On the Fixed Common Step-Size of the Frequency-Domain Normalized Generalized Multidelay Adaptive Filter

Junghsi Lee and Hsu Chang Huang

***Abstract*—In this paper, we analyze the bounds of the fixed common step-size parameter μ_{GMDF} for the generalized multidelay adaptive filter (GMDF). Frequency domain adaptive filters are attractive in applications requiring a large number of coefficients such as acoustic echo cancellation (AEC). However, the very restrictive convergence bound for block LMS has limited its usefulness. Derivations on step-size bounds for the partitioned frequency-domain block LMS have been reported recently, but are not consistent with each other. Contrary to other researchers' work, this paper derives a not-so-restrictive step-size bound that supports a stable GMDF. We also derive relations of step-size parameters for NLMS and GMDF to have similar convergence properties. The results of extensive simulation experiments are included in the paper. These results show the bounds and the convergence behavior predicted by the analysis is in very good agreement with the experimental results.**

***Index Terms*—Acoustic Echo Cancellation, Block LMS, Convergence Analysis, Frequency Domain LMS,**

I. Introduction

The normalized least-mean-square (NLMS) scheme has been the most popular adaptive filtering algorithm in many applications. There are quite a number of variations of the NLMS algorithm being developed for certain specific applications. For example, frequency-domain fast block LMS (FBLMS) adaptive filters are attractive alternatives for acoustic echo cancellation (AEC), which may need thousands of filter coefficients to reach the desired level of performance [1]-[6]. The great reduction in computational complexity associated with FBLMS is due to the usage of fast Fourier transform (FFT). In the literature, the FBLMS is also referred to as the block frequency-domain adaptive filters (BFDAF).

It is well known that the normalized block LMS (NBLMS), with block length N, and the NLMS algorithms converge at the same rate and achieve the same mis-adjustment if the step-size parameter of NBLMS is N times as large as that of the NLMS [7]. However, both algorithms have the same convergence bounds for the step-size parameter. Therefore, even for a moderate block length N, the NBLMS has to employ a fairly small step-size parameter to meet the very restrictive convergence bound. This limitation greatly reduces the usefulness of the NBLMS and its frequency-domain variations FBLMS.

A major problem of FBLMS in AEC application is the long delay associated with the large filter size. Soo proposed a variation of FBLMS, which was referred to as the multidelay block frequency domain adaptive filter (MDF) to alleviate this delay trouble [4]. MDF segments the filter into several partitions and employ as many sub-filters as well. The MDF belongs to the class of partitioned FBLMS (PFBLMS) algorithms. The PFBLMS is most commonly implemented by normalizing its input in frequency domain (known as frequency bins) and is referred to here as the NPFBLMS. Some researchers thought that the frequency-bin normalization procedure resolves the problem of slow modes of the NLMS algorithm and the resulting algorithm converges faster than the NLMS [10]. However, there were researchers reasoned that because of the restriction on the step-size bounds that supports a stable filter, the frequency domain algorithms actually do not perform better than the NLMS in convergence and tracking properties [5]. And they have introduced a hybrid coefficients update scheme that performs comparable to that of the NLMS with a computational complexity comparable to that of the standard frequency domain algorithms [5].

Moulines [1] proposed the generalized MDF (GMDF) that allows one to select FFT size and the block delay separately. This advantage is owing to the controlling of the overlap between the successive input blocks. Because of this flexibility, the GMDF is more general than the NPFBLMS. However, like we just pointed out, researchers presented different views on the convergence performance of NPFBLMS, and the derivations of step-size bounds in the literature are not consistent [1], [6], [9]. In this paper, we make a further study on the step-size bounds of the generalized multidelay adaptive filter. The step-size bounds we derived in this paper is N times larger than that of the NBLMS, and is much bigger than the one reported in [1] for GMDF, and a recent paper [6] as well. Because of this new analysis, we can now choose proper step-size so that the well-designed GMDF maintains good

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tracking and convergence performance and has great saving in computations as well. The rest of the paper is organized as follows. Section 2 summarizes the GMDF algorithm. Section 3 investigates the range of step-size that supports a stable GMDF filter. We then compare our derivations with other work currently available in the literature. Extensive simulation results confirming our performance analysis are presented in Section 4. The conclusions are made in the last section of the paper.

