Co-optimization of Metro Train Timetable and Freight Fitting Scheme under Trailer Mode

Jianghong Feng, Cunjie Dai, Yao Yu, Haijun Li, Junjie Li, Bohan Ji

Abstract—Utilizing the redundant capacity of metro lines during non-peak passenger hours for freight transportation is an effective means to reduce urban logistics pressure and alleviate road congestion. This paper studies the problem of jointly optimizing train timetable and freight loading plans under the scenario of dynamically attaching freight carriages to passenger trains during non-peak hours. Based on a dual model of shared passenger and freight trains and shared tracks, the study considers constraints such as train stops, safe operation, and carriage capacity. With the goal of minimizing the costs of adding freight carriages, penalties for freight detention, and total dwell time, a mixed-integer programming model for the collaborative optimization of train timetable and freight loading plans is constructed. Using the Beijing metro Batong Line as an example, a case study is constructed, and the Gurobi solver is used for solving. Based on the calculation results, the impact of different passenger transport scenarios, the cost of adding freight carriages, the simultaneous loading and unloading queue for a single freight carriage, and the freight cost coefficient on the number of added freight carriages, train timetables, and freight loading plans is analysed. Research results indicate that the dynamic towing mode designed in this paper can effectively address freight transportation tasks under different passenger transport scenarios and can provide technical support for optimising train operation plans and freight loading decisions in the metro operation department.

Index Terms—Metro Logistics, Passenger and Freight Coordination, Train Timetable, Train Formation, Freight Fitting Scheme.

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I. INTRODUCTION

hina's economic expansion and the annual rise in social logistics volume have resulted in a total social logistics increase from 219.2 trillion CNY to 360.6 trillion CNY, reflecting a growth of 64.51% from 2015 to 2024 [1]. The flourishing logistics industry presents both opportunities and challenges for the transportation sector. Dampier et al. [2] and Qian [3] advocated the construction and development of an Underground Logistics System (ULS) to optimize urban subterranean area and mitigate transportation congestion associated with urban freight. Nonetheless, the intricate geological characteristics of the subterranean environment significantly elevate the development costs and building challenges of the ULS, rendering its execution quite arduous. A proposed solution to this issue is the implementation of an underground logistics system (Metro-based ULS, M-ULS) that utilizes existing metropolitan metro lines for freight transit. Intended to utilize the underused capacity of metro passenger services during off-peak hours for freight transport, capitalizing on the system's timeliness and energy efficiency.

Various researchers have undertaken study and analysis from diverse angles to assess the feasibility of implementing M-ULS. Hu et al. [4] indicated that China currently has the capacity to execute it regarding policy, market demand, and technology advancement. Liu [5] and Chen [6] examined the viability of establishing M-ULS in Beijing and Nanjing from many perspectives, including policy formulation and market positioning. Kikuta et al. [7] examined the influence of weather on ground transportation in Sapporo, Japan, during winter, and executed a public opinion survey alongside a trial project on metro freight transit. Motraghi et al. [8] assessed freight trains and specialized operational modes inside the current metro rail system of the Newcastle-Tyne region utilizing ARENA software. Behiri et al. [9] investigated the scheduling problem of freight rail transport with respect to resource sharing (vehicles, routes, stops) and performed simulated experiments. Wang et al. [10] performed a prediction analysis of the logistical capacity of metro lines. Comprehensive research integrating qualitative analysis of official policies, market conditions, and public sentiment, alongside quantitative analysis via simulation and forecasting, establishes a robust theoretical foundation for the advancement of M-ULS.

To fully use the surplus capacity of trains and attain effective coordinated movement of passengers and freight, numerous academics have undertaken comprehensive study on the optimization of M-ULS infrastructure and layout. Zhou et al. [11] indicated that the covering area of transfer points directly influences the ideal quantity of transfer

points. Shu et al. [12] developed a bilateral matching model for the positioning of metro transfer stations, enhancing the satisfaction of the alignment between logistics nodes and demand points. Wang et al. [13] examined the collinear separation method, employing several optimization models to individually ascertain freight zones, logistics supply and demand, and appropriate metro station sites. Zheng et al. [14] employed Voronoi diagrams to address an enhanced P-median model, indicating that metro stations in proximity to residential zones and business hubs are more likely to be designated as distribution stations. Wang et al. [15] developed a metro carriage suitable for both people and freight, with the objective of alleviating the burden of freight movement during off-peak hours on the metro. Kelly developed three freight arrangement et al. [16] configurations for the interiors of metro freight carriages. Chen et al. [17] independently devised the spatial configuration of the platform for both mixed and segregated metro passenger-freight transportation lines. Hu et al. [18] systematically organized the functional zones of the freight platform to maximize the collaborative capabilities of each region. By employing judicious site selection and optimizing freight space design within train carriages, it is feasible to prioritize passenger transport while facilitating speedy freight transport organization and enhancing the efficiency of train transport operations.

