Multi-Period Heterogeneous Vehicle Green Location-Routing Optimization Considering Uncertain Demand

Zhang Jian, Cunjie Dai, Runyu Wu, Tingyu Wang, Jianghong Feng

Abstract—To lessen the amount of money spent on logistics and the amount of carbon dioxide that is produced during delivery, as well as to investigate the interconnected and highly influential factors that facility location and vehicle delivery have on delivery efficiency, a multi-period heterogeneous vehicle delivery center location-path optimization model under uncertainty has been developed. With the aim of reducing the costs associated with the logistics system, this model takes into account the limitations imposed by the capacity of distribution centers and a method of logistics that concurrently manages delivery and returns. The issue was developed to be solved using the NSGA-II algorithm, which was included with tabu search and recombination techniques. The efficiency of the model and algorithm was shown by numerical tests. Examine the implications that optimistic, expected, and pessimistic values have on site selection and vehicle service route tactics within the context of the planning model for uncertain opportunity constraints. Additionally, a sensitivity analysis should be carried out to have a better understanding of how credibility and the range of unknown factors influence the results of site selection and route optimization. The results indicate the model's ability to guide decision-making regarding site selection and vehicle transportation route determination.

Index Terms—Location-Routing Problem, Facility Location, Vehicle distribution, Uncertainty Analysis, Service Path.

I. INTRODUCTION

The Location-Routing Problem (LRP) combines two problems: the Location Allocation Problem (LAP) and the Vehicle Routing Problem (VRP), both of which are complex problems that involve making many decisions. The goal of building a distribution center is to optimize the

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logistics distribution system and allocate resources more effectively; hence, the distribution center's location is critical to the logistics system. The goal of the VRP, on the other hand, is to transport the commodities requested by consumers from the distribution center to the customers while adhering to specific limits, hence lowering logistical costs. LAP and VRP are two connected difficulties that have a substantial influence on delivery efficiency. Studying both may prevent the local optimization issues that arise from examining site selection or vehicle routing problems separately, leading to a reduction in the overall logistics system costs[1].

Because real-world transportation is complex and constantly changing, which greatly impacts how well networks operate, it's more practical to study transportation issues in uncertain situations rather than in predictable ones. Researchers have used effective methodologies to investigate transportation under uncertain circumstances.

The Vehicle Routing Problem with Simultaneous Pick-up and Delivery (VRPSPD) pertains to the scenario in which delivery and pick-up services are executed concurrently during each client visit. In contrast to solely addressing individual pick-up or delivery requirements for distribution services, simultaneous pick-up and delivery optimize the utilization of residual vehicle capacity during transit, thereby integrating remanufacturing and resource recycling. This approach significantly diminishes vehicle empty load rates, lowers distribution costs, and enhances distribution efficiency, leading to its widespread implementation.

Logistics expenses are a primary emphasis of LRP research, including elements such as facility development expenses, transportation costs, and vehicle deployment costs[2]. Costs serve as a crucial metric for transportation efficiency and economic gain, enhancing firms by augmenting economic efficacy. In the current context of sustainable development, green logistics focused on "low energy consumption and emissions" has received significant attention. A primary objective of VRP research is to minimize logistical expenses while also decreasing carbon emissions produced during the transportation process. Some scholars are currently researching it from broader perspectives, including reducing logistics costs, and promoting low-carbon environmental practices [3].

This manuscript presents the following contributions: (1) We have developed a mathematical model for the Fuzzy Low-Carbon LRP with Simultaneous Pick-up and Delivery Heterogeneous Fleet (FLCLRPSPDHF), which addresses simultaneous pickup and delivery within a sustainable multi-period, multi-vehicle framework characterized by uncertain consumer demand. (2) An uncertain

chance-constrained programming model was established based on the notion of uncertain variables. Transform the indeterminate model, which cannot be immediately resolved, into a corresponding deterministic model for resolution. (3) Created an improved NSGA-II algorithm with numerous strategic improvements, scrutinizing the model parameters to gauge the impact of reliability and fluctuating demand range on the placement of facilities and the optimization of vehicle routes. The performance was corroborated using numerical examples, demonstrating the algorithm's practicality and efficacy, with the intention of serving as a reference for the design of corporate logistics systems.

The following part is titled "Literature Review." This part offers a thorough examination of the current literature on LRP, the synchronized pickup and delivery issue, and uncertainty, while also outlining the research goals of this article. The "Problem Description" section outlines the study's objectives and clarifies the problem hypotheses. The following section is entitled "Model Formulation." This article will formulate the FLCLRPSPDHF mathematical model and transform it into a deterministic model based on the specification of uncertain variables. The following section is the "Solution Approach," in which this study presents the improved NSGA-II algorithm. The following is "Case Study" examining the influence of believability and the breadth of uncertainty intervals on the ideal configuration of facility and vehicle route placements. This study accomplishes this objective via a numerical example, serving as a reference for the design of corporate logistics systems. The findings of this paper were ultimately summarized.

II. LITERATURE REVIEW

Numerous experts have undertaken a comprehensive study of the LRP. Tai-Hi [4] developed a multi-site LRP model and formulated a simulated annealing approach to address it. Hansen [5] created an LRP model that includes limits on how much can be carried, while Prins[6] developed a Hybrid Metaheuristics that uses local search and Genetic Algorithms to solve the LRP problem with these vehicle capacity limits. Wang [7] established a capacitated LRP model and designed a two-stage hybrid heuristic algorithm for its solution, combining Tabu search and a dual-population Ant Colony algorithm. Yuan [8] developed a bi-level programming model for LRP, employing an Immune algorithm for facility location in the upper layer and a particle swarm optimization algorithm for route planning in the lower layer.

Given the existence of unpredictable variables in real-world transportation, examining transportation issues under uncertain contexts is more pragmatically relevant than in deterministic contexts. Zarandi [9] developed the Fuzzy Location Routing Problem (FLRP) model in which customer demand and trip time are represented as fuzzy numbers. Wang T [10]developed a multimodal transportation model that accounts for transit time uncertainty and resolved it with Gurobi. Tian[11] developed a secondary distribution scheduling model for refined oil that accounts for customer service time and demand uncertainty and devised an MMA algorithm for its resolution; Wu [12] created an emergency evacuation system for subway stations in response to sudden surges in passenger flow, grounded in uncertainty theory. Pekel [13] created a Hybrid Metaheuristics that combines

Variable Neighborhood Search to tackle the location-path issue with uncertain demand; Nassab [14] designed a model for location and routing that includes uncertain demand and job limits and developed an ant colony algorithm to solve it. These studies combine where to place facilities and how to route trucks, avoiding problems that arise when these parts are looked at separately, which helps lower the overall cost of the logistics system[15]. Nevertheless, little research has examined the influence of product returns on facility locations. Zhang [16] examined the influence of ambiguous customer return requests and the concurrent pick-up and delivery method on facility placement within the B2C logistics facility location issue, but his research does not address how fuzzy variables are managed by stochastic algorithms. Stochastic algorithms manage fuzzy variables; however, the impact of credibility on the model's outcomes remains understudied. Credibility denotes the risk tolerance of the decision-maker. While several research studies elucidate ambiguous models grounded on credibility, few examine the influence of credibility and various categories of decision-makers on the outcomes.