II. SUMMARY OF THE GMDF

Let $u(n)$ and $d(n)$ represent the reference input and desired output signal, respectively, to the adaptive filter with order M. The NLMS is formulated by

$$e(n) = d(n) - \boldsymbol{w}(n)^T \boldsymbol{u}(n) \,, \tag{1}$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \frac{\mu}{Mr_0} \boldsymbol{u}(n) e(n) \,, \tag{2}$$

where $\boldsymbol{u}(n)$ and $\boldsymbol{w}(n)$ are input and coefficient vectors, respectively, and r_0 is an estimate of the variance of $u(n)$. The step-size μ is bounded in $(0,2)$ for convergence.

The NBLMS with block size N is formulated by

$$e(kN+i) = d(kN+i) - \mathbf{w}_{\mathbf{k}}^{\mathbf{T}} \boldsymbol{u}(kN+i) \,,$$
$$i = 0,1,\cdots,N-1, \tag{3}$$

$$\mathbf{w}_{\mathbf{k+1}} = \mathbf{w}_{\mathbf{k}} + \frac{\mu_B}{Mr_0 N} \sum_{i=0}^{N-1} \boldsymbol{u}(kN+i) e(kN+i) \,, \tag{4}$$

where $\mathbf{w}_k$ denotes coefficient vector at block iteration k. The step-size μ_B is bounded in $(0,2)$ for convergence [7].

The GMDF segments the filter into L sub-filters, each with an order N and is implemented in frequency domain with FFT size $2N$. Without loss of generality, we assume that $M = NL$. The GMDF uses a positive integer α to control the overlap between the successive input blocks. Consequently, it updates the coefficients every $R = N/\alpha$ samples. In the k^{th} iteration, define reference input vector $\mathbf{x}_k$ and desired response vector $\mathbf{d}_k$, respectively, as

$$\mathbf{x}_k = \left[u(kR), u(kR+1), \cdots, u(kR+N-1) \right]^T \,, \tag{5}$$

$$\mathbf{d}_k = \left[d(kR), d(kR+1), \cdots, d(kR+N-1) \right]^T \,. \tag{6}$$

Frequency-domain input vector for l^{th} sub-filter, denoted as $\mathbf{X}_{l,k}$, $l = 1,2,\cdots,L$ is computed as

$$\mathbf{X}_{l,k} = FFT\left[\mathbf{x}_{k-l\alpha}^T, \mathbf{x}_{k-(l-1)\alpha}^T \right]^T \,. \tag{7}$$

The corresponding frequency-domain coefficient vector $\mathbf{H}_{l,k}$ is defined accordingly as

$$\mathbf{H}_{l,k} = FFT\left[\mathbf{h}_{l,k}^T, \mathbf{0}^T \right]^T \,, \tag{8}$$

where $\mathbf{h}_{l,k}$ is the l^{th} sub-filter's time-domain coefficient vector. Filter output vector $\hat{\mathbf{d}}_k$ is calculated as

$$\hat{\mathbf{d}}_k = \text{second part of } FFT^{-1}\left[\sum_{l=1}^{L} \mathbf{H}_{l,k} \otimes \mathbf{X}_{l,k} \right], \tag{9}$$

where $\otimes$ denotes element-wise multiplication. In practice, any weighted overlap and add reconstruction algorithm could be used to construct the final R-sample output at each block iteration. Frequency-domain error vector $\mathbf{E}_k$ is obtained as follows.

