Study on the Logistic Model of Mosquito Population Involving Caputo-Fabrizio Fractional Derivative

Zhongyuan Wang, Wei Zhang and Di Zhang

Abstract—**This study extends the logistic model, which describes the population dynamics of wild female mosquitoes in the absence of intervention by Wolbachia-infected male mosquitoes, to the case involving fractional derivative. A novel fractional logistic equation is formulated utilizing the Caputo-Fabrizio fractional derivative. By using the properties of the Caputo-Fabrizio fractional calculus, the implicit analytical solution of the proposed model is derived. Additionally, numerical simulations are performed to explore the influence of the fractional order and various parameter values on mosquito population dynamics.**

Index Terms—**Fractional logistic equation, Caputo-Fabrizio fractional derivative, Analytical solution, Mosquito population dynamics**

I. Introduction

THE classical logistic equation

$$\frac{dx}{dt} = x(t)(1 - x(t)), \quad t \geq 0, \tag{1}$$

has been widely employed to characterize population growth dynamics, particularly within biological and ecological frameworks. Its solution reflects the influence of environmental carrying capacity, exhibiting an initial phase of rapid growth followed by stabilization. In recent years, the scope of the logistic model has substantially broadened to encompass various disciplines, including epidemiology, economics, biomedical sciences, cyber-physical systems, and the modeling of opinion dynamics on networks [1, 2]. Accordingly, the logistic equation has emerged as a fundamental analytical tool in the study of nonlinear behavior in complex systems. As a representative application, Zhang et al. [3] introduced a modified logistic model to examine the population dynamics of wild female mosquitoes in the absence of Wolbachia-infected male mosquito intervention, formulated as

$$\frac{dx}{dt} = ax(t) - (\mu + \xi x(t))x(t), \quad t \geq 0, \tag{2}$$

where $a > \mu$ corresponds to the intrinsic birth rate of wild mosquitoes, and μ, ξ denote the density-independent and density-dependent mortality rates, respectively. The model predicts that, in the absence of intervention, the mosquito population stabilizes at an equilibrium level of $(a - \mu)/\xi$. As model (2) describes the dynamic evolution of populations over time, it provides a theoretical foundation and framework for further research on how external interventions, such as releasing Wolbachia-infected male mosquitoes, can effectively suppress the population of wild female mosquitoes.

It is well recognized that the classical logistic model of integer order exhibits inherent limitations when applied to systems with pronounced memory effects and nonlocal behavior. In recent years, fractional calculus has attracted significant attention due to its enhanced flexibility and broad scope of applicability [4, 5]. The principal advantage of fractional derivatives lies in their capacity to effectively characterize the historical dependence and nonlocal features intrinsic to complex dynamical systems [6]. The application of fractional derivatives provides a more generalized mathematical framework for the classical logistic equation, enabling it to describe non-local effects and time-delay characteristics inherent in complex systems. Recently, researchers have extensively and comprehensively studied the solutions of fractional logistic equations under different types of fractional derivative frameworks [7–19]. For instance, Izadi and Srivastava [8] developed approximate solutions for a fractional logistic equation involving the LiouvilleCCaputo derivative via a collocation scheme based on fractional-order Bessel and Legendre basis functions. Jornet and Nieto [9], based on the L-fractional derivative framework, analyzed fractional logistic equation through a power series expansion method and derived results concerning the local analytic solutions. El-Saka et al. [10] discussed the stability and Hopf bifurcation phenomena of Caputo fractional logistic equation with two distinct time delays using the method of key curve analysis methods. Abreu-Blaya et al. [17] investigated the qualitative behavior of conformable fractional logistic equations, formulating generalized derivatives with a kernel of the form $T(t, \alpha) = e^{(\alpha-1)t}$, and further examined solution stability and inverse problem modeling with empirical data. Abdeljawad et al. [19] explored higher-order nonlinear extensions, including quadratic and cubic formulations of fractional logistic equations under the ABC-type fractional derivative framework, and proposed numerical schemes utilizing multi-parameter Mittag-Leffler kernels to establish existence, uniqueness, and stability results.