Min [17] built a public library distribution system capable of delivering and collecting various goods. Privé [18] established a VRPSPD model for beverage distribution that necessitates the delivery of full bottles and the collection of empty ones, while Soysal [19] formulated a VRPSPD model for food distribution that entails the transportation of food and the retrieval of expired or damaged products for appropriate disposal. At present, the majority of research is on VRPSPD, although there is a paucity of studies that integrate the pickup and delivery problem with the location-routing problem.

Recently, as the significance of sustainable development has grown, green logistics has garnered considerable attention. Wang [20] suggested a multi-objective urban routing model for green vehicles transporting hazardous chemicals, taking into account carbon emissions and examining the effects of transportation hazards, costs, and emissions on delivery routes. Zhao [21] suggested a Low-Carbon LRP issue that incorporates numerous vehicle types and simultaneous pickup and delivery, and they developed an evolutionary hyper-heuristic algorithm to address it. Li [22] proposed a detailed LRP model that includes the carbon trading system and created an improved NSGA-II algorithm to put it into action. The construction of facilities for the LRP entails considerable time and financial investment, and once a decision is reached, it necessitates long-term maintenance. Customer demand uncertainty over extended decision cycles; thus, addressing a multi-period LRP is more pragmatic. Wang [23] developed a two-phase green Location-Routing model incorporating time windows for pickup and delivery and formulated a Heuristic Algorithm utilizing Lagrange relaxation for its resolution. Tang [24] investigated the characteristics of the green LRP with fuzzy demand by formulating a multi-period fuzzy chance-constrained optimization model and creating a Hybrid Genetic Algorithm to solve it. Saffarian [25] formulated a multi-objective Location-Routing problem model that incorporates disaster management and developed a GA-SA algorithm for its solution.

TABLE I LITERATURE REVIEW

Article	Objective	Uncertainty	Multi-Cycle	Heterogeneous Vehicles	Risk attitude	Solution methodology
Zarandi[9]	Single-objective	Yes	No	No	No	SA
Pekel[13]	Single-objective	Yes	No	No	No	HVNS
Nassab[14]	Single-objective	Yes	No	No	No	ACO
Li[22]	Bi-objective	No	No	No	No	NSGA-II
Saffarian[25]	Bi-objective	Yes	Yes	No	No	GA-SA
This paper	Bi-objective	Yes	Yes	Yes	Yes	NSGA-II

This paper presents a two-part integer programming method for eco-friendly multi-period, multi-vehicle pickup and delivery, even when client demand is unpredictable. Taking into account the varying risk tolerance of different decision-makers, the uncertain model is transformed into a corresponding deterministic model based on the definition of uncertain variables. This analysis uses numerical examples to look at how credibility and product return rates affect the best locations for facilities and truck routes, aiming to provide advice for designing logistics systems in companies.

TABLE I delineates the distinctive characteristics of the current investigation in comparison to other pertinent research.

III. PROBLEM DESCRIPTION

The FLCLRPSPDHF problem is defined in the following way: The information regarding all potential distribution centers and customers indicates that customer demands are uncertain. Each distribution center possesses a capacity limit and is outfitted with k varieties of transportation vehicles. The demand from customers allocated to a specific distribution center must not exceed its operational capacity. Multiple delivery cycles exist. During each delivery cycle, the distribution center must dispatch vehicles to serve customers. The vehicles must also collect returned goods throughout the delivery process. The delivery mode, characterized by simultaneous pickup and delivery of goods, results in a dynamic fluctuation of vehicle capacity. Throughout the delivery process, the dynamic load of the vehicle must consistently adhere to the vehicle's capacity limit. It is essential to select an appropriate portion from the alternative distribution centers for construction and to establish the vehicle scheduling plan. The FLCLRPSPDHF model is developed to minimize the total cost of the logistics system while ensuring that the established plan for distribution center locations and vehicle scheduling adequately meet customer demands.

The FLCLRPSPDHF model is based on the following assumptions:

- 1) The construction costs and operational capacities of the alternative distribution centers are established.
- 2) The quantity, location, and demand of customers are established, with demand is regarded as an uncertain variable. Each customer is served exclusively by a single vehicle from a designated distribution center.
- 3) The quantities of each vehicle type, dispatch costs, transportation capacity, and unit transportation expenses are established, with each vehicle assigned a singular service route that begins and concludes at the same distribution center.

- 4) The unit fuel cost is established.
- 5) Select delivery vehicles according to the criteria of maximum load capacity and minimal carbon emissions.

IV. MODEL FORMULATION

A. Model Parameters

TABLE II presents the symbols utilized in the model along with their definitions.

TABLE II SYMBOL DESCRIPTION

Symbol	Description Description
Set	Description
N_D	The set of alternative distribution centers
Nc	The set of customers
K	The set of vehicle types
Sk	The set of vehicles of type <i>K</i>
V	$V = N_C \cup N_D$ is the set of all nodes in the network
Parameters	
f_i	Construction cost of distribution center i
CD_i	Capacity limit of distribution center i
d_{ij}	Distance from node <i>i</i> to node <i>j</i>
r	Transportation cost per unit of product per unit distance
$ ilde{d}_{i^{'}}$	Customer i's delivery demand in cycle t
$ ilde{p}_{i^{'}}$	Customer i's return demand in cycle t
k	Vehicle type
m_k	Number of vehicles of type <i>K</i>
C_k	Fixed cost of vehicle k
Q_k	Capacity limit of vehicle k
U_{ijks}	Capacity of the <i>s</i> th <i>k</i> -vehicle during cycle <i>t</i> when it departs from point <i>i</i> en route to point <i>j</i>
$p_k{}^{\scriptscriptstyle 0}$	Fuel consumption factor of vehicle type <i>k</i> in unloaded condition
p_k^{-1}	Fuel consumption factor of vehicle type k in fully loaded condition
η	Factors associated with the emission of carbon dioxide.
Decision va	riables
	0-1 decision variables, if Vehicle s, type k driving from
X_{ijks}'	point <i>i</i> to point j in cycle <i>t</i> , $x_{ijks'} = 1$, or $x_{ijks'} = 0$
y_i	0-1 decision variables, if distribution center <i>i</i> being selected, $y_i = 1$, or $y_i = 0$
Zij	0-1 decision variables, if distribution center <i>i</i> delivering to customer <i>j</i> , $z_{ij} = 1$, or $z_{ij} = 0$

