Arithmetic Mean-Based Approach for Solving Fuzzy Multi-Objective Transportation Problems Using Triangular Fuzzy Numbers

Ramakant Sharma, Sohan Lal Tyagi

Abstract— **The transportation problem (TP) is a classical optimization problem in operations research and logistics. The transportation problem is a particular type of Linear Programming Problem (LPP). In the present time scenario, the decision maker handles several objectives simultaneously. This paper presents an approach based on the arithmetic mean technique that solves the Fuzzy Multi-Objective Transportation Problem (FMOTP) in which all the transportation objectives are imprecise and represented by triangular fuzzy numbers. This approach first formulates the given FMOTP into mathematical form and then decomposes it into three levels (lower, middle, and upper) Crisp Multi-Objective Transportation Problems (CMOTPs). Then we converted these CMOTPs into Single Objective Transportation Problems (SOTPs) using the Fuzzy Arithmetic Mean (FAM) technique. The combined fuzzy optimal solution is obtained by solving these three SOTPs. TORA software is used to solve these SOTPs. The incentre point is used to defuzzify the fuzzy optimal value to get a crisp optimal value. The proposed approach is elaborated using two numerical examples, and the results are compared with those obtained by other methods.**

Index Terms— **Transportation problem, fuzzy multi-objective transportation problem, arithmetic mean, triangular fuzzy number**

I. Introduction

The Transportation Problem (TP) is defined as a specific kind of Linear Programming Problem (LPP). It involves determining the most cost-effective way to distribute a product from various providers to various consumers to satisfy requirements and availabilities. The goal of TP is to reduce the overall cost of transportation. Traditional methods, including the Vogel approximation method, the Matrix minima approach, and the Northwest Corner method, are used to solve the TP. In real-world scenarios, decision makers can now handle multiple objectives simultaneously, but are unsure about the accurate values of requirements, transportation costs, and availability. In the Multi-Objective Transportation Problem (MOTP), several objectives have different parameters, and equality constraints must be satisfied. Fuzzy concepts often deal with such types of uncertainty and vagueness in the exact cost of transportation, availabilities, and requirements. The Concept of Fuzzy Transportation Problems (FTPs) was developed to solve the TP's unpredictable parameters, such as fuel prices, weather conditions, product supply, demands, etc. Triangular fuzzy numbers are useful when modeling uncertain parameters in transportation problems, such as requirements and availabilities quantities or transportation costs, which are not precisely known but have a range of potential values. F.L. Hitchcock [1] first formulated the transportation problems as LPP in 1941. An LPP form was used to represent the TP, which can be solved by the simplex algorithm. The Stepping Stone approach offers an alternative approach for obtaining the information from the simplex method developed by Charnes and Cooper [2]. Lotfi A. Zadeh [3] was given the concept of fuzziness in 1965. Zimmermann H.J. [4] was the first to use an appropriate membership function to solve an LP problem with multiple objectives. Ringuest et al. [5] provide two interactive algorithms to tackle MOTP. Bit et al. [6] solved TP problems with several criteria by using fuzzy programming. SJ Chen and SM Chen [7] proposed an algorithm to solve multicriteria fuzzy decision problems using the Ordered Weight Averaging Operator (IOWA). SM Chen and SJ Chen [8] proposed a novel method to solve fuzzy risk analysis problems by using ranking functions for generalized trapezoidal fuzzy numbers. M. et al. [9] presented the DEA approach to solving FMOTP. Karthy, T., & Ganesan, K. [10] presented a new approach to solving FMOTP using a genetic approach. SK Bharti and SR Singh [11] proposed a novel method to solve fuzzy MOTP using trapezoidal fuzzy numbers based on a new distance function. Srikanth Gupta et. al. [12] investigated the MOTP with capacitated restrictions in which some objective functions are linear and some are fractional. G. Krishnaveni and K. Ganeshan [13] developed a fuzzy DEA-based approach for solving FMOTP. M.A. Sayed et.al. [14] developed a novel approach to solve intuitionistic fuzzy fractional MOTP. Ahmed J.S. [15] et al. modified the Centre of Gravity (COG) approach using a Multi-Objective Linear Programming (MOLP) model to provide a novel technique for defuzzifying fuzzy integers. This method used triangular fuzzy numbers to describe the observed data, with crisp values inside each fuzzy number's range. Kamal, M. et.al. [16] proposed a method for solving MOTP under Type-2 Trapezoidal Fuzzy Numbers (T2TpFN) is presented. T2TpFN is first converted into a crisp form using the two-phase defuzzification method. Next, multi-choice and probabilistic random variables are converted into equivalent

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values using the binary variable and the Stochastic Programming (SP) approach. The Fuzzy Goal Programming (FGP) approach is applied for the best decision-making process. Hamiden Abd El-Waheed Khalifa et.al. [17] proposed a fuzzy geometric programming approach to obtain an optimal compromise solution for two-stage MOTP. Y Kacher and P Singh [18] proposed a method based on the Fuzzy Harmonic Mean (FHM) to solve FFMOTP. They used FHM as a tool to convert CMOTP to CSOTP. Sharma M.K. et al. [19] proposed a novel approach to solve MOTP using the Fermatian fuzzy technique. Akram, M. et.al. [20] presented a method using data envelope analysis, for decision-making units with multiple inputs and outputs ranked according to their relative efficiency P. Indira and M. Jayalakshmi [21] proposed an approach to find the minimum transportation cost of triangular TP using the alpha cut, and row-column minima method. Kokila A. and Deepa G.[22] proposed a new method based on the harmonic mean to solve fuzzy multi-objective transportation problems.

This paper presents a novel approach to solving the Fuzzy Multi-Objective Transportation Problem (FMOTP). Initially, the FMOTP is transformed into a Crisp Single Objective Transportation Problem (CSOTP) using the fuzzy arithmetic mean technique. The resulting CSOTP is then solved using the TORA optimization software to obtain a compromised optimal solution to the original fuzzy problem.

II. Preliminaries

A. *Membership Function (MF)*

Let X be a universal set. The Membership Function (MF) of a fuzzy set $\tilde{A}$ in X is denoted by $\mu_{\tilde{A}}$; i.e.

$$\mu_A : X \to [0,1]$$

$$\mu_{\tilde{A}} : X \to [0, 1]$$

For each x ∈ X, $\mu_{\tilde{A}}(x)$ represents the membership grade of element x in the fuzzy set $\tilde{A}$.

B. *Fuzzy Number GF*

A fuzzy set $\tilde{A}$, with a membership function $\tilde{A} : \mathbb{R} \longrightarrow [0,1]$ defined on the set of real numbers is called a fuzzy number if it satisfies the following properties:

(i) $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{A}(x_1), \tilde{A}(x_2)\}$

(ii) ∃ a x ∈ $\mathbb{R}$ such that $\tilde{A}(x) = 1$.

(iii) $\tilde{A}$ is piece-wise continuous.

C. *Triangular fuzzy number (TFN)*

A fuzzy set $\tilde{A} = (f, g, h)$ such that $f \leq g \leq h$ is said to be TFN with the centre g, left width $g - f > 0$, and right width $h - g > 0$, If its MF is as follows:

$$\tilde{A}(y) = \begin{cases} (1 - \dfrac{(g-y)}{(g-f)}) & f \leq y \leq g \\ (1 - \dfrac{(g-y)}{(g-f)}) & g \leq y \leq h \\ 1 & y = g \\ 0 & otherwise \end{cases} \quad [1]$$

The triangular fuzzy number $\tilde{A} = (f, g, h)$ such that $f \leq g \leq h$. It is said to be a non-negative TFN if $f \geq 0$.

D. *Fuzzy arithmetic operations for Triangular Fuzzy Numbers (TFNs)*

Let $\tilde{A} = (f_1, g_1, h_1)$ and $\tilde{B} = (f_2, g_2, h_2)$ be any two TFNs. Then arithmetic operations on triangular fuzzy numbers are defined as follows:

- Addition: $\tilde{A} + \tilde{B} = (f_1 + f_2, g_1 + g_2, h_1 + h_2)$
- Subtraction: $\tilde{A} - \tilde{B} = (f_1 - h_2, g_1 - g_2, h_1 - f_2)$
- Multiplication: Let $\tilde{A} = (f_1, g_1, h_1)$ be any arbitrary TFN and $\tilde{B} = (f_2, g_2, h_2)$ any non-negative TFN.

$$\text{Then, } .\tilde{B} = \begin{cases} (f_1 f_2, g_1 g_2, h_1 h_2) & if\ f_1 \geq 0 \\ (f_1 h_2, g_1 g_2, h_1 h_2) & if\ f_1 < 0, h_1 \geq 0 \\ (f_1 h_2, g_1 g_2, h_1 f_2) & if\ h_1 < 0 \end{cases}$$

If k is any real number, then

$$k.\tilde{A} = \begin{cases} (kf_1, kg_1, kh_1) & if\ k_1 \geq 0 \\ (kh_1, kg_1, kf_1) & if\ k_1 < 0 \end{cases}$$

- Division: Let $\tilde{A} = (f_1, g_1, h_1)$ be any arbitrary TFN and $\tilde{B} = (f_2, g_2, h_2)$ any non-zero TFN. Then

$$\tilde{A} / \tilde{B} = (f_1 / h_2, g_1 / g_2, h_1 / f_2)$$

E. *Ranking of generalized triangular fuzzy numbers by the incentre point method*

The fuzzy numbers are compared using the ranking method. The ranking function, denoted as $\Re : T(\mathbb{R}) \longrightarrow \mathbb{R}$, maps each fuzzy number to a real number where the natural order is present, $T(\mathbb{R})$, is the set of fuzzy numbers defined on the set of real numbers. A fuzzy ranking function changes these optimal solutions into crisp values.

If $\tilde{A} = (f_1, g_1, h_1)$ is any TFN. Let ABD be a triangle formed by a triangular fuzzy number .