The Caputo-Fabrizio fractional derivative is a novel form of fractional-order derivative introduced by Caputo and Fabrizio in 2015 [20]. Its primary aim is to

Manuscript received May 12, 2025; revised July 17, 2025.

This work was supported in part by the Key Project of Graduate Education and Teaching Reform of Anhui Province (No. 2024jyjxggyjY180).

Z. Wang is a postgraduate student of the School of Mathematics and Big Data, Anhui University of Science and Technology, Huainan, 232001, China (e-mail: zywang_equations@163.com).

W. Zhang is an associate professor of the School of Mathematics and Big Data, Anhui University of Science and Technology, Huainan, 232001, China (corresponding author, phone: +8605546663050; e-mail: zhangwei_azyw@163.com).

D. Zhang is an assistant lecturer of the Department of Basic, Jiangsu Urban and Rural Construction Vocational College, Changzhou, 213147, China (e-mail: zhangdi80303@163.com).

address challenges encountered in traditional fractional derivatives, such as the Riemann-Liouville and Caputo derivatives, particularly the complexity and computational difficulties arising from singular kernels. This fractional derivative is distinguished by its incorporation of a non-singular exponential-type kernel and its linear growth characteristics, rendering it well suited for modeling systems exhibiting memory and hereditary properties. Owing to its desirable analytical features, the CaputoCFabrizio derivative has garnered substantial interest across diverse scientific and engineering disciplines, with applications spanning epidemiological modeling and disease control [21–24], food processing technologies [25], electrical circuit analysis [26], biomedical systems [27, 28], plant pathology [29], climate and ecological dynamics [30], as well as financial systems [31, 32]. Given the extensive application of the Caputo-Fabrizio fractional derivative in modeling complex systems, the Logistic-type equations formulated under the CaputoCFabrizio framework have recently become a focal point of study.

For instance, Khalouta [33] investigated the existence and uniqueness of solutions to a nonlinear fractional logistic differential equation by employing a novel decomposition transformation technique alongside Banach's fixed point theorem. Kumar et al. [34] adopted the fixed point approach for analyzing a class of CaputoCFabrizio fractional logistic model, further supplemented by numerical simulations using iterative schemes. Fahel et al. [35] extended the analysis to quadratic and cubic logistic systems, leveraging Lagrange polynomial-based fractional iterative methods to establish exact solutions. EL-Fassi et al. [36] examined a Richards-type fractional equation under the CaputoCFabrizio framework and successfully derived analytical solutions, thereby expanding the theoretical foundation for logistic-type systems in fractional contexts. Of particular note, Nieto [37] formulated a fractional-order extension of the classical logistic equation (1) by employing the CaputoCFabrizio derivative, and considered the initial value problem:

$$\begin{cases} {}^{CF}D^{\alpha}x(t) = x(t)[1-x(t)], \quad t>0, \\ x(0)=x_0, \end{cases} \tag{3}$$

where $\alpha \in (0,1)$, ${}^{CF}D^{\alpha}$ denotes the Caputo-Fabrizio fractional derivative of order α. By utilizing the properties of Caputo-Fabrizio fractional calculus, the author derived that (3) possesses an analytical solution in the following implicit form

$$\frac{x(t)-x^2(t)}{(1-x(t))^{2/\alpha}} = \frac{x_0-x_0^2}{(1-x_0)^{2/\alpha}} \cdot e^t.$$

Motivated by reference [37], the present work employs the Caputo-Fabrizio fractional derivative to extend the model (2) to the fractional-order form and investigates the initial value problem for the following fractional logistic equation

$$\begin{cases} {}^{CF}D^{\alpha}x(t) = ax(t)-(\mu+\xi x(t))x(t), \quad t>0, \\ x(0)=x_0, \end{cases} \tag{4}$$

where $\alpha \in (0,1)$, $a,\mu,\xi \in \mathbf{R}^+$ and $a>\mu$, ${}^{CF}D^{\alpha}$ denotes the Caputo-Fabrizio fractional derivative of order α. By applying the relevant properties of Caputo-Fabrizio fractional calculus, we will derive the analytical solution of the fractional logistic model (4). The significance of exploring the analytical solution of the fractional logistic model (4) can be summarized as follows:

- Diversity in dynamic behaviors: The fractional logistic model (4) exhibits more complex and diverse dynamic behaviors compared to integer-order model (2). This provides new perspectives for understanding the population dynamics of male mosquitoes without Wolbachia infection.
- Flexibility in parameter control: The analytical solution offers explicit expressions that can be used to investigate the specific effects of the fractional order α, parameters a, μ and ξ on the system's behavior. This is crucial for optimizing the system model and fine-tuning its parameters.
- Universality of the Model: If the parameters a, μ, and ξ take specific values, such as $a\to 1$, $\mu=0$, $\xi=1$, model (4) degenerates into the fractional logistic model (3), thereby uncovering the hierarchical structure and universal applicability of the model.

The remainder of this paper is structured as follows: In Section 2, we recall the fundamental definitions and properties of Caputo-Fabrizio fractional calculus. In Section 3, we analysis of model (4), deriving its implicit analytical solution and demonstrating through a corollary that the result presented in [37] is a special case of this study. Additionally, numerical simulations are performed for various values of the fractional order α and the parameters a, μ, and ξ. Finally, Section 4 concludes with a concise summary of the principal findings of this study.

II. Preliminaries

In this section, we first recall some definitions and properties of the Caputo-Fabrizio fractional calculus.

Definition 1. ([38]) The Caputo-Fabrizio fractional integral of order $\alpha \in (0,1]$ of an integrable function $x:[0,+\infty)\to \mathbf{R}$ is defined by

$${}^{CF}I^{\alpha}x(t) = (1-\alpha)[x(t)-x(0)] + \alpha\int_0^t x(s)ds, \quad t>0.$$

Definition 2. ([20]) Let $T>0$, $\alpha \in (0,1)$. The Caputo-Fabrizio fractional derivative of order α for a function $x\in C^1(0,T)$ is defined by

$${}^{CF}D^{\alpha}x(t) = \frac{1}{1-\alpha}\int_0^t e^{-\frac{\alpha}{1-\alpha}(t-s)}x'(s)ds, \quad t\in(0,T).$$

Remark 1. ([20]) Let $\alpha\in[0,1)$ and $x\in C^1(0,T)$, then

$${}^{CF}D^{0}x(t) = x(t)-x(0), \qquad \lim_{\alpha\to 1^-} {}^{CF}D^{\alpha}x(t) = x'(t).$$

Lemma 1. ([39]) Let $\alpha\in[0,1)$ and $x\in C^1(0,T)$, then

$${}^{CF}D^{\alpha CF}I^{\alpha}x(t) = x(t)-x(0)e^{-\frac{\alpha}{1-\alpha}t},$$
$${}^{CF}I^{\alpha CF}D^{\alpha}x(t) = x(t)-x(0).$$

III. Main results

In this section, we provide the analytical solution to the initial value problem (4).

Theorem 1. Let $x(t)\in C^1(0,T)$ be the solution of the initial value problem (4), then there holds

(i) For $\alpha\in(0,1]$, if $x_0=0$, then $x(t)=0$.

(ii) For $\alpha \in (0,1]$, if $x_0 = (a-\mu)\xi^{-1}$, then $x(t) = (a-\mu)\xi^{-1}$.

(iii) For $\alpha \in (0,1)$, if $x(t) \neq 0, (a-\mu)\xi^{-1}$, then $x(t)$ can be expressed in the following implicit form

$$\frac{\xi^m [x(t)]^{m+1}[(a-\mu)-\xi x(t)]}{[(a-\mu)-\xi x(t)]^{m+(2/\alpha)}} = \frac{\xi^m x_0^{m+1}[(a-\mu)-\xi x_0]}{[(a-\mu)-\xi x_0]^{m+(2/\alpha)}} e^t,$$

where $m = [1-(a-\mu)][\alpha(a-\mu)]^{-1}$.