B. Mathematical Model

B.1 Objective function

This study looks closely at how choosing facility locations and planning vehicle routes are connected, trying to prevent problems that can happen when these two are looked at separately. The primary objective is to minimize logistics expenses and carbon emissions. The objective function 1 aims to minimize the total logistics cost, which encompasses facility construction costs, vehicle scheduling costs, and transportation expenses.

$$\operatorname{MinZ}_{1} = \sum_{i \in ND} f_{i} y_{i} + \sum_{t \in T} \sum_{i \in ND} \sum_{j \in NC} \sum_{k \in K} \sum_{s \in sk} C_{k} x_{ijks}^{t}$$

$$+ \sum_{t \in T} \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{s \in sk} r U_{ij}^{t} d_{ij} x_{ijks}^{t}$$

$$(1)$$

Objective Function 2 aims to minimize carbon dioxide emissions produced during transportation.

$$MinZ_{2} = \sum_{t \in T} \sum_{i \in ND} \sum_{j \in NC} \sum_{k \in K} \sum_{s \in sk} \eta d_{ij} (p_{k}^{0} + (p_{k}^{1} - p_{k}^{0}) \frac{U_{ijks}^{t}}{Q_{k}})$$
(2)

B.2 Constrain

$$\sum_{i \in V} \sum_{k \in V} \sum_{c \in V} x_{ijks}^{t} \le 1, \forall j \in N_{C}, i \ne j, \forall t \in T$$
 (3)

$$\sum_{i \in I'} x_{ijks}^{\ \ t} - \sum_{i \in I'} x_{jiks}^{\ \ t} = 0, \forall j \in N_C, \forall k \in K, \forall s \in s_k, \forall t \in T$$
 (4)

$$\sum_{i \in N_C} \sum_{s \in s_t} x_{ijks}^{\ t} \le m_k, \forall i \in N_D, \forall k \in K, \forall t \in T$$
 (5)

$$MAX\left\{\sum_{j \in NC} d_{jt}^{t}, \sum_{j \in NC} p_{jt}^{t}\right\} \leq \sum_{i \in ND} CD_{i}y_{i}, \forall t \in T$$
 (6)

$$MAX\left\{\sum_{j\in NC} z_{ij}d_{jt}^{t}, \sum_{j\in NC} z_{ij}p_{jt}^{t}\right\} \leq CD_{i}y_{i}, \forall i\in N_{D}, \forall t\in T \quad (7)$$

$$\sum_{i \in N_D} z_{ij} = 1, \forall j \in N_C \tag{8}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijks}^{t} \le |A| - 1 \tag{9}$$

 $\forall k \in K, \forall s \in s_k, \forall i \neq j, A \neq \emptyset, \forall t \in T$

$$\sum_{i \in N_D} \sum_{j \in N_C} U_{ijks}^{t} = \sum_{i \in V} \sum_{j \in N_C} x_{ijks}^{t} d_{jt}^{t}, \forall k \in K, \forall s \in s_k, \forall t \in T \quad (10)$$

$$\sum_{i \in N_D} \sum_{j \in N_C} U_{jiks}^{t} = \sum_{i \in V} \sum_{j \in N_C} x_{ijks}^{t} p_{jt}^{t}, \forall k \in K, \forall s \in s_k, \forall t \in T \quad (11)$$

$$\sum_{i \in V} \sum_{k \in K} U_{ijks}^{t} - d_{jt}^{t} = \sum_{i \in V} \sum_{k \in K} U_{jiks}^{t} - p_{jt}^{t}$$

$$\forall i \in N_{C}, \forall s \in s_{k}, \forall t \in T$$

$$(12)$$

$$0 \le U_{ijks}^t \le Q \tag{13}$$

 $\forall i \in N_D, \forall k \in K, \forall s \in s_k, \forall i, j \in V, \forall t \in T$

$$\sum_{i \in \mathcal{I}} z_{ij} \ge y_i, \forall i \in N_D$$
 (14)

$$z_{ij} \le y_i, \forall i \in N_D, \forall j \in N_C$$
 (15)

$$z_{ij} \leq y_i, \forall i \in N_D, \forall j \in N_C$$

$$\sum_{k \in K} x_{ijks}^t + z_{mj} + \sum_{g \in ND, g \neq m} z_{gj} \leq 2$$

$$\forall j \in N_C, \forall s \in s_k, \forall t \in T$$
(15)

$$\sum_{i \in NC} x_{ijks}^{t} \le y_i, \forall i \in N_D, \forall k \in K, \forall t \in T, \forall s \in s_k, \quad (17)$$

$$x_{ijks}^t \le z_{ij}, \forall i \in N_D, \forall j \in N_C, \forall k \in K, \forall s \in s_k, \forall t \in T$$
 (18)

$$y_i \ge z_{ij}, \forall i \in N_D, \forall j \in N_C$$
 (19)

$$\sum_{i \in V} \sum_{k \in K} \sum_{s \in sk} \sum_{t \in T} x_{ijks}^{t} \ge \sum_{i \in ND} z_{ij}, \forall j \in N_{C}, i \neq j$$
 (20)

$$y_i \le \sum_{i \in NC} z_{ij}, \forall i \in N_D$$
 (21)

$$x_{ijks'} \in \{0,1\}, \forall i, j \in V, \forall k \in K, \forall s \in s_k, \forall t \in T$$
 (22)

$$y_i \in \{0,1\}, \forall i \in N_D$$
 (23)

$$z_{ij} \in \{0,1\}, \forall i \in N_D, \forall j \in N_C$$
 (24)

Eq (3) indicates that each consumer can only be serviced once throughout a cycle; Eq (4) depicts the vehicle in-and-out balance constraint at customer points, which ensures that the number of cars arriving and departing each

customer point is equal. Eq (5) guarantees that the number of cars dispatched from the distribution center is fewer than the number of available vehicles. Eq (6) assures that the overall capacity of the chosen distribution centers is sufficient to satisfy all customers' entire demands. Eq (7) guarantees that the capacity of each distribution center is sufficient to satisfy the total demands of the customers served by it. Eq. (8) guarantees that each customer is served by a single distribution center. Eq (9) removes constraints for sub-branches, with S representing the set of customers served by vehicle k. Eq (10) demonstrates that the vehicle's load upon leaving the distribution center is equivalent to the aggregate of the delivery demands from the served customers. Eq (11) demonstrates that the vehicle's load upon returning to the distribution center is equivalent to the aggregate of the return demands from the serviced customers. Eq (12) illustrates the dynamic change in the vehicle's capacity. Eq (13) establishes the constraint on the vehicle's capacity. Eqs (14), (15), and (16) demonstrate that the activated distribution centers are required to serve customers. Eq (17) stipulates that these centers must have vehicles departing. Eq (18) demonstrates that the unconstructed distribution centers lack vehicle departures in each period. Eqs (19), (20), and (21) delineate the constraints among the decision variables. Eqs (22), (23), and (24) denote the binary decision variables.