Di et al. [19] investigated the optimization problem of synchronized passenger and freight transportation on metro lines, focusing on rigid time constraints for passenger transport and flexible time constraints for freight transport, to promptly address freight transportation demands and attain coordinated optimization of passengers, freight, and trains. The findings indicate that this model can significantly diminish system delay penalties and enhance the efficiency of passenger and freight transport on metro lines. Hou et al. [20][21][22] studied passenger and freight collaborative transportation based on virtual formation, virtual formation + carbon emission policies, and virtual formation + urban rail transit large and small routes, which not only better respond to passenger flow changes but also improve carriage utilisation and reduce various costs. Ye et al. [23] developed a metro freight model that incorporates restrictions including train arrival and departure schedules, freight capacity, and headway, resulting in an optimal operational timetable and freight loading strategy given the specified parameters. Di et al. [24] executed a synchronized optimization of train schedules and capacity distribution, maintaining constant passenger and freight flow arrival rates during off-peak hours. Cacchiani et al. [25] increased the number of freight trains under both fixed and non-fixed timetables, which aids in minimizing train arrival and departure delays during operations. Yao et al. [26] investigated the train scheduling dilemma, considering both passenger and freight demand, and offered technical assistance for practical train scheduling through the co-optimization of train timetables and rolling stock rotation strategies. Ozturk et al. [27] proposed adding dedicated freight trains between passenger trains to achieve more efficient freight transportation. Ye et al. [28] proposed distinct metro and freight train transportation modes to accommodate diverse passenger and freight transport scenarios under non-peak, variable passenger flow situations. Huber et al. [29] examined the collaborative passenger-freight transportation mode utilizing shared infrastructure and train models, highlighting that service time for freight transport is a critical aspect influencing this mode.

The optimization of transportation organization ensures alignment between capacity and operational demand. Nevertheless, current research has not accounted for the actual loading and unloading durations of freight at stations in the timetable optimization process. Furthermore, there has been an increase in study about the mixed passenger and freight transit of fixed formation trains and the operation of specialized freight trains, although research on the towing mode has been rather limited. This study proposes a freight transportation model utilizing the dynamic trailer mode to fulfill the varied freight transportation requirements throughout the city, mitigate the challenges of mixed passenger and freight loads, and alleviate the operational complexities associated with dedicated freight trains. This paper's contributions, in comparison to existing studies, are as follows:

- (i) The synchronized optimization of metro train schedules and freight loading strategies within the towing mode complements existing theories of metro logistics services, so augmenting the variety of transportation modes and the adaptability of freight loading in metro logistics.
- (ii) Leveraging freight transportation demand to determine the requisite number of freight carriages for trains helps mitigate delays and optimize transportation capacity.
- (iii) Utilizing the total number of freight carriages to establish the simultaneous loading and unloading queue of the train as the research focal point, and integrating it with the loading and unloading requirements of the freight stations to ascertain the actual dwell time at each station, thereby achieving a coordinated optimization of freight allocation and scheduling.

TABLE I displays the distinctions between this paper's research and the related literature.

II. PROBLEM DESCRIPTION

To enhance urban cargo transportation services, this paper focuses on a single subway line during off-peak hours. It aims to optimize the scheduling and cargo allocation schemes within a trailer mode while ensuring passenger transportation needs are met. This is achieved by adjusting the arrival and departure intervals, stopping times at stations, and incorporating additional freight cars.

Fig. 1 illustrates the train operations and freight loading arrangements. To facilitate more flexible freight loading while ensuring safe train operations, Trains 2 and 3 have been adjusted to the dashed line position. This adjustment changes the arrival and departure intervals of the three trains at the station. Manifest 1 was loaded onto the adjusted Train 3 at Station 3, resulting in an increased dwell time for Train 3 compared to its original schedule. Manifest 2 was loaded onto Train 2, which remained at Station 2 for the same amount of time as before; Train 2 only needed to maintain its original dwell time. It is evident that although the arrival

and departure intervals and dwell times at the station for the three trains have been modified, these changes do not affect the operation of subsequent trains. Fig. 2 is a schematic diagram illustrating the metro freight co-line towing mode. It is important to determine whether additional freight carriages should be added to the original fixed train formation. First, the passenger carriages are arranged based on the original fixed formation to satisfy passenger transportation needs. Afterward, a decision is made regarding the addition of freight carriages based on actual freight demand. If the available carriages are sufficient to accommodate the freight, only the remaining carriages need to be modified. If not, additional freight carriages must be added. This dynamic addition (or towing) method effectively utilizes the surplus capacity of metro passenger services during non-peak periods.

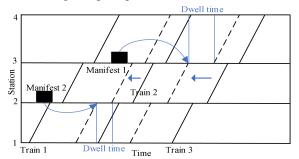


Fig. 1. Schematic diagram of train operation and freight distribution

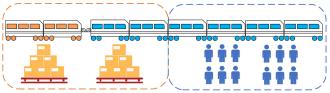


Fig. 2. Schematic diagram of metro freight co-location trailer

III. MODEL FORMULATION

A. Model assumptions

In light of the complexities involved in coordinating passenger and freight transport within urban metro systems, this paper outlines the following assumptions:

- (i) The train operation plan is established and known.
- (ii) The number of passenger carriages assigned to each train is predetermined, with a minimum of two-thirds of the total carriages allocated for passengers.
- (iii) Passengers and freight must be assigned to separate carriages.
- (iv) Freight is loaded, unloaded, and delivered in standardized freight boxes.
- (v) The train has completed the shunting operation at the starting station.

B. Model Parameters

The sets, indices, parameters and variables involved in the modelling process are shown in TABLE II.

B. 1 Constraints

(i) Train formation constraints

This paper addresses the need to increase the number of train formations in order to meet the growing demand for freight transportation. Considering the actual operational characteristics of the metro system, it is essential to establish limits on both the maximum number of train formations and the maximum number of spare carriages.

