will eventually converge to $E_4(1,1)$, indicating that substantial government subsidies can incentivize both firms in the game to adopt the equipment renewal strategy.

(2)When $K_i^{'} > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$ 、 $K_i < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, corresponding to the initial table state ① in the table (1 < 0, 2 < 0, 3 < 0, 4 < 0). The system has five local equilibrium points, and the stable points are $E_1(0,0)$ and $E_4(1,1)$, implying that both firms will adopt the same decision. This indicates that, under a lower level of government subsidy but with greater cost savings from collaborative renewal, both firms are inclined to adopt the same decision. As the cost savings from collaborative renewal increase, the probability that the system converges to $E_4(1,1)$ also increases, indicating that the enhanced synergies from equipment renewal can promote firms to adopt the renewal strategy.

(3)When $K_i^{'} < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, the initial state ⑥ (1 > 0, 2 > 0, 3 < 0, 4 < 0) . The system has four local equilibrium points, with the stable point being $E_1(0,0)$, where neither firm A nor firm B adopts the equipment renewal strategy. This indicates that, when the level of government subsidy is low and the cost savings from collaborative renewal are moderate, neither firm is willing to adopt equipment renewal.

(4)When $K_i^{'} < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, the initial state ⑫ (1 > 0, 2 > 0, 3 < 0, 4 > 0). The system has four local equilibrium points, with the stable points being $E_2(0,1)$ and $E_3(1,0)$, where only one firm chooses to renew the equipment. This indicates that, under a low level of market competition and with substantial government subsidies, but moderate cost $-\Delta K_i$ savings from collaborative renewal, the system will eventually converge to a stable state where only one firm adopts the equipment renewal strategy after continuous decision-making and learning.

When comparing Scenario ② and Scenario ③ horizontally, it can be observed that the net cost of equipment renewal $-K_i$ is higher in both scenarios, indicating that the intensity of government subsidies is relatively weak, and that firms face greater cost pressures when undertaking equipment renewal. However, the net cost of collaborative renewal $-K_i^{'}$ in Scenario ② is lower than that in Scenario ③. This implies that, as the cost savings from collaborative renewal $-\Delta K_i$ increase, the system stability points shift from $E_1(0,0)$ to $E_1(0,0)$ and $E_4(1,1)$, with $E_4(1,1)$ increasing. This suggests that improvements in the synergistic effect can effectively reduce the marginal cost of renewal for firms, thereby enhancing their willingness to undertake the renewal of the equipment. Comparing Scenarios ① and ④, the net cost of replacement $-K_i$ is lower in both scenarios, reflecting stronger government subsidies. In this context, the synergistic effect in Scenario ① is further enhanced compared to Scenario ④, and the corresponding system stabilization strategy shifts from $E_2(0,1)$ and $E_3(1,0)$ to $E_4(1,1)$, indicating that the outcome of the evolution changes from only one firm renewing its equipment to both firms renewing their equipment. This also confirms that the enhancement of the synergistic effect can significantly increase the willingness of manufacturing firms to renew equipment, thereby forming a more stable cooperative renewal situation.

Comparing Scenarios ③ and ④, under the condition of keeping other factors constant, as the net cost $-K_i$ decreases and the intensity of government subsidies rises, the system's evolutionary result shifts from both firms not renewing equipment to one firm renewing equipment. This further demonstrates that government subsidies can motivate firms to undertake equipment renewal.

Discussion II When $\frac{2\lambda_b U_0-\lambda_a U_0}{6} < (a-b)^2 < \frac{2\lambda_a U_0-\lambda_b U_0}{6}$, the intensity of competition in the market becomes greater, the system evolution path is as follows:

(1)When $K_i^{'} > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, as shown in Table 3, the initial state ⑩(1 < 0, 2 > 0, 3 > 0, 4 > 0). The system has four local equilibrium points, and the stability point is $E_3(1,0)$, under larger government subsidies, but with moderate cost savings from firms collaborating on equipment renewal $-\Delta K_i$, the system eventually converges to $E_3(1,0)$, indicating that only Firm A adopts the strategy. This suggests that, at this point, government subsidies are ineffective for firms with weaker financial capabilities.