C. Model Analysis

 d_{jt}^{t} and p_{jt}^{t} is an indeterminate variable and cannot be resolved directly. To render the model solvable, it is essential to transform it into an equivalent deterministic model. Let $\xi = \mathcal{Z}(a,b,c)$ be a zigzag-type uncertain variable, and consider $0 \le a \le b \le c$ [26].

Theorem 1[27]: The uncertainty distribution Φ of an uncertain variable, \mathcal{M} is an uncertain measure, \Re is the whole number of real numbers, ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}, \forall x \in \Re$$
 (25)

Theorem 2[28]: Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(a)$ is called the inverse uncertainty distribution of ξ .

Theorem 3[29]: Let ξ be an uncertain variable. \mathcal{M} is an uncertainty measure of ξ with confidence level α for any real number r.

(1) The optimistic value of ξ_{sup} is defined by:

$$\xi \sup(\alpha) = \sup\{r \mid \mathcal{M}\{\xi \ge r\} \ge \alpha\}, \alpha \in (0,1]$$
 (26)

(2) The expected value of ξ_E is defined by:

$$\xi_E = \int_0^{+\infty} \mathcal{M}\left\{\xi \ge r\right\} dr - \int_{-\infty}^0 \mathcal{M}\left\{\xi \le r\right\} dr \tag{27}$$

(3) The pessimistic value of ξ_{inf} is defined by:

$$\xi_{\inf}(\alpha) = \inf \{ r \mid \mathcal{M} \{ \xi \le r \} \ge \alpha \}, \alpha \in (0,1]$$
 (28)

Definition 1[27]: When uncertain variable ξ has a continuous uncertain distribution $\Phi(r)$, there exists $1 - \Phi(r) = \mathcal{M}\{\xi \ge r\}$. Combining Theorems 1, 2, and 3 can obtain:

$$\xi_{\text{sup}}(\alpha) = \sup\{r \mid 1 - \Phi(r) \ge \alpha\}, \alpha \in (0,1]$$
 (29)

$$\xi_{E} = \int_{0}^{+\infty} (1 - \Phi(r)) dr - \int_{-\infty}^{0} \Phi(r) dr$$
 (30)

$$\xi_{\inf}(\alpha) = \inf \{ r \mid \Phi(r) \ge \alpha \}, \alpha \in (0,1]$$
 (31)

Definition 2[27]: The mathematical expression for the confidence level α of the unknown variable an is shown in equation (32); the graphical representation of the distribution function is illustrated in Figure 1. Where α are the model's confidence levels. The decision maker must ascertain its worth prior to resolution, and the acceptance of risk by the decision maker dictates the value of r.

$$\Phi(\xi \le \mathbf{r}) = \begin{cases}
0, r \le a \\
\frac{r-a}{2(b-a)}, a < r \le b \\
\frac{r+c-2b}{2(c-b)}, b < r \le c
\end{cases}$$
(32)

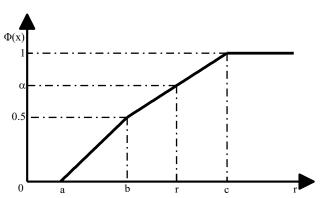


Figure 1. Zigzag confidence function for uncertainty variable

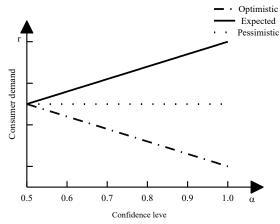


Figure 2. The amount of trust and the trend of customer demand

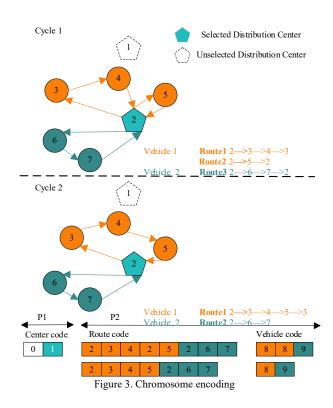
When the uncertain variable is designated as a, as seen in Figure 2, the optimistic value diminishes with increasing believability when α ranges from 0.5 to 1, whereas the escalates. pessimistic value The decision-maker's predetermined confidence level fundamentally indicates their varying attitudes about risk. When α is elevated, the decision maker exhibits risk aversion and aims to diminish actual demand below the anticipated demand with increased probability. Conversely, when α assumes a lower value, the decision maker exhibits risk preference and opts for the less expensive alternative, confident in their ability to manage the risks associated with the uncertain environment.

V. SOLUTION APPROACH

The LRP problem encompasses both the location issue and the vehicle routing problem, both of which are classified as NP-hard; hence, it is categorized under NP-hard problems[30]. For extensive NP-hard problems, precise algorithms often encounter difficulties in achieving computing results within a constrained timeframe; hence, several heuristic methods have been extensively used to address these issues.

NSGA-II is a problem-solving method that is effective at finding the best solutions for complex issues, and it has been successfully used in many situations where there are multiple goals to achieve. The tabu search algorithm, recognized for its robust search capabilities and efficiency, is extensively examined as a method for addressing the VRP problem[6]. The taboo search algorithm is integrated as a strategy within NSGA-II to formulate a two-stage approach for addressing the LRP problem. Customers are allocated to the nearest distribution center according to the location scheme, resulting in chromosome encoding. The initial issue is segmented into various distribution centers and their corresponding customers. Each distribution center and its corresponding customers can be considered a VRPSPD problem. The VRPSPD problem is addressed through the application of the taboo search strategy, incorporating all distribution centers along with their associated construction costs. A recombination strategy is introduced to prevent the population from becoming trapped in local optima, thereby diversifying the population distribution and enhancing its search capability.