Proof. Integrating both sides of equation (4), we obtain

$$^{CF}I^{\alpha CF}D^{\alpha}x(t) = {}^{CF}I^{\alpha}(ax(t) - (\mu + \xi x(t))x(t)).$$

This, together with Lemma 1, implies

$$\begin{aligned} &x(t)-x(0) \\ &= (1-\alpha)[ax(t)-(\mu+\xi x(t))x(t) \\ &\quad - ax(0) + (\mu + \xi x(0))x(0)] \\ &\quad + \alpha \int_0^t [ax(s) - (\mu + \xi x(s))x(s)]ds. \end{aligned}$$

Differentiating both sides of the above equation with respect to the variable t, it follows that

$$\begin{aligned} x'(t) = &(1-\alpha)[ax'(t) - \xi x'(t)x(t) - (\mu + \xi x(t))x'(t)] \\ &+ \alpha[ax(t) - (\mu + \xi x(t))x(t)], \end{aligned}$$

that is,

$$\begin{aligned} &(1-(a-\mu))x'(t) + \alpha(a-\mu)x'(t) \\ &\quad +2\xi x(t)x'(t) - 2\alpha\xi x(t)x'(t) \\ &= \alpha x(t)[(a-\mu) - \xi x(t)]. \end{aligned} \tag{5}$$

Note that as $\alpha \to 1$, equation (5) degenerates into the logistic equation (2). It is easy to see that equation (5) has the trivial solutions $x(t) = 0$ and $x(t) = (a-\mu)\xi^{-1}$. Thus, (i) and (ii) hold. For $\alpha \in (0,1)$, if $x(t) \neq 0, (a-\mu)\xi^{-1}$, then equation (5) can be rewritten as

$$\begin{aligned} &\frac{[1-(a-\mu)]x'(t)}{\alpha x(t)[(a-\mu)-\xi x(t)]} + \frac{[(a-\mu)-2\xi x(t)]x'(t)}{x(t)[(a-\mu)-\xi x(t)]} \\ &+ \frac{2\xi x'(t)}{\alpha[(a-\mu)-\xi x(t)]} = 1, \end{aligned}$$

which then yields

$$\begin{aligned} &\frac{(1-(a-\mu))x'(t)}{\alpha x(t)[(a-\mu)-\xi x(t)]} + \frac{d}{dt}\ln|x(t)[(a-\mu)-\xi x(t)]| \\ &- \frac{2}{\alpha}\frac{d}{dt}\ln|(a-\mu)-\xi x(t)| = 1. \end{aligned}$$

By integrating both sides of the above equation with respect to the variable t, we obtain

$$\begin{aligned} &\frac{1-(a-\mu)}{\alpha}\int \frac{d(\xi x(t))}{\xi x(t)[(a-\mu)-\xi x(t)]} \\ &+ \ln\frac{|x(t)[(a-\mu)-\xi x(t)]|}{|(a-\mu)-\xi x(t)|^{2/\alpha}} = t + C_1, \quad C_1 \in \mathbf{R}. \end{aligned}$$

Further computation of the above expression gives

$$\begin{aligned} &\frac{1-(a-\mu)}{\alpha(a-\mu)}\ln\frac{|\xi x(t)|}{|(a-\mu)-\xi x(t)|} + \ln\frac{|x(t)[(a-\mu)-\xi x(t)]|}{|(a-\mu)-\xi x(t)|^{2/\alpha}} \\ &= t + C, \quad C \in \mathbf{R}. \end{aligned}$$

Therefore, we have

$$\left[\frac{\xi x(t)}{(a-\mu)-\xi x(t)}\right]^{\frac{1-(a-\mu)}{\alpha(a-\mu)}} \cdot \frac{x(t)[(a-\mu)-\xi x(t)]}{[(a-\mu)-\xi x]^{2/\alpha}} = e^t \cdot e^C,$$

that is,

$$\frac{\xi^{\frac{1-(a-\mu)}{\alpha(a-\mu)}}[x(t)]^{\frac{1-(1-\alpha)(a-\mu)}{\alpha(a-\mu)}}[(a-\mu)-\xi x(t)]}{[(a-\mu)-\xi x(t)]^{\frac{1+(a-\mu)}{\alpha(a-\mu)}}} = e^t \cdot e^C. \tag{6}$$