TABLE I LITERATURE REVIEW

Literature	Research Target	Model	Optimization Objective	Solution Method
Di et al [19]	Grouping, flow control	MILP	Minimum total cost	I-BDA
Hou et al [20]	Virtual formation	MILP	Minimum waiting time and maximum load rate	NSGA-II
Hou et al [21]	Virtual formation, carbon emission	MINLP	Minimum waiting time and carbon emission	NSGA-II
Hou et al [22]	Virtual formation, large and small routes	MINLP	Minimum operation cost, passenger travel cost	NSGA-II
Ye et al [23]	Timetable, freight fitting	MILP	Minimum freight cost	I-VNSA
Di et al [24]	Timetable, capacity allocation	INLP	Minimum total cost	GP-VNSA
Cacchiani et al [25]	Timetable, freight train	ILP	Maximum number of additional freight trains	LR
Yao et al [26]	Timetable, rolling stock turnover	MILP	Minimum waiting time and operation cost	NSGA-II+GUROBI
Ozturk et al [27]	Freight train	MILP	The shortest total delay time	DSA+PPDPA
Ye et al [28]	Fixed/Flexible loading solutions	ILP	Minimum total cost	I-VNSA+CPLEX
Lena et al [29]	Shared facilities, shared trains	MILP	Minimum waiting time and delay penalties	GUROBI
This paper	Timetable, trailer mode, freight fitting	MILP	Minimum grouping, dwell time, penalty costs	GUROBI

^{**}Note: ILP: Integer Linear Programming; MILP: Mixed Integer Linear Programming; MINLP: Mixed Integer Non-Linear Programming; INLP: Integer Nonlinear Programming; I-BDA: Improved Benders Decomposition Algorithm; NSGA-II: Non-dominated Sorting Genetic Algorithm; I-VNSA: Improved Variable Neighborhood Search Algorithm; GP-VNS: Gradient Projection Based Variable Neighborhood Search Algorithm; LR: Lagrange Relaxation Algorithm; DSA: Demand Splitting Algorithm; PPDPA: Pseudo-Polynomial Dynamic Programming Algorithm.

TABLE II SYMBOL DESCRIPTION

Symbol	Description
Set	
M	The set of trains, $M = \{1, 2,, M \}, m \in M$
F	The set of manifest, $F = \{1, 2,, F \}, f \in F$
S	The set of stations, $S = \{1, 2,, S \}, o_f, d_f, s \in S$, o_f, d_f indicates the origin and destination of the manifest f , respectively.
T	The set of discrete-time, $t \in T$
K	The set of available freight carriages for train, $K = \{0,1,2, K \}, k \in K$
Parameters	
C_m	Fixed carriage formation
C_m^{max}	Maximum train formation
C^{max}	Maximum number of spare freight carriage
C^p_m	Number of passenger carriages available on train m
C_m^f	Number of freight carriage available on train m
Q_f	Number of freight boxes that can be accommodated in a single freight carriage
q^f	Number of freight boxes in manifest f (unit:box)
v	Simultaneous loading and unloading queues for single freight carriage
ho	Average loading and unloading time per freight box(unit: s)
$[t_f^e,t_f^l]$	Departure time window of the manifest f
tr_m^s	Running time of the train from station s to station $s + 1$ (unit: s)
$t_{m,s}^{min} / t_{m,s}^{max}$	Minimum/maximum dwelling time of the train m at station s (unit: s)
$[h_{\min}^d,h_{\max}^d]$	Train safety running interval (unit:s)
δ	Unit dwell time penalty charge (unit: CNY/s)
σ	Cost of assembling a single freight carriage (unit: CNY/carriage)
φ	Penalty cost for unserved unit freight box (unit: CNY/box)
M	A very large number
Decision variable	
t_m^s	Continuous variables, dwelling time of the train m at station s
ta_m^s	Continuous variables, arrival time of the train m at station s
td_m^s	Continuous variables, departure time of the train m at station s
C_m^{add}	Integer variables, number of freight carriage additions
x_m^f	0-1 variables, if manifest f is assigned to the train m , $x_m^f = 1$ or $x_m^f = 0$.
\mathcal{Y}_m^k	0-1 variables, if train m chooses to use k freight carriages, $y_m^k = 1$, or $y_m^k = 0$.

$$C_m + C_m^{add} \le C_m^{\max}, \forall m \in M$$
 (1)

$$\sum_{m \in M} C_m^{add} \le C^{\max} \tag{2}$$

Eq. (1) and Eq. (2) represent the maximum limit for train formations and the maximum number of available carriages.

(ii) Train capacity constraint

All trains must adhere to the following capacity constraints:

$$C_m^f = C_m + C_m^{add} - C_m^p, \forall m \in M$$
 (3)

$$\sum_{f \in F: o_f \le s < d_f} x_m^f q^f \le Q^f C_m^f, \forall m \in M, s \in S \setminus \{ |S| \}$$
 (4)

Eq. (3) shows that the actual number of freight carriages in use is determined by subtracting the number of carriages

assigned to passengers from the total number of train carriages. Eq. (4) states that the amount of freight loaded onto the train when departing from a non-terminal station must not exceed the train's capacity limit.

(iii) Freight loading constraints

Due to time constraints on freight departures and the indivisibility of the manifest, the limitations related to freight loading can be defined as follows:

$$td_m^{o_f} \ge t_f^e - M\left(1 - x_m^f\right), \forall m \in M, f \in F$$
 (5)

$$td_{m}^{o_{f}} \leq t_{f}^{l} + M\left(1 - x_{m}^{f}\right), \forall m \in M, f \in F$$
 (6)

$$\sum_{m \in M} x_m^f \le 1, \forall f \in F \tag{7}$$

Eqs. (5) and (6) outline the time limits for transporting manifests f; manifests must be transported promptly upon arrival at the station to avoid incurring penalty costs. To

better illustrate the connection between trains and manifests, parameters t_f^e and t_f^l for the earliest transportation time and the latest departure time of the manifests are introduced. Eq. (7) establishes the uniqueness constraint for manifest allocation, indicating that each manifest cannot be divided and must be assigned to a single train.