A. Encode



The chromosomes employ a multi-layer encoding structure, consisting of two components, p_1 and p_2 . The p_1 component denotes the site selection scheme, while the gene value signifies the selection status of the distribution center. p_1 gene value of 1 indicates the selection of the

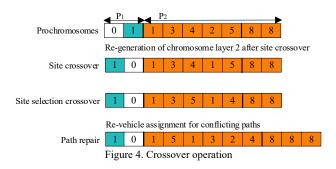
distribution center, while a value of 0 indicates its non-selection. Part p_2 comprises multiple strings of differing lengths, with the quantity of segments aligned with the number of delivery cycles. Each string denotes the vehicle routing plan for a specific cycle, comprising multiple substrings, each of which represents an individual vehicle route. For instance, there are three candidate distribution centers and seven customers, supported by two types vehicles, where numbers 1-2 denote distribution centers numbers 3-7 indicate customers and numbers 8-9 indicate vehicles, as illustrated in a specific chromosome in Figure 3.

B. Degree of Congestion

Individuals may be classified into distinct distribution centers and clients according to the chromosomal encoding. Each distribution center and its related consumers may be seen as a VRPSPD issue throughout each distribution cycle. The VRPSPD issue is addressed using a tabu search technique, yielding objective functions Z_1 and Z_2 for the person. We acquire the objective functions of all individuals in the population, normalize them, and determine each individual's crowding degree T_d .

C. Crossover

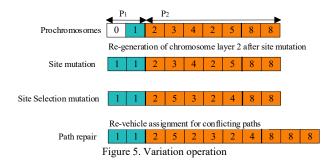
The crossover process comprises site selection crossover and allocation crossover. Figure 4 illustrates that the site selection crossover includes chromosome part p_1 , which represents the site selection scheme and uses the uniform crossover method. The new site selection scheme is evaluated following the crossover procedure. If the new site selection scheme satisfies customer requirements, p_2 is regenerated; otherwise, the site selection scheme is revised. Customer crossover refers to segment p_2 of the chromosome, which represents the customer allocation scheme. Employing the partially matched crossover method, the vehicle routes in p_2 are examined post-crossover, and routes with customer demand surpassing vehicle capacity are subsequently reallocated.



D. Mutation

The mutation process consists of two types: location mutation and allocation mutation. Following the mutation operation, it is essential to evaluate the new solution post-mutation, as illustrated in Figure 5. Location mutation entails the random selection of mutation sites within part p_1 of the chromosome, followed by the execution of a flip operation. Allocation mutation entails executing mutation operations on a randomly chosen segment of chromosome p_2 . Following the execution of the mutation, an inspection of the vehicles in p_2 is conducted, leading to the

reallocation of routes where customer demand surpasses vehicle capacity.



E. Tabu Search Strategy

This study utilizes a Tabu Search approach to address the deterministic VRPSPD problem. The algorithmic strategy is outlined as follows:

- 1) Neighborhood search: The 2-opt approach is used for neighborhood search, which entails the random selection of two gene sites and the subsequent exchange of the customers at these spots.
- 2) Establishment of taboo objects and tabu length: The conclusive solution of each iteration is included in the taboo list as a taboo object; the tabu length denotes the number of iterations during which the tabu item is prohibited from selection, with the tabu length established accordingly.
- 3) The ambition criteria are used based on evaluative values. When the current solution in the candidate set is prohibited but superior to the optimal solution, it is expunged from the taboo list, and both the current and optimal solutions are revised.
- 4) Termination criterion: Conclude the loop when reaching the specified number of iterations.

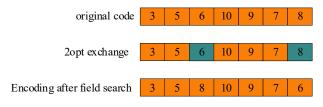


Figure 6. Tabu Search strategy

F. Elite Retention Strategy

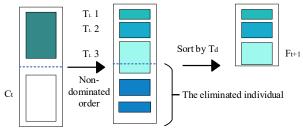


Figure 7. Elite retention strategy

Subsequent to merging the parental population with the progeny population, execute non-dominated sorting. Currently, all individuals in the population possess non-dominated sorting T_L and crowding degree T_d . Prioritize the selection of individuals with lower indices, and among those with identical indices, choose individuals with lesser crowding distances to preserve population variety.

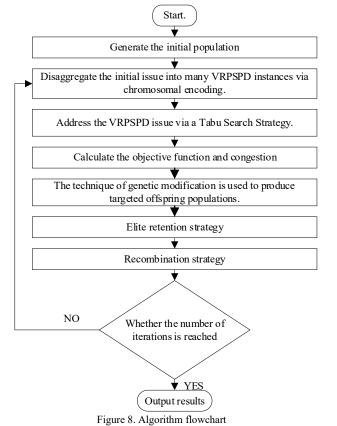
The leading N people will be chosen to constitute the subsequent generation population. This technique facilitates the retention of superior individuals from the parent population, directing subsequent genetic operations and assuring the conservation of exceptional people.

G. Reorganization Strategy

Establish the number of iterations N_a and the concentration threshold P_a to prevent premature convergence in algorithm. Premature convergence happens when the best solution stops getting better or when too many individuals in the population are the same, exceeding the concentration threshold $p_a = m/N$, m represent the count of individuals sharing an identical fitness value and N denotes the total population size. Employ the initial population generation method outlined in section 2.1 to create a subset of new individuals that will substitute the identical components within the population, thus enhancing population diversity and broadening the solution space.

H. Algorithmic Process

The main process of the improved NSGA-II algorithm, illustrated in Figure 8:



VI. CASE STUDY

A. Parameters Setting

This article evaluates the Gaskell67-21x5 case by incorporating customer return demands and adjusting the construction costs and capacity of the distribution center, ensuring that both costs and capacity are proportional. Customer demand underwent uncertainty processing. Utilizing the methodology outlined in reference [31], parameters p = ad, a = 0.2 were established to produce customer return demands. Subsequently, customer demand was further analyzed for uncertainty by expanding β to the left and right by a specific proportion $\beta = 0.2$, resulting in uncertain delivery demands $d_i = ((1-\beta)d, d, (1+\beta)d)$. was configured to produce uncertain return demands, leading to customer pick-up and delivery demands $d_{i'}$ and $p_{i'}$ for the initial cycle. Subsequently, β_t was randomly generated within the interval [-0.5, 0.5] to derive customer demands for the second and third cycles. Each center is equipped with two vehicle types, as detailed in TABLE III. TABLE IV and TABLE V present the information regarding customer nodes and distribution center nodes.

Perform numerical experiments utilizing the optimistic, expected, and pessimistic values. The experiments will be executed on a PC with Python 3.0 to compile the algorithm, specifically using an i7-8565U 1.8GHz CPU, 8GB of memory, and running on Windows 11. The primary parameters of the algorithm include a population size of N=200 for the genetic algorithm and a maximum iteration count of MAXGEN=200. The candidate set of TS contains $A=3\times B$ individuals, with B representing the customer scale of the VRPSPD problem. Python software optimizes the tabu length \sqrt{A} and caps the maximum number of iterations at A.