Using the initial condition $x(0) = x_0$, we can derive

$$e^C = \frac{\xi^{\frac{1-(a-\mu)}{\alpha(a-\mu)}} x_0^{\frac{1-(1-\alpha)(a-\mu)}{\alpha(a-\mu)}}[(a-\mu)-\xi x_0]}{[(a-\mu)-\xi x_0]^{\frac{1+(a-\mu)}{\alpha(a-\mu)}}}. \tag{7}$$

Substituting (7) into (6), we find that equation (4) has the following implicit solution

$$\begin{aligned} &\frac{\xi^{\frac{1-(a-\mu)}{\alpha(a-\mu)}}[x(t)]^{\frac{1-(1-\alpha)(a-\mu)}{\alpha(a-\mu)}}[(a-\mu)-\xi x(t)]}{[(a-\mu)-\xi x(t)]^{\frac{1+(a-\mu)}{\alpha(a-\mu)}}} \\ &= \frac{\xi^{\frac{1-(a-\mu)}{\alpha(a-\mu)}} x_0^{\frac{1-(1-\alpha)(a-\mu)}{\alpha(a-\mu)}}[(a-\mu)-\xi x_0]}{[(a-\mu)-\xi x_0]^{\frac{1+(a-\mu)}{\alpha(a-\mu)}}} e^t. \end{aligned} \tag{8}$$

This completes the proof.

Corollary 1. The initial value problem for the fractional logistic equation

$$\begin{cases} ^{CF}D^{\alpha}x(t) = x(t)[1-x(t)], \quad t > 0, \\ x(0) = x_0, \end{cases} \tag{9}$$

where $\alpha \in (0,1)$. The initial value problem (9) has the following implicit analytical solution

$$\frac{x(t)-x^2(t)}{(1-x(t))^{2/\alpha}} = \frac{x_0 - x_0^2}{(1-x_0)^{2/\alpha}} \cdot e^t. \tag{10}$$

Proof. Taking $a \to 1, \mu = 0, \xi = 1$, from equation (8), we directly obtain (10). Clearly, this result coincides with that presented in reference [37]. The proof is complete.

IV. Numerical simulation

Currently, under the initial condition $x_0 = 1$, we plot the solution curves of equation (4) for different fractional orders α and varying values of the parameters a, μ and ξ. Meanwhile, we compare the solutions of the integer-order logistic equation (2) (corresponding to α= 1) with those of the Caputo-Fabrizio fractional logistic equation (4), which include cases where α = 0.9, α = 0.7, α= 0.5, α= 0.1 (see Fig. 1, Fig. 2 and Fig. 3). The results indicate that the solution of the integer-order logistic equation is significantly greater than the solutions of the Caputo-Fabrizio fractional logistic equation.

V. Conclusion

In this work, the traditional integer-order logistic model was extended by incorporating the Caputo-Fabrizio fractional derivative, resulting in the construction of a generalized fractional logistic equation. Numerical simulations were performed, illustrating the significant impacts of the fractional order and model parameters on population dynamics, thereby providing new insights into mosquito

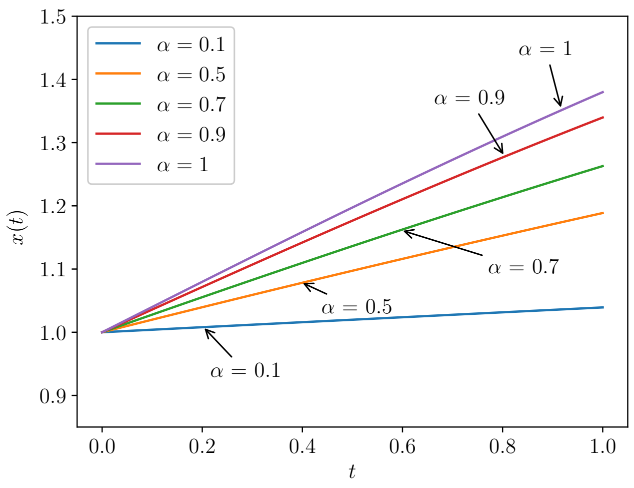

Fig. 1. Illustrates the solution curves of the logistic equation (4) under the parameters $a = 1.5$, $\mu = 0.5$ and $\xi = 0.5$.