$$\mathbf{e}_k = \mathbf{d}_k - \hat{\mathbf{d}}_k \,, \tag{10}$$

$$\mathbf{E}_k = FFT\left[\mathbf{0}^T, \mathbf{e}_k^T \right]^T \,. \tag{11}$$

Most NPFBLMS algorithms employ, although might be slightly different, frequency bin power normalization. We present a sub-filter based normalization scheme as follows. The frequency power of the l^{th} subfilter at k^{th} iteration is calculated as

$$\mathbf{Z}_{l,k} = \beta \mathbf{Z}_{l,k-1} + (1-\beta) \bar{\mathbf{X}}_{l,k} \otimes \mathbf{X}_{l,k} \,, \tag{12}$$

where $\bar{\mathbf{X}}_{l,k}$ denotes the complex conjugate of $\mathbf{X}_{l,k}$, and β is a forgetting factor. The coefficient vector $\mathbf{H}_{l,k}$ is updated as

$$\mathbf{H}_{l,k+1} = \mathbf{H}_{l,k} + \frac{2\mu_{GMDF}}{M} \mathbf{\Phi}_{l,k} \,, \tag{13}$$

where μ_{GMDF} is a fixed common un-normalized step-size parameter of the GMDF filter. In (13), $\mathbf{\Phi}_{l,k}$, the new information for updating, is obtained as

$$\mathbf{\Phi}_{l,k} = FFT\left[\phi_{l,k}^T, \mathbf{0}^T \right]^T \,, \tag{14}$$

where

$$\phi_{l,k} = \text{ first part of}$$
$$FFT^{-1}\left[\left(\mathbf{E}_k \otimes \bar{\mathbf{X}}_{l,k} \right) \odot \mathbf{Z}_{l,k} \right], \tag{15}$$

and $\odot$ denotes element-wise division.

III. STEP-SIZE BOUNDS ANALYSIS

We present a range of step-size that supports a stable GMDF filter in this section. A recent paper claimed that for sufficiently large block size *N*, NPFBLMS algorithm ought to perform similarly, regardless of whether the input process is white or correlated [10]. That is, as a consequence of the frequency-bin normalization process, the eigenvalue spread of the autocorrelation matrix would approach unity. However, [10] did not derive a range of the fixed common un-normalized step-size μ_{GMDF}, it simply referred to the bounds presented in [1].

In this paper, we derive a bound for the fixed common step-size parameter μ_{GMDF}. Note the Parseval's theorem relates power computed in time domain and in frequency domain. Now assuming $u(n)$ is a white process with zero mean and variance σ_u^2, the expected value of averaged frequency bin power would equal $2N\sigma_u^2$. Using this $2N\sigma_u^2$ for each frequency bin power associated with (13), and noting that (13) is a frequency-bin normalized version of (4), we have the following equality for bounding step-size parameter

$$\frac{\mu_B}{M\sigma_u^2} = \frac{2\mu_{GMDF}}{M(2N\sigma_u^2)} . \quad (16)$$

Transforming the bound on μ_B to μ_{GMDF} yields

$$0 < \mu_{GMDF} < 2N . \quad (17)$$

The bound in (17) is good for supporting a stable GMDF algorithm for all types of input processes provided the FFT size is large enough to de-correlate the transformed input samples. Contrary to a recent paper that explicitly states the limitation of frequency domain filters owing to the very restrictive step-size bounds [5], our results give a very good range of values that support a stable GMDF.

In the following, we compare our step-size bound with other researchers' work. Since the structures of the NPFBLMS algorithms presented in the literature are somewhat different, we rewrite the associated coefficient updating equations in the form of (13) for comparison. The updating equation in [1] is rewritten as

$$\tilde{\mathbf{H}}_{k+1} = \tilde{\mathbf{H}}_k + \frac{2\mu_{B1}}{M}\frac{L}{4}\tilde{\mathbf{\Phi}}_k , \quad (18)$$

where $\tilde{\mathbf{H}}_k$ is the $2M \times 1$ coefficient vector, and μ_{B1} is the fixed common un-normalized step-size. [1] derived the range of convergence as

$$0 < \mu_{B1} < 4/(1+L) . \quad (19)$$

This is equivalent to confine μ_{GMDF} as

$$0 < \mu_{GMDF} < L/(1+L) . \quad (20)$$

Obviously, this bound is too restrictive compared to our derivation in (17).