TABLE III
VEHICLE INFORMATION

Vehicles	Composites	city Num Fix Co		Fuel Cons	nsumption	
Type	Capacity	Num	Fix Cost	Empty Load	Full Load	
v_1	400	2	100	0.26	0.37	
v_2	600	2	260	0.27	0.38	

TABLE IV DISTRIBUTION CENTER NODE INFORMATION

Distribution Center	1	2	3	4	5
Coordinates (x, y)	(136,194)	(143,237)	(136,216)	(137,204)	(128,197)
Capacity (item)	1200	1350	1500	1650	1800
Cost (CNY)	40000	15000	50000	55000	60000

TABLE V
CUSTOMER NODE INFORMATION

Customer	1	2	3	4	5	6	7
Coordinates	(151,264)	(159,261)	(130,254)	(128,252)	(163,247)	(146,246)	(161,242)
(x, y)							
				Cycle 1			
Delivery (item)	(68.2,77.4,86.7)	(43.4,49.3,55.2)	(49.6,56.3,63.1)	(86.7,98.6,110.4)	(130.1,147.8,165.6)	(24.8,28.2,31.6)	(49.6,56.3,63.1)
Return (item)	(13.6,15.5,17.3)	(8.7,9.9,11)	(9.9,11.3,12.6)	(17.3,19.7,22.1)	(26,29.6,33)	(5,5.63,6.3)	(9.9,11.3,12.6)
				Cycle 2			
Delivery (item)	(85.2,96.8,108.4)	(54.2,61.6,69)	(62,70.4,78.9)	(108.4,123.2,138)	(162.6,184.8,207)	(31,35.2,39.4)	(62,70.4,78.8)

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Return (item)	(17,19.4,21.7)	(10.8,12.3,13.8)	(12.4,14.1,15.8)	(21.7,24.6,27. 6)	(32.5,37,41.4)	(6.2,7,7.9)	(12.4,14,15.8)
(Item)				Cycle 3			
Delivery (item)	(102,116.2,130.1)	(65.1,73.9,82.8)	(74.3,84.5,94.6)	(130.1,147.8,165.6)	(195.1,221.7,2.4)	(37.2,42.2,47.3)	(74.3,84.5,94.6)
Return (item)	(20.4,23.23,26)	(13,14.8,16.6)	(14.9,16.9,18.9)	(26,29.6,33.1)	(39,44.35,49.7)	(7.4,8.5,9.5)	(14.9,16.9,18.9)
Customer	8	9	10	11	12	13	14
Coordinates (x, y)	(142,239)	(163,236)	(148,232)	(128,231)	(156,217)	(129,214)	(146,208)
				Cycle 1			
Delivery (item)	(6.2,7.04,7.9)	(31,35.2,39.4)	(37.2,42.4,47.3)	(74.3,84.48,94.6)	(80.5,91.5,102.5)	(80.6,91.5,102.5)	(18.6,21.1,23.7)
Return (item)	(1.2,1.41,1.6)	(6.2,7,7.9)	(7.4,8.5,9.5)	(14.9,16.9,18.9)	(16.1,18.3,20.5)	(16.1,18.3,20.5)	(3.7,4.2,4.7)
				Cycle 2			
Delivery (item)	(7.7,8.8,9.9)	(38.7,44,49.3)	(46.5,52.8,59.1)	(92.9,105.6,118.3)	(100.7,114.4,128.2)	(100.7,114.4,128.1)	(23.2,26.4,29.6)
Return (item)	(1.5,1.8,2)	(7.7,8.8,9.9)	(9.3,10.6,11.8)	(18.6,21.12,2.7)	(20.1,22.9,25.6)	(20.1,22.8,25.6)	(4.6,5.28,5.9)
()				Cycle 3			
Delivery	(9.3,10.6,11.8)	(46.5,52.8,59.1)	(55.8,63.36,71)	(111.6,126.7,141.9)	(120.8,137.3,153.8)	(120.8,137.3,153.8)	(27.9,31.7,35.5)
(item) Return	(1.9,2.11,2.4)	(9.3,10.6,11.8)	(11.1,12.7,14.2)	(22.3,25.3,28.4)	(24.2,27.5,30.8)	(24 2 27 46 20 8)	(5 6 6 2 7 1)
(item)	(1.9,2.11,2.4)	(9.5,10.0,11.8)	(11.1,12.7,14.2)	(22.3,23.3,26.4)	(24.2,27.3,30.8)	(24.2,27.46,30.8)	(5.6,6.3,7.1)
Customer	15	16	17	18	19	20	21
Coordinates (x, y)	(164,208)	(141,206)	(147,193)	(164,193)	(129,189)	(155,185)	(139,182)
() 3)				Cycle 1			
Delivery (item)	(55.8,63.4,71)	(130.1,147.8,165.6)	(62,70.4,78.8)	(55.8,63.4,71)	(154.9,176,197.1)	(111.5,126.7,141.9)	(43.4,49.3,55.2)
Return (item)	(11.1,12.7,14.2)	(26,29.6,33.1)	(12.4,14.1,15.8)	(11.1,12.7,14.2)	(31,35.2,39.4)	(22.3,25.34,28.4)	(8.7,9.9,11)
(Helli)				Cycle 2			
Delivery (item)	(69.7,79.2,88.7)	(162.6,184.8,207)	(77.4,88,98.6)	(69.7,79.2,88.7)	(193.6,220,246.4)	(139.4,158.4,177.4)	(54.2,61.6,69)
Return (item)	(13.9,15.8,17.7)	(32.5,37,41.4)	(15.5,17.6,19.7)	(13.9,15.8,17.7)	(38.7,44,49.3)	(27.8,31.68,35.5)	(10.8,12.3,13.8)
,				Cycle 3			
Delivery (item)	(83.6,95,106.4)	(195.1,221.7,248.4)	(92.9,105.6,118.3)	(83.6,95,106.5)	(232.3,264,295.7)	(167.2,190.1,212.9)	(65,73.9,82.8)
Return (item)	(16.7,19,21.3)	(39,44.4,49.7)	(18.6,21.1,23.7)	(16.7,19.1,21.3)	(46.5,52.8,59.1)	(33.5,38,42.6)	(13,14.8,16.6)

B. Numerical Results

Figure 9 illustrate the Pareto front solution set across optimistic, expected, and pessimistic scenarios. Identify three solutions for examination, with Pareto 1 and 3 representing extreme solutions, while Pareto 2 serves as a non-extreme point. Results are presented in TABLE VI.