$I = (I_x, I_y)$ is the incentre point of triangle ABD as shown in Fig.1. I_x, and I_y is given as:

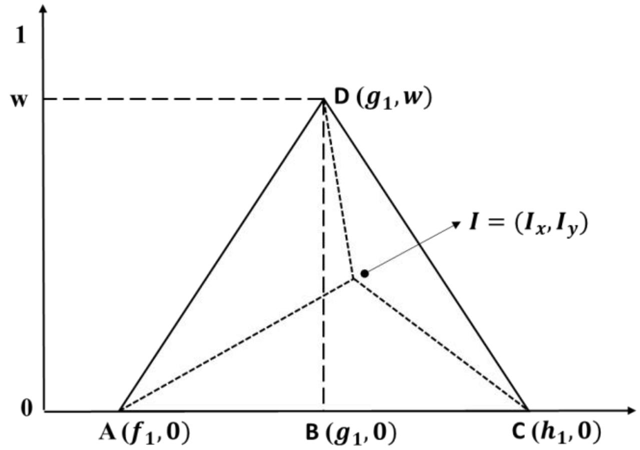

Figure 1. The triangular fuzzy number with incentre point

$$I_x = \frac{\alpha f_1 + \beta g_1 + \gamma h_1}{\alpha+\beta+\gamma}, \quad I_y = \frac{w\beta}{\alpha+\beta+\gamma}$$

Here α, β, and γ is the length of the sides of the triangle ABC. α, β, and γ is given as:

$$\alpha = \sqrt{(h_1-g_1)^2+w^2}, \qquad \beta = h_1 - f_1, \text{ and}$$

$$\gamma = \sqrt{(g_1-f_1)^2+w^2}$$

Here ranking of $\tilde{A} = (f_1, g_1, h_1)$ is given by only the X-coordinate of the incentre point I.

$$\Re(\tilde{A}) = I_x = \frac{\alpha f_1 + \beta g_1 + \gamma h_1}{\alpha+\beta+\gamma}$$

F. Fuzzy Arithmetic Mean (FAM)

The fuzzy arithmetic mean is a concept in fuzzy logic, a branch of logic that deals with uncertainty and imprecision. The fuzzy arithmetic mean can be used to determine the average of a set of fuzzy values or sets. If $y_1, y_2, \ldots\ldots, y_{n_0}$ are the n_0 quantities. Then the arithmetic means of these n_0 quantities are defined as:

$$\text{A.M. } (y_1, y_2, \ldots\ldots, y_{n_0}) = \frac{y_1 + y_2 + \ldots\ldots + y_{n_0}}{n_0}$$

If $A_{j_0} = (f^l_{j_0}, f^m_{j_0}, f^u_{j_0})$ for $j_0 = 1,2,\ldots\ldots,n_0$ are n_0 triangular fuzzy numbers, then the fuzzy arithmetic mean of $\tilde{A}_1, \tilde{A}_2, \ldots\ldots, \tilde{A}_{n_0}$ is defined as:

$$FAM(\tilde{A}_1, \tilde{A}_2, \ldots\ldots, \tilde{A}_{n_0}) = \frac{\sum_{j_0=1}^{n_0} \tilde{A}_{j_0}}{n_0} = \frac{\sum_{j_0=1}^{n_0} (f^l_{j_0}, f^m_{j_0}, f^u_{j_0})}{n_0}$$

$$= \left(\frac{\sum_{j_0=1}^{n_0} f^l_{j_0}}{n_0}, \frac{\sum_{j_0=1}^{n_0} f^m_{j_0}}{n_0}, \frac{\sum_{j_0=1}^{n_0} f^u_{j_0}}{n_0}\right) \qquad [2]$$

III. The Mathematical Formulation of Fuzzy Multi-Objective Transportation Problem (FMOTP)

The fuzzy MOTP in mathematical form can be formulated as:

$$Min\ \tilde{Z}_u(y) = \sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0} \tilde{\alpha}^{(k)}_{i_0 j_0} \tilde{y}_{i_0 j_0} \qquad [3]$$

Subject to

$$\sum_{i_0=1}^{m_0} \tilde{y}_{i_0 j_0} = \tilde{d}_{j_0} \text{ : for fixed } j_0 \text{ such that } j_0 = 1,2,\ldots.,n_0 \quad [4]$$

$$\sum_{j_0=1}^{n_0} \tilde{y}_{i_0 j_0} = \tilde{s}_{i_0} \text{ : for fixed } i_0 \text{ such that } i_0 = 1,2,\ldots.,m_0 \quad [5]$$

$$\tilde{y}_{i_0 j_0} \geq 0 \qquad [6]$$

here,

m_0 = total no. of origins

n_0 = total no. of destinations

$\tilde{y}_{i_0 j_0} \geq 0$ = Transportable fuzzy quantity of goods transported from i_0-th origins to j_0-th destinations

$\tilde{s}_{i_0}$ = The fuzzy availabilities of goods at i_0 -th origins

$\tilde{d}_{j_0}$ = Fuzzy requirements of goods at j_0-th destinations.

$\alpha^{(k)}_{i_0 j_0}$ = The fuzzy cost for transporting one unit of the given good from the i-th origin to the j-th destination.

The above LPP (3-6) can be written as:

$$Min\ \tilde{Z}_u(y) = \min_{\tilde{y}}(\tilde{F}_{11}(y), \tilde{F}_{12}(y), \ldots\ldots, \tilde{F}_{1p}(y)) \qquad [7]$$

Subject to the constraints [4]-[6] [8]

Here,

$$\left.\begin{aligned} \tilde{F}_{11}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0}^{n_0} \alpha^{(1)}_{i_0 j_0} \tilde{y}_{i_0 j_0} \\ \tilde{F}_{12}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0}^{n_0} \alpha^{(2)}_{i_0 j_0} \tilde{y}_{i_0 j_0} \\ &\vdots \\ &\vdots \\ \tilde{F}_{1p}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0}^{n_0} \alpha^{(p)}_{i_0 j_0} \tilde{y}_{i_0 j_0} \end{aligned}\right\} \qquad [9]$$

If $\sum_{i_0=1}^{m_0} \tilde{s}_{i_0} = \sum_{j_0=1}^{n_0} \tilde{d}_{j_0}$ so, then it is called balanced FMOTP.

Here, we assume that FMOTP is balanced without losing generality.

Remark 1: In the real world, transportation parameters such as requirements, availabilities, and transportation costs of the products are non-negative quantities. So, we assume that the triangular fuzzy numbers $A = (a^l, a^m, a^u)$ are non-negative quantities in the above FMOTP.

Regarding remark 1, $\tilde{Z}_u$, $\tilde{F}_{uv}(y)$, $\tilde{s}_{i_0}$, $\tilde{d}_{j_0}$, $\alpha^{(k)}_{i_0 j_0}$ and $\tilde{y}_{i_0 j_0}$ do triangular fuzzy numbers represent all as $\tilde{Z}_u = (\mu^l_k, \mu^m_k, \mu^u_k)$, $\tilde{F}_{uv}(y) = (\vartheta^l_{rk}, \vartheta^m_{rk}, \vartheta^u_{rk})$ $\tilde{s}_{i_0} = (\tilde{s}^l_{i_0}, \tilde{s}^m_{i_0}, \tilde{s}^l_{i_0})$, $\tilde{d}_{j_0} = (\tilde{d}^l_{j_0}, \tilde{d}^m_{j_0}, \tilde{d}^u_{j_0})$ $\tilde{\alpha}^{(k)}_{i_0 j_0} = (\tilde{\alpha}^{kl}_{i_0 j_0}, \tilde{\alpha}^{km}_{i_0 j_0}, \tilde{\alpha}^{ku}_{i_0 j_0})$, $\tilde{y}_{i_0 j_0} = (y^l_{i_0 j_0}, y^m_{i_0 j_0}, y^u_{i_0 j_0})$ respectively.

Thus (7)-(9) can be restated by using fuzzy arithmetic as:

$$\min(\mu^l_k, \mu^m_k, \mu^u_k) = \min_{\tilde{y}}[(\vartheta^l_{11}, \vartheta^m_{11}, \vartheta^u_{11}), (\vartheta^l_{12}, \vartheta^m_{12}, \vartheta^u_{12}), \ldots\ldots, (\vartheta^l_{1p}, \vartheta^m_{1p}, \vartheta^u_{1p})] \qquad [10]$$

subject to

$$\sum_{j_0=1}^{n_0} (y^l_{i_0 j_0}, y^m_{i_0 j_0}, y^u_{i_0 j_0}) = (\tilde{s}^l_{i_0}, \tilde{s}^m_{i_0}, \tilde{s}^l_{i_0}) \qquad [11]$$

$$\sum_{i_0=1}^{m_0} (y^l_{i_0 j_0}, y^m_{i_0 j_0}, y^u_{i_0 j_0}) = (\tilde{d}^l_{j_0}, \tilde{d}^m_{j_0}, \tilde{d}^u_{j_0}) \qquad [12]$$

$$(y^l_{i_0 j_0}, y^m_{i_0 j_0}, y^u_{i_0 j_0}) \geq 0.\ \forall\ (i_0 = 1,2,\ldots\ldots,m_0,\ j_0 = 1,2,\ldots\ldots,n_0) \qquad [13]$$

here,

$$\left.\begin{aligned}
\tilde{F}_{11}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}(\alpha_{i_0j_0}^{1l},\alpha_{i_0j_0}^{1m},\alpha_{i_0j_0}^{1u})\otimes(y_{i_0j_0}^{l},y_{i_0j_0}^{m},y_{i_0j_0}^{u}) \\
\tilde{F}_{12}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}(\alpha_{i_0j_0}^{2l},\alpha_{i_0j_0}^{2m},\alpha_{i_0j_0}^{2u})\otimes(y_{i_0j_0}^{l},y_{i_0j_0}^{m},y_{i_0j_0}^{u}) \\
&\vdots \\
\tilde{F}_{1p}(y) &= \sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}(\alpha_{i_0j_0}^{pl},\alpha_{i_0j_0}^{pm},\alpha_{i_0j_0}^{pu})\otimes(y_{i_0j_0}^{l},y_{i_0j_0}^{m},y_{i_0j_0}^{u})
\end{aligned}\right\} \quad [14]$$

Now, using fuzzy arithmetic mean above MOTP can be reformulated as

$$\min(\mu_k^l,\mu_k^m,\mu_k^u) = \min_{\tilde{y}}\sum_{i_0=1}^{m_0}\sum_{j_{0=1}}^{n_0}(\alpha_{i_0j_0}^{kl}y_{i_0j_0}^{l},\alpha_{i_0j_0}^{km}y_{i_0j_0}^{m},$$
$$\alpha_{i_0j_0}^{ku}y_{i_0j_0}^{u}) \quad [15]$$

$k = 1,2,......,p$

Subject to

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{l} = s_{i_0}^{l} \quad (i_0 = 1,2,......,m_0) \quad [16]$$

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{m} = s_{i_0}^{m} \quad (i_0 = 1,2,......,m_0) \quad [17]$$

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{u} = s_{i_0}^{u} \quad (i_0 = 1,2,......,m_0) \quad [18]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{l} = d_{j_0}^{l} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [19]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{m} = d_{j_0}^{m} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [20]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{u} = d_{j_0}^{u} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [21]$$

$$y_{i_0j_0} \ge 0 \quad (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [22]$$

$$y_{i_0j_0}^{m} - y_{i_0j_0}^{l} \ge 0\ (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [23]$$

$$y_{i_0j_0}^{u} - y_{i_0j_0}^{m} \ge 0\ (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [24]$$

Remark 2: Formulation (3)-(6) is equivalent to (15)-(24).