(iv) Dwell time constraint

To accommodate the boarding and alighting needs of passengers, trains must meet the minimum time requirements for passengers to safely get on and off at each station. Additionally, to better align with actual operating conditions, the dwell time at each station is adjusted based on the number of freight boxes being loaded and unloaded by the train.

$$t_m^s \ge t_{m,s}^{min}, \forall m \in M, s \in S$$
 (8)

$$t_{m}^{s} \geq \rho \cdot \frac{\sum_{f \in F: o_{f} = s} \left| d_{f} = s \right|}{v \cdot C_{m}^{f}}, \forall m \in M, s \in S$$
 (9)

$$t_m^s \le t_{m,s}^{\text{max}}, \forall m \in M, s \in S \tag{10}$$

Eq. (8) states that the dwell time must not be shorter than the dwell time specified in the original fixed timetable. Eq. (9) introduces a parameter related to the simultaneous loading and unloading queue for a single freight carriage. The queue for both loading and unloading of the train depends on the number of available freight carriages, and the train's dwell time must accommodate the actual time required for loading and unloading freight. To prevent excessively long dwell times due to extended loading and unloading requests, which could disrupt passenger travel, Eq. (10) sets a limit on the maximum allowable dwell time for the train.

(v) Train operation constraints

To determine the arrival and departure times, as well as the dwell durations of each train at each station, the following train operation constraints have been introduced:

$$ta_m^{s+1} = td_m^s + tr_m^s, \forall m \in M, s \in \mathcal{S} \setminus \{ |\mathcal{S}| \}$$
 (11)

$$t_m^s = td_m^s - ta_m^s, \forall m \in M, s \in S \setminus \{|S|\}$$
 (12)

Eq. (11) shows that a train's arrival time at the station is calculated by adding the departure time from the previous station to the interval running time. Meanwhile, Eq. (12) states that the train's dwell time at the station is determined by subtracting the arrival time from the departure time at the station.

(vi) Train safety interval constraint

Safety in train operations is a crucial aspect of metro processes. To ensure the new timetable is feasible, it is essential to determine the safety interval constraints for train operations.

$$h_{min}^d \le ta_m^s - td_{m-1}^s \le h_{max}^d, \forall m \in M, s \in S \setminus \{|S|\}$$
 (13)

The fixed running times of the metro train sections, as indicated in Eq. (13), show that the arrival and departure times of trains at the same station must comply with safety

time interval requirements. This is essential to ensure operational safety.

(vii) Variable constraints:

$$td_m^s, ta_m^s \in R, \forall m \in M, s \in S$$
 (14)

$$C_m^{add} \in N, \forall m \in M \tag{15}$$

$$x_m^f \in \{0,1\}, \forall m \in M, f \in F$$
 (16)

Eqs. (14)-(16) constrain the values of the decision variables. Among these, the arrival and departure times of the trains are continuous variables; the marshalling variables are non-negative integers; and the decision factor variables are binary.

B.2 Objective function

To maximize the number of freight transport tasks completed, this paper establishes the optimization objective as the weighted sum of three costs: the increased cost of freight carriages, the penalty cost for unsatisfied freight boxes, and the total dwell time cost.

$$C_1 = \sum_{m \in M} \sigma \cdot C_m^{add} \tag{17}$$

$$C_2 = \sum_{f \in F} \varphi \cdot q^f \left(1 - \sum_{m \in M} x_m^f \right) \tag{18}$$

$$C_3 = \sum_{m \in M} \sum_{s \in S} \delta t_m^s \tag{19}$$

$$\min \alpha \cdot (C_1 + C_2) + \beta \cdot C_3 \tag{20}$$

Among these, the weight coefficient are α, β , and C_1 , C_2 , and C_3 represent the costs of adding freight carriages, the penalty costs for unserved freight boxes, and the costs associated with total dwell time. Eqs. (18–20) correspond to these three costs.

In summary, this paper develops a collaborative optimization model for coordinating metro train timetables and freight loading plans under the towing mode:

$$\begin{cases}
\min \alpha \cdot (C_1 + C_2) + \beta \cdot C_3 \\
\text{s.t.}(1) \sim (16)
\end{cases}$$
(21)

Since C_m^f , t_m^s , and x_m^f in Eq. (9) are variables, the model constructed above is nonlinear. To facilitate the solution of the model, Eq. (9) needs to be linearised.