In a recent work [6], Sommen presented the equivalent updating equation

$$\tilde{\mathbf{H}}_{l,k+1} = \tilde{\mathbf{H}}_{l,k} + \frac{2\mu_{B2}}{M}\tilde{\mathbf{\Phi}}_{l,k} , \quad (21)$$

that gave a range of convergence

$$0 < \mu_{B2} < 1 . \quad (22)$$

Transform this bound to our version will result in

$$0 < \mu_{GMDF} < 1 . \quad (23)$$

It's obvious to see that the bounds derived in [1] and [6] are very restrictive compared to our bound presented in (17).

Regarding the convergence properties, it is well known that for the NBLMS that adjusts coefficients once per N samples of data, the NBLMS and NLMS algorithms exhibit similar convergence performance if $\mu_B = N\mu$ [7]. Follow the work in [7], and note that the GMDF modifies the weights every R samples, we can show that NLMS and GMDF would perform similarly if

$$\mu_{GMDF} = \mu R = \mu N/\alpha \quad (24)$$

for small value of μ and N is large enough. Even though the GMDF was first presented over a decade ago, to our best knowledge, our paper is the first one that derives relations of step-size parameters for NLMS and GMDF to have similar convergence properties.

IV. SIMULATION RESULTS

In this section, we present the results of several experiments that validate our analysis on step-size bounds as well as verify the convergence analysis. The adaptive filter was used to identify a 512-tap acoustic echo system, $\mathbf{h}_{opt}$, measured in a small office. The acoustic echo system was kept unchanged for the first 2.4 seconds. During the next 4 seconds, the system turned to be time-varying. The evolution of coefficients is described by

$$\mathbf{h}_{opt}(n) = \mathbf{h}_{opt} + \mathbf{g}(n) , \quad (25)$$

where $\mathbf{g}(n)$ is uniformly distributed in $(\mathbf{-10^{-4}}, \mathbf{10^{-4}})$. The system was switched back to be time-invariant with coefficients $\mathbf{h}_{opt}$ for the final 3.6 seconds. Several types of input signals (white Gaussian processes, moving average (MA) processes and autoregressive (AR) processes) were used. For all experiments, the power of the acoustic echo system was set to be unit (during the first 2.4 seconds and the last 3.6 seconds) and the additive white Gaussian noise has variance 0.01. The reported mean squared error (MSE) curves are results of ensemble averages over 20 independent runs, followed by 64-sample time averages.

A. *Example 1: Step-size bounds*

We have performed extensive simulations to validate our stable step-size bounds in (17). We observed satisfactory results with $\mu_{GMDF} = 1.6\times 128$ for all experiments with parameters $L = 4$, $N = 128$, and $\alpha = 1$. We also performed GMDF with L equals 8, 16, 32, and 64, respectively. We observed stable step-size bounds getting closer to $2N$, where N is the block size associated with that particular L . The extreme case would be $L = 512$, $N = 1$, and stable step-size seem will approach to 2. Therefore, we conclude that Moulines [1] and Sommen [6] gave a too restrictive bound. The MSE curves associated with AR processes input signal for $L = 32,\ \alpha = 2$ and $L = 64,\ \alpha = 2$ are depicted in Fig. 1 and Fig. 2, respectively. Fig. 3 shows the MSE curves of white input signal for $L = 32,\ \alpha = 2$. We never had any good result with step-size larger than $2N$.

B. *Example 2: Convergence properties*

Extensive experiments were conducted to support the usefulness of our convergence analysis in (24). Due to the limitation of space, we only provide some MSE plots of the setup: $L = 4$, $N = 128$, NLMS with $\mu = 0.4$, GMDF with $\alpha = 1, 2, 4$ and the corresponding fixed common step-size parameter $\mu_{GMDF} = 0.4\times 128/\alpha$. The MSE curves associated with AR, MA, and white Gaussian inputs are depicted in Figs. 4, 5, and 6, respectively. We observed the MSE curves of GMDF algorithms are fairly close in all experiments. The results verified our performance analysis presented in the previous section. The NLMS performed comparably to the GMDF for AR and white Gaussian inputs. However, the NLMS converged quite slowly for MA inputs.