IV. Proposed Method to solve fuzzy Multi-objective transportation problem (FMOTP)

Step I: In this step, we write all objectives in the minimization form and reduce them into the problem in (7)-(9).

Step II: In this step, we write the problem (7)– (9) in the form of equations (10)– (14), replacing the fuzzy parameter values with the non-negative triangular fuzzy number.

Step III: This step divides the entire problem (9)-(13) into three levels of crisp MOTP.

Lower-level Multi-objective Transportation Problem (L-MOTP):

$$\min_{\tilde{y}}(\vartheta_{11}^{l},\vartheta_{12}^{l},......,\vartheta_{1p}^{l}) = \min_{\tilde{y}}(\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{1l}\otimes y_{i_0j_0}^{l},$$
$$\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{2l}\otimes y_{i_0j_0}^{l},......,\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{pl}\otimes y_{i_0j_0}^{l}) \quad [25]$$

Subject to

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{l} = s_{i_0}^{l} \quad (i_0 = 1,2,......,m_0) \quad [26]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{l} = d_{j_0}^{l} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [27]$$

$$y_{i_0j_0} \ge 0 \quad (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [28]$$

Here, equations (25)-(28) represent a crisp MOTP of a lower level with p-objectives. These equations can be solved by TORA.

Middle-level Multi-Objective Transportation Problem (M-MOTP):

$$\min_{\tilde{y}}(\vartheta_{11}^{m},\vartheta_{12}^{m},......,\vartheta_{1p}^{m}) = \min_{\tilde{y}}(\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{1m}\otimes y_{i_0j_0}^{m},$$
$$\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{2m}\otimes y_{i_0j_0}^{m},......,\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{pm}\otimes y_{i_0j_0}^{m}) \quad [29]$$

Subject to

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{m} = s_{i_0}^{m} \quad (i_0 = 1,2,......,m_0) \quad [30]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{m} = d_{j_0}^{m} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [31]$$

$$y_{i_0j_0} \ge 0 \quad (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [32]$$

Here, equations (29)-(32) represent a crisp MOTP of middle level with p-objectives. These equations can be solved by TORA.

U upper-level multi-objective transportation problem (U-MOTP):

$$\min_{\tilde{y}}(\vartheta_{11}^{u},\vartheta_{12}^{u},......,\vartheta_{1p}^{u}) = \min_{\tilde{y}}(\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{1u}\otimes y_{i_0j_0}^{u},$$
$$\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{2u}\otimes y_{i_0j_0}^{u},......,\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}\alpha_{i_0j_0}^{pu}\otimes y_{i_0j_0}^{u}) \quad [33]$$

Subject to

$$\sum_{j_0=1}^{n_0} y_{i_0j_0}^{u} = s_{i_0}^{u} \quad (i_0 = 1,2,......,m_0) \quad [34]$$

$$\sum_{i_0=1}^{m_0} y_{i_0j_0}^{u} = d_{j_0}^{u} \quad (\mathrm{j}_0 = 1,2,......,n_0) \quad [35]$$

$$y_{i_0j_0} \ge 0 \quad (i_0 = 1,2,......,m_0,\ \mathrm{j}_0 = 1,2,......,n_0) \quad [36]$$

Here, equations (33)-(36) represent a crisp MOTP of the upper level with p-objectives. TORA can solve these equations.

Step IV: Find each objective function's optimal solution, denoted as ϑ_{rk}^{s} (for $r=1,k=1,2,......,p,s=l,m,u$), that is involved in three levels receptively, denoted as ϕ_{rk}^{s} (for $r=1,k=1,2,......,p,s=l,m,u$).

Step V: In this step, we calculate the fuzzy arithmetic mean (FAM) for each optimal solution for these three levels. The FAM can be calculated as:

$$FAM_1(\phi_{rk}^{s})=(AM_1^{l},AM_1^{m},AM_1^{u})$$

$$=\left(\frac{\sum_{k=1}^{p}\phi_{1k}^{l}}{p},\frac{\sum_{k=1}^{p}\phi_{1k}^{m}}{p},\frac{\sum_{k=1}^{p}\phi_{1k}^{u}}{p}\right) \quad [37]$$

Here

$AM_1^{l}=\frac{\sum_{k=1}^{p}\phi_{1k}^{l}}{p}$ = Arithmetic Mean (AM) lower level's optimal solutions

$AM_1^{m}=\frac{\sum_{k=1}^{p}\phi_{1k}^{m}}{p}$ = AM of middle level's optimal solutions

$AM_1^{u}=\frac{\sum_{k=1}^{p}\phi_{1k}^{u}}{p}$ = AM of the upper level's optimal solutions

Step VI: Three-level crisp MOTP is changed into three-level crisp Single Objective Transportation Problems (SOTP) in this step.

L- Lower level SOTP

$$\min_{\tilde{y}}\xi_1^{l}=\frac{\min\vartheta_{11}^{l}+\min\vartheta_{12}^{l}+\min\vartheta_{13}^{l}+......+\min\vartheta_{1p}^{l}}{AM_1^{u}} \quad [38]$$

Subject to

$$\sum_{j_0=1}^{n_0}y_{i_0j_0}^{l}=s_{i_0}^{l} \quad (i_0=1,2,......,m_0) \quad [39]$$

$$\sum_{i_0=1}^{m_0}y_{i_0j_0}^{l}=d_{j_0}^{l} \quad (\mathrm{j}_0=1,2,......,n_0) \quad [40]$$

$$y_{i_0j_0}\geq 0 \quad (i_0=1,2,......,m_0,\ \mathrm{j}_0=1,2,......,n_0) \quad [41]$$

We solve equations (38)-(41) by TORA. We get the optimal solution $(y_{i_0j_0}^{l*})_{mn\times 1}$ for lower levels of the problem.

M- Middle level SOTP:

$$\min_{\tilde{y}}\xi_1^{m}=\frac{\min\vartheta_{11}^{m}+\min\vartheta_{12}^{m}+\min\vartheta_{13}^{m}+......+\min\vartheta_{1p}^{m}}{AM_1^{m}} \quad [42]$$

Subject to

$$\sum_{j_0=1}^{n_0}y_{i_0j_0}^{m}=s_{i_0}^{m} \quad (i_0=1,2,......,m_0) \quad [43]$$

$$\sum_{i_0=1}^{m_0}y_{i_0j_0}^{m}=d_{j_0}^{m} \quad (\mathrm{j}_0=1,2,......,n_0) \quad [44]$$

$$y_{i_0j_0}\geq 0 \quad (i_0=1,2,......,m_0,\ \mathrm{j}_0=1,2,......,n_0) \quad [45]$$

We solve equations (42)-(45) by TORA. We get the optimal solution $(y_{i_0j_0}^{m*})_{mn\times 1}$ for the middle level of the problem.

U- Upper level SOTP:

$$\min_{\tilde{y}}\xi_1^{u}=\frac{\min\vartheta_{11}^{u}+\min\vartheta_{12}^{u}+\min\vartheta_{13}^{u}+......+\min\vartheta_{1p}^{u}}{AM_1^{l}} \quad [46]$$

Subject to

$$\sum_{j_0=1}^{n_0}y_{i_0j_0}^{u}=s_{i_0}^{u} \quad (i_0=1,2,......,m_0) \quad [47]$$

$$\sum_{i_0=1}^{m_0}y_{i_0j_0}^{u}=d_{j_0}^{u} \quad (\mathrm{j}_0=1,2,......,n_0) \quad [48]$$

$$y_{i_0j_0}\geq 0 \quad (i_0=1,2,......,m_0,\ \mathrm{j}_0=1,2,......,n_0) \quad [49]$$

We solve equations (46)-(49) by TORA. We get the optimal solution $(y_{i_0j_0}^{u*})_{mn\times 1}$ for the upper level of the problem.

Step VII: To solve these three different levels of crisp SOTPs, we will get the values of $\tilde{y}_{i_0j_0}$, $\tilde{y}_{i_0j_0}=(y_{i_0j_0}^{l*},y_{i_0j_0}^{m*},y_{i_0j_0}^{u*})$ FMOTP (2)-(5) will be the fuzzy optimal values.

Step VIII: In this step, by using the proposed ranking function, defuzzify the fuzzy optimal value obtained in step VII. By defuzzification, we get the crisp value of the problem (2)-(5). This crisp value helps us compare the results obtained by our proposed method (FAM technique) and some other existing methods.

To elaborate on our proposed method, two examples are considered

Example 1: A multi-objective fuzzy transportation problem is considered, involving triangular fuzzy numbers to represent the fuzzy cost, time, and supply and demand parameters, as presented in Tables 1(i) and 1(ii). This problem has previously been solved by Y. Kacher and P. Singh [18], as well as A. Kokila and G. Deepa [22].