(2)When $K_i^{'} < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, as shown in Table 3,the initial state ⑪(1 > 0, 2 > 0, 3 > 0, 4 > 0). The system has five local equilibrium points, and the stable points are $E_2(0,1)$ and $E_3(1,0)$. This result suggests that under conditions of relatively large government subsidies but limited cost savings $-\Delta K_i$ from collaborative equipment renewal, the system will eventually stabilize at a state where only one firm adopts the equipment renewal strategy after repeated strategy adjustment and learning.

(3)When $K_i^{'} > \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, as shown in Table 3, the initial state ⑦(1 < 0, 2 > 0, 3 < 0, 4 < 0). The system has four local equilibrium points, and the stability point is $E_1(0,0)$. This indicates that, under low levels of government subsidies, firms exhibit limited willingness to undertake equipment renewal, even when the cost savings from collaborative renewal $-\Delta K_i$ are considerable.

(4)When $K_i^{'} < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0-2\lambda_j U_0)}{18(a-b)}$, $K_i < \frac{\lambda_i U_0(6(a-b)^2+\lambda_i U_0)}{18(a-b)}$, as shown in Table 3, the initial state ⑥ （1 > 0, 2 > 0, 3 < 0, 4 < 0） . The system exhibits four local equilibrium points, and the stable point is also $E_1(0,0)$, implying that neither firm A nor firm B adopts the equipment renewal strategy. This result indicates that neither firm is willing to undertake equipment renewal when the government subsidies are low and the cost savings from collaborative renewal $-\Delta K_i$ are moderate.

Comparing Scenarios ③ and ④, it can be observed that the level of government subsidy remains low in both scenarios, while the cost savings $-\Delta K_i$ from collaborative equipment renewal increases are higher in scenario ③ than in scenario ④. Nevertheless, the equilibrium point in both scenarios is $E_1(0,0)$, indicating that neither firm opts for equipment renewal. This suggests that even with a strong synergistic effect from collaborative renewal, its incentive effect on firms' renewal strategies may be significantly diminished in a highly competitive market environment. For further cross-sectional comparison, in Scenario ① and Scenario ③, where all other conditions are held constant, the level of government subsidy is relatively higher in scenario ①. Under this condition, the stability point of the system shifts from $E_1(0,0)$ to $E_3(1,0)$, indicating that the outcome changes from neither firm undertaking equipment renewal to only firm A undertaking equipment renewal. Similarly, in the comparison between Scenario ④ and Scenario ②, the subsidy level in Scenario ④ is higher, and the corresponding stability point of the system shifts from $E_1(0,0)$ to $E_2(0,1)$ and $E_3(1,0)$, meaning that the evolution result changes from neither of the two firms undertake the equipment renewal. This further supports the positive incentive effect of government subsidies in encouraging firms to undertake equipment renewal.

Discussion III When $0 \le (a-b)^2 < \frac{2\lambda_b U_0 - \lambda_a U_0}{6}$, $\frac{\lambda_i U_0 (6(a-b)^2 + \lambda_i U_0 - 2\lambda_j U_0)}{18(a-b)}$ is constant greater than 0 and $K_i < K_i^{'} < 0$, then $\frac{\lambda_i U_0 (6(a-b)^2 + \lambda_i U_0 - 2\lambda_j U_0)}{18(a-b)} > 0 > K_i^{'}$ ($1>0, 2>0$ is constant) is constant and when $(a-b)^2$ is smaller and the intensity of competition in the market is further increased, the system evolution path is as follows:

(1) When $K_i > \frac{\lambda_i U_0 (6(a-b)^2 + \lambda_i U_0)}{18(a-b)}$, the stability analysis of the system equilibrium point is as shown in Table 3, the initial state ⑪ (1 > 0, 2 > 0, 3 > 0, 4 > 0). There are five local equilibrium points in the system, and the stability points are $E_2(0,1)$ and $E_3(1,0)$, indicating that only one firm will choose to make investment in equipment renewal. This suggests that under a relatively high level of government subsidy, but limited cost saving from collaborative renewal $-\Delta K_i$, the system will ultimately converge through repeated strategy learning to a stable state where only one firm adopts the equipment renewal strategy.