B.3 Model linearisation

To streamline the model's processing, the quantity of accessible freight carriages C_m^f for the train m in Eq. (9) is managed by inserting a decision variable y_m^k , so reformulating it as follows:

$$C_m^f = \sum_{k \in K} k \cdot y_m^k, \forall m \in M$$
 (22)

$$\sum_{k \in K} y_m^k = 1, \forall m \in M$$
 (23)

Eq. (22) denotes the quantity of freight carriages utilized by the selected train m from the set K, Eq. (23) stipulates that the train must select a certain number of freight

carriages from the set K. At this juncture, Equation (9) assumes the subsequent form:

$$v \cdot t_m^s \cdot \sum_{k \in K} k \cdot y_m^k, \ge \rho \cdot \sum_{f \in F : |\mathcal{D}_f = s|} |d_f = s| x_m^f \cdot q^f$$

$$, \forall m \in M, s \in S$$

$$(24)$$

Given that the dwell time t_m^s is a continuous variable and y_m^k is a binary variable, this product exhibits nonlinearity. Consequently, auxiliary variables $z_{m,s}^k$ and Ct_m^s are introduced to delineate:

$$z_{m,s}^{k} = y_{m}^{k} \cdot t_{m}^{s}, \forall m \in M, s \in S, k \in K$$
 (25)

$$Ct_m^s = \sum_{k \in K} z_{m,s}^k \cdot k, \forall m \in M, s \in S$$
 (26)

The linearization process of z_{m}^{k} is as follows:

$$z_{m,s}^{k} \le t_{m,s}^{\max} \cdot y_{m}^{k}, \forall m \in M, s \in S, k \in K$$
 (27)

$$z_{m,s}^{k} \le t_{m}^{s}, \forall m \in M, s \in S, k \in K$$
 (28)

$$z_{m s}^{k} \ge t_{m}^{s} - t_{m s}^{max} \cdot (1 - y_{m}^{k}), \forall m \in M, s \in S, k \in K$$
 (29)

Thus Eq. (24) can be transformed into the following form:

$$v \cdot Ct_m^s \ge \sum_{f \in \mathcal{F}} \rho \cdot x_m^f \cdot q^f, \forall m \in M, s \in S$$
 (30)

$$y_m^k \in \{0,1\}, \forall m \in M, k \in K$$
 (31)

The collaborative optimization model for the metro train timetable and freight loading plan under the tow mode presented in this research can be articulated as follows:

$$\begin{cases} \min \alpha \cdot (C_1 + C_2) + \beta \cdot C_3 \\ \text{s.t.}(1) \sim (8), (10) \sim (16), \\ (22) \sim (23), (25) \sim (31) \end{cases}$$
(32)

IV. CASE STUDY

All tests in this work were conducted on a personal computer equipped with an i5-8250U CPU and 8GB of RAM, programmed in Python 3.11 and solved using Gurobi optimization solver version 11.0.2.

A. Parameters Setting

This study focuses on the descending segment of Beijing Metro Line Batong (Tuqiao—Sihui), consecutively numbering the 13 stations along this route and presenting the line diagram seen in Fig. 3. The train operation chart is shown in Fig. 5 (a). This study particularly designates the non-peak period of metro passenger travel (9:33-10:56) as the focus of investigation. Given that metro operations prioritize passenger travel over freight transport, it is essential to aggregate the passenger flow card data from the Automatic Fare Collection (AFC) system on designated weekdays to ascertain the number of carriages designated for passenger transit. The remaining available carriages of fixed formation trains may thereafter be utilized for freight loading distribution. For comprehensive research, please

consult reference [30]. This research assigns random integers to denote the number of carriages filled by passengers during non-peak hours, especially 5, 4, 4, 5, 5, 6, 5, 4, and 5, acknowledging the daily variability of card-swipe data in public transport. Subsequently, according to the freight transportation requirements at each station, the surplus carriages and supplementary freight carriages are allocated for freight transport. The remaining carriages and supplementary freight carriages are employed for freight transportation. The freight specifications required for the numerical trials are presented in TABLE III. To facilitate calculation and analysis, the parameter values in the model are established as follows:

$$\begin{split} &C_m = 6 \;, C_m^{max} = 8 \;, C^{max} = 18 \;, Q_f = 20 \;, v = 2 \;, \rho = 12s \; [31] \\ &, \; h_{\min}^d = 180s, h_{\max}^d = 480s \;\;, \; t_{m,s}^{\max} = 120s \;\; \sigma = 200 \;\;, \; \varphi = 50 \;\;, \\ &\delta = 1.5 \;/\; s \;, \alpha = 0.9, \beta = 0.1 \;. \end{split}$$

B. Numerical Results

Utilizing the aforementioned data and parameter configurations, with the train timetable information, the computation yielded a solution with a discrepancy of 3% between the upper and lower bounds, resulting in an objective function value of 2,211.84 CNY. The total number of carriages designated for freight transport is 20, accommodating 30 manifests (606 freight boxes). The comprehensive formation scheme and freight loading scheme are presented in TABLE IV, the train's in-transit load is illustrated in Fig. 4(a), and the train timetable is depicted in Fig. 5(b). Trains 2, 3, 8, and 9 have incorporated 2 freight carriages into their formations; train 6 has incorporated 1 freight carriage, but trains 4, 5, and 7 have not incorporated any freight carriages. This is the outcome of the synergistic influence of freight departure time intervals and train operational times. By synchronizing departure time windows and consolidating freight lists with continuous transport segments on the same train, the coordinated optimization of train scheduling and freight scheme is attained.

Comparing Fig. 5(a) and Fig. 5(b), it is evident that to accommodate the transportation requirements of various manifests, the total dwell time of the modified train has increased by 988 seconds relative to the original total dwell time, while the arrival time of the final train has been expedited by 8 minutes. The transportation model presented in this paper effectively synchronizes train departure times within the designated timeframe, ensuring safe operations while optimizing departure intervals. It allows adequate time for freight loading and unloading without disrupting the arrival and departure of subsequent trains.