In the scenario of optimistic, the total cost of Pareto 1 was reduced by 39% compared to Pareto 2, while carbon emissions rose by 15% and driving distance increased by 9.9%. Conversely, the total cost of Pareto 3 increased by 48.8%, carbon emissions decreased by 10.7%, and driving distance decreased by 3.6%. Comparative analysis of these three solutions indicates that while an increase in the number of distribution centers raises total costs, it simultaneously decreases vehicle driving distances, resulting in lower carbon emissions and transportation expenses.

The site selection plan for the expected value scenario aligns with that of the optimistic value scenario; however, the vehicle dispatch plan and vehicle routing differ. The customer demand in the expected value scenario increases relative to the optimistic value scenario; however, it remains within the capacity limit of the location plan. Consequently, only the vehicle dispatch and routing plans undergo modification. Using Pareto 1 as a case study, the distribution centers identified in both the expected and optimistic scenarios are Distribution Centers 1 and 2. In the anticipated scenario, the total cost rose by 1%, carbon emissions increased by 1.3%, and the total distance grew by 0.3%.

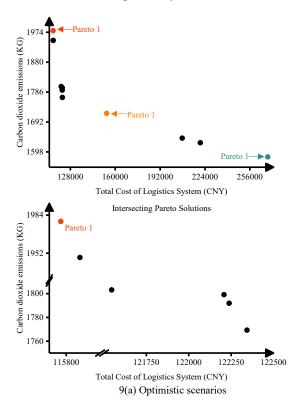
Distribution Center 2 dispatched an additional v_2 vehicle over three cycles, leading to a 65% increase in vehicle dispatch costs. The costs in the expected value scenario exceed those in the optimistic scenario.

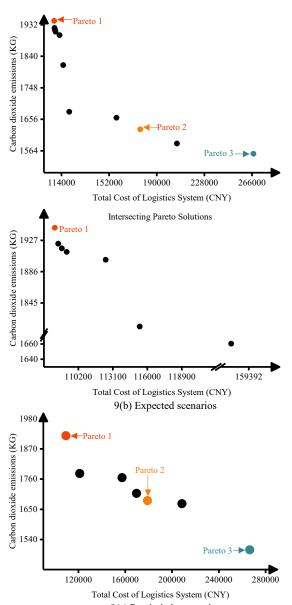
When comparing Pareto 1 and 3 to Pareto 2, Pareto 1 exhibited a total cost reduction of 38.6%, alongside a 19.5% increase in carbon emissions and a 10.4% rise in total distance. In contrast, Pareto 3 showed a total cost increase of 48.5%, a 4.4% decrease in carbon emissions, and a 9.2% reduction in total distance. The site selection plan in the pessimistic scenario diverges from those in the optimistic and expected scenarios due to elevated customer demand, which surpasses the operational capacity of Distribution Centers 1 and 2, necessitating a revision of the site selection plan. In comparing extreme and non-extreme solutions, Pareto 1 exhibited a total cost reduction of 24.8%, an increase in carbon emissions by 15.1%, and a total distance increase of 14.3%. Conversely, Pareto 3 showed a total cost increase of 74.5%, a decrease in carbon emissions by 8%, and a total distance decrease of 13.4%.

C. Credibility Analysis

Maintain other parameters constant, thereby allowing for a credibility α variation between (0.5, 1] in increments of 0.1. Perform numerical experiments utilizing both optimistic and pessimistic customer demand values. Select the extreme value solution, specifically Pareto solution 1, for analysis under various conditions and assess the influence of α on

the outcomes across different scenarios, as presented in TABLE VII. The expected value of the zigzag uncertain variable does not change with α , meaning that customer demand is not influenced by changes in α , the location selection results and paths stay the same.





9(c) Pessimistic scenarios Figure 9. Spatial distribution of the pareto frontier

TABLE VI
PARETO SOLUTION INDICATORS

Scenario Optimistic			Expected			Pessimistic			
Indicators (CNY)	Pareto 1	Pareto 2	Pareto 3	Pareto 1	Pareto 2	Pareto 3	Pareto 1	Pareto 2	Pareto 3
Aggregate expenditures (CNY)	109325.4	179119.1	266503.3	110416.9	179901.7	267178.1	115768.2	153974.8	268777.8
Distribution center	1, 2	2, 4, 5	1, 2, 3, 4, 5	1, 2	2, 4, 5	1, 2, 3, 4, 5	2, 3	1, 2, 3	1, 2, 3, 4, 5
Construction Costs (CNY)	85000	160000	250000	85000.0	160000	250000	95000	135000	250000
Transportation expenses (CNY)	23125.4	17619.1	14703.3	23436.9	16841.7	14598.1	18008.2	16394.8	16197.8
vehicle dispatch Costs (CNY)	1200	1500	1800	1980	3060	2580	2760	2580	2580
Carbon dioxide emissions (KG)	1917.7	1681.8	1502.6	1943.4	1626.5	1555	1978. 8	1719.1	1582.4
Total distance (KM)	1754.5	1595.8	1539.1	1835.9	1663.3	1510	1988.53	1740.26	1507.7

TABLE VII CREDIBILITY IMPACT ON CENTER SELECTION AND COSTS

Scenario	Credibility	Distribution center	Construction costs (CNY)	Total costs (CNY)	Carbon dioxide emissions (KG)	Vehicle dispatch costs (CNY)	Transport costs (CNY)
	0.6	1 2	85000	107709.9	1949.2	1980	20729.9
Optimistic	0.7	1 2	85000	110797.2	1939.5	1200	24597.2
	0.8	1 2	85000	109325.4	1917.7	1200	23125.4
	0.9	1 2	85000	108203.9	1916	1200	22003.9
	1	1 2	85000	107666.7	1820.2	1200	21466.7
	0.6	1 2	85000	105884.4	1840.4	2760	21124.4
	0.7	1 3	90000	114256.9	2027.3	1980	22276.9
Pessimistic	0.8	2 3	95000	115768.2	1978. 8	2760	18008.2
	0.9	2 3	95000	117894.76	2082.4	2760	20134.8
	1	2 4	100000	120954	1780	3060	17894

TABLE VIII

IMPACT OF COMMODITY CREDIBILITY ON CENTER SELECTION AND COSTS

Expansion	Scenario	Distribution	Construction costs	Total costs	Carbon dioxide	Vehicle dispatch	Transport costs
ratio	Scenario	center	(CNY)	(CNY)	emissions (KG)	costs (CNY)	(CNY)
0	Optimistic	1 2	85000	109068	1981.6	1980	22088
U	Pessimistic	1 2	85000	110416.9	1943.4	1980	23436.9
0.1	Optimistic	1 2	85000	108385.3	1898.2	1980	21405.3
0.1	Pessimistic	1 2	85000	111326.8	1983.2	1980	24346.8
0.2	Optimistic	1 2	85000	109325.4	1917.7	1200	23125.4
0.2	Pessimistic	2 3	95000	115768.2	1978. 8	2760	18008.2
0.3	Optimistic	1 2	85000	108564.4	1830. 9	1200	22364.4
0.3	Pessimistic	2 3	95000	116243.6	1815	2760	18483.6
0.4	Optimistic	1 2	85000	108184.1	1829.8	1200	21984.1
0.4	Pessimistic	3 4	105000	127703.7	1950.6	2760	19943.7
0.5	Optimistic	5	60000	95850.9	2392. 2	1380	34470.9
0.5	Pessimistic	1 2 5	145000	167266	1783.5	3060	19206