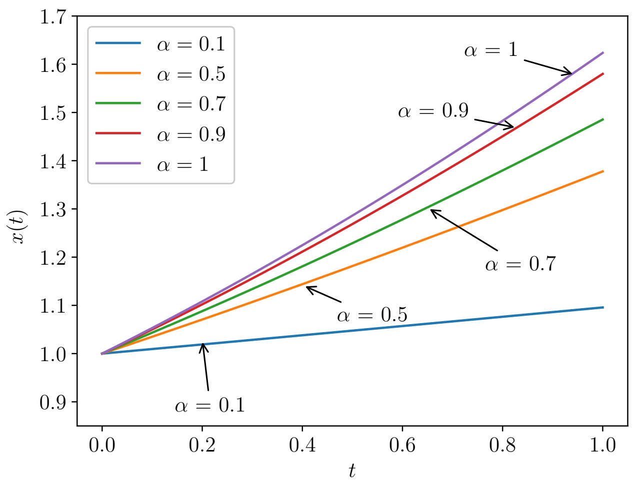

Fig. 2. Represents the solution curves of the equation (4) under the parameters $a = 1$, $\mu = 0.2$ and $\xi = 0.15$.

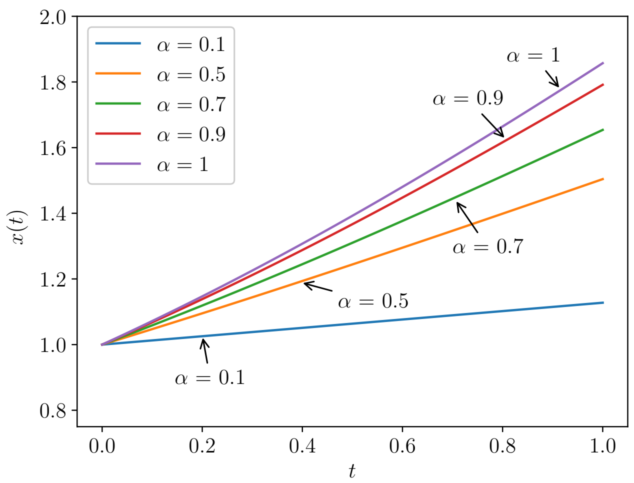

Fig. 3. Demonstrates the solution curves for the equation (4) under the parameters $a = 1$, $\mu = 0.1$ and $\xi = 0.2$.

population control strategies and ecological modeling. Future investigations will aim to extend model (4) to stochastic fractional logistic systems and undertake a comprehensive analysis of the existence and stability properties of their solutions.

REFERENCES

[1] J. P. Chauhan, R. K. Jana, J. J. Nieto, P. V. Shah, and A. K. Shukla, Fractional calculus approach to logistic equation and its application. Fixed point theory and fractional calculus-recent advances and applications, pp. 261–274. Forum Interdiscip. Math. Springer, Singapore, 2022.

[2] K. Cao, D. Ding, and Y. Fan, "Delicate modeling of network opinion and its application based on fractional calculus," *Journal of Intelligence* (Chinese Ed.) Vol. 41, no. 1, pp. 113–116, 2022.

[3] Z. Zhang, L. Chang, Q. Huang, R. Yan, and B. Zheng, "A mosquito population suppression model with a saturated Wolbachia release strategy in seasonal succession," *J. Math. Biol.*, Vol. 86, no.4, 51, 2023.

[4] X. Tian, X. Hu, J. Gu, C. Man, and S. Fei, "Finite-time control of a class of engineering system with input saturation via fractional-order backstepping strategy," *IAENG International Journal of Computer Science*, vol. 50, no.3, pp. 941–946, 2023.

[5] K. Kankhunthodl, V. Kongratana, A. Numsomran, and V. Tipsuwanporn, "Self-balancing Robot Control Using Fractional-Order PID Controller," *Lecture Notes in Engineering and Computer Science: Proceedings of The International MultiConference of Engineers and Computer Scientists 2019*, 13–15 March, 2019, Hong Kong, pp. 77–82.