C. Sensitivity Analysis

(i) Cost of adding freight carriages σ sensitivity analysis. This study examines the influence of varying additional freight carriage costs for freight transport on the solution outcomes. Drawing from the literature [34], which cites an additional cost of 500 CNY, this paper establishes supplementary costs at 200 CNY, 300 CNY, 400 CNY, and 500 CNY, respectively, and analyzes the correlation

between these supplementary costs and freight loading. The results of the solution are presented in TABLE V, the interval loading capacity is illustrated in Fig. 4, and the train operating charts are depicted in Fig. 5 (b), (c), and (d).

TABLE V illustrates that the objective function's value progressively rises with an increase in σ , from 2,211.84 CNY to 4,438.64 CNY, reflecting a 100.68% increase. When the additional freight carriage costs values σ are 400 CNY and 500 CNY, the train's loading plan, overall dwell time, and arrival time remain unchanged. The total number of additional freight carriages is decreased by one in comparison to σ at 200 CNY and 300 CNY, this results from the minimal freight amount of the 15 manifests and the freight detention penalty price being merely 300 CNY, which is less than the expense of extra freight carriages.

When the additional freight carriage cost σ is established at 300 CNY, the penalty cost resulting from non-transportation of manifest 15 equals the supplementary cost. Nevertheless, the introduction of more freight carriages can augment the concurrent loading and unloading queue, consequently diminishing the overall stay time and associated costs; thus, the quantity of additional freight carriages will remain unchanged.

In summary, fluctuations in the expenses associated with augmenting freight carriage σ influence the train formation strategy and the freight loading plan. Simultaneously, as the transportation of goods is intricately linked to the penalty $\cos t \varphi$, for undelivered freight, the penalty $\cos t$ for goods delay in practical operations must also be evaluated alongside the expense of augmenting freight carriages σ .

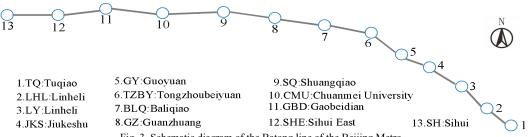


Fig. 3. Schematic diagram of the Batong line of the Beijing Metro

TABLE III FREIGHT TRANSPORT DEMAND

No.	Originati on Station	Destinati on Station	Demand (box)	Earliest Departur e Time	Latest Departur e Time	No.	Originati on Station	Destinati on Station	Demand (box)	Earliest Departur e Time	Latest Departure Time
1	1	5	30	9:51	10:25	16	5	7	12	9:45	10:02
2	1	5	37	9:34	9:50	17	5	9	12	9:31	10:52
3	1	8	19	10:16	10:31	18	5	9	38	9:37	10:23
4	1	13	15	9:39	10:46	19	5	11	38	9:41	10:04
5	2	7	25	9:34	9:44	20	6	8	19	9:50	10:24
6	2	8	7	9:40	9:59	21	6	9	20	9:59	10:46
7	2	8	17	9:41	10:05	22	6	12	13	9:37	9:52
8	2	10	13	9:55	10:19	23	7	10	17	9:43	10:16
9	3	6	29	10:05	10:57	24	7	10	15	9:41	10:32
10	3	8	32	9:32	10:40	25	7	13	15	9:42	10:51
11	3	11	6	10:10	10:25	26	8	11	6	10:01	10:34
12	3	9	19	9:44	10:53	27	8	13	30	10:00	10:45
13	4	7	27	10:02	10:24	28	9	12	19	9:51	10:07
14	4	10	36	9:30	10:30	29	10	12	13	9:42	10:16
15	4	12	6	10:22	10:57	30	10	13	21	9:59	10:44

TABLE IV FREIGHT TRANSPORT PROGRAMMES

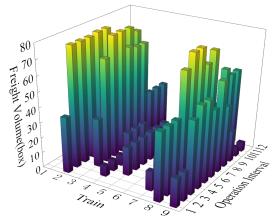
Train No.	Additional Freight Carriages	Freight Carriages	Manifest	Train Additional Freight No. Freight Carriages			Manifest		
1	0	1	22	6	1	1	20		
2	2	4	2、5、7、19、23、28、29	7	0	1	8、11		
3	2	4	10、14、26、27	8	2	4	1, 4, 13, 18, 25, 26, 30		
4	0	1	6, 17	9	2	3	3、9、15、21、24		
5	0	1	12						

TABLE V

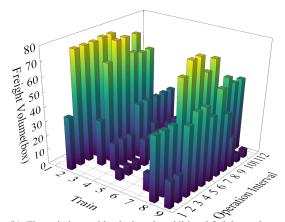
COST SENSITIVITY ANALYSIS OF INCREASING
FREIGHT CARRIAGE CAPACITY

σ	Objective Value	TDT(-)	UM	NAEC
(CNY/carriage)	(CNY)	TDT(s)	UM	NAFC
200	2211.84	4948		9
300	2997.30	4939		9
400	3740.69	5292	15	8
500	4438.64	5292	15	8

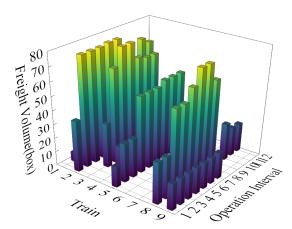
* Note: TDT: Total dwell time; UM: Unserved manifest; NAFC: Number of additional freight carriage.