(2) When $K_i < \frac{\lambda_i U_0 (6(a-b)^2 + \lambda_i U_0)}{18(a-b)}$, the system equilibrium point stability analysis as shown in Table 3, the initial state ⑥ (1 > 0, 2 > 0, 3 < 0, 4 < 0). The system has four local equilibrium points, and with a stable point at $E_1(0,0)$, indicating that under a low level of government subsidy, firms are unwilling to undertake equipment renewal, even when the cost savings from collaborative renewal $-\Delta K_i$ are substantial.

Finally, comparing Scenario ③ in Discussion II with Scenario ② in Discussion I vertically, under otherwise identical conditions, reveals that as market competition intensifies, firms tend to abandon their equipment renewal strategy. This suggests that increasing market competition weakens the incentive effect of government subsidies, thereby reducing firms' willingness to invest in equipment renewal.

Similarly, comparing Discussion II with ① in Discussion I shows that the evolutionary outcome shifts from both firms undertaking equipment renewal to only firm A doing so. This further confirms that the effectiveness of government subsidy policies in promoting equipment renewal among manufacturing firms diminishes with stronger market competition. Moreover, this inhibitory effect is more pronounced for firm B, which faces more severe financing constraints.

IV. Simulation Analysis

This study conducts simulation analysis using MATLAB to verify the validity of the game-theoretical conclusions, while also providing a more intuitive comparison of the evolutionary paths of equipment renewal choices by firms A and B under varying subsidy intensities, competition levels, and degrees of synergistic effect.

A. Impact Analysis of Market Competition Intensity

Let the initial assignment of the variables be: $U_0 = 4, \lambda_a = 0.8, \lambda_b = 0.4, C_a = 7, C_b = 5, -\Delta K_i = 1$

The following figure illustrates the evolution path of firms' strategies regarding equipment renewal under different levels of market competition intensity, assuming both manufacturing firms A and B initially choose a 0.5 proportion of equipment renewal. As competition intensity increases, that is, when $b - a = \Delta$ decreases from 0.9 to 0.1, while holding the initial values of other parameters constant, it is observed that both firms are willing to undertake equipment renewal when market competition is low. However, as competition intensity rises, the evolutionary

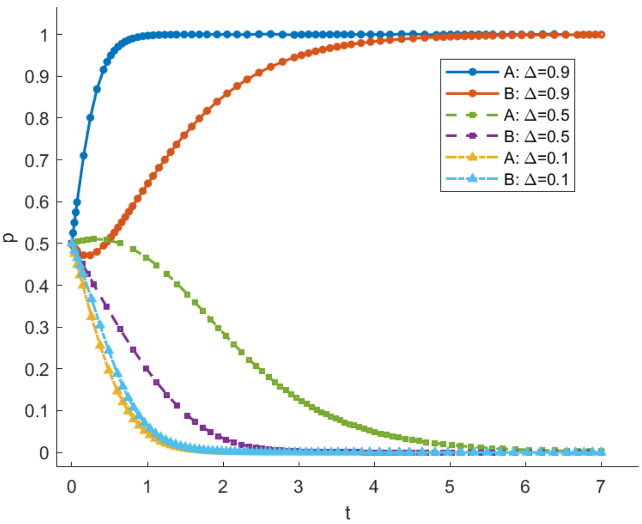
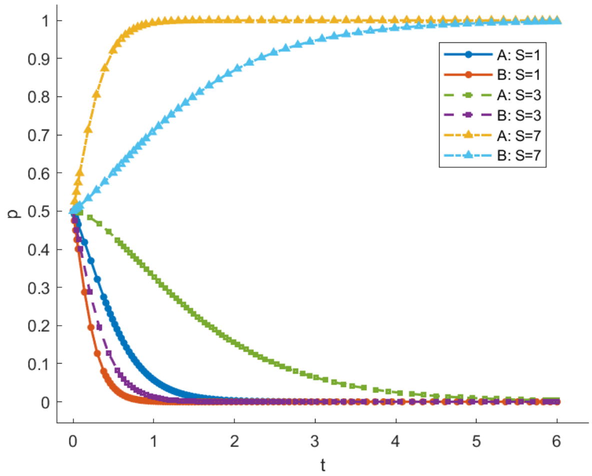