[6] X. J. Yang, F. Gao, and Y. Ju, General fractional derivatives with applications in viscoelasticity. Elsevier/Academic Press, London, 2020.

[7] K. Chen and W. Zhang, "Study on a New Fractional Logistic Model via Deformable Fractional Derivative," *IAENG International Journal of Applied Mathematics*, vol. 54, no.8, pp. 1540–1544, 2024.

[8] M. Izadi and H. M. Srivastava, "Numerical approximations to the nonlinear fractional-order logistic population model with fractional-order Bessel and Legendre bases," *Chaos Solitons Fractals*, Vol. 145, 110779, 2021.

[9] M. Jornet and J. J. Nieto, "Power-series solution of the L-fractional logistic equation," *Appl. Math. Lett.*, Vol. 154, 109085, 2024.

[10] H. A. A. El-Saka, D. El. A. El-Sherbeny, and A. M. A. El-Sayed, "Dynamic analysis of the fractional-order logistic equation with two different delays," *Comput. Appl. Math.*, Vol. 43, no. 6, 369, 2024.

[11] I. Area and J. J. Nieto, "On the fractional Allee logistic equation in the Caputo sense," *Ex. Countex.*, Vol. 4, 100121, 2023.

[12] J. J. Nieto, "Fractional Euler numbers and generalized proportional fractional logistic differential equation," *Fract. Calc. Appl. Anal.*, Vol. 25, no. 3, pp. 876–886, 2022.

[13] S. Ahmed, S. Jahan, and K. S. Nisar, "Hybrid Fibonacci wavelet method to solve fractional-order logistic growth model," *Math. Methods Appl. Sci.*, Vol. 46, no. 15, pp. 16218–16231, 2023.

[14] A. H. Ganie, A. Khan, G. Alhamzi, A. M. Saeed, and M. B. Jeelani, "A new solution of the nonlinear fractional logistic differential equations utilizing effcient techniques," *AIP Advances*, Vol. 14, 035134, 2024.

[15] L. N. Kaharuddin, C. Phang, and S. S. Jamaian, "Solution to the fractional logistic equation by modified Eulerian numbers," *Eur. Phys. J. Plus*, Vol. 135, 229, 2020.

[16] B. Shiri, Z. Alijani, and Y. Karaca, "A power series method for the fuzzy fractional logistic differential equation," *Fractals*, Vol. 31, no. 10, 2340086, 2023.

[17] R. Abreu-Blaya, A. Fleitas, J. E. Nápoles Valdés, R. Reyes, J. M. Rodríguez, and J. M. Sigarreta, "On the conformable fractional logistic models," *Math. Methods Appl. Sci.*, Vol. 43, no. 7, pp. 4156–4167, 2020.

[18] T. Abdeljawad, Q. M. Al-Mdallal, and F. Jarad, "Fractional logistic models in the frame of fractional operators generated by conformable derivatives," *Chaos Solitons Fractals*, Vol. 119, pp. 94–101, 2019.

[19] T. Abdeljawad, M. A. Hajji, Q. M. Al-Mdallal, and F. Jarad, "Analysis of some generalized ABC-fractional logistic models," *Alex. Eng. J.*, Vol. 59, pp. 2141–2148, 2020.

[20] M. Caputo and M. Fabrizio, "A new definition of fractional derivative without singular kernel," *Prog. Fract. Differ. Appl.*, Vol. 1, pp. 73–78, 2015.

[21] J. Dianavinnarasi, R. Raja, J. Alzabut, J. Cao, M. Niezabitowski, and O. Bagdasar, "Application of Caputo-Fabrizio operator to suppress the Aedes Aegypti mosquitoes via Wolbachia: an LMI approach,", *Math. Comput. Simulation*, Vol. 201, pp. 462–485, 2022.

[22] P. Verma and M. Kumar, "Analysis of a novel coronavirus (2019-nCOV) system with variable Caputo-Fabrizio fractional order," *Chaos Solitons Fractals*, Vol. 142, 110451, 2021.