Table 1(i) First objective function (Cost)

Source→ Destination↓	D_1	D_2	D_3	D_4	Supply
S_1	(1,1.5,2)	(1,2,3)	(5,7,9)	(4,6,8)	(7,8,9)
S_2	(1,1.5,2)	(7,8.5,10)	(2,4,6)	(3,4,5)	(17,19,21)
S_3	(7,8,9)	(7,9,11)	(3,4,5)	(5,6,7)	(16,17,18)
Demand	(10,11,12)	(2,3,4)	(13,14,15)	(15,16,17)	

Table 1(ii) Second objective function (Time)

Source→ Destination↓	D_1	D_2	D_3	D_4	Supply
S_1	(3,4,5)	(2,4,6)	(2,3,4)	(1,3,5)	(7,8,9)
S_2	(4,5,6)	(7,8,9)	(7,8.5,10)	(9,10,11)	(17,19,21)
S_3	(4,6,8)	(1,2,3)	(3,4.5,6)	(1,1.5,2)	(16,17,18)
Demand	(10,11,12)	(2,3,4)	(13,14,15)	(15,16,17)	

Here,

$$\tilde{F}_{11}(y)=\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}(\alpha_{i_0j_0}^{1l},\alpha_{i_0j_0}^{1m},\alpha_{i_0j_0}^{1u})\otimes(y_{i_0j_0}^{l},y_{i_0j_0}^{m},y_{i_0j_0}^{u})$$
$$=(y_{11}^l+y_{12}^l+5y_{13}^l+4y_{14}^l+y_{21}^l+7y_{22}^l+2y_{23}^l$$
$$+3y_{24}^l+7y_{31}^l+7y_{32}^l+3y_{33}^l+5y_{34}^l,1.5y_{11}^m+2y_{12}^m$$
$$+7y_{13}^m+6y_{14}^m+1.5y_{21}^m+8.5y_{22}^m+4y_{23}^m+4y_{24}^m+$$
$$8y_{31}^m+9y_{32}^m+4y_{33}^m+6y_{34}^m,2y_{11}^u+3y_{12}^u+9y_{13}^u+8y_{14}^u$$
$$+2y_{21}^u+10y_{22}^u+6y_{23}^u+5y_{24}^u+9y_{31}^u+11y_{32}^u+5y_{33}^u+7y_{34}^u) \quad [50]$$

$$\tilde{F}_{12}(y)=\sum_{i_0=1}^{m_0}\sum_{j_0=1}^{n_0}(\alpha_{i_0j_0}^{2l},\alpha_{i_0j_0}^{2m},\alpha_{i_0j_0}^{2u})\otimes(y_{i_0j_0}^{l},y_{i_0j_0}^{m},y_{i_0j_0}^{u})$$

$$=(3y_{11}^l+2y_{12}^l+2y_{13}^l+y_{14}^l+4y_{21}^l+7y_{22}^l+7y_{23}^l+9y_{24}^l$$
$$+4y_{31}^l+y_{32}^l+3y_{33}^l+y_{34}^l,4y_{11}^m+4y_{12}^m+3y_{13}^m+3y_{14}^m+5y_{21}^m$$
$$+8y_{22}^m+8.5y_{23}^m+10y_{24}^m+6y_{31}^m+2y_{32}^m+4.5y_{33}^m+1.5y_{34}^m,$$
$$5y_{11}^u+6y_{12}^u+4y_{13}^u+5y_{14}^u+6y_{21}^u+9y_{22}^u+10y_{23}^u+11y_{24}^u+$$
$$8y_{31}^u+3y_{32}^u+6y_{33}^u+2y_{34}^u) \quad [51]$$

Now the whole problem is divided into three levels crisp multi-objective transportation problems.

L- Lower level

$$\min_{\tilde{y}}(\vartheta^l)=\min_{\tilde{y}}(u_{11}^l,u_{12}^l,u_{13}^l)$$
$$=(y_{11}^l+y_{12}^l+5y_{13}^l+4y_{14}^l+y_{21}^l+7y_{22}^l+2y_{23}^l+3y_{24}^l+7y_{31}^l$$
$$+7y_{32}^l+3y_{33}^l+5y_{34}^l,3y_{11}^l+2y_{12}^l+2y_{13}^l+y_{14}^l+4y_{21}^l+7y_{22}^l+$$
$$7y_{23}^l+9y_{24}^l+4y_{31}^l+y_{32}^l+3y_{33}^l+y_{34}^l) \quad [52]$$

Subject to

$$\left.\begin{array}{l} y_{11}^l+y_{12}^l+y_{13}^l+y_{14}^l=7\\ y_{21}^l+y_{22}^l+y_{23}^l+y_{24}^l=17\\ y_{31}^l+y_{32}^l+y_{33}^l+y_{34}^l=16\\ y_{11}^l+y_{21}^l+y_{31}^l=10\\ y_{12}^l+y_{22}^l+y_{32}^l=2\\ y_{11}^l+y_{21}^l+y_{31}^l=13\\ y_{11}^l+y_{21}^l+y_{31}^l=15\\ y_{ij}^l\geq 0\ \forall i,j \end{array}\right\} \quad [53]$$

M-Middle level:

$$\min_{\tilde{y}}(\vartheta^m)=\min_{\tilde{y}}(u_{11}^m,u_{12}^m,u_{13}^m)$$
$$=(1.5y_{11}^m+2y_{12}^m+7y_{13}^m+6y_{14}^m+1.5y_{21}^m+8.5y_{22}^m+4y_{23}^m+4y_{24}^m$$
$$+8y_{31}^m+9y_{32}^m+4y_{33}^m+6y_{34}^m,4y_{11}^m+4y_{12}^m+3y_{13}^m+3y_{14}^m+5y_{21}^m$$
$$+8y_{22}^m+8.5y_{23}^m+10y_{24}^m+6y_{31}^m+2y_{32}^m+4.5y_{33}^m+1.5y_{34}^m) \quad [54]$$

Subject to

$$\left.\begin{array}{l} y_{11}^m+y_{12}^m+y_{13}^m+y_{14}^m=8\\ y_{21}^m+y_{22}^m+y_{23}^m+y_{24}^m=19\\ y_{31}^m+y_{32}^m+y_{33}^m+y_{34}^m=17\\ y_{11}^m+y_{21}^m+y_{31}^m=11\\ y_{12}^m+y_{22}^m+y_{32}^m=3\\ y_{11}^m+y_{21}^m+y_{31}^m=14\\ y_{11}^m+y_{21}^m+y_{31}^m=16\\ y_{ij}^m\geq 0\ \forall i,j \end{array}\right\} \quad [55]$$

U -Upper level

$$\min_{\tilde{y}}(\vartheta^u)=\min_{\tilde{y}}(u_{11}^u,u_{12}^u,u_{13}^u)$$
$$=(2y_{11}^u+3y_{12}^u+9y_{13}^u+8y_{14}^u+2y_{21}^u+10y_{22}^u+6y_{23}^u+5y_{24}^u$$
$$+9y_{31}^u+11y_{32}^u+5y_{33}^u+7y_{34}^u,5y_{11}^u+6y_{12}^u+4y_{13}^u+5y_{14}^u+$$
$$6y_{21}^u+9y_{22}^u+10y_{23}^u+11y_{24}^u+8y_{31}^u+3y_{32}^u+6y_{33}^u+2y_{34}^u) \quad [56]$$

Subject to

$$\left.\begin{array}{l} y_{11}^u+y_{12}^u+y_{13}^u+y_{14}^u=8\\ y_{21}^u+y_{22}^u+y_{23}^u+y_{24}^u=19\\ y_{31}^u+y_{32}^u+y_{33}^u+y_{34}^u=17\\ y_{11}^u+y_{21}^u+y_{31}^u=11\\ y_{12}^u+y_{22}^u+y_{32}^u=3\\ y_{11}^u+y_{21}^u+y_{31}^u=14\\ y_{11}^u+y_{21}^u+y_{31}^u=16\\ y_{ij}^u\geq 0\ \forall i,j \end{array}\right\} \quad [57]$$

Solving the equations (52)-(57) by TORA software, Table 2 is obtained.

Table 2 Optimal values for three levels

Lower level			Middle level			Upper level		
y_{11}^l	5	0	y_{11}^m	5	0	y_{11}^u	5	0
y_{12}^l	2	0	y_{12}^m	3	0	y_{12}^u	4	0
y_{13}^l	0	6	y_{13}^m	0	8	y_{13}^u	0	9
y_{14}^l	0	1	y_{14}^m	0	0	y_{14}^u	0	0
y_{21}^l	5	10	y_{21}^m	6	11	y_{21}^u	7	12
y_{22}^l	0	0	y_{22}^m	0	2	y_{22}^u	0	3
y_{23}^l	0	7	y_{23}^m	0	6	y_{23}^u	0	6
y_{24}^l	12	0	y_{24}^m	13	0	y_{24}^u	14	0
y_{31}^l	0	0	y_{31}^m	0	0	y_{31}^u	0	0
y_{32}^l	0	2	y_{32}^m	0	1	y_{32}^u	0	1
y_{33}^l	13	0	y_{33}^m	14	0	y_{33}^u	15	0
y_{34}^l	3	14	y_{34}^m	3	16	y_{34}^u	3	17
	102	118		148.50	172		202	232

Fuzzy Arithmetic Mean (FAM)

$FAM_1 = (AM_1^l, AM_1^m, AM_1^u)$

$= (110, 160.25, 217)$

here,

$AM_1^l = \frac{102+118}{2}$ = 110 (AM lower level's optimal solutions)

$AM_1^m = \frac{148.50+172}{2}$ = 160.25 (AM of middle level's optimal solutions),

$AM_1^u = \frac{202+232}{2}$ = 217 (AM of upper level's optimal solutions),

The Three-Level Crisp Multi-Objective Linear Programming Problem (TL-CMOLPP) is converted into the Three-Level Crisp Single-Objective Linear Programming Problem (TL-CSOLPP) in the following manner:

L-CSOLPP:

$$\min_{\tilde{y}}(\gamma^l) = \frac{\min(u_{11}^l) + \min(u_{12}^l)}{AM_1^u}$$

$$= (0.01843y_{11}^l + 0.01382y_{12}^l + 0.03226y_{13}^l + 0.02304y_{14}^l + 0.02304y_{21}^l + 0.06452y_{22}^l + 0.04147y_{23}^l + 0.05530y_{24}^l + 0.05069y_{31}^l + 0.03687y_{32}^l + 0.02765y_{33}^l + 0.02765y_{34}^l) \quad [58]$$

Subject to

$$\left.\begin{array}{l} y_{11}^l + y_{12}^l + y_{13}^l + y_{14}^l = 7 \\ y_{21}^l + y_{22}^l + y_{23}^l + y_{24}^l = 17 \\ y_{31}^l + y_{32}^l + y_{33}^l + y_{34}^l = 16 \\ y_{11}^l + y_{21}^l + y_{31}^l = 10 \\ y_{12}^l + y_{22}^l + y_{32}^l = 2 \\ y_{11}^l + y_{21}^l + y_{31}^l = 13 \\ y_{11}^l + y_{21}^l + y_{31}^l = 15 \\ y_{ij}^l \geq 0 \ \forall i, j \end{array}\right\} \quad [59]$$

M-CSOLPP:

$$\min_{\tilde{y}}(\gamma^m) = \frac{\min(u_{11}^m) + \min(u_{12}^m)}{AM_1^m}$$

$$= (0.03432y_{11}^m + 0.03744y_{12}^m + 0.06240y_{13}^m + 0.04056y_{14}^m + 0.04056y_{21}^m + 0.10296y_{22}^m + 0.07800y_{23}^m + 0.08736y_{24}^m + 0.08736y_{31}^m + 0.06864y_{32}^m + 0.05304y_{33}^m + 0.04680y_{34}^m) \quad [60]$$

Subject to

$$\left.\begin{array}{l} y_{11}^m + y_{12}^m + y_{13}^m + y_{14}^m = 8 \\ y_{21}^m + y_{22}^m + y_{23}^m + y_{24}^m = 19 \\ y_{31}^m + y_{32}^m + y_{33}^m + y_{34}^m = 17 \\ y_{11}^m + y_{21}^m + y_{31}^m = 11 \\ y_{12}^m + y_{22}^m + y_{32}^m = 3 \\ y_{11}^m + y_{21}^m + y_{31}^m = 14 \\ y_{11}^m + y_{21}^m + y_{31}^m = 16 \\ y_{ij}^m \geq 0 \ \forall i, j \end{array}\right\} \quad [61]$$

U-CSOLPP:

$$\min_{\tilde{y}}(\gamma^u) = \frac{\min(u_{11}^u) + \min(u_{12}^u)}{AM_1^u}$$

$$= (0.6364y_{11}^u + 0.08182y_{12}^u + 0.11818y_{13}^u + 0.11818y_{14}^u + 0.07273y_{21}^u + 0.17272y_{22}^u + 0.14545y_{23}^u + 0.14545y_{24}^u + 0.015454y_{31}^u + 0.12727y_{32}^u + 0.1y_{33}^u + 0.08182y_{34}^u) \quad [62]$$

Subject to

$$\left.\begin{array}{l} y_{11}^u + y_{12}^u + y_{13}^u + y_{14}^u = 8 \\ y_{21}^u + y_{22}^u + y_{23}^u + y_{24}^u = 19 \\ y_{31}^u + y_{32}^u + y_{33}^u + y_{34}^u = 17 \\ y_{11}^u + y_{21}^u + y_{31}^u = 11 \\ y_{12}^u + y_{22}^u + y_{32}^u = 3 \\ y_{11}^u + y_{21}^u + y_{31}^u = 14 \\ y_{11}^u + y_{21}^u + y_{31}^u = 16 \\ y_{ij}^u \geq 0 \ \forall i, j \end{array}\right\} \quad [63]$$

The final optimal solution (Table 3) is obtained by solving equations (58)-(63) using the TORA software.

Table 3. Final Solution for Example 1.

Lower Level		Middle Level		Upper Level	
y_{11}^l	0	y_{11}^m	0	y_{11}^u	0
y_{12}^l	2	y_{12}^m	3	y_{12}^u	4
y_{13}^l	0	y_{13}^m	5	y_{13}^u	5
y_{14}^l	5	y_{14}^m	0	y_{14}^u	0
y_{21}^l	10	y_{21}^m	11	y_{21}^u	12
y_{22}^l	0	y_{22}^m	0	y_{22}^u	0
y_{23}^l	7	y_{23}^m	8	y_{23}^u	9
y_{24}^l	0	y_{24}^m	0	y_{24}^u	0
y_{31}^l	0	y_{31}^m	0	y_{31}^u	0
y_{32}^l	0	y_{32}^m	0	y_{32}^u	0
y_{33}^l	6	y_{33}^m	1	y_{33}^u	1
y_{34}^l	10	y_{34}^m	16	y_{34}^u	17

Example 2: An FMOTP with fuzzy objectives, delivery time, loss, and profit are considered, where all parameters are represented by triangular fuzzy numbers as given in Tables 4(i)–4(iii). The problem was previously solved by Y. Kacher & P. Singh [18] and A. Kokila & G. Deepa [22]

Table 4(i) Objective values for the first objective function (Delivery time)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Supply ($\tilde{s}_i$)
T_1	(7.5,8,9)	(4.5,5,6)	(5.5,6,7)	(7,9,9.5)	(8.5,10,11)	(3.5,4,5)	(7.6,8,9.5)	(6.5,7,8)	(105,120,140)
T_2	(6.8,7,7.5)	(3.8,4,5)	(9,10,10.5)	(5,6,7.5)	(7.8,8,8.5)	(6,6,8)	(4.5,5,7)	(9,10,11.5)	(72,85,108)
T_3	(9,10,11)	(8,9,9.5)	(8.5,9,9.5)	(10.5,11,11.5)	(10,11,12)	(7,8,10)	(8.6,9,10.5)	(10.4,11,12)	(73,75,83)
Demand ($\tilde{d}_j$)	(26,30,33)	(22,25,35)	(32,35,40)	(37,40,49)	(50,55,60)	(22,25,29)	(24,34,40)	(37,40,55)	

Table 4(ii) Objective values for the second objective function (loss)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Supply ($\tilde{s}_i$)
T_1	(2.5,3,3.5)	(1,1,2)	(1.5,2,2.5)	(2,3,3.5)	(1,2.5,3)	(1,1.5,3)	(2,2.6,3)	(1,1.5,2)	(105,120,140)
T_2	(1.5,2,2.8)	(1,1.5,2.5)	(0.5,1,1.25)	(5,6,8)	(2.5,3,3.5)	(2,2.6,3.5)	(1,1.7,3.5)	(2.5,2.5,4)	(72,85,108)
T_3	(3.5,4,5)	(2.5,3.5,4.5)	(2.5,3.5,4.5)	(1,2,4)	(4,4.2,6)	(2.8,3,3.5)	(2,2.8,3)	(3.5,4,5)	(73,75,83)
Demand ($\tilde{d}_j$)	(26,30,33)	(22,25,35)	(32,35,40)	(37,40,49)	(50,55,60)	(22,25,29)	(24,34,40)	(37,40,55)	

Table 4(iii) Objective values for the third objective function (profit)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Supply ($\tilde{s}_i$)
T_1	(115,125,130)	(82,85,98)	(71,75,83)	(92,100,108)	(60,65,69)	(90,95,96)	(40,45,60)	(86,90,110)	(105,120,140)
T_2	(60,65,72)	(42,50,51)	(135,145,150)	(180,205,210)	(50,59,63)	(59,60,70)	(62,65,75)	(95,105,120)	(72,85,108)
T_3	(132,135,150)	(129,130,140)	(129,130,140)	(130,165,170)	(79,80,83)	(70,79,90)	(80,82,85)	(70,93,110)	(73,75,83)
Demand ($\tilde{d}_j$)	(26,30,33)	(22,25,35)	(32,35,40)	(37,40,49)	(50,55,60)	(22,25,29)	(24,34,40)	(37,40,55)	

The final solution is obtained using the proposed approach, which is given in Table 5.

Table 5. Final solution table for Example 2.

Lower level	Middle level	Upper level
$y_{11}^l = 0$	$y_{11}^m = 30$	$y_{11}^u = 33$
$y_{12}^l = 22$	$y_{12}^m = 25$	$y_{12}^u = 35$
$y_{13}^l = 0$	$y_{13}^m = 0$	$y_{13}^u = 0$
$y_{14}^l = 0$	$y_{14}^m = 0$	$y_{14}^u = 0$
$y_{15}^l = 27$	$y_{15}^m = 10$	$y_{15}^u = 17$
$y_{16}^l = 22$	$y_{16}^m = 25$	$y_{16}^u = 29$
$y_{17}^l = 0$	$y_{17}^m = 0$	$y_{17}^u = 0$
$y_{18}^l = 34$	$y_{18}^m = 30$	$y_{18}^u = 26$
$y_{21}^l = 0$	$y_{21}^m = 0$	$y_{21}^u = 0$
$y_{22}^l = 0$	$y_{22}^m = 0$	$y_{22}^u = 0$
$y_{23}^l = 32$	$y_{23}^m = 35$	$y_{23}^u = 40$
$y_{24}^l = 37$	$y_{24}^m = 40$	$y_{24}^u = 49$
$y_{25}^l = 0$	$y_{25}^m = 0$	$y_{25}^u = 0$
$y_{26}^l = 0$	$y_{26}^m = 0$	$y_{26}^u = 0$
$y_{27}^l = 0$	$y_{27}^m = 0$	$y_{27}^u = 0$
$y_{28}^l = 3$	$y_{28}^m = 10$	$y_{28}^u = 19$
$y_{31}^l = 26$	$y_{31}^m = 0$	$y_{31}^u = 0$
$y_{32}^l = 0$	$y_{32}^m = 0$	$y_{32}^u = 0$
$y_{33}^l = 0$	$y_{33}^m = 0$	$y_{33}^u = 0$
$y_{34}^l = 0$	$y_{34}^m = 0$	$y_{34}^u = 0$
$y_{35}^l = 23$	$y_{35}^m = 45$	$y_{35}^u = 43$
$y_{36}^l = 0$	$y_{36}^m = 0$	$y_{36}^u = 0$
$y_{37}^l = 24$	$y_{37}^m = 30$	$y_{37}^u = 40$
$y_{38}^l = 0$	$y_{38}^m = 0$	$y_{38}^u = 0$

Example 3: An FMOTP with time and cost objectives is considered, where availability, demand, and all parameters are represented by triangular fuzzy numbers, as given in Table 6. This problem was previously solved by Admasu et. al.[23].