Fig. 2. Evolutionary path of firms' equipment renewal strategies under different market competition intensities

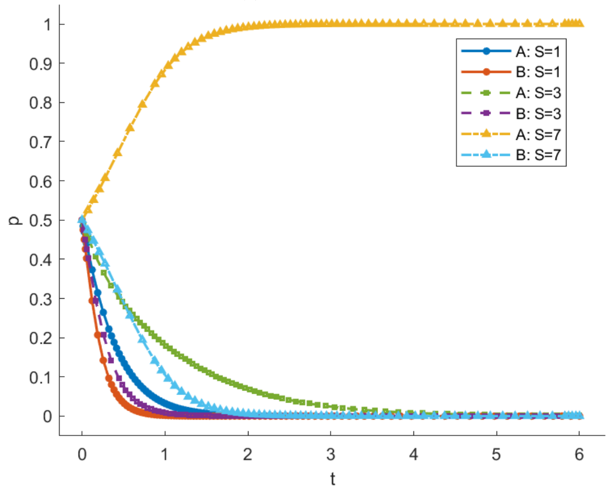
outcome shifts to a state where neither firm opts for equipment renewal.

B. Impact analysis of government subsidies

Let the initial assignment of the variable be $U_0 = 4, \lambda_a = 0.8, \lambda_b = 0.4, C_a = 7, C_b = 5, -\Delta K_i = 1$. Figure 3 illustrates the evolution path of firms' strategies regarding equipment renewal under different levels of government subsidy level, assuming both manufacturing firms A and B initially choose a 0.5 proportion of equipment renewal. As shown in (a) , when the intensity of market competition is low ($\Delta = 0.7$),an increase in government subsidy level leads the system to gradually evolve to the state (1,1), where both firms choose to invest in equipment renewal. This suggests that the government subsidy can enhance the willingness of the manufacturing firms to undertake equipment renewal. In contrast, when the intensity of market competition is high ($\Delta = 0.1$), as shown in (b), with the increase of the intensity of market competition, the system gradually evolves to the point that only firm A undertakes equipment renewal. This further indicates that government subsidy, while initially promoting renewal, can lose their effectiveness as competition intensifies. Additionally, the intensity of market competition in diminishing the incentive of government subsidies is more pronounced for firms with stronger financing constraints.



(a) $\Delta = 0.7$

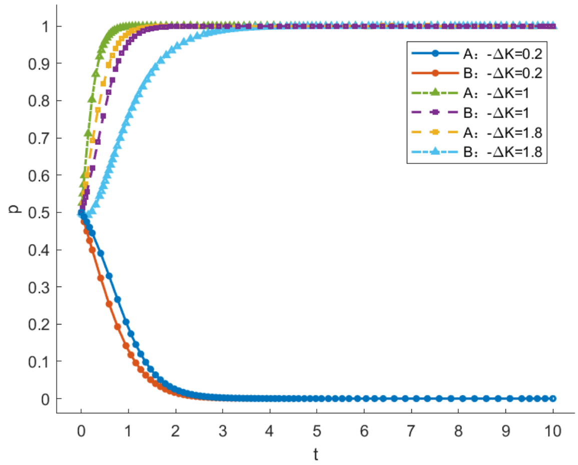


(b) $\Delta = 0.1$

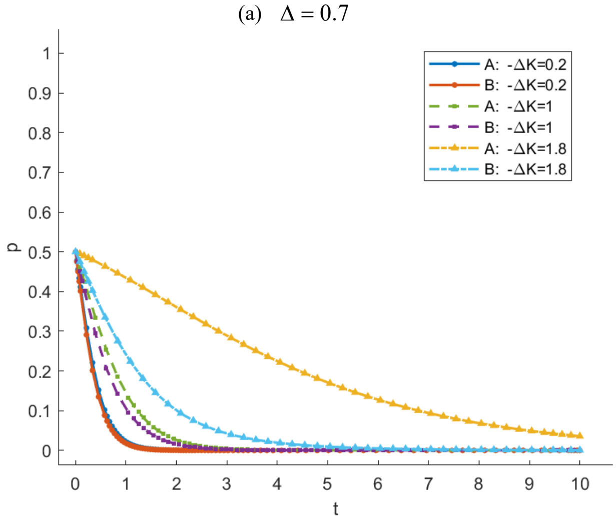
Fig. 3. Evolutionary path of firms' equipment renewal strategy choices under different government subsidies