[23] A. S. Shaikh and K. Sooppy Nisar, "Transmission dynamics of fractional order typhoid fever model using Caputo-Fabrizio operator," *Chaos Solitons Fractals*, Vol. 128, pp. 355–365, 2019.
[24] S. A. Khan, K. Shah, G. Zaman, and F. Jarad, "Existence theory and numerical solutions to smoking model under Caputo-Fabrizio fractional derivative," *Chaos*, Vol. 29, 013128, 2019.
[25] F. Ali, F. Ali, N. A. Sheikh, I. Khan, and K. S. Nisar, "Caputo-Fabrizio fractional derivatives modeling of transient MHD Brinkman nanoliquid: applications in food technology," *Chaos Solitons Fractals*, Vol. 131, 109489, 2020.
[26] D. Lin, X. Liao, L. Dong, R. Yang, S. S. Yu, H. H. C. Iu, F. Tyrone, and Z. Li, "Experimental study of fractional-order RC Circuit model using the Caputo and Caputo-Fabrizio derivatives," *IEEE Trans. Circuits Syst. I. Regul. Pap.*, Vol. 68, no. 3, pp. 1034–1044, 2021.
[27] D. Baleanu, A. Jajarmi, H. Mohammadi, and S. Rezapour, "A new study on the mathematical modelling of human liver with Caputo-Fabrizio fractional derivative," *Chaos Solitons Fractals*, Vol. 134, 109705, 2020.
[28] S. Doley, A. V. Kumar, K. R. Singh, and L. Jino, "Study of Time Fractional Burgers' Equation using Caputo, Caputo-Fabrizio and Atangana-Baleanu Fractional Derivatives," *Engineering Letters*, vol. 30, no.3, pp. 1017–1024, 2022.
[29] A. Sajjad, M. Farman, A. Hasan, and K. S. Nisar, "Transmission dynamics of fractional order yellow virus in red chili plants with the Caputo-Fabrizio operator," *Math. Comput. Simulation*, Vol. 207, pp. 347–368, 2023.
[30] Y. Sekerci, "Climate change effects on fractional order prey-predator model," *Chaos Solitons Fractals*, Vol. 134, 109690, 2020.
[31] R. P. Chauhan, S. Kumar, B. S. T. Alkahtani, and S. S. Alzaid, "A study on fractional order financial model by using Caputo-Fabrizio derivative," *Results Phys.*, Vol. 57, 107335, 2024.
[32] A. Al-khedhairi, "Dynamical analysis and chaos synchronization of a fractionalorder novel financial model based on Caputo-Fabrizio derivative," *Eur. Phys. J. Plus*, Vol. 134, 532, 2019.
[33] A. Khalouta, "A new decomposition transform method for solving nonlinear fractional logistic differential equation," *J. Supercomput.* Vol. 80, pp. 8179–8201, 2024.
[34] D. Kumar, J. Singh, M. Al-Qurashi, and D. Baleanu, "Analysis of logistic equation pertaining to a new fractional derivative with non-singular kernel," *Adv. Mech. Eng.*, Vol. 9, no. 2, pp. 1–8, 2017.
[35] S. A. Fahel, D. Baleanu, Q. M. Al-Mdallal, and K. M. Saad, "Quadratic and cubic logistic models involving Caputo-Fabrizio operator," *Eur. Phys. J. Spec. Top.*, Vol. 232, pp. 2351–2355, 2023.
[36] I. EL-Fassi, J. J. Nieto, and M. Onitsuka, "A new representation for the solution of the Richards-type fractional differential equation," *Math. Methods Appl. Sci.*, Vol. 48, no. 2, pp. 1519–1529, 2025.
[37] J. J. Nieto, "Solution of a fractional logistic ordinary differential equation," *Appl. Math. Lett.*, Vol. 123, 107568, 2022.
[38] J. Losada and J. J. Nieto, "Properties of a new fractional derivative without singular kernel," *Progr. Fract. Differ. Appl.*, Vol. 1, no. 2, pp. 87–92, 2015.
[39] J. Losada and J. J. Nieto, "Fractional integral associated to fractional derivatives with nonsingular kernels," *Progr. Fract. Differ. Appl.*, Vol. 7, no. 3, pp. 137–143, 2021.