Experimental results of NLMS with $\mu = 0.8$, and GMDF filters with $\mu_{GMDF} = 0.8 \times 128/\alpha$ are shown in Figures 7, 8, and 9 for AR, MA, and white Gaussian inputs, respectively. The GMDF filters perform similarly, but the MSE curves are not that close as for the case $\mu_{GMDF} = 0.4 \times 128/\alpha$. This is probably because the associated μ is not small enough.

V. CONCLUSIONS

In this paper, we derived the bounds of the fixed common step-size parameter μ_{GMDF} for the generalized multidelay adaptive filter. Contrary to most work currently available in the literature, our results gave a very good range of values that support a stable GMDF. Extensive simulation results were provided to validate the analysis. We also derived relations of step-size parameters for NLMS and GMDF to have similar convergence properties. The performance analysis was verified by extensive simulations.

REFERENCES

[1] E. Moulines, O. Ait Amrane, and Y. Grenier, "The generalized multidelay adaptive filter: structure and convergence analysis," IEEE Transactions on Signal Processing, Vol.43, No.1, pp. 14-28, Jan. 1995.

[2] E. Ferrara, "Fast implementations of LMS adaptive filters," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. 28, pp. 474-475, Aug. 1980.

[3] J. J. Shynk, "Frequency-domain and multirate adaptive filtering," IEEE Signal Processing Magazine, pp. 14-37, Jan. 1992.

[4] J. S. Soo and K. K. Pang, "Multidelay block frequency domain adaptive filter," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. 38, No. 2, pp. 373-376, Feb. 1990.

[5] Y. Bendel, D. Burshtein, O. Shalvi and E. Weinstein, "Delayless frequency domain acoustic echo cancellation," IEEE Transactions on Speech and Audio Processing, Vol. 9, No. 5, pp. 589-597, July 2001.

[6] R. M. M. Derkx, G. P. M. Egelmeers and P. C. W. Sommen, "New constraining method for partitioned block frequency-domain adaptive filters," IEEE Transactions on Signal Processing, Vol. 50, No. 9, pp. 2177-2186, Sep. 2002.

[7] G. A. Clark, S. A. Mitra and S. R. Parker, "Block implementation of adaptive digital filters," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, No. 3, pp. 744 - 752, June 1981.

[8] J. C. Lee and C. K. Un, "Performance analysis of frequency domain block LMS adaptive digital filters," IEEE Transactions on Circuits and Systems, Vol. 36, No.2, pp. 173-189, Feb. 1989.

[9] K. S. Chan and B. Farhang-Boroujeny, "Analysis of the partitioned frequency-domain block LMS (PFBLMS) algorithm," IEEE Transactions on Signal Processing, Vol. 49, No. 9, pp. 1860 -1874, Sep. 2001.

[10] B. Farhang-Boroujeny and K. S. Chan, "Analysis of the frequency-domain block LMS algorithm," IEEE Transactions on Signal Processing, Vol. 48, No. 8, pp. 2332 -2342, Aug. 2000.

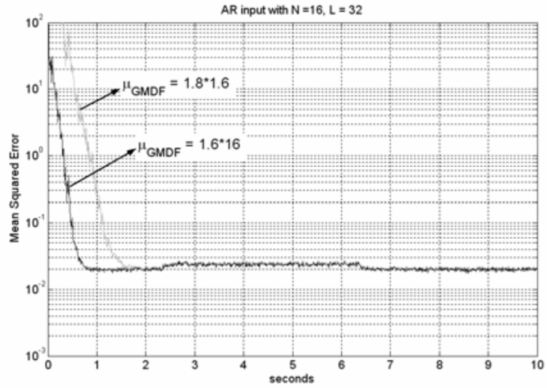

Figure 1, MSE curves (Example 1) of GMDF algorithms with step-size $\mu_{GMDF} = 1.8 \times 16$ and 1.6×16. AR input signal. $(L = 32,\ \alpha = 2)$.