Table 6: The Input data for Example 3

Source→ Destination↓		S_1	S_2	S_3	Availability (s_i)
D_1	Cost	(15,25,35)	(55,65,85)	(85,95,105)	(75,95,125)
	Time	(4,6,8)	(6,8,10)	(7,9,11)	
D_2	Cost	(65,75,85)	(80,90,110)	(30,40,50)	(45,65,95)
	Time	(3,5,7)	(5,7,9)	(11,13,15)	
Requirement (d_j)		(35,45,65)	(25,35,45)	(60,80,110)	

Now, we solve Example 3 using the proposed method and TORA software. The Final Optimal Solution is given in Table 7.

Table 7: The optimal solution for Example 3.

Lower level	Middle level	Upper level
$y_{11}^{l}=35$	$y_{11}^{m}=45$	$y_{11}^{u}=65$
$y_{12}^{l}=25$	$y_{12}^{m}=35$	$y_{12}^{u}=45$
$y_{13}^{l}=15$	$y_{13}^{m}=15$	$y_{13}^{u}=15$
$y_{21}^{l}=0$	$y_{21}^{m}=0$	$y_{21}^{u}=0$
$y_{22}^{l}=0$	$y_{22}^{m}=0$	$y_{22}^{u}=0$
$y_{23}^{l}=45$	$y_{23}^{m}=65$	$y_{23}^{u}=95$

V. RESULTS AND DISCUSSION

In this part, we analyzed the results of both solved examples and compared the results with the existing methods. The fuzzy compromised solutions for the time and cost objectives of Example 1 in the graphical form are displayed in Fig 2(i), and Fig 2(ii) respectively. The defuzzification technique described in Section 2.3 is used to obtain crisp compromised solutions for both objectives. The results of this process, along with a comparison of fuzzy and crisp results for Example 1 with the existing methods, are presented in Table 8. In comparison to Y. Kacher and P. Singh [18], there is a 6.761% increase in cost and a 9.16% decrease in time. Compared to the results reported by A. Kokila and G. Deepa [22], the proposed method results in a 12.578% increase in cost and a 30.588% reduction in time. As illustrated in Figure 3(i), the fuzzy cost membership function exhibits a broader spread, indicating a higher degree of uncertainty in cost estimation. Conversely, the fuzzy time membership function depicted in Figure 3(ii) displays a narrower triangular profile, suggesting a lower level of uncertainty in time prediction. This comparative analysis highlights that although the cost objective demonstrates a marginal increase, attributable to the trade-offs inherent in multi-objective optimization, the time objective achieves a substantial improvement, offering more efficient performance relative to existing methods.

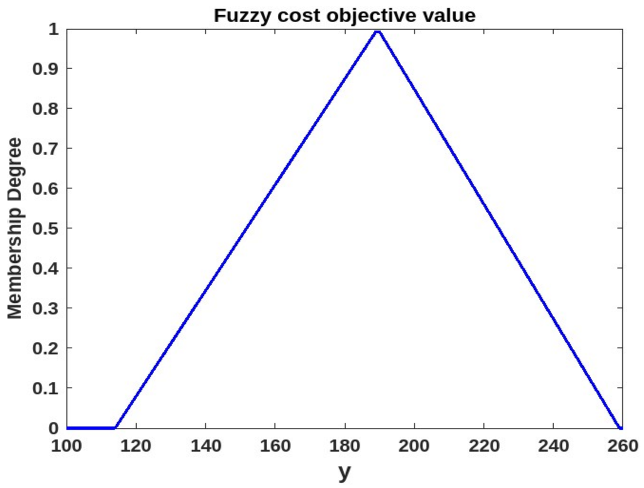

Fig. 2(i) Optimal fuzzy cost value for Example 1

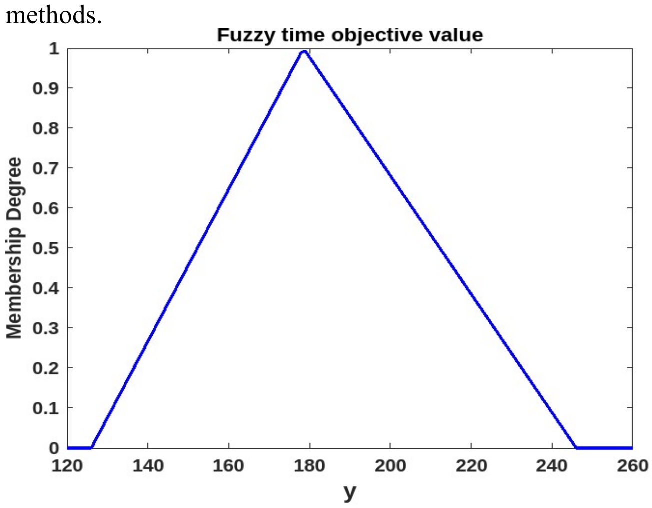

Fig. 2(ii) Optimal fuzzy time value for Example 1

Table 8 Comparison of results of Example 1

Methods	Fuzzy compromised solution		Crisp compromised solution	
	Cost	Time	Cost	Time
Y. Kacher and P. Singh [18]	(114,174.5,244)	(126,193.5,271)	177.5	196.5
A. Kokila and G. Deepa [22]	(107,168,230)	(154,268.5,349)	168.33	257.16
Proposed approach	(114,189.5,259)	(126,178.5,246)	189.5	178.5

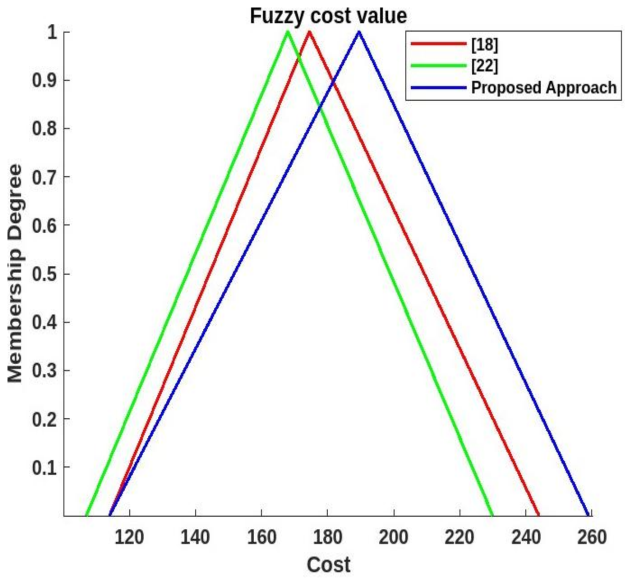

Fig. 3(i) Fuzzy cost values across different approaches for Example 1.

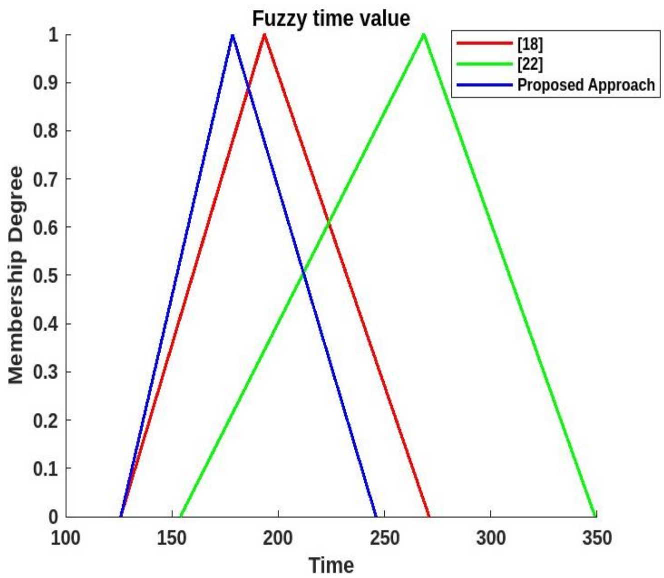

Fig. 3(ii) Fuzzy time values across different approaches for Example 1.

Similarly, for Example 2, Fig. 4(i), Fig. 4(ii), and Fig. 4(iii) represent the fuzzy compromised solution for the delivery time, loss, and profit objective values, respectively.

Compared to the method in [18], the proposed approach exhibits the same spread in delivery time, a narrower spread in the loss objective, crucial for reducing uncertainty, and a slightly narrower spread in the profit objective, indicating a marginal increase in uncertainty when estimating profit, as shown in Fig. 5(i),5(ii), and 5(iii).

Compared to [22], the proposed method shows a narrower spread in both delivery time and loss, which contributes to reduced uncertainty. However, the spread of the profit objective is broader, which is beneficial for accurately estimating and optimizing profit.

Compared to [24], the proposed method has a slightly broader spread in delivery time, suggesting increased uncertainty in delivery time estimation. However, for the profit objective, the spread is broader, while for the loss objective, it is narrower, which is essential for reducing uncertainty in both objectives.