(a) The train interval load when the additional freight carriage's cost is 200 CNY

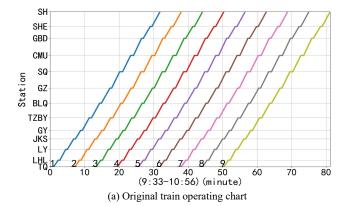


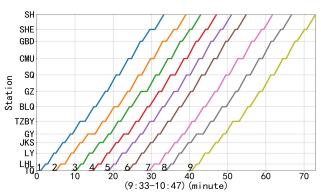
(b) The train interval load when the additional freight carriage's cost is 300 CNY



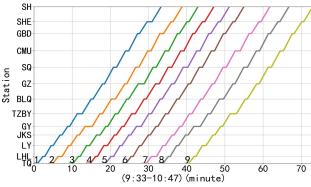
(c) The train interval load when the additional freight carriage's cost is 500 CNY

Fig. 4. Train interval load

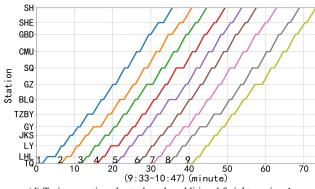




(b) Train operating chart when the additional freight carriage's cost is 200 CNY



(c) Train operating chart when the additional freight carriage's cost is 300 CNY



(d) Train operating chart when the additional freight carriage's cost is 500 CNY

Fig. 5. Train operating charts

(ii) Simultaneous loading and unloading queue sensitivity analysis of a single freight carriage

Given that the train model analyzed in this study is the B-type train, which features four pairs of doors on one side of each carriage, the maximum number of concurrent loading and unloading queues for a single carriage is four. To analyze the effect of variations in the quantity of simultaneous loading and unloading queues on the solution outcomes, the results corresponding to different loading and unloading queues, as illustrated in the example presented in this paper, are displayed in TABLE VI.

As the value v increases, TABLE VI shows that the incersed dwell time and the model's objective value gradually decrease. This is due to the fact that a greater value v means that less time is spent at the station loading and unloading freight. The only method to transfer all the freight when v=1 is to increase the number of freight carriages in the train, which will increase the train's simultaneous loading and unloading queue. But the carriages will have to pay more as a consequence. Because the trains in that time frame are able to carry these freight, changes in the value won't affect the number of additional freight carriages in the train when v=2, 3, or 4. Additionally, as the number of loading and unloading lines increases, the train's total stay time will gradually decrease, reducing overall expenses. Therefore, the decision to use simultaneous loading and unloading queues in actual operations should be informed by the operational department's focus on the costs of additional freight carriages, the time costs associated with adding freight carriages, and the station's actual loading and unloading

(iii) Analysis of the effectiveness of transportation modes in different passenger transport scenarios.

TABLE VII, we have included the train formation plans for the various scenarios.

TABLE VII indicates that a total of 20 freight cars were utilized in situations 1, 2, 3, 4, and 8; 21 freight carriages were employed in scenarios 5 and 6; and 19 freight carriages were utilized in scenarios 7 and 5. Consequently, although the train formation initiatives will vary, the total quantity of freight carriages utilized will remain relatively stable under identical freight demand but differing passenger circumstances. The dynamic formation mode facilitates the operational department's scientific decision-making.

(iv) Cost coefficient sensitivity analysis

A sensitivity analysis experiment is undertaken to further examine the influence of coefficients α and β on the final decision outcomes, based on the example presented in this study. The findings are presented in TABLE VIII. As the value of α rises and the value of β declines, the number of additional freight carriage progressively diminishes, resulting in cost reduction. Nonetheless, the overall dwell time in relation to a train timetable progressively escalates, which will inevitable impact passengers' travels.

As the value of β escalates and the value of α diminishes, the number of additional freight carriage progressively increases, accompanied by a gradual rise in cost. Nevertheless, the dwell time will increasingly approximate the fixed timetable. Refer to Fig. 6 for a visual representation of the coefficient modifications' outcomes. In conclusion, to get optimal efficiency, the operations department must adaptively modify the values of coefficients α and β during actual operations.

TABLE VI
SOLUTION RESULTS OF DIFFERENT LOADING
AND UNLOADING QUEUES

Single freight carriage loading and unloading queue	v=1	v = 2	v=3	v = 4
Increased dwelling time (s)	2581	988	508	240
Number of additional freight carriage (carriage)	15	9	9	9
Costs (CNY)	3394.70	2211.8	2149.66	2114.32

TABLE VII
TRAIN CARRIAGE ADDITION PLANS FOR DIFFERENT PASSENGER TRANSPORT SCENARIOS

Scenario		1	l	2	2	3	3	۷	1	:	5	(5	(7	8	3
train			TN	NA	TN												
train		FC															
	1	1	3	2	4	0	0	2	2	0	2	1	2	2	3	2	4
	2	2	4	2	2	2	4	2	2	2	4	2	3	2	4	0	0
	3	0	0	0	0	2	3	2	2	0	2	2	3	0	0	0	0
	4	2	4	0	0	2	3	2	4	0	2	1	2	0	2	2	4
	5	0	1	2	4	0	0	0	0	0	2	2	3	2	4	2	3
	6	0	1	2	4	2	3	2	2	0	2	1	2	2	4	1	2
	7	2	4	2	3	2	4	2	4	1	3	2	3	0	0	2	4
	8	0	2	1	2	0	0	0	0	0	2	0	1	0	0	0	1
	9	0	1	0	1	1	3	2	4	0	2	1	2	1	2	0	2
	合计	7	20	11	20	11	20	14	20	3	21	12	21	9	19	9	20

*Note: NAFC: Number of additional freight carriage; TNFC: Total number of freight carriage.