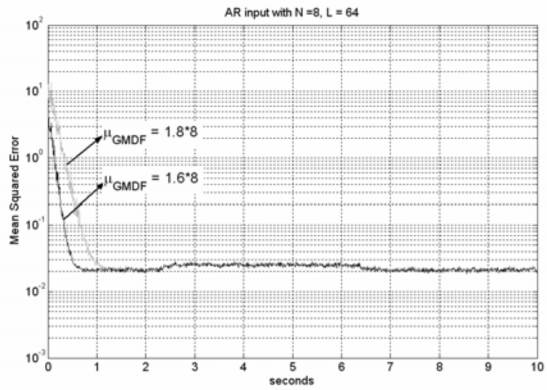

Figure 2, MSE curves (Example 1) of GMDF algorithms with step-size $\mu_{GMDF} = 1.8 \times 8$ and 1.6×8. AR input signal. $(L = 64,\ \alpha = 2)$.

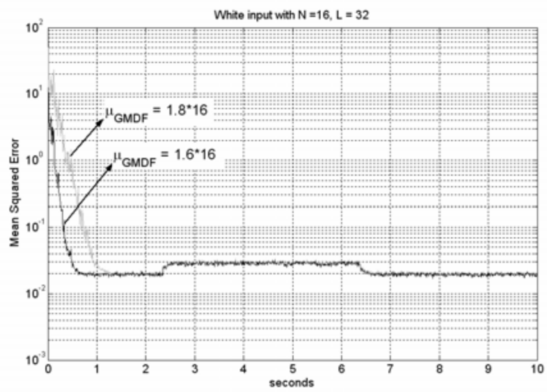

Figure 3, MSE curves (Example 1) of GMDF algorithms with step-size $\mu_{GMDF} = 1.8 \times 16$ and 1.6×16. White Gaussian input signal. $(L = 32,\ \alpha = 2)$.

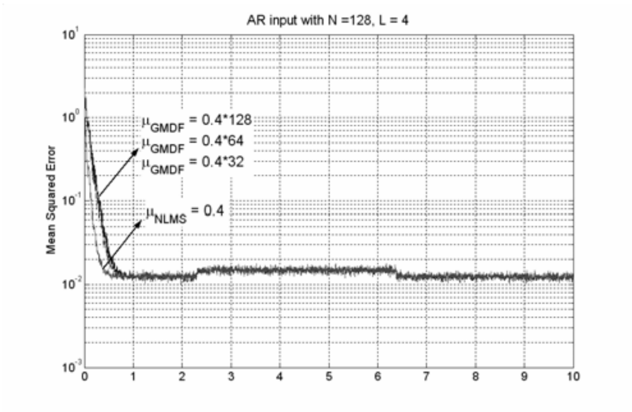

Figure 4, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.4\times 32$, 0.4×64, 0.4×128), and NLMS (with $\mu = 0.4$). (AR input, $L = 4$, $N = 128$)

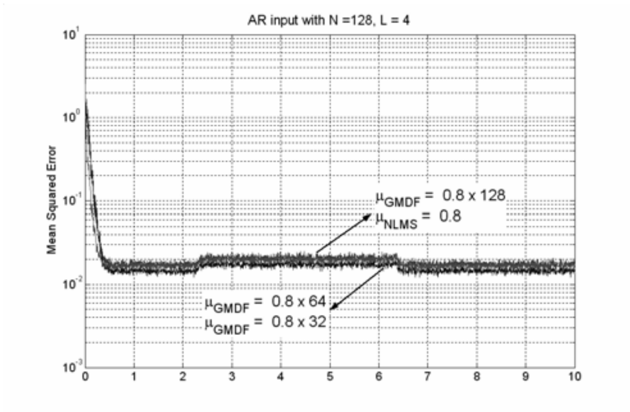

Figure 7, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.8\times 32$, 0.8×64, 0.8×128), and NLMS (with $\mu = 0.8$). (AR input, $L = 4$, $N = 128$)