TABLE IX
COMPARISON OF ALGORITHM INDICATORS

Example	Customer number	Centers number	Algorithm	Total costs (CNY)	Carbon dioxide emissions (KG)	Solution time (s)
			INSGA-II	121988.96	2346.62	254
Gaskell67-21x5	21	5	NSGA-II	122181.41	1496.11	259
			GA-SA	122419	1493.71	263
	32	5	INSGA-II	122845.24	1486.299	535
Gaskell67-32x5			NSGA-II	127382.95	1541.209	515
			GA-SA	128668.87	1632.01	561
			INSGA-II	134628.33	1407.28	2072
Gaskell67-50x5	50	0 5	NSGA-II	137977.14	1387.92	2058
			GA-SA	112381.58	1623.52	2101

In the optimistic scenario, α change did not alter the location plan. This result arises because, despite a decrease in customer demand with increased reliability, it remains above the operational capacity of a single distribution center. This situation does not necessitate a change in the location plan but rather results in modifications to the vehicle routing plan. At $\alpha > 0.6$, the decline in customer demand allowed for the completion of deliveries from Distribution Center 1 using fuel vehicles. This resulted in a 2% reduction in vehicle dispatch costs, a total cost decrease of up to 1.3%, and a reduction in carbon emissions of up to 0.4%.

In the context of pessimistic values Upon the occurrence of α =0.7, the site selection outcome transitions from Distribution Centers 1 and 2 to Distribution Centers 1 and 3; upon the occurrence of $\alpha \in [0.8,0.9]$, the outcome is Distribution Centers 2 and 3; and upon the occurrence of $\alpha > 0.9$, it is Distribution Centers 2 and 4. This is due to the fact that when $\alpha \in (0.5,1]$ happens, the pessimistic value of the zigzag uncertain variable is a monotonically rising function of α . As α rises, the gloomy estimate of consumer demand correspondingly grows in a monotone manner. When consumer demand surpasses the operating capability of the initial distribution centers, it necessitates modifications to the site selection strategy.

In conclusion, within a specified range, when the site selection plan and vehicle routing plan adequately address customer requirements, variations in α will not affect the outcomes of the site selection process. Changes in α that surpass this range will result in modifications to the site selection plan or the vehicle routing plan.

D. Uncertainty Interval Width Analysis

This paper examines the influence of customer demand uncertainty intervals on facility location and associated cost parameters within the model. The expansion ratio β , with

other parameters held constant, ranges from 0 to 0.5 in increments of 0.1. Customer demand remains unaffected by the β in the expected value scenario, leading to consistent location results and paths.

The findings presented in TABLE VIII demonstrate that in the optimistic scenario, the occurrence of $\beta > 0.2$ leads to alterations in the vehicle routing plan as the uncertainty interval widens. Conversely, $\beta \in [0.2, 0.4]$ results in a gradual reduction in total costs and carbon emissions. Additionally, the occurrence of $\beta = 0.5$ causes a decline in customer demand to a level that can be adequately addressed by Distribution Center 5, prompting a modification in the location plan. In the optimistic scenario, it can be concluded that as the uncertainty interval widens, customer demand progressively diminishes. In the pessimistic scenario, the occurrence of $\beta \in [0,0.2]$ leads to a gradual decrease in total costs and carbon emissions as the uncertainty interval expands. When $\beta > 0.2$ occurs, customer demand surpasses the operational capacity of distribution centers 1 and 2, resulting in a modification of the location plan. As $\beta \ge 0.3$ occurs and the uncertainty interval expands, the location plan is adjusted accordingly. In the pessimistic scenario, an expansion of the uncertainty interval correlates with a gradual increase in customer demand.

In conclusion, customer demand exhibits fluctuations within a limited fuzzy range, while the site selection outcomes remain consistent. The costs associated with vehicle dispatch and delivery vary slightly in response to alterations in vehicle capacity. Exceeding a specific range of fluctuations will result in modifications to the site selection plan. The width of the customer uncertainty interval correlates with variations in customer demand, total costs, and carbon emissions. In optimistic scenarios, an increased range of customer uncertainty correlates with reduced

customer demand, leading to lower total costs and carbon emissions. Conversely, in pessimistic scenarios, a wider customer uncertainty range is associated with heightened customer demand, resulting in increased total costs and carbon emissions.

E. Algorithm Comparison

Look at the results and the time taken for the improved NSGA-II method discussed in this article, and compare them to the regular NSGA-II without improvements and the GA-SA results found in other studies[25]. TABLE IX presents the results.

Improved NSGA-II solutions exhibit superior quality and enhanced search capabilities when compared to NSGA-II and GA-SA solutions. improved NSGA-II has a longer runtime compared to NSGA-II and GA-SA due to its utilization of multiple strategies; however, the additional time is merely 20 seconds, representing a 3.9% increase, thus maintaining a reasonable overall computing time. The improved NSGA-II algorithm shows better quality and greater ability to find the best solutions for the FLRP problem.

VII. CONCLUSION

This study seeks to reduce total logistics costs and carbon emissions in transportation by developing a multi-objective model for multi-period, multi-vehicle simultaneous pickup and delivery in uncertain conditions. The primary tasks accomplished include the following:

- 1) The model incorporating uncertain variables is equivalently transformed to derive location and vehicle service path schemes under optimistic, expected, and pessimistic conditions, based on the definitions of these values.
- 2) The proposed model and methods were validated through numerical examples, demonstrating the influence of the confidence level, indicative of the decision-maker's risk preference, on the results. The examples showed that the model and algorithm work well in real situations and highlighted how confidence levels and uncertain variables affect location and vehicle routing results.
- 3) The results of the improved NSGA-II algorithm in this study were compared to the results of the NSGA-II and GA-SA algorithms for solving the LRP problem. The improved NSGA-II algorithm has been shown to effectively address the LRP problem.
- 4) This study focuses exclusively on the location-routing optimization problem for a single product, presenting certain limitations. The next research direction will focus on optimizing the location-routing problem for multiple products, which holds significant practical relevance.

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