 $\label{eq:table_viii} \text{Sensitivity analysis of cost coefficient } \alpha, \beta \text{ values}$

				or coerrien	ω,ρ	
α	β	Objective Value (CNY)	Dwell Time(s)	Percentage Increase	Number of Additions (carriage)	Gap
1.0	0	1800.00	7021	77.30%	9	0.00%
0.9	0.1	2241.08	5176	30.71%	9	2.99%
0.8	0.2	2677.37	4948.5	24.96%	9	2.98%
0.7	0.3	3157.32	4984	25.86%	9	2.97%
0.6	0.4	3602.90	4939.5	24.73%	9	2.99%
0.5	0.5	4160.06	4783	20.78%	11	3.00%
0.4	0.6	4473.67	4665.5	17.82%	11	2.97%
0.3	0.7	4905.92	4583.5	15.74%	13	3.00%
0.2	0.8	5145.29	4446	12.27%	14	2.96%
0.1	0.9	5488.81	4453.5	12.46%	15	2.74%

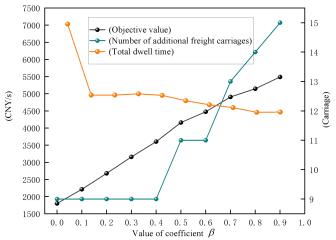


Fig. 6. Sensitivity analysis of coefficient α, β values

TABLE IX
SOLUTION RESULTS FOR DIFFERENT SCALE EXAMPLES

NM	LUQ	NAFC	UB	CR	Objective	FCQ	NM	LUQ	NAFC	UB	CR	Objective	FCQ
8	2	0	0	100.00%	609.30	163	30	3	8	0	100.00%	1973.69	606
9	2	1	0	100.00%	785.57	192	40	3	15	0	100.00%	3082.01	814
10	2	3	0	100.00%	1177.07	224	50	3	18	51	94.80%	6032.76	980
20	2	7	0	100.00%	1938.27	437	10	4	2	0	100.00%	883.90	224
30	2	9	0	100.00%	2059.54	606	20	4	7	0	100.00%	1751.86	437
40	2	14	0	100.00%	3119.43	814	30	4	8	0	100.00%	1939.66	606
50	2	18	53	94.59%	6120.01	980	40	4	14	0	100.00%	3058.15	814
10	3	2	0	100.00%	909.06	224	50	4	18	51	94.80%	6000.29	980
20	3	7	0	100.00%	1778.99	437	60	4	18	131	89.19%	11357.14	1212

**Note: NM: Number of manifests; LUQ: Loading and unloading queues; NAFC: Number of additional freight carriage; CR: Completion rate; FBQ: Freight box quantity.

(v) Comparison of different scale cases

This paper modifies the freight departure time window, based on the initial freight demand, to validate the model's performance and effectiveness, establishing it from the departure time of the first train in the original timetable at the starting station to the arrival time of the ninth train at the destination station, in order to calculate the maximum freight transportation capacity within the specified period. The maximum solving time is established at 1200 seconds, with the gap value modified to 3% for resolution. The results are presented in TABLE IX.

TABLE IX indicates that when the freight manifests (freight volume) are small, the train can transport goods without increasing the number of carriages, thereby transforming the problem under study into an optimization problem regarding the allocation of freight loading between passenger and freight carriages. When the number of simultaneous loading and unloading queues for single freight carriages is the same, as the number of freight manifests (freight volume) increases, the number of additional wagons and the associated costs will gradually rise. At the same time, due to the limited supply of additional freight carriages, once the freight volume reaches a certain threshold, the shortage of additional assembled carriages will result in some manifests being unable to be loaded and transported. When the number of

simultaneous loading orders remains constant, the cost will gradually decrease as the number of simultaneous loading and unloading queues for individual freight carriages increases. According to the previous context, this is because the increase in simultaneous loading and unloading queues can reduce the cost of dwell time. Given that simultaneous loading and unloading queues involve the service level of station infrastructure, it is necessary to adaptively determine the quantity of goods transported and the number of train cars in actual operations based on the station's service capacity.

V. CONCLUSION

This paper comprehensively considers constraints such as freight transportation modes, freight time windows, and freight loading, and formulates an optimisation problem for coordinated passenger and freight transport on a metro line as a mixed-integer nonlinear programming model. After linearising the model into a mixed-integer linear programming model, it is solved using the commercial solver Gurobi. Experimental results show that, under the condition of known passenger and freight demand:

(i) The metro train timetable and freight loading plan under the co-line towing mode can effectively serve the freight needs of different passenger scenarios and different departure times, ensuring timely delivery of freight to their destination and enhancing customer service satisfaction. At the same time, this mode can adjust capacity based on freight demand, achieving maximum utilisation of resources.

- (ii) The number of freight carriages can determine the number of train loading and unloading queues, but the number of simultaneous loading and unloading queues is also related to the service level of the station infrastructure. Therefore, during operations, it is necessary to make decisions based on the number of freight carriages and the station's service capacity to provide reasonable freight transportation plans for the operations department.
- (iii) The co-line trailer mode has a maximum capacity limit within a period, necessitating a restriction on the scale of freight transportation. Since the cost of increasing freight carriages and the penalty cost for freight boxes jointly affect the freight transportation plan, it is essential to coordinate the relationship between the two to leverage the advantages of this transportation mode.
- (iv) In future research, consideration can be given to the transportation of freight under uncertain passenger and freight flow conditions; studying the freight transfer transportation model for networked line operations to provide diverse freight transportation organisation modes, offering a variety of choices for the operations department.

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