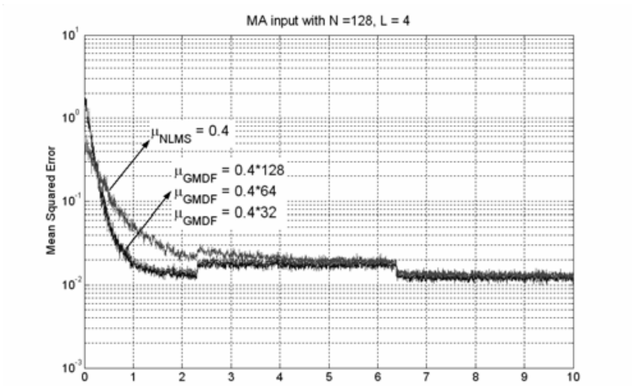

Figure 5, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.4\times 32$, 0.4×64, 0.4×128), and NLMS (with $\mu = 0.4$). (MA input, $L = 4$, $N = 128$)

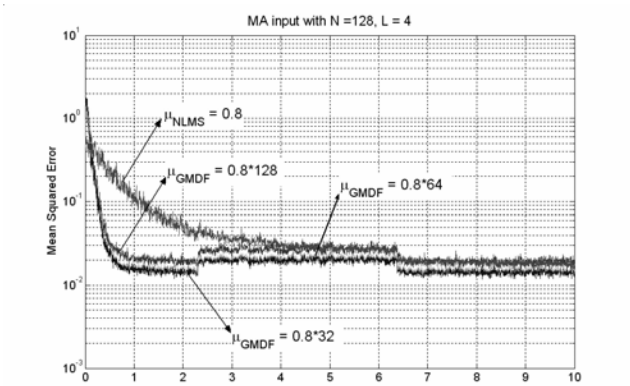

Figure 8, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.8\times 32$, 0.8×64, 0.8×128), and NLMS (with $\mu = 0.8$). (MA input, $L = 4$, $N = 128$)

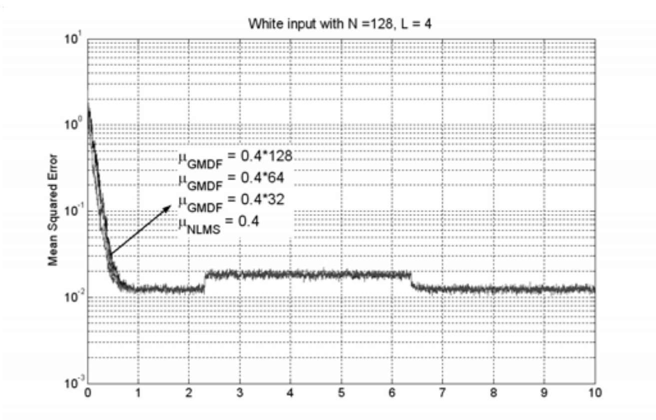

Figure 6, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.4\times 32$, 0.4×64, 0.4×128), and NLMS (with $\mu = 0.4$). (White Gaussian input, $L = 4$, $N = 128$)

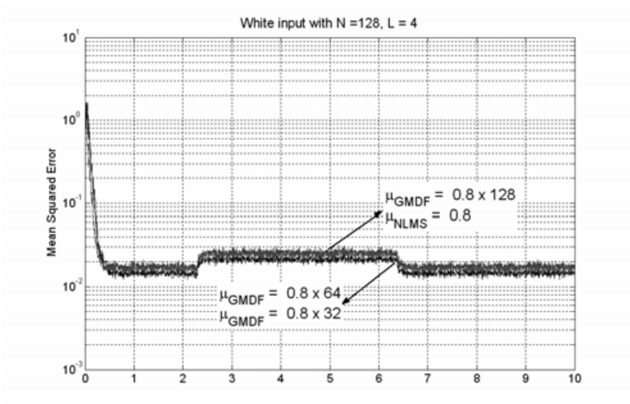

Figure 9, MSE curves of GMDF algorithms (with step-size $\mu_{GMDF} = 0.8\times 32$, 0.8×64, 0.8×128), and NLMS (with $\mu = 0.8$). (White Gaussian input, $L = 4$, $N = 128$)