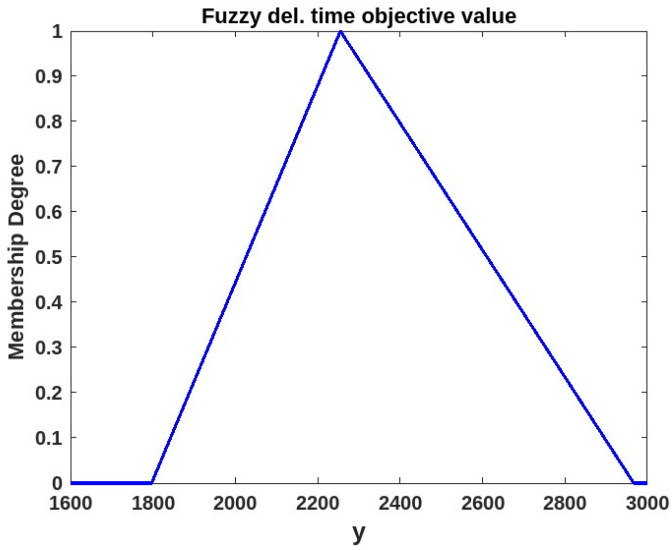

Fig. 4(i) Optimal fuzzy delivery time value for Example 2.

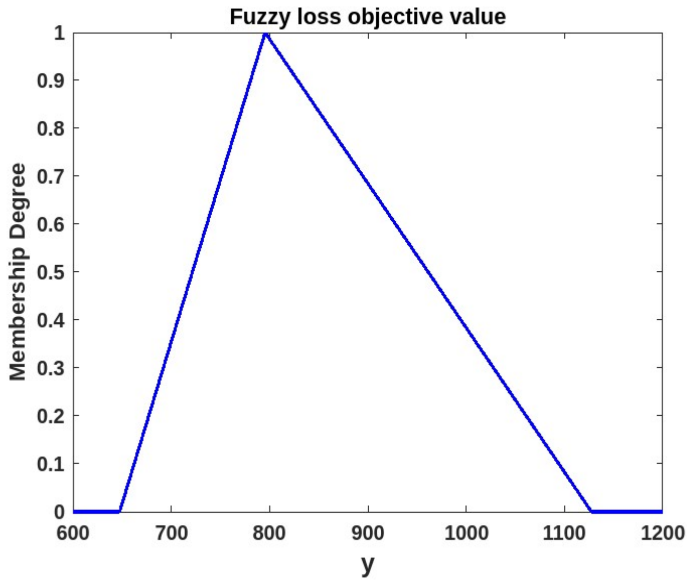

Fig. 4(ii) Optimal fuzzy loss value for Example 2.

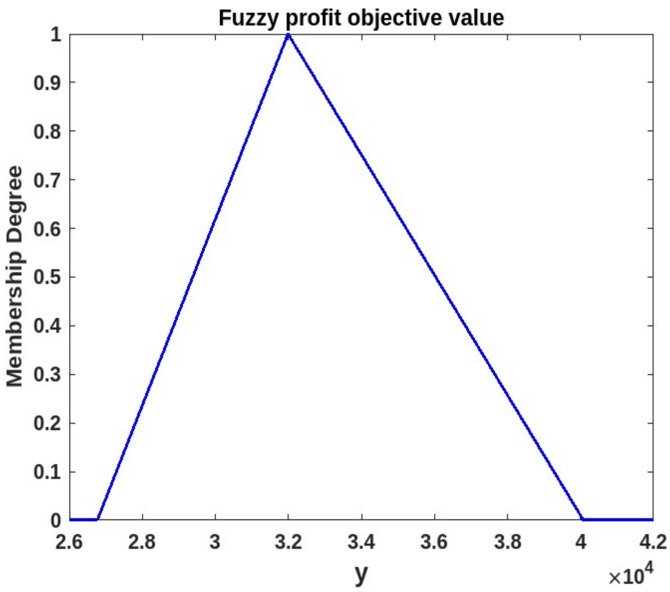

Fig. 4(iii) Optimal fuzzy profit value for Example 2.

Table 9. Comparison of fuzzy optimal values for Example 2.

Methods	Delivery time	Loss	Profit
M. Bagheri, and A. Ebarhimnejad [24]	(1793.7,2233,2893)	(595,792,1285.4)	(26715,31741,39256)
Y. Kacher and P. Singh [18]	(1796.9,2255,2967)	(544.5,775.7,1294.5)	(26762,31787,40184)
A. Kokila and G. Deepa [22]	(1945.2,2265,3048.5)	(322,842,967.4)	(21908,29195,34151)
Malihe Niksirat [25]	(1877.32,2377.9,2655.28)	(543.16,798.1,987.07)	(16402.4,21449.12,23998.56)
Proposed approach	(1796.9,2255,2967)	(647.5,795.5,1127.5)	(26762,31985,40076)

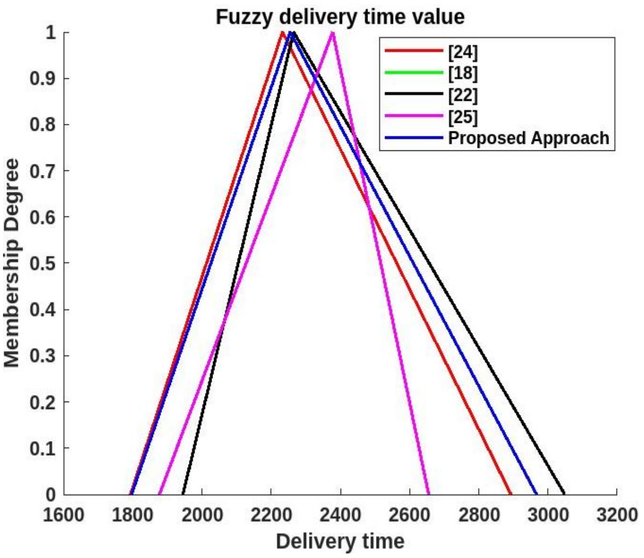

Fig. 5(i) Fuzzy delivery time values across different approaches for Example 2.

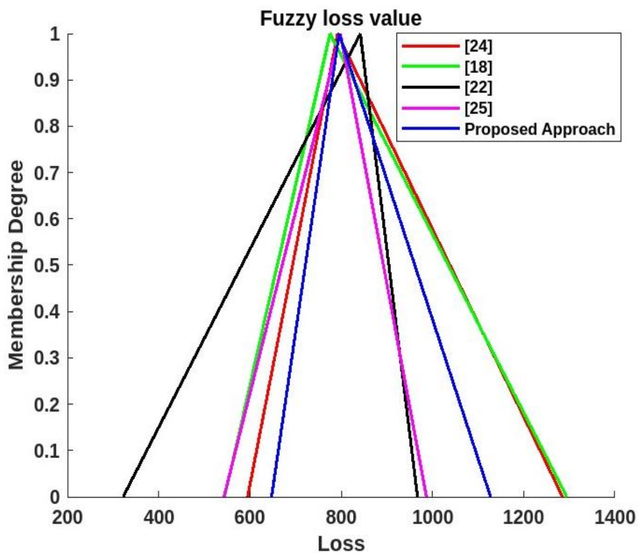

Fig. 5(ii) Fuzzy loss values across different approaches for Example 2.

Compared to [25], the proposed method shows a slightly wider spread in both delivery time and loss, reflecting greater uncertainty in these objectives. In contrast, the profit objective demonstrates a broader spread, indicating improved consistency and reduced uncertainty in profit estimation.

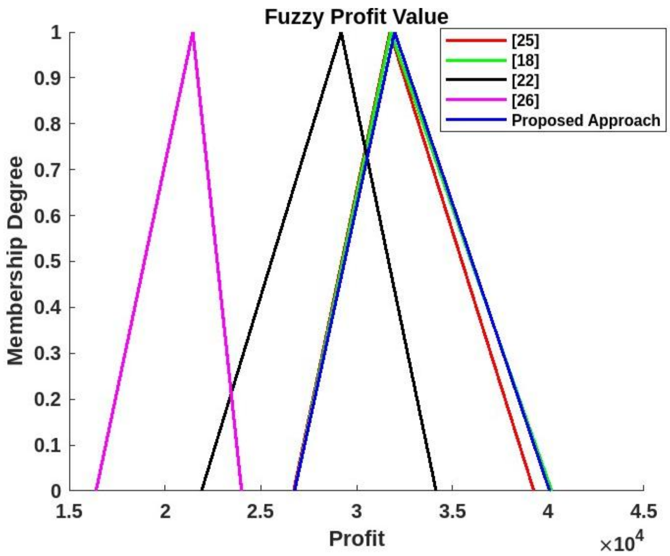

Fig.5(iii) Fuzzy profit values across different approaches for Example 2.

Tables 9 and 10 present a comparison of fuzzy and crisp composite solutions, respectively, using the fuzzy arithmetic mean technique (proposed approach) by Y. Kacher and P. Singh [18], and A. Kokila and G. Deepa [22]. Compared to M. Bagheri, and A. Ebarhimnejad [25], delivery time decreased by 2.235787%, loss decreased by 10.69824%, and profit decreased by 2.33787%. Compared to Y. Kacher and P. Singh [18], the delivery time decreased by 3.6616%, the loss decreased by 8.7276%, and the profit decreased by 2.81364%.

Table 10. Comparison of crisp optimal values for Example 2.

Methods	Delivery time	Loss	Profit
M. Bagheri, and A.Ebarhimnejad[24]	2306.57	890.8	32750.67
Y. Kacher and P. Singh [18]	2339.6	871.567	32911
A. Kokila and G. Deepa [22]	2419.5	710.46	28418
Malihe Niksirat [25]	2303.5	776.11	20616.69
Proposed approach	2255	795.5	31985

Table 11 Comparison of optimal results of Example 3

Methods	Fuzzy compromised solution		Crisp compromised solution	
	Cost	Time	Cost	Time
Admassu Tadesse [23]	(6875.31,10341.77,16075)	(748.32,1355.13,2335)	10341.77	1355.13
Proposed approach	(4525,7425,12425)	(890,1530,2560)	7425	1530

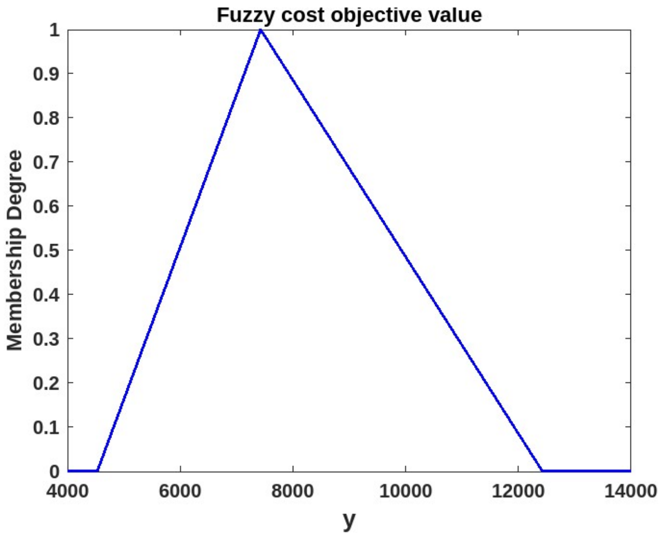

Fig. 6(i) Optimal fuzzy cost value for Example 3.