C. Impact analysis of Synergies

Let the initial assignment of the variable be: $U_0 = 4, S = 7, \lambda_a = 0.8, \lambda_b = 0.4, C_a = 7, C_b = 5$. When the synergy effect is stronger, it reduces more trial-and-error costs for the firms, and Fig. 4 illustrates the evolution path of the strategy choices by the two manufacturing firms under different synergy effects. When market competition is low, a greater synergy effect leads the system to evolve toward (1,1), with both manufacturing firms increasingly willing to undertake equipment renewal. However, when competition intensity is higher, although neither firm is willing to renew equipment, the system shifts toward (0,0) at a slower pace, suggesting that a stronger synergy effect can enhance firms' willingness to invest in equipment renewal to some extent. This also confirms that higher market competition can weaken the role of the synergy effect.



(a) $\Delta = 0.7$



(b) $\Delta = 0.1$

Fig. 4. Evolutionary path of firms' equipment renewal strategy selection under different synergies

V. Conclusion

This paper examines the role of government subsidies as an incentive for firms to renew their equipment, and explores its relationship with the competitive market environment in the context of equipment renewal investments by manufacturing firms. The study draws the following main conclusions:

First, government subsidies have a significant incentive effect in promoting equipment renewal in manufacturing firms, and the incentive effect is further enhanced as the subsidy increases. However, there exists a threshold effect such that when the subsidy is low, its incentive effect on investment in equipment renewal is weak, making it difficult to effectively encourage firms to undertake equipment renewal. Second, the intensity of market competition is an important external factor influencing the effectiveness of government subsidies. As market competition intensifies, the likelihood of firms choosing to undertake equipment renewal declines, suggesting that, in a competitive market environment, firms are more inclined to improve production efficiency by optimizing existing equipment rather than investing in large-scale renewal. This effect is particularly pronounced for firms with high capital constraints and strong financing limitations, where the negative impact of intense competition on the effectiveness of government subsidies is more apparent. Additionally, the synergistic effects arising from collaborative equipment renewal among manufacturing firms play a crucial role in their investment decisions. The sharing of technology, equipment, and experience through collaborative upgrading can reduce costs and enhance the appeal of equipment renewal. However, in a more competitive market, these synergies are diminished, reducing the overall incentive for firms to engage in equipment renewal.

Based on the above conclusions, the following policy recommendations are made: First, the government should increase subsidies for investment in equipment renewal, particularly when subsidies are low. The government could establish special funds and raise subsidy standards to enhance firms' willingness to renew their equipment and facilitate technological upgrading and industrial transformation. Second, the market competition environment should be comprehensively considered when implementing differentiated subsidy strategies. In industries or regions with low market competition, the incentive effect of government subsidies on equipment renewal investment is more pronounced, and therefore, increasing the input of subsidy resources could be considered. In contrast, in more competitive environments, the effect of government subsidies is more limited. In such cases, direct subsidies should be appropriately reduced, and policy guidance should be emphasized to encourage firms to undertake equipment renewal based on their own strengths, thereby reducing the inhibitory effect of homogeneous competition on subsidy effectiveness. Finally, the government should play a guiding role in promoting cooperation among firms and establishing a platform for shared equipment renewal. Particularly in the early stages of the equipment renewal market, the government can set up a public service platform to integrate equipment, technology, and experience from various firms, reducing the cost of repeated investments and enhancing overall industry efficiency in terms of equipment renewal. At the same time, the government can provide technical support and financial guidance to promote synergistic cooperation among firms, thereby optimizing the overall effectiveness of equipment renewal.

This study is subject to several limitations. The model simplifies the market structure to a duopoly and assumes homogeneous consumer preferences, which may not fully capture the complexity of real-world manufacturing sectors. Future research could extend this framework by considering multi-firm competition and heterogeneous consumer demand to enhance the generalizability of the findings.

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