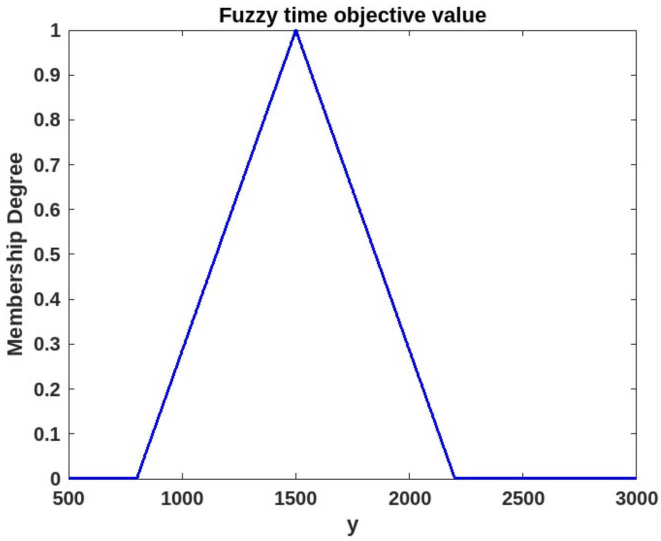

Fig. 6(ii) Optimal fuzzy delivery time value for Example 3.

In comparison to A. Kokila and G. Deepa [22], the delivery time decreases by 6.7988%, the profit increases by 12.5519%, and the loss increases by 11.9697% due to the conflict of multiple objectives. In comparison to Malihe Niksirat [25], the delivery time decreases by 2.1036%, the profit increases by 55.1413%, and the loss increases by 2.4983% due to the conflict of multiple objectives. Here, two objectives are more optimized out of the three. Based on the comparison, it is clear that the fuzzy arithmetic mean technique (Proposed approach) yields a more optimized and better optimal solution than the existing methods. The Graphical representation of the comparison for crisp compromised values is given in Fig. 8.

Similarly, for Example 3, Fig. 6(i) and Fig. 6(ii) represent the fuzzy compromised solution for the cost and time objective values, respectively. The fuzzy time graph in Fig 7(i) demonstrates that the proposed approach maintains a narrower triangular shape, reflecting reduced uncertainty in time predictions. However, the fuzzy cost graph is shown in Figure 7(ii) indicates that the proposed method exhibits a slightly broader spread, suggesting higher uncertainty in cost estimation.

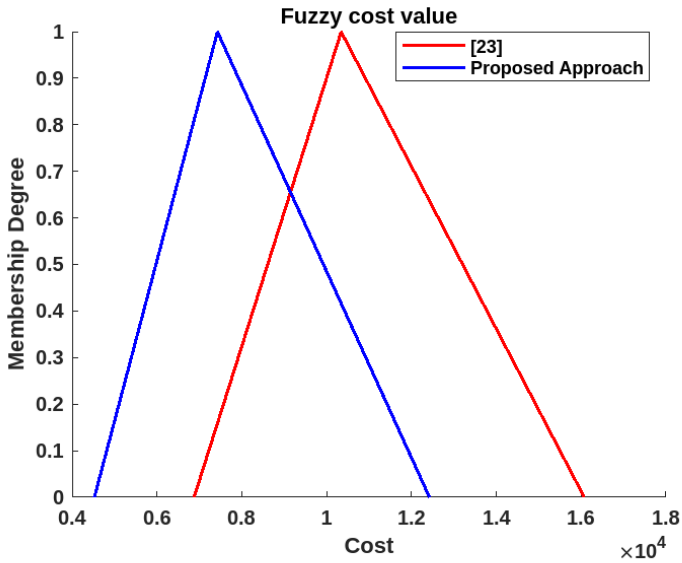

Fig. 7(i) Fuzzy cost values across different approaches for Example 3.

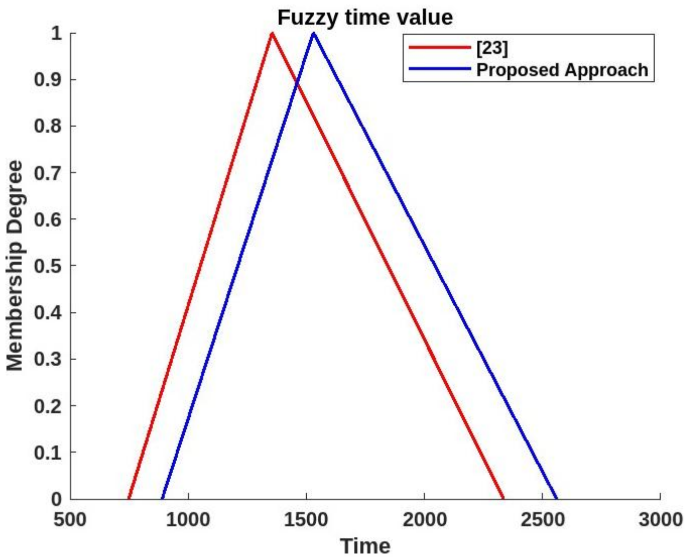

Fig. 7(ii) Fuzzy time values across different approaches for Example 3.

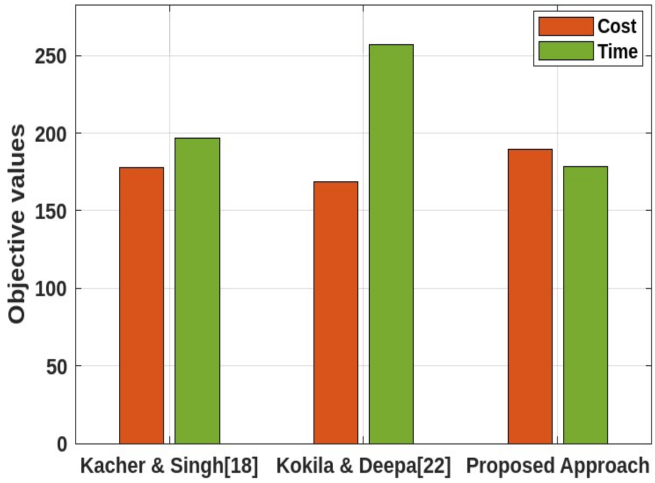

Figure 8. The Crisp Objective values comparison for Example 1.

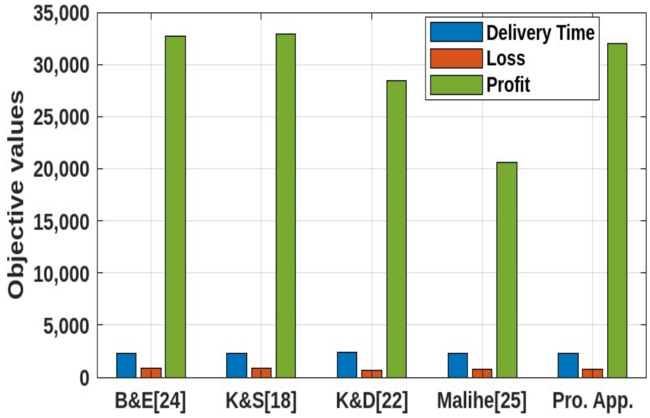

Figure 9. The Crisp Objective values comparison for Example 2

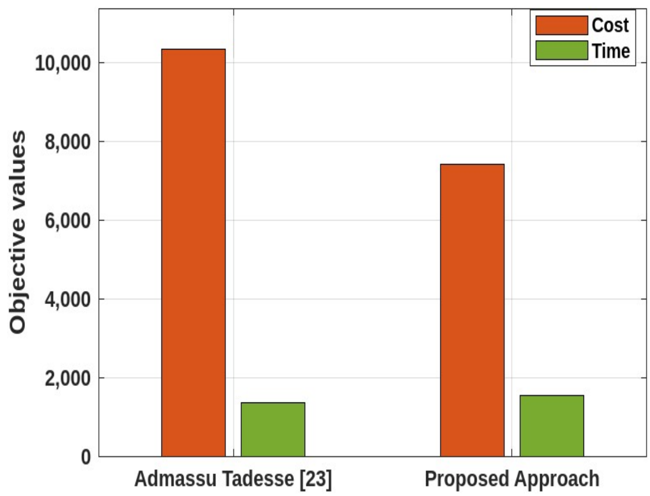

Figure 10. The Crisp Objective values comparison for Example 3

For Example, 3, Table 11 shows that the proposed approach reduces cost by 28.2%, but time increases by 12.9% due to conflicting objectives compared to Admasu Tadesse et al [23]. This is a significant improvement in cost minimization. The graphical presentation of the comparison of crisp objective values for Example 2 is given in Fig.9.

The graphical representation of the comparison for Example 3 is given in Fig. 10. These findings highlight the effectiveness of the proposed approach in achieving a balanced trade-off between conflicting objectives.

VI. Conclusion

This research paper presented a new approach to solving fuzzy MOTP with triangular fuzzy numbers by using a fuzzy arithmetic mean approach. This approach first decomposed the FMOTP into three-level crisp MOTP using fuzzy arithmetic operations. Each of the three crisp single-objective transportation problems is solved independently by using TORA software to get the optimum values, by which we calculate the fuzzy arithmetic mean. The FAM proposed approach is validated by solving three numerical examples. The fuzzy compromise solution provided by our method is better than the optimal solution provided by some other methods in each instance. The proposed approach is useful for solving transportation problems in which the decision-makers are unsure about the exact value of the transportation objectives in real-life